Analysis and Optimization of Supermarket Operation Mode Based on Queuing Theory: Queuing and Pricing of Personalized Service

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ABSTRACT
As that large supermarkets combine cashier and weighing into one or use self-service all-in-one machine to alleviate the problem of customer queuing, this paper studies two queuing models: one is the queuing system with separate cashier and weighing, which can be represented by a mixed queuing system consisting of a two-stage series queuing and a single-stage queuing. The other is a queuing system combining the cashier and the weighing, which can be expressed as M/M/1. This paper fist analyzes the behavior of the customers, get the balance of the customer arrival rate in these two queueing systems. Further, this paper analyzes the optimal pricing problem of supermarket, and gets the average optimal price the supermarket shall set up. Finally, through comparing two kinds of queuing system, this paper gives the supermarket the determination when the cashier weighing mergers, and when to separate the two conditions.

CCS Concepts
• **Applied computing → Operations research → Decision analysis → Multi-criterion optimization and decision-making**

Keywords
Queuing system, Supermarket, Customized service, Gift package.

1. INTRODUCTION
Supermarket shopping and leisure is an essential part of people's life. However, on the one hand, the long queues in supermarkets, especially in the rush hours of weekends and holidays, often make customers abandon their purchased goods and leave the supermarket empty-handed. On the other hand, many ordinary customers complain about the various expensive gift boxes put forward by supermarkets on holidays. Reporter three-phase, Chen Heng [1] had visited Beijing Jingkelong supermarket carrefour stores and found that not only is at the feast, in the daily alcohol and tobacco, health food and fruit, candy, dried fruits, vegetables and other ordinary commodity has a variety of gift boxes for sale, the most exquisite packaging gift boxes goods, parcels sealed, box quantity is little, but the price is the same "lite" goods more expensive. The reporter visited and found that the company's group purchase is the main source of these gift boxes, and for the vast majority of ordinary customers, beautiful packaging once opened will become worthless waste, better than buying bulk cost-effective, so most ordinary customers are not willing to buy. This not only affects the turnover of the supermarket, but also affects the shopping experience of customers. With the continuous promotion of personalized and customized services [2], many supermarkets have launched the service of "customers choose New Year goods by themselves". Customers can freely match goods and then the supermarkets provide packaging services. For example, Hangzhou Citylife century Lianhua life supermarket launched this customized service in 2015, providing nearly 20 kinds of packages at different prices, which largely meets the personalized needs of customers. Another example is Ole's boutique supermarket in Hangzhou, Tiangong supermarket and rt-mart supermarket in Haikou, which offer similar customized services. Although this kind of service is popular with many customers, it also brings about some problems. On the one hand, from the perspective of supermarkets, higher requirements are put forward for the ability of employees. Employees not only need to be proficient in packaging skills, but also need to have the ability to effectively communicate with customers. For this reason, supermarkets may need to hire new employees or train old employees, which will undoubtedly incur certain costs. If supermarkets continue to employ untrained staff, the quality and speed of packaging will also make the customer experience worse. On the other hand, from the perspective of customers, customers need to wait in line twice, one time for packaging, the other time for payment. In holidays, too long queues greatly affect customers' shopping experience.[3] Therefore, how to find a balance in terms of customized service, cost and customer queuing is an important problem for supermarkets to solve.

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Based on the above analysis, this paper from the customer to buy packaged gift box and homemade gift bag two perspective, in a single counter series queuing system and queuing system on the basis of the establishment of a homemade gift bag service mix queuing system and purchase for the performance of M/M / 1 queue system box, respectively of the queuing system to build the game process between two phases: first by the supermarket to determine the optimal price, then determine its equilibrium can be reached by the customer, in order to determine the supermarket's biggest profit under two kinds of queuing system, so as to determine when to launch a service homemade gift bag, when direct selling packaged gift box.

There is no lack of research on supermarket queuing system at home and abroad and Lucey [4] pointed out that in supermarkets, too long waiting time for checkout will reduce customers' satisfaction in the whole shopping activity. Xiong Jun magnitude [5] through the questionnaire survey to get the customer can tolerate the longest queue length and waiting for the longest time, through the actual data collection in the supermarket, got the number of customers to reach a minute, and each customer's service time, on the basis of the supermarket checkout service/up/up of M/M/c queuing system to get the corresponding line indicators, and queue indexes were optimized by using SPSS software, it is concluded that the best open the register number. Lin Chenqi [6] used Arena simulation software to conduct simulation experiments on the cashier service system of Hangzhou Wu Mart supermarket (Wenze road store), and obtained the optimal ratio between the number of customers and the number of service counter development. From the perspective of customer experience affected by excessive queuing in supermarket cashier service, this paper analyzes the optimization of the number of cashier service counters by using queuing theory. There are many factors that affect customers' shopping experience in supermarkets [7]. Checkout line is just one of them. Therefore, for the supermarket, whether it is suitable to launch homemade gift package service has become a key issue. However, there is a lack of research on this aspect.

