Building a model for a collision between two disks with a constant radial force

1. Geometric representation of the force vector during the collision

Two disks are moving past each other on a table. Figure 1 shows the disks a few moments before the collision, and arrows representing the velocity vector of each disk. Assume that a **constant radial force** acts between the disks during the collision.

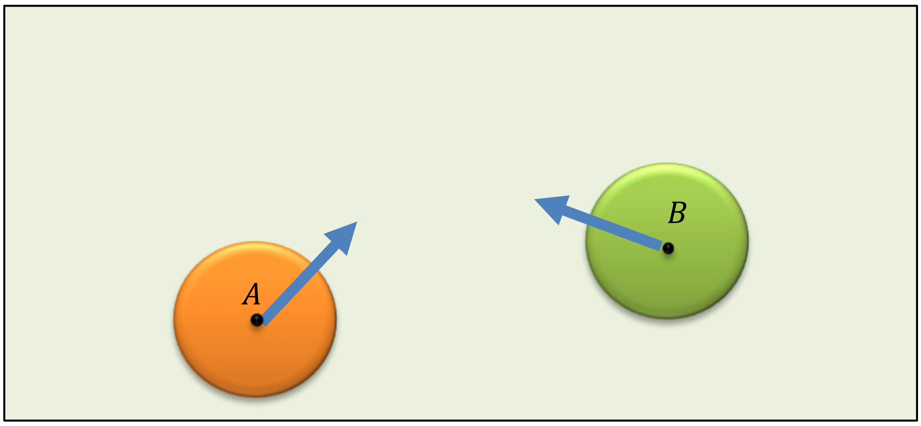


Figure 1: *Two disks moving past each other with constant velocity. The arrows represent the velocity vector of each disk.*

We'll want to sketch the vectors of the forces that the disks exert on each other during the collision.

A. Add to Figure 1 a sketch of the two disks at the moment when they come into contact. Find the point of contact between the two disks.
B. Draw the normal force vectors acting on each disk at the point of contact.

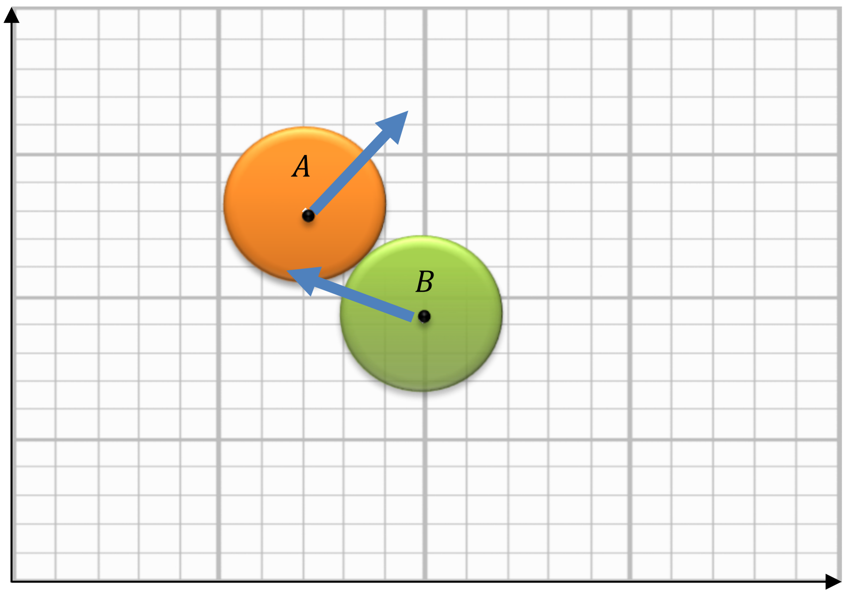
2. Calculating the force vector using a unit vector

We'll want to calculate the vectors of the forces that the disks exert on each other during the collision. Let's use a two-dimensional coordinate system to calculate the direction of the force vector (see Figure 2 on the next page).

