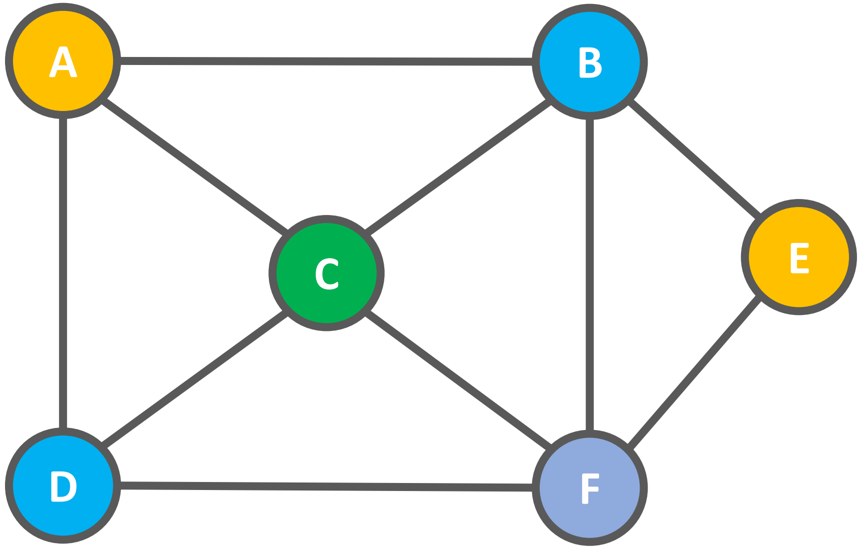
Coloring Problem

(Algorithmic Problems)

Coloring Problem

- the problem is that we have to color the vertices of a **G(V, E)** graph such that no two adjacent vertices share the same color
- we use integer representation for the given colors
- the smallest number of colors needed to color a graph **G(V, E)** is called its **chromatic number**
- there may be more than one solution - for example we can color a graph with **4** vertices in **12** ways with **3** colors
- there are several applications of this problem

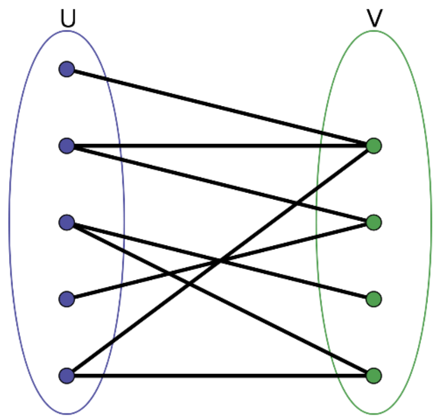
Coloring Problem



Coloring Problem

- the problem is that we have to color the vertices of a **G(V, E)** graph such that no two adjacent vertices share the same color
- it is again an **NP-complete** problem
- there is an exponential number of possible states
- if we have k colors and want to assign colors to vertices in a **G(V, E)** graph then the running time is **$O(k^V)$**
- so with brute-force approach we can achieve **exponential running** time complexity

Applications - Coloring Problem



1.) BIPARTITE GRAPHS

- determining if a graph can be colored with **2** colors is equivalent to determining whether or not the graph is bipartite
- this problem is computable in **O(N)** linear time using **breadth-first search**
- **bipartite graph**: a graph whose vertices can be divided into two disjoint sets **U** and **V** (**U** and **V** are independent sets) such that every edge connects a vertex in **U** to one in **V**

Applications - Coloring Problem

2.) SCHEDULING

- the aim is to make an **exam schedule** for a university
- we have different subjects and different students enrolled on every subject - many subjects would have common students
- **how do we schedule the exam so that no two exams with a common student are scheduled at the same time?**
- **how many minimum time slots are needed to schedule all exams?**

Applications - Coloring Problem

2.) SCHEDULING

- this problem can be represented as a **G(V, E)** graph where every vertex is a subject and an edge between two vertices means there is a common student
- so this is a **G(V, E)** graph **coloring problem** where minimum number of time slots is equal to the **k chromatic number** of the graph

Applications - Coloring Problem

3.) RADIO FREQUENCY ASSIGNMENT

- when frequencies are assigned to towers - frequencies assigned to all towers at the same location must be different because of the interference
- **how to assign frequencies with this constraint? what is the minimum number of frequencies needed?**
- this problem is also an instance of graph coloring problem where every tower represents a vertex
- an edge between two towers represents that they are in range of each other

Applications - Coloring Problem

4.) REGISTER ALLOCATION (COMPUTERS)

- **compiler optimization** relies heavily on the coloring problem
- register allocation is the process of assigning a large number of target program variables onto a small number of **CPU** registers

Applications - Coloring Problem

5.) MAP COLORING

- we want to construct a **map of countries** or states where adjacent countries or states can not be assigned the same color

- this is one of the widely known algorithmic problems

- four colors are enough to color any map – this is the so-called **four color theorem**

Algorithms - Coloring Problem

There are several **approaches** to solve this problem:

1.) greedy approach – it finds a solution but not necessarily the best one possible (it may use more colors)

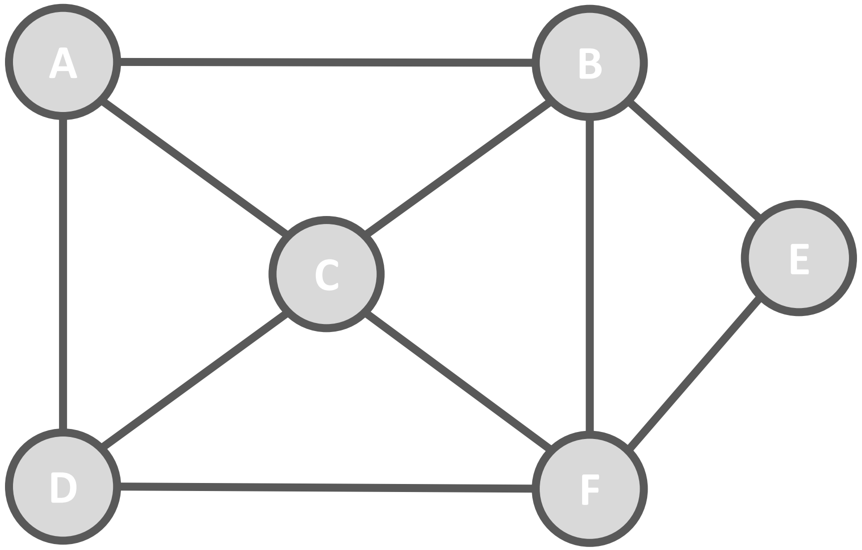
2.) backtracking – it can discard and reject multiple bad states within a single iteration (or recursive function call)

3.) Powell-Welsh algorithm - relies on sorting the nodes based on the degree (number of edges)

Coloring Problem

- we assign **colors** one by one to different vertices starting from the first vertex (for example with index **0**)
- before assigning a color: we check for safety by considering already assigned colors to the adjacent vertices
- if we find a color assignment which is feasible: we mark the color assignment as part of solution
- if we do not a find color due to clashes: we **BACKTRACK**

Coloring Problem



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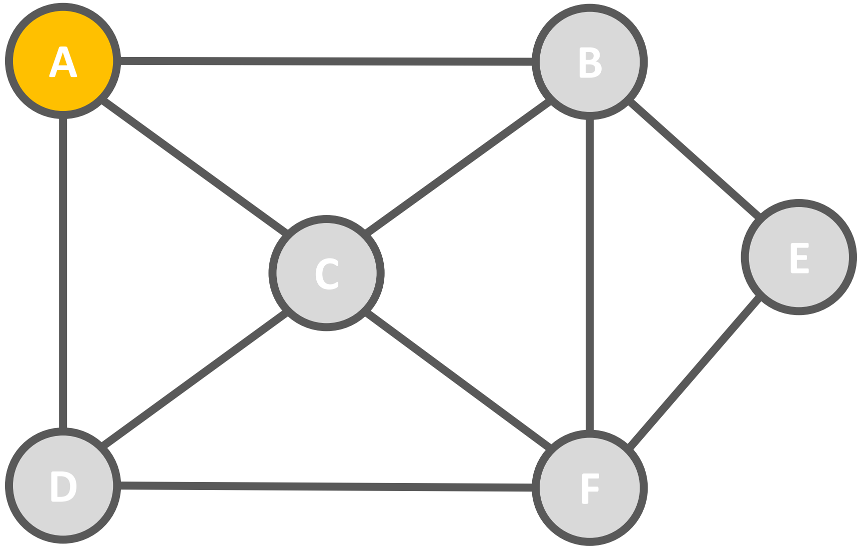

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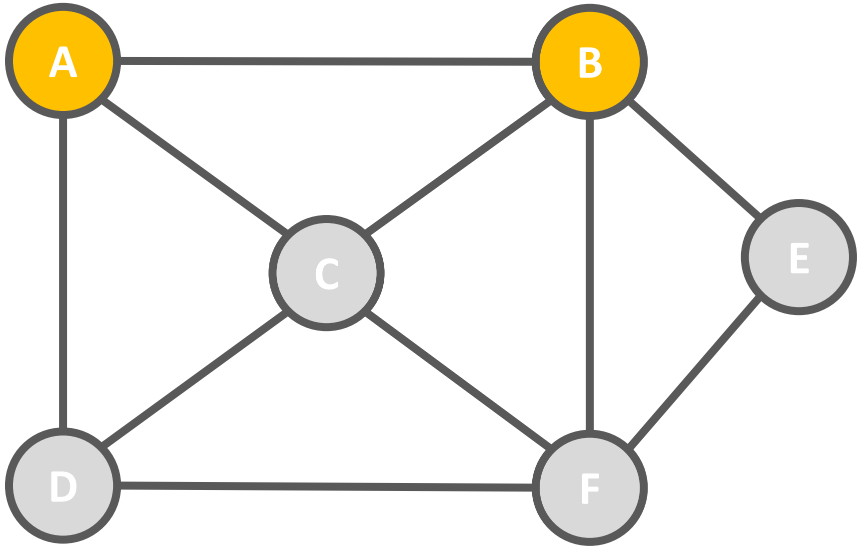

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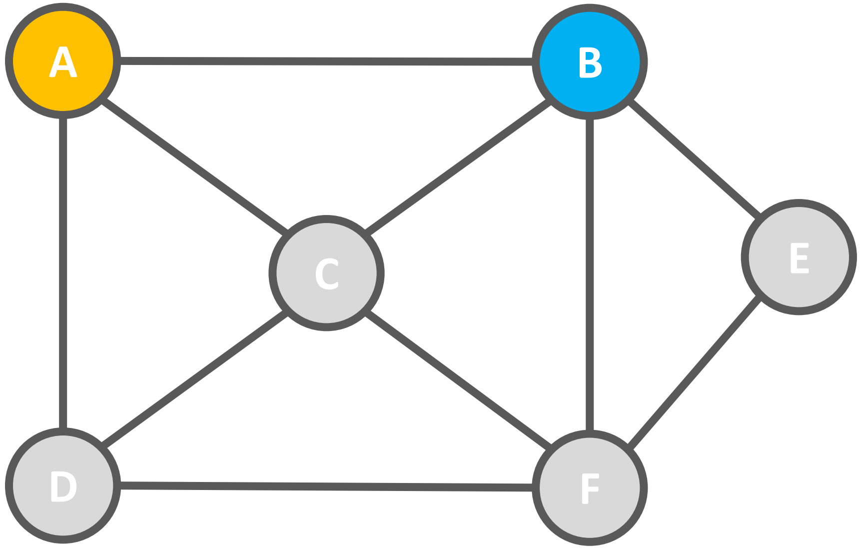

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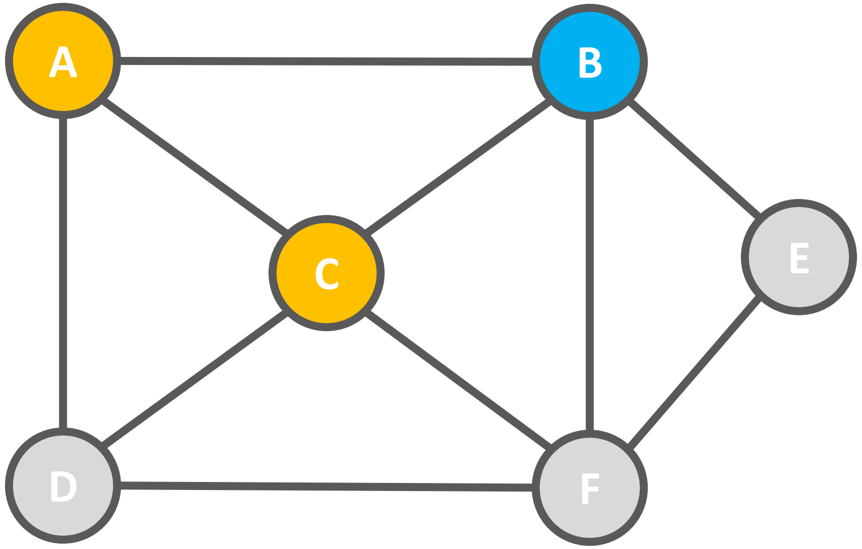

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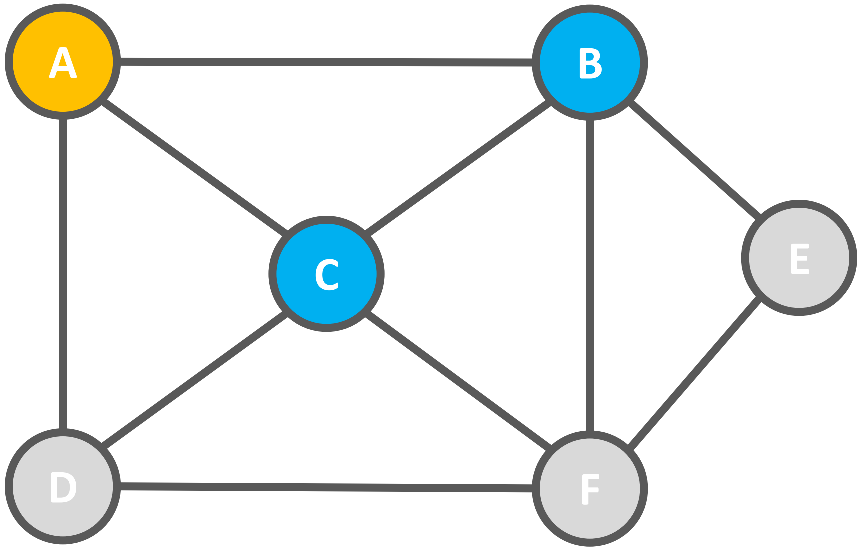

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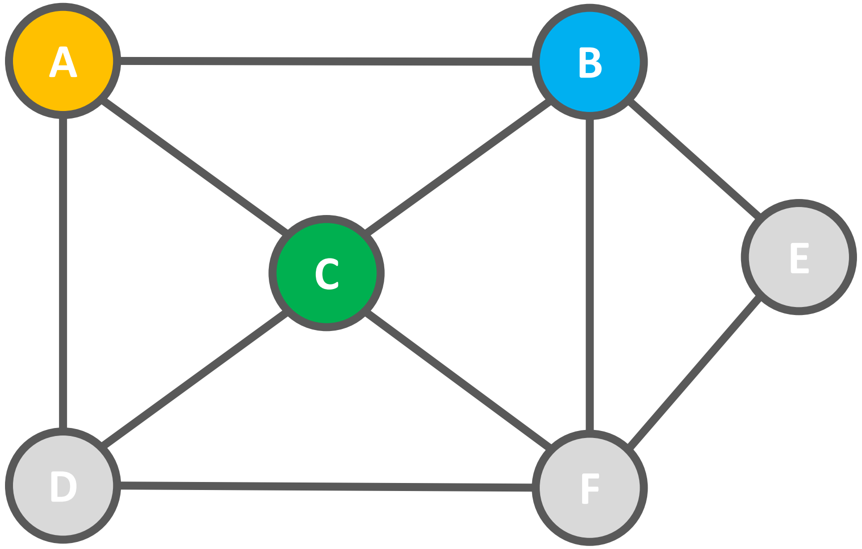

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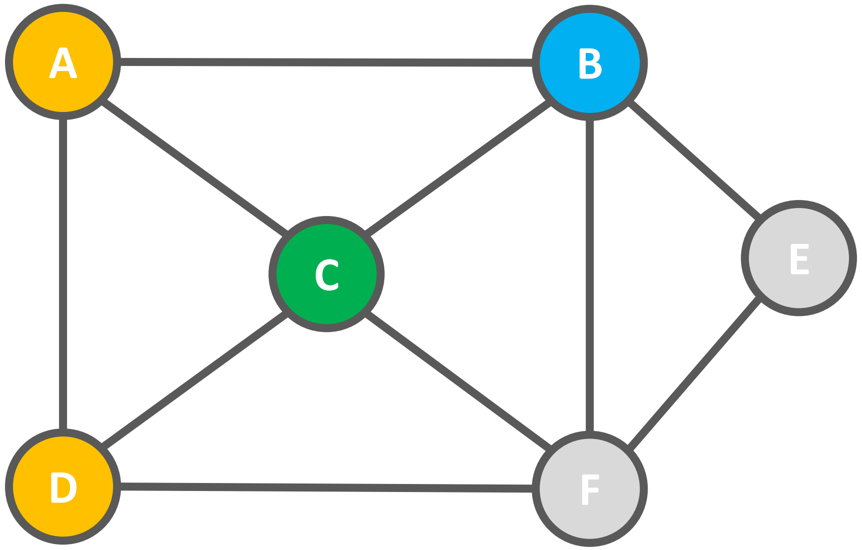

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