The model discussed in this paper can also be extended to other cases of personality service and common service classification. For example, many Banks now set up self-service teller machines. For some standardized financial services, customers can complete on the self-service teller machine, but some special services still need to be completed by human. Usually, after the service of special needs, the staff will ask the customers to operate the ATM by themselves if they have common needs, such as printing, inquiry, etc.

2. TRADITIONAL SUPERMARKET QUEUING SYSTEM

2.1 Model Description and Performance Indicators

We start from a real case, such as the homemade New Year gift package launched by Hangzhou Citylife Century Lianhua Supermarket. We set up a queuing system as shown in figure 1. We divide customers into two categories: one is the customer who needs gift box packing. After purchasing the goods, they first queue up at the service counter 1 for gift box packing, and then queue up at the service counter 2 for payment. The second type of customers are those who don't need to pack gift boxes. They go directly to the service counter 2 and queue for payment. λ_1 and λ_2, the service time is exponential, with a service rate of μ_1 and μ_2. In this paper, we assume that μ_1 and μ_2 are constant, because as workers' proficiency tends to stabilize over time, so does the number of people served per hour, so it makes sense to assume that μ_1 and μ_2 are constant.

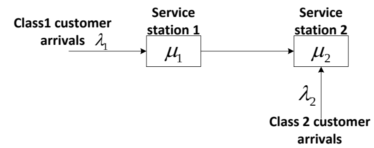

Figure 1. Traditional supermarket queuing system

Introduction 1. The necessary and sufficient condition for stable service counter 1 is $\frac{\lambda_1}{\mu_1} < 1$, and the necessary and sufficient condition for stability of service counter 2 is $\frac{\lambda_1+\lambda_2}{\mu_2} < 1$. The duration of a customer's stay at counter 1 is $w_1 = \frac{1}{\mu_1-\lambda_1}$, and the staying time of the second type of customers and the first type of customers at the service counter 2 is $w_1 = \frac{1}{\mu_2-\lambda_1-\lambda_2}$, The expected utility of type two customers per unit time is $u_2 = R_2 - P_2 - \frac{c}{\mu_2-\lambda_1-\lambda_2}$.

2.2 Customer Equilibrium Queuing Decision Analysis

2.2.1 Model Description and Performance Indicators

Firstly, we give the arrival rate of two kinds of customers and analyze the queuing equilibrium decision of one kind of customers. Suppose that the expected revenue obtained by a class of customers in unit time is R_1, the cost paid is P_1 (including the purchase cost of the goods and the gift box packaging cost), and the waiting cost per unit time is c, so the expected utility of a class of customers in unit time is $u_1 = R_1 - P_1 - \frac{c}{\mu_1-\lambda_1} - \frac{c}{\mu_2-\lambda_1-\lambda_2}$. Equilibrium arrival rate for a class of customers λ_1 shall meet $u_1 = 0$, we have

$$\lambda_1 = \arg\left\{R_1 - P_1 - \frac{c}{\mu_1-\lambda_1} - \frac{c}{\mu_2-\lambda_1-\lambda_2} = 0\right\}$$
$$= \frac{\alpha_1(\mu_1+\mu_2-\lambda_2)-2c+\sqrt{\alpha_1^2(\mu_1-\mu_2+\lambda_2)^2+4c^2}}{2\alpha_1} \quad (1)$$

where $\alpha_1 = R_1 - P_1$.

Note: when the arrival rate of the second type of customers λ_2 and the average selling price of the goods P_1 are determined, the arrival rate of the first type of customers λ_1 cannot be higher than that of the first type of customers, otherwise they will get negative returns due to the long waiting time, so they will leave directly without purchasing any goods.

