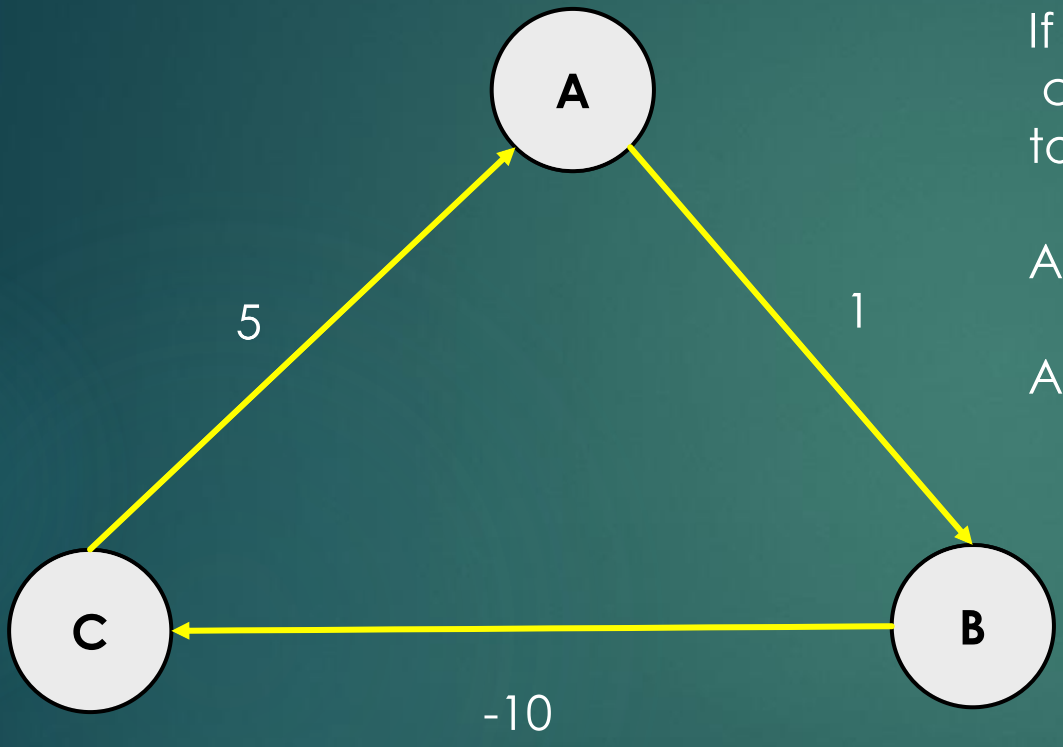
SHORTEST PATH

BELLMAN-FORD ALGORITHM

- Invented in 1958 by Bellman and Ford independently
- Slower than Dijkstra's but more robust: it can handle negative edge weights too
- Dijkstra algorithm choose the edge greedely, with the lowest cost: Bellman-Ford relaxes all edges at the same time for V-1 iteration
- Running time is O(V*E)
- Does V-1 iteration + 1 to detect cycles: if cost decreases in the V-th iteration, than there is a negative cycle, because all the paths are traversen up to the V-1 iteration !!!

Negative cycle:



What is the problem?

If we would like to find a path with the minimum cost we have to go A → B → C → A to decrease the overall cost

And a next cycle: decrease the cost again

And again ...

Real life scenarios: no negative cycles at all ... but sometimes we transform a problem into a graph with positive / negative edge weights and looking for some negative cycles !!!

Bellman-Ford: pseudocode

```
function BellmanFordAlgorithm(vertices, edges, source)

        distance[source] = 0

        for v in Graph
                distance[v] = inf
                predecessor[v] = undefined  // previous node in the shortest path

        for i=1...num_vertexes-1
                for each edge (u,v) with weight w in edges

                        tempDist = distance[u] + w

                        if tempDist < distance[v]
                                distance[v] = tempDist
                                predecessor[v] = u

        for each edge (u,v) with weight w in edges
                if distance[u] + w < distance[v]
                        error: „Negative cycle detected"
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For all edges, if the distance to the destination can be shortened by taking the edge, the distance is updated to the new lower value

V-1 times → we make relaxation

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Since the longest possible path without a cycle can be V-1 edges, the edges must be scanned V-1 times to ensure the shortest path has been found for all nodes

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A final scan of all the edges is performed and if any distance is updated → means *there is a negative cycle !!!*

1970: Yen optimization

- Yen algorithm: it is the Bellman-Ford algorithm with some optimization.
- We can terminate the algorithm if there is no change in the distances between two iterations !!!
- (we use the same technique in bubble sort)

Applications

- Cycle detection can prove to be very important
- Negative cycles as well → we have to run the Bellman-Ford algorithm that can handle negative edge weights by default
- On the FOREX market it can detect arbitrage situations !!!