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Contingency and Choice: The Implications of Matching Theory for Classroom Instruction

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This paper describes a mathematical account of behavior known as matching theory. Matching theory evolved out of basic operant research and assumes that individuals can engage in a variety of behaviors at any moment, but they choose one to the exclusion of others. According to the matching equation, choices in behavior match the relative amount of reinforcement provided for each alternative. Although the principles of matching theory have proven useful in developing novel treatment strategies, few data exist validating the matching equation in natural human environments. Recent applications of matching theory to children's classroom behavior are described, and the implications of matching theory for classroom management and effective teaching are discussed.

KEY WORDS: matching theory; classroom behavior; effective teaching.

Several authors have observed that applied behavior analysts are becoming less concerned with basic experimental principles, and are focusing instead on the clinical applications of behavioral technology (McDowell, 1988; Pierce & Epling, 1980). This situation would be desirable if we had a full understanding of how basic experimental principles operated in applied settings. Quite the opposite, however, this situation has been viewed as problematic because current behavioral technology emphasizes only the most rudimentary principle of behavior, the law of effect (Pierce & Epling, 1980).

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The growing distinction between experimental and applied behavior analysis has been related to a variety of factors including economic contingencies on the activity of researchers, increasing adoption of behavioral technology by professionals outside the field (e.g., educators), and editorial policies of major journals reporting behavioral research. Increased support in the applied literature for what Pierce and Epling (1980) have termed a "cure-help approach" (p. 3) has resulted in few attempts to extend basic science principles to behavior in applied settings (cf. Mace, Hock, Lalli, West, Belfiore, Pinter, & Brown, 1988; Martens, Lochner, & Kelly, 1992). Particularly overlooked by applied researchers have been mathematical accounts of behavior which frequently enjoy extensive laboratory support.

The purpose of this article is to describe one such mathematical account of behavior known as matching theory (Herrnstein, 1961, 1970), and to present research demonstrating how matching theory can be applied to children's classroom behavior. As the article will show, the characteristics of teacher-student interactions make classroom behavior particularly suited to matching theory analysis. These characteristics are identified, and the implications of matching theory for classroom management and effective teaching are discussed.

THE MATCHING EQUATION

Matching theory (Herrnstein, 1961, 1970) is a mathematical description of choice behavior. A central premise of matching theory is that, at any given moment, individuals have a variety of alternative behaviors in which to engage, and they select one behavior to the exclusion of others. In the classroom, for example, a child who has just been given a seatwork assignment may choose to begin working quietly, leave her seat to sharpen a pencil, or turn to visit with a neighbor. Choices among behaviors occur continuously (e.g., the child may switch to a different activity at any time), and consequences (either programmed or naturally occurring) are associated with each selection (McDowell, 1988). Choosing to visit with a neighbor may result in the child described above meeting her friend for lunch. Conversely, choosing to work quietly at her seat may result in teacher praise and extra free time after school.

Matching theory evolved out of basic concurrent schedule research in the 1950's and 1960's (e.g., Catania, 1963; Findley, 1958). In a concurrent schedule experiment, subjects are presented with a choice between two response alternatives (e.g., pigeons allowed to peck a key on the left or right), and each response is associated with an independent schedule of reinforcement. Variable-interval (VI) schedules are typically used which reinforce the first response following brief periods of time that average to some predetermined value (e.g., an average of every 2 min). Other common features of concurrent schedule research include the use of substitutable reinforcers, identical response alternatives, and a delay in reinforcement when the subject switches to a different schedule. Subjects are typically maintained under a given pair of schedules (e.g., VI2 min for one behavior and VI4 min for the second behavior) until their distribution of responding stabilizes across the two alternatives.