2.2.2 Equilibrium Decision Analysis for Two Types of Customers

Given the arrival rate of a class of customers, we analyze the queuing equilibrium decision of a class of customers. Suppose that the expected revenue obtained by type two customers in unit time is R_2, the cost paid is $P_2(P_1 \geq P_2)$, Unit time waiting cost is c. The expected utility of type two customers per unit time is $u_2 = R_2 - P_2 - \frac{c}{\mu_2-\lambda_1-\lambda_2}$. Equilibrium arrival rate of two types of customers λ_2 should have $u_2 = 0$. We have

$$\lambda_2 = \arg\left\{R_2 - P_2 - \frac{c}{\mu_2-\lambda_1-\lambda_2} = 0\right\} = -\frac{c}{\alpha_2} + \mu_2 - \lambda_1 \quad (2)$$

where $\alpha_2 = R_2 - P_2$.

Note: when the arrival rate of the first type of customers λ_1 and the average selling price of the goods P_2 are determined, the arrival rate of the second type of customers λ_2 cannot be higher than that, or the waiting time will be too long, and negative returns will be obtained.

2.2.3 Customer Equilibrium Queuing Decision Analysis

Next, we will consider the equilibrium queuing decision for both type 1 and type 2 customers. Equations (1) and (2) are set up in parallel

$$\begin{cases} \lambda_1 = \dfrac{\alpha_1(\mu_1+\mu_2-\lambda_2)-2c+\sqrt{\alpha_1^2(\mu_1-\mu_2+\lambda_2)^2+4c^2}}{2\alpha_1} \\ \lambda_2 = -\dfrac{c}{\alpha_2}+\mu_2-\lambda_1 \end{cases} \tag{3}$$

We can get the unique solution

$$\begin{cases} \lambda_1^e = \mu_1 - \dfrac{c}{\alpha_1-\alpha_2} \\ \lambda_2^e = \mu_2 - \mu_1 - \dfrac{c}{\alpha_2} + \dfrac{c}{\alpha_1-\alpha_2} \end{cases} \tag{4}$$

λ_1^e and λ_2^e are equilibrium queuing decisions of one type customer and two type customers.

Property 1:

$\frac{\partial \lambda_1^e}{\partial P_1} = -\frac{c}{(\alpha_1-\alpha_2)^2} < 0$, $\frac{\partial^2 \lambda_1^e}{\partial P_1^2} = -\frac{2c}{(\alpha_1-\alpha_2)^3} < 0$, $\frac{\partial \lambda_1^e}{\partial P_2} = \frac{c}{(\alpha_1-\alpha_2)^2} > 0$, $\frac{\partial^2 \lambda_1^e}{\partial P_2^2} = -\frac{2c}{(\alpha_1-\alpha_2)^3} < 0$

Property 2:

$\frac{\partial \lambda_2^e}{\partial P_1} = \frac{c}{(\alpha_1-\alpha_2)^2} > 0$, $\frac{\partial^2 \lambda_2^e}{\partial P_1^2} = \frac{2c}{(\alpha_1-\alpha_2)^3} > 0$, $\frac{\partial \lambda_2^e}{\partial P_2} = -\frac{c}{\alpha_2^2} - \frac{c}{(\alpha_1-\alpha_2)^2} < 0$, $\frac{\partial^2 \lambda_2^e}{\partial P_2^2} = -\frac{2c}{\alpha_2^3} + \frac{2c}{(\alpha_1-\alpha_2)^3}$.

2.2.4 Supermarket Optimal Pricing Problem

The supermarket aims to select the optimal price P_1 and P_2 to maximize its profit per unit time, we have

$$\max_{P_1,P_2} \overline{\Pi} = P_1\lambda_1^e + P_2\lambda_2^e - \omega_1\mu_1 - \omega_2\mu_2 \tag{5}$$

ω_1 and ω_2 are the unit service cost of service counter 1 and service counter 2 in the mixed system respectively.

Theorem 1: here is a unique best price $P_1^* = R_1 - \sqrt{\frac{cR_2}{\mu_2}} - \sqrt{\frac{c(R_1-R_2)}{\mu_1}}$ and $P_2^* = R_2 - \sqrt{\frac{cR_2}{\mu_2}}$ to make the supermarket's expected profit per unit time largest. The maximized profit $\overline{\Pi}^* = R_1\mu_1 + R_2(\mu_2-\mu_1) - 2\sqrt{c(R_1-R_2)\mu_1} - 2\sqrt{cR_2\mu_2} + 2c - \omega_1\mu_1 - \omega_2\mu_2$.