A. Write an algebraic expression for the position vectors of the disks at the moment of collision (the positions of the centers of the disks). Assume that each square equals one unit.

Answer:

B. On Figure 2, represent the position vectors of the disks (with arrows).



.***Figure 2:*** *Two disks at the moment of collision*

C. Calculate the separation vector between the two disks $\vec{r}_{AB}$, whose tail is at the center of disk B and whose tip is at the center of disk A. Add a drawing of the vector that you get on Figure 2.

Answer:

D. Calculate the unit vector corresponding to the separation vector.

Answer:

E. Calculate the force vector acting on disk A, given that the magnitude of the force is constant during the collision and equal to 40 newtons.

Answer:

F. Calculate the velocity of disk A after the collision, given that the mass of the disk is 200 grams, its velocity before the collision is $\vec{v}_A = (3,3,0)\frac{\mathrm{m}}{\mathrm{s}}$, and the duration of the collision is 0.1 sec.

Answer:

G. Check your answer: Does the direction of the vector that you found represent the velocity of the disk after the collision? Explain.

Answer:

3. Building the simulation – computational method

Below is incomplete code for a simulation of two identical disks moving on a table and colliding with each other. Complete the missing lines of code in order to get a working simulation that fits the parameters that appear in the earlier parts. Add missing lines if needed.

```
from visual import *

###Initial Conditions###

v1 =
v2 =
m1 =
m2 =
R = 2.0
L = 10.0
dt = 0.001

### System Creation ###

table = box(pos=vector(0,0,0), size=(L,L,0))
disk1 = cylinder(pos=vector(-4,0,0), radius=R, axis=(0,0,0.1), color=color.green)
disk2 = cylinder(pos=vector(4,1,0), radius=R, axis=(0,0,0.1), color=color.blue)

### Time Evolution ###

t=0
while t < 1:
    rate(100)

    r =
    r_hat =

    if
    else:

    v1 =
    v2 =

    disk1.pos =
    disk2.pos =

    t = t + dt
```

4. Setting up the simulation

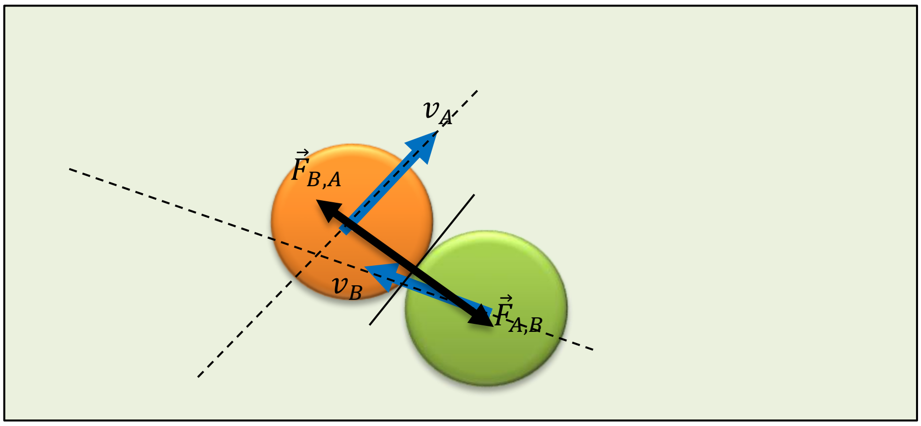
A. Change the constant radial force to an elastic radial force (choose a spring constant as you wish). What do you think is the difference between the simulations with different forces?
B. Add a (blue) arrow to each of the disks to represent the velocity vector.
C. Add a (black) arrow to each of the disks to represent the force vector.