Research using concurrent schedules of reinforcement has found consistently that subjects tend to distribute their behavior across the two response alternatives in proportion to reinforcement provided for those same alternatives (Davison & McCarthy, 1988; Herrnstein, 1961). For example, if twice as much reinforcement is provided for pecking the key on the left, rates of responding to that key will be double that of the key on the right. The finding that *relative* amounts of behavior "match" *relative* amounts of reinforcement in contrived choice situations has been obtained with a wide variety of species, behaviors, and reinforcers (Baum & Rachlin, 1969; de Villiers, 1977; McDowell, 1988), giving rise to a mathematical statement of choice behavior termed the matching law. The matching law is represented by the equation

$$\frac{R_1}{R_1 + R_2} = \frac{r_1}{r_1 + r_2} \tag{1}$$

where R_1 and R_2 represent rates of responding to the two alternatives and r_1 and r_2 represent rates of reinforcement for those same alternatives. The matching equation is a simple equivalency which states that proportions of behavior equal or match proportions of reinforcement in a two-choice situation.

In 1970, Herrnstein proposed a mathematically equivalent form of Equation 1 to describe situations when only one response was under experimenter control. This version of the matching equation relates *absolute* rates of the target behavior to *absolute* rates of reinforcement (Herrnstein, 1970). Whereas the notion of choice was explicit in the original matching equation, the revised version depicts choice as occurring implicitly between the target behavior, for which reinforcement is known, and all other competing behaviors, for which reinforcement is unknown (Millenson & Leslie, 1979). This form of the matching equation has often been referred to as a quantitative statement of the law of effect because it describes in mathematical terms the effects of contingent reinforcement on rates of responding. The single-alternative form of the matching equation is written as

$$R = \frac{kr}{r + r_e} \quad (2)$$

where R is the rate of the target behavior, k is the maximum possible rate of this behavior, r is the rate of reinforcement contingent on the target behavior, and r_e is extraneous or background reinforcement. The parameter r_e in the equation is particularly noteworthy because it represents all reinforcement obtained by the subject either spontaneously, independent of responding (i.e., noncontingent reinforcement), or for other, competing behaviors. According to Herrnstein's equation, contingent reinforcement (r) increases behavior (R) but does so relative to all other reinforcement available to the subject (r_e). Thus, when there is little reinforcement available for competing behavior, increasing reinforcement for the target behavior produces large effects. Conversely, efforts to reinforce the target behavior yield smaller results when reinforcement for competing behavior is abundant (McDowell, 1982). Since Equation 2 was first introduced in 1970, it has undergone extensive scrutiny. Although several changes in the equation have been proposed (e.g., the generalized matching equation), many of the assumptions which underlie strict matching continue to receive support (Davison & McCarthy, 1988, pp. 45-46). (The interested reader is referred to Davison & McCarthy for a complete discussion of the evolution of the matching equation).

In contrast to previous theories of reinforcement (e.g., Ferster & Skinner, 1957), Herrnstein's equation describes a curvilinear relationship between reinforcement and behavior. The shape of this curve, which takes the form of a hyperbola, corresponds to how rapidly behavior reaches its maximum rate (k) as a function of reinforcement. It is interesting to note here that hyperbolic functions have been observed in other branches of science including, for example, the volume of air expired from the lungs, the action of enzymes on a substrate (Dowd & Riggs, 1965), and the rate at which the universe is believed to be expanding (Hawking, 1988). Just as mathematical accounts have advanced understanding in these areas, matching theory advances our understanding of reinforcement effects by (a) specifying the shape of the function between reinforcement and behavior, and (b) showing mathematically how this function is affected by the context in which it occurs.

THE APPLIED RELEVANCE OF MATCHING THEORY

Matching theory acknowledges the existence of choice by taking into account reinforcement available for other behaviors when predicting the rate of a single behavior. Stated more generally, Herrnstein's equation assumes that, in any setting, there exists a context of reinforcement against which reinforcement for a target behavior must be evaluated (Martens, 1990; Martens & Witt, 1988; Schoenfeld & Farmer, 1970). Given that reinforcers are available in natural human environments for a variety of competing behaviors (Myerson & Hale, 1984), this assumption has a certain intuitive appeal and offers a more ecologically valid description of applied behavior than previous accounts.