Proof: based on $u_1(\lambda_1^e,\lambda_2^e)$=0 and $u_2(\lambda_1^e,\lambda_2^e)$=0, we have $0 = R_1 - P_1 - \frac{c}{\mu_1-\lambda_1^e} - (R_2-P_2) < (R_1-P_1) - (R_2-P_2) = (\alpha_1-\alpha_2)$, so that $\alpha_1 > \alpha_2$, therefore, $\frac{\partial^2\overline{\Pi}}{\partial P_1^2} = -\frac{2c}{(\alpha_1-\alpha_2)^2} - \frac{2c(P_1-P_2)}{(\alpha_1-\alpha_2)^3} < 0$, $\frac{\partial^2\Pi}{\partial P_2^2} = -\frac{2c(P_1-P_2)}{(\alpha_1-\alpha_2)^3} - \frac{2c}{\alpha_2^2} - \frac{2c}{(\alpha_1-\alpha_2)^2} - \frac{2cP_2}{\alpha_2^3} < 0$ and $\frac{\partial^2\overline{\Pi}}{\partial P_1\partial P_2} = \frac{\partial^2\overline{\Pi}}{\partial P_2\partial P_1} = \frac{2c}{(\alpha_1-\alpha_2)^2} + \frac{2c(P_1-P_2)}{(\alpha_1-\alpha_2)^3}$. Hessin matrix is less than 0, we have

$$H = \begin{pmatrix} \dfrac{\partial^2\overline{\Pi}}{\partial P_1^2} & \dfrac{\partial^2\overline{\Pi}}{\partial P_1\partial P_2} \\ \dfrac{\partial^2\overline{\Pi}}{\partial P_2\partial P_1} & \dfrac{\partial^2\overline{\Pi}}{\partial P_2^2} \end{pmatrix} = -\frac{4c^2(P_2+\alpha_2)(\alpha_1-\alpha_2+P_1-P_2)}{(\alpha_1-\alpha_2)^3\alpha_2^3} < 0,$$

Therefore, there exists a unique optimal price P_1^* and P_2^* to maximize the expected profit of the supermarket in unit time, and P_1^* and P_2^* are the roots of the following equations:

$$\begin{cases} \dfrac{\partial\overline{\Pi}}{\partial P_1} = \mu_1 - \dfrac{c}{\alpha_1-\alpha_2} - \dfrac{c(P_1-P_2)}{(\alpha_1-\alpha_2)^2} = 0 \\ \dfrac{\partial\overline{\Pi}}{\partial P_2} = \mu_2 - \mu_1 + \dfrac{c(P_1-P_2)}{(\alpha_1-\alpha_2)^2} + \dfrac{c}{\alpha_1-\alpha_2} - \dfrac{c}{\alpha_2} - \dfrac{cP_2}{\alpha_2^2} = 0 \end{cases}.$$

We have

$$\begin{cases} P_1^* = R_1 - \sqrt{\dfrac{cR_2}{\mu_2}} - \sqrt{\dfrac{c(R_1-R_2)}{\mu_1}} \\ P_2^* = R_2 - \sqrt{\dfrac{cR_2}{\mu_2}} \end{cases} \tag{6}$$

We can update customers' equilibrium queuing decisions:

$$\begin{cases} (\lambda_1^e)^* = \mu_1 - \sqrt{\dfrac{c\mu_1}{R_1-R_2}} \\ (\lambda_2^e)^* = \mu_2 - \mu_1 - \sqrt{\dfrac{c\mu_2}{R_2}} + \sqrt{\dfrac{c\mu_1}{R_1-R_2}} \end{cases} \tag{7}$$

Note: In short term, supermarkets can get the highest profits by quantifying effective demand forecasts and satisfaction surveys and pricing according to P_1^* and P_2^*.At this point, if the arrival rates of the two types of customers is less than $(\lambda_1^e)^*$ and $(\lambda_2^e)^*$, they can have a satisfying shopping experience.

3. WHEN SHOULD WE OFFER CUSTOMIZED GIFT BOX PACKING SERVICE?

In the past, many supermarkets directly provided packaged gift boxes for sale. Therefore, we first established a queuing model of the previous situation, and then compared it with the customized gift box packaging service launched now, to analyze when to provide customized gift box packaging service, and when not to provide. The past service situation can be described as the M/M/1 queuing system shown in figure 2. Customers arrive at the system with Possion distribution with arrival rate of Λ, and service counter 0 follows the index service time with a service rate of μ_0.