Useful formulas

Physical quantity	Definition
Displacement	$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$
Velocity	$\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$
Acceleration	$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$
Calculating position (by the Euler approximation)	$x_{i+1} = x_i + v \cdot dt$
Calculating velocity (by the Euler approximation)	$v_{i+1} = v_i + a \cdot dt$
Newton's 2nd Law	$\Sigma\vec{F} = m\vec{a}$
Hooke's Law (elastic force)	$\vec{F} = -k\Delta\vec{x}$
Unit vector	$\hat{r} = \frac{\vec{r}}{\lvert\vec{r}\rvert}$

Solution

1. **Geometric representation of the force vector during the collision**

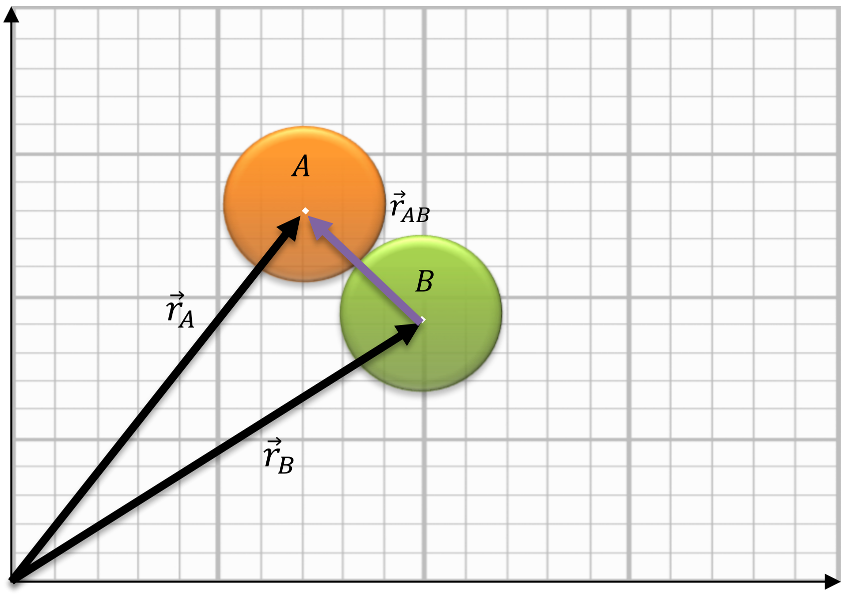


2. **Calculating the force vector with a unit vector – algebraic method**

   A. Algebraic expression for the position vectors of the disks at the moment of collision:

Answer: $$r_A = (7,13,0)$$ $$r_B = (10,9,0)$$

B. On Figure 2, represent the position vectors of the disks (with arrows).



C. Separation vector between the two disks $\vec{r}_{AB}$:

Answer:
$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$
$\vec{r}_{AB} = (7{,}13{,}0) - (10{,}9{,}0)$

D. Unit vector corresponding to the separation vector:

$$\hat{r} = \frac{\Delta\vec{r}}{|\Delta\vec{r}|}$$

$$\hat{r} = \frac{(-3{,}4{,}0)}{\sqrt{(-3)^2 + 4^2}} = \frac{(-3{,}4{,}0)}{5}$$

$$\hat{r} = (-0.6{,}0.8)$$

E. Force vector acting on disk A:

$$\vec{F} = |\vec{F}|\hat{r}$$

$$\vec{F} = 40 \cdot (-0.6{,}0.8{,}0)$$

$$\vec{F} = (-24{,}32{,}0)N$$

F. Velocity of disk A after the collision:

First we calculate the acceleration vector:

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\vec{a} = \frac{(-24,32,0)}{0.2}$$

$$\vec{a} = (-120,160,0)m/s^2$$

We find the velocity after the collision from the definition of acceleration:

$$\vec{v}_f = \vec{v}_i + \vec{a} \cdot \Delta t$$

$$\vec{v}_f = (3,3,0) + (-120,160,0) \cdot 0.1$$

$$\vec{v}_f = (-8,19,0)m/s$$

G. Check your answer: Does the direction of the vector that you found represent the velocity of the disk after the collision? Explain.

The disk moves up and to the right before the collision, and after the collision with a disk from the left, it is logical that the disk will move up and to the left.