An important implication of matching theory is that rates of behavior are determined by the relative proportion rather than absolute amount of reinforcement for responding. Because matching theory emphasizes the role of reinforcement context, the theory also has several treatment implications (McDowell, 1982, 1988). Desired behavior can be increased directly by increasing reinforcement for that behavior, or indirectly by decreasing the amount of reinforcement available for competing behavior. The latter approach suggests a means of increasing the potency of reinforcement-based treatment procedures. According to matching theory, these procedures can be made more effective by reducing the total amount of reinforcement available in a given setting prior to their implementation. In support of this argument, Sajwaj, Twordosz, and Burke (1972) found that ignoring a preschool child's initiated speech to the teacher (i.e., decreasing the amount of reinforcement available in the setting) was accompanied by increases in cooperative play and initiated speech to children.

Matching theory also suggests that aberrant behavior can be decreased indirectly by increasing reinforcement for competing behaviors or by increasing rates of noncontingent reinforcement. The former set of procedures, or reducing inappropriate behavior through the differential reinforcement of incompatible (DRI) or alternative (DRA) behaviors, has been a popular intervention strategy for some time in the behavior therapy literature (Lentz, 1988). For example, Russo, Cataldo, and Cushing (1981) and Parrish, Cataldo, Kolko, Neef, and Egel (1986) reduced children's inappropriate behavior by reinforcing compliance. Additional studies have found that children's inappropriate classroom behavior decreased following reinforcement of academic performance (Ayllon, Layman, & Burke, 1972; Ayllon, Layman, Kandel, 1975; Ayllon & Roberts, 1974; Ferritor, Buckholdt, Hamblin, & Smith, 1972; Witt, Hannafin, & Martens, 1983).

Although findings such as these are easily explained by matching theory, they have typically been interpreted as side-effects of treatment or attributed to incompatibility among behaviors (Kazdin, 1982; McDowell, 1982). As noted by Parrish et al. (1986), however, these accounts are inadequate since basic concurrent schedule research has shown that behaviors are related, not by virtue of their occurrence, but because of their consequences. Only after discussions of matching theory emerged in the applied literature (e.g., Martens & Witt, 1988; McDowell, 1982, 1988, 1989; Myerson & Hale, 1984) did researchers begin to examine relationships among socially significant behaviors as a function of their rates of reinforcement (e.g., Mace, McCurdy, Quigley, 1990; Martens, Halperin, Rummel, & Kilpatrick, 1990; Martens et al., 1992; Parrish et al., 1986; Simon, Ayllon, & Milan, 1982). To date, research in this area has been concerned with the more basic issue of whether experimental principles can be applied in naturalistic settings. With the exception of DRA and DRI procedures (see also McDowell, 1988), there have been no systematic attempts to develop a technology of behavior change based explicitly on the principles of matching theory.

In an ingenious extension of basic research, Simon, Ayllon, and Milan (1982) examined the interactive effects of token reinforcement at three learning stations. The goal of the study was to determine if a specific effect, known as "behavioral contrast," would be obtained using token reinforcement. Behavioral contrast occurs when behavior that is reinforced in one situation increases after reinforcement for that behavior in a second situation is withdrawn (Reynolds, 1975). Contrast effects often occur when individuals learn to discriminate between two or more schedules of reinforcement presented in succession. Behavioral contrast is similar to matching theory in that it provides a means of changing behavior indirectly. In the case of behavioral contrast, however, this change is accomplished by manipulating reinforcement obtained in a different setting rather than for a competing response.

Seven hearing-impaired middle-school students were allowed to work on math problems for 20-min periods in succession at each of three learning stations. Following an initial baseline period, token reinforcement was made available at all three stations or at only one station in an alternating fashion. Results indicated that withdrawing token reinforcement at two of the stations resulted in more problems being completed at the third station where reinforcement remained constant. Similar to previous studies, with increases in academic performance came decreases in disruptive behavior at each station and vice versa.

In an explicit examination of the indirect effects predicted by matching theory, Mace et al. (1990) reinforced students for correct completion of academic and vocational tasks. In Experiment 1, a 16-year-old male student of average intelligence was given a choice between two types of math problems (multiplication or division). In Experiment 2, a 12-year-old profoundly retarded boy chose between two vocational tasks (assembling pens or bagging silverware). In both experiments, independent schedules of reinforcement were arranged for each response alternative. Continuous and variable ratio 2 (VR2) schedules were used in which candy or chips were provided to subjects following each correct response or following two correct responses, respectively. When both response alternatives were reinforced according to the VR2 schedule, subjects engaged in the two behaviors at approximately equal rates. Switching to a continuous schedule increased the rate of responding to one alternative while rates of the other alternative decreased even though reinforcement remained constant.

MATCHING THEORY APPLIED TO CLASSROOM BEHAVIOR

The evidence available to date supports the heuristic value of matching theory in developing and refining intervention strategies. Despite the theory's conceptual utility, however, validation of the matching equation in applied settings has been difficult to accomplish. As noted previously in the description of concurrent schedule research, matching has been obtained in the laboratory by observing certain procedural requirements (e.g., the use of equivalent reinforcers). To the extent that these requirements are absent in applied settings, the validity of Equation 2 for socially significant behavior may be limited. According to Fuqua (1984), " . . . empirical validations of the matching law . . . are noticeably lacking and any attempts at such validation in applied settings will need to grapple with several quantification problems" (p. 382). Notable among these problems are the use of measures along which different responses can be equated and the ability to identify multiple reinforcers contingent on a variety of competing behaviors (Herrnstein, 1979; Martens & Houk, 1989). Recently, procedures have been developed that address some of these methodological concerns, enabling matching theory to be applied to children's classroom behavior (e.g., Martens & Houk, 1989).

Contingency and Choice in the Classroom

The classroom setting has several characteristics that make it particularly suited to matching theory analysis. First, most activities in the classroom are directed toward the goal of learning academic material. Toward this goal, curricula are typically arranged in small units which build on each other in a hierarchical fashion. Each unit, in turn, consists of several, relatively self-contained activities that introduce a new skill and provide opportunities for drill. This structure requires both teacher-directed and independent learning and, from the student's perspective, results in numerous decision points throughout the school day (e.g., to work quietly or 'interact with peers, which task to complete first). In addition, because skills must be acquired at a certain pace determined by the curriculum demands (Gettinger, 1988), consequences for engaging in appropriate (e.g., learning-directed) or inappropriate (e.g., disruptive) behavior are often immediate. From the perspective of matching theory, therefore, classrooms represent settings in which choices between alternative behaviors are abundant.

Second, a significant portion of social reinforcement in the classroom is under teacher control. This is especially true in the elementary grades where younger children view adult attention and approval as highly desirable (White, 1975). Not only have the various forms of teacher attention been extensively cataloged (e.g., Atwater & Morris, 1988), but a large body of research exists demonstrating the effects of teacher attention on children's classroom behavior. Both verbal (e.g., praise, individual instruction) and nonverbal (e.g., smiles, proximity) attending behaviors have proven effective in altering such student behaviors as time on-task (Broden, Bruce, Mitchell, Carter, & Hall, 1970; Kazdin & Klock, 1973; Madsen, Becker, & Thomas, 1968), compliance (Parrish et al, 1986; Schutte & Hopkins, 1970), and classroom disruption (Thomas, Becker, & Armstrong, 1968). Although interaction with peers has been shown to both facilitate and impede learning (e.g., Greenwood, Carta, & Hall, 1988; Hoge & Andrews, 1987), the traditional classroom structure provides teachers with a great deal of social control. Again, from a matching theory perspective, this arrangement greatly simplifies the task of identifying social reinforcers available for various classroom behaviors.