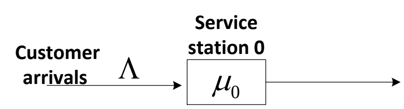

Figure 2: M/M/1 queuing system

Introduction 2: The necessary and sufficient conditions for the stability of the system are $\frac{\Lambda}{\mu_0} < 1$. In this queuing system, the expected utility of the customer per unit time is $\mathrm{u} = R - P - \frac{c}{\mu_2 - \Lambda}$, In this queuing system, the expected utility of customers per unit time is $\mathrm{u} = R - P - \frac{c}{\mu_2 - \Lambda}$, where R is the average revenue of customers and c is the waiting cost per unit time of customers. Because $\mathrm{u} = 0$, we have equilibrium arrival rate of customers $\Lambda^e = \mu_0 - \frac{c}{R-P}$. At this time, the service provider's goal is to find the best price P^* to maximize the expected revenue per unit of time, so we have

$$\max_P \Pi = P\Lambda^e - \omega_0\mu_0, \qquad (8)$$

where ω_0 is the unit cost a service provider provides to a customer in this system. We solute planning model (8) and have optimal price $P^* = \mathrm{R} - \sqrt{\frac{cR}{\mu_0}}$,the maximized profit $\Pi^* = (R - \omega_0)\mu_0 - 2\sqrt{cR\mu_0} + c$ and updated equilibrium arrival rate $(\Lambda^e)^* = \mu_0 - \sqrt{\frac{c\mu_0}{R}}$.

Theorem 2: When customer satisfaction, customer waiting cost per unit time, employee service rate and supermarket service cost per customer meet the following condition: $(\sqrt{(R_1 - R_2)\mu_1} - \sqrt{c})^2 + (\sqrt{R_2\mu_2} - \sqrt{c})^2 - \left(\sqrt{R\mu_0} - \sqrt{c}\right)^2 > \omega_1\mu_1 + \omega_2\mu_2 - \omega_0\mu_0$, supermarket should offer customized gift box packaging service. Otherwise if $(\sqrt{(R_1 - R_2)\mu_1} - \sqrt{c})^2 + (\sqrt{R_2\mu_2} - \sqrt{c})^2 - \left(\sqrt{R\mu_0} - \sqrt{c}\right)^2 \le \omega_1\mu_1 + \omega_2\mu_2 - \omega_0\mu_0$, supermarket should not offer customized gift box packaging service. At this point, customers who purchase customized gift boxes for packaging and those who do not purchase customized gift boxes for packaging should enter the queuing system with the arrival rate no higher than $(\lambda_1^e)^*$ and $(\lambda_2^e)^*$ to get a non-negative return.

Notice: According to customer satisfaction survey, combined with supermarket's own service rate, if $(\sqrt{(R_1 - R_2)\mu_1} - \sqrt{c})^2 + (\sqrt{R_2\mu_2} - \sqrt{c})^2 - \left(\sqrt{R\mu_0} - \sqrt{c}\right)^2 > \omega_1\mu_1 + \omega_2\mu_2 - \omega_0\mu_0$, supermarket offer customized gift box packaging service and set average price to P_1^* and P_2^* and earn maximized profit. If the supermarket speculates that $(\sqrt{(R_1 - R_2)\mu_1} - \sqrt{c})^2 + (\sqrt{R_2\mu_2} - \sqrt{c})^2 - \left(\sqrt{R\mu_0} - \sqrt{c}\right)^2 \le \omega_1\mu_1 + \omega_2\mu_2 - \omega_0\mu_0$, supermarket should not provide customized gift box packaging service, but directly sell packaged gift boxes and set the average price to P^*. At this point, the customer should enter the queuing system with an arrival rate no higher than $(\Lambda^e)^*$.

4. CONCLUSION

The combination of cashier and weighing is a method adopted by many large supermarkets to solve the problem of customers queuing, which is either completed by cashiers or completed by customers themselves. In this way, although the queuing time of some customers is reduced to a certain extent, a lot of new problems are created. From the perspective of queuing theory, this paper discusses when to combine weighing and cashier, and when to separate weighing and cashier when the service rate of supermarket staff is stable. This paper not only effectively alleviates the problem of queuing, but also provides a theoretical basis for how to price supermarket under two ways (weighing and cashier merging or separating).The analysis results of the model have great practical reference significance to Banks, airports, stations and hospitals.

5. ACKNOWLEDGMENTS

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