Third, although teacher attention can be a potent determinant of student behavior, the frequency and timing of student contacts is a function of numerous factors (e.g., class size, curriculum demands), many of which are beyond teacher control (Witt & Martens, 1988). In addition, teachers tend to rely on instructional or managerial statements directed toward children in groups, making it difficult to attend selectively to the behavior of an individual child (Atwater & Morris, 1988; Martens et al., 1990). Even the most disciplined teacher will find it impossible attending to children only when they are behaving appropriately (e.g., Madsen et al., 1968). Evidence of this fact was provided in a recent study by Martens (1990). Using a computer-assisted observation system, measures were taken of the overlap in real time between six categories of teacher attention (e.g., individual and group instruction, praise) and appropriate student behavior. Results indicated that the majority of teacher attention was indeed provided to students while they were appropriately engaged. However, no category of teacher attention was perfectly contingent on desired behavior, with almost 30% of some attending behaviors occurring while students were behaving inappropriately. These findings are consistent with matching theory in suggesting that a larger context of social reinforcement exists in the classroom against which attention for desired behavior must be evaluated.

Validation of the Matching Equation

Using both naturalistic observation and programmed social reinforcement, a series of recent investigations has attempted to apply matching theory as a description of children's classroom behavior with promising results (Martens, 1990; Martens & Houk, 1989; Martens et al., 1990, 1992). In an initial effort, Martens (1990) collected detailed observational data in two classrooms comparing the relationship of time on-task to three measures of contingent teacher attention. The first measure, total amount (TA), was an index in seconds per minute of all attention (both instructional and managerial) received by subjects regardless of their behavior. The second measure, contingent amount (CA), represented the amount of teacher attention delivered to subjects contingent only on appropriate behavior. The third measure, proportion of the total amount contingent on behavior (CA/TA), was calculated as attention for appropriate behavior taken as a percentage of all attention received by the subjects. This latter measure represented the amount of attention for desired behavior *relative* to attention provided for other, competing behaviors.

According to matching theory, the proportional measure (CA/TA) should best predict time on-task because the relative proportion rather than absolute amount of teacher attention is viewed as the more potent determinant of behavior. Results confirmed this prediction, with CA/TA accounting for an average 69% of the variance in time on-task, CA accounting for an average 18% of the variance in time on-task, and TA accounting for a mere 5% of the variance in time on-task. Because the study was correlational in nature, it cannot be inferred that increases in teacher attention caused the observed increases in on-task behavior. However, the finding that time on-task more closely "matched" the proportional measure of contingent teacher attention is consistent with Equation 1.

In a direct application of matching theory, Martens et al. (1990) attempted to fit Equation 2 to the behavior of a first-grade student in a remedial summer school class. Again, minute-by-minute indices of student behavior and naturally occurring teacher attention were obtained using a computer-assisted observation system. These data were used to examine the relationship between time on-task and contingent teacher attention at four levels of background reinforcement (i.e., attention for inappropriate behavior). As specified by the matching equation, increases in contingent

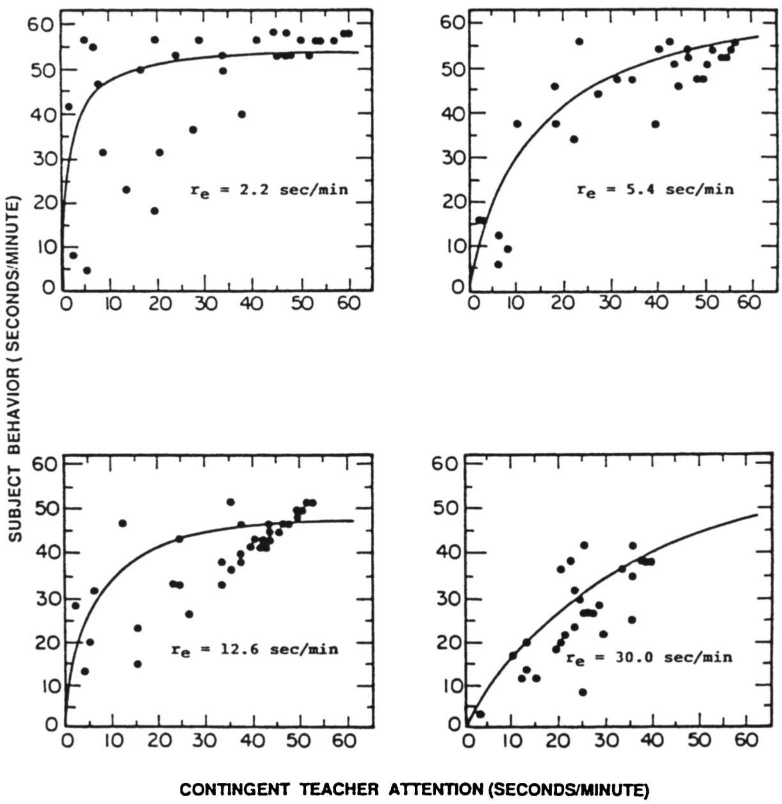

Fig. 1. Hyperbolic functions describing the relationship between on-task student behavior and contingent teacher attention at each of four levels of naturally occurring background reinforcement. From "Matching Theory Applied to Contingent Teacher Attention" by B. K. Martens, S. Halperin, J. E. Rummel, and D. Kilpatrick, 1990, *Behavioral Assessment*, 12, p. 150. Copyright 1990 by Pergamon Press. Reprinted by permission.

teacher attention (r) should produce larger increments in time on-task (R) as the amount of attention concurrently available for other, competing behaviors decreases (r_e). Presented in Figure 1 are the fitted hyperbolic functions describing the relationship between the student's on-task behavior and contingent teacher attention at each of four levels of naturally occurring background reinforcement. Consistent with Herrnstein's equation, increases in contingent teacher attention were associated with more rapid increases and higher sustained levels of on-task behavior when less teacher attention was available for competing behaviors (i.e., the top left of the figure). Moreover, the matching equation provided a significantly better fit to the data than an absolute model of reinforcement, accounting for 51% of the variance in the student's behavior.

In a recently completed study, Martens et al. (1992) exposed two fourth-grade students to VI schedules of social reinforcement contingent on academic engagement. An experimenter noted whether the students were engaged in schoolwork using a 20-sec momentary time sampling procedure and delivered praise at scheduled intervals. Following an initial baseline period, the richness of the VI reinforcement schedules was increased in four phases (VI5 min, VI4 min, VI3 min, VI2 min). Throughout the experiment, six categories of adult and peer behavior and seven categories of student behavior were recorded by independent observers. These observational data were used to obtain independent estimates of teacher and peer attention contingent on inappropriate behavior.

The left-hand portion of Figure 2 depicts the percentage of intervals in which engagement was observed for each of the two subjects across all phases of the experiment. Shown in the right-hand portion of the figure are mean percentages of engagement plotted against mean rates of obtained reinforcement for each child, the fitted hyperbolic functions, and the estimated equation parameters. As shown in the figure, Herrnstein's equation provided an excellent fit to the data, accounting for 99% and 88% of the variance in the students' behavior, respectively. In addition, the fitted hyperbolic functions varied in shape with the observation-based estimates of concurrently available attention in the manner predicted by the matching law. In contrast to the studies reported above, social reinforcement in this study was under experimenter control thereby yielding a more powerful demonstration of matching theory.

IMPLICATIONS OF MATCHING THEORY FOR CLASSROOM INSTRUCTION

A basic tenet of this paper is that, at any moment in the classroom, children are free to engage in a variety of alternative behaviors. The data reported previously suggest the choices in behavior that children make can be predicted by the amount of teacher attention contingent on the various alternatives. These data suggest further that the principles of matching theory which govern choice behavior in the laboratory may also describe choice behavior in the classroom.

The apparent relevance of matching theory for children's classroom behavior has important implications for identifying the principles that underlie effective teaching. A popular trend in research on teaching has been to characterize classroom instruction as an information-processing task in

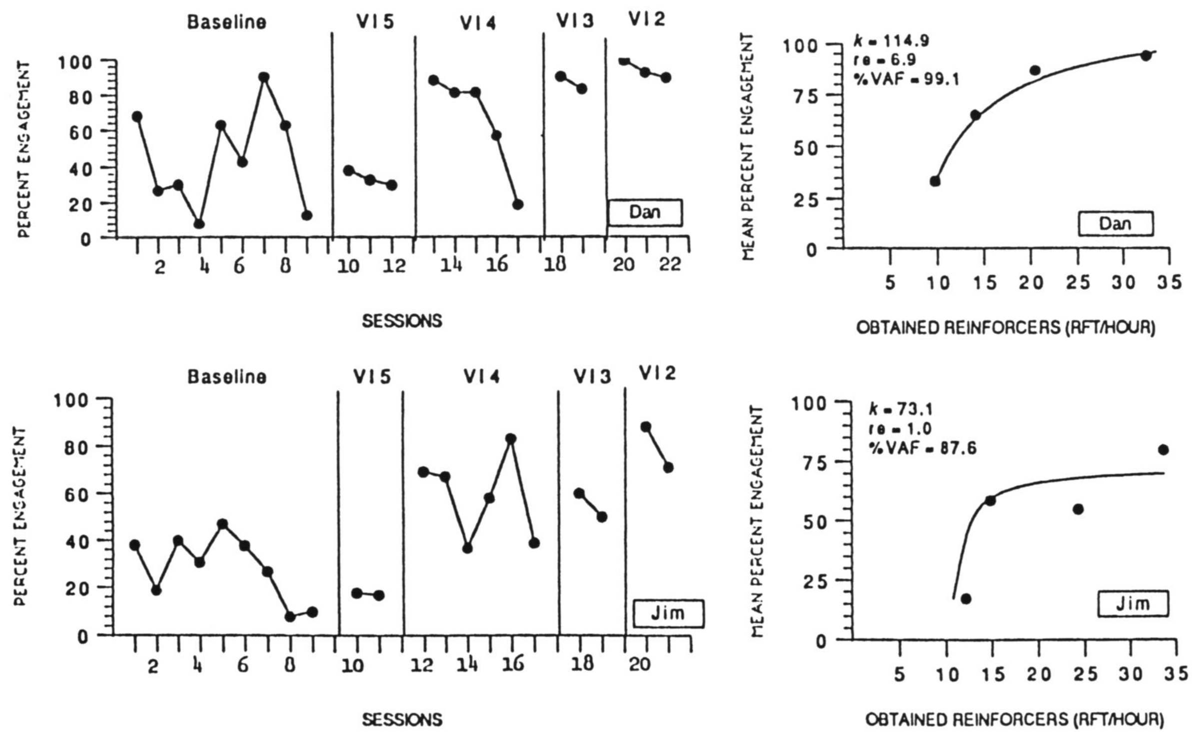

Fig. 2. Percentages of academic engagement for two children under conditions of increasing VI reinforcement (left side). Mean percentages of academic engagement plotted against mean numbers of obtained reinforcers for each VI treatment condition, the fitted hyperbolic functions, and the estimated equation parameters (right side). From "The Effects of Variable-Interval Reinforcement on Academic Engagement: A Demonstration of Matching Theory" by B. K. Martens, D. G. Lochner, and S. Q. Kelly, 1992, *Journal of Applied Behavior Analysis, 25,* p. 147. Copyright 1992 by the Society for the Experimental Analysis of Behavior, Inc. Reprinted by permission.

which teachers recognize, interpret, and respond appropriately to students' instructional needs (Doyle, 1986; Gettinger, 1988). This article proposes an alternative perspective which characterizes effective teaching as the management of behavioral choice.

According to matching theory, sound teaching techniques are also likely to serve as effective management tools because they reduce the amount of reinforcement available to students for inappropriate behavior. As argued by Gettinger (1988), effective teachers are able to prevent most discipline problems from occurring. The available evidence supports this argument by suggesting that effective teachers interact with students in ways that maintain reinforcement for inappropriate behavior at low levels. In such environments, children are continually encouraged to make the choice of engaging in appropriate or desired behavior, thereby greatly reducing the need to deal with inappropriate or problem behavior.

One obvious way that teachers could *increase* the relative amount of attention for appropriate behavior would be to interact with students *only* when they are appropriately engaged and simply ignore children when they misbehave (Brophy, 1983; Harris, 1985; Martens & Meller, 1990). Although these strategies may be effective on occasion, they are likely to prove impractical in the long run because of limitations in the degree of structure teachers can impose on their interactions with children. Remember, teacher attention for appropriate student behavior occurs within a larger reinforcement context (Martens, 1990). Moreover, the teacher's instructional practices and managerial activities contribute to this reinforcement context in sometimes obvious, sometimes subtle ways. Keeping this in mind, effective teaching strategies also serve as effective management tools because they (a) increase the frequency of teacher contact when students are academically engaged, and (b) enable teachers to redirect children who are misbehaving with a minimum of interaction.

Several effective teaching strategies seem well suited to accomplishing the first goal because they place teachers in close proximity to students or increase the duration of teacher-student contact during academic lessons. Included among these strategies are frequent monitoring and supervision of student seatwork, instructing students in small groups, providing detailed feedback for completed assignments, soliciting oral responses from all students during group discussions, moving around the classroom to provide additional assistance, and engaging in high levels of interactive teaching. (The interested reader is referred to Gettinger, 1986 or 1988, for a complete description of effective teaching strategies.) In a similar fashion, various instructional techniques have been suggested for quickly redirecting students who are misbehaving. These strategies include signaling and alerting students prior to giving instructions, withholding instructions until all students are attending, restricting the use of teacher reprimands, and using gestures, demanded eye contact, or verbal prompts as cues to resume working (Gettinger, 1988; Kounin, 1970; Wyne & Stuck, 1982).

CONCLUSIONS

Within the past decade, behaviorism has been the target of severe criticism for its dogmatic belief in environmental determinism, its denunciation of group-comparison methodology, and its failure to assimilate findings from other branches of psychology (Neuringer, 1991). During the heyday of behaviorism in the 1960s and 1970s, criticisms such as these were overshadowed by the simplicity and power with which behavioral principles could be applied to socially significant problems (Baer, Wolf, & Risley, 1968). In the intervening years, applied behavior analysts continued to refine behavioral technology but showed less regard for advances in basic experimental research. Perhaps this lack of interest by applied researchers in more complex models of behavior has contributed in part to behaviorism's reversal of fortune from 20 years ago.

Mathematical descriptions of behavior such as matching theory have been shown to yield more accurate predictions of responding than the simple law of effect (Herrnstein, 1990; Mace et al., 1988; Martens et al., 1990; Pierce & Epling, 1980). The evidence reviewed here and elsewhere (McDowell, 1988) indicates that matching theory can explain unanticipated treatment effects where other approaches have failed, and that it can provide a unifying account of the principles underlying effective classroom instruction. These findings suggest that a technology of behavior may ultimately benefit from continued attempts to extend basic experimental principles to natural human environments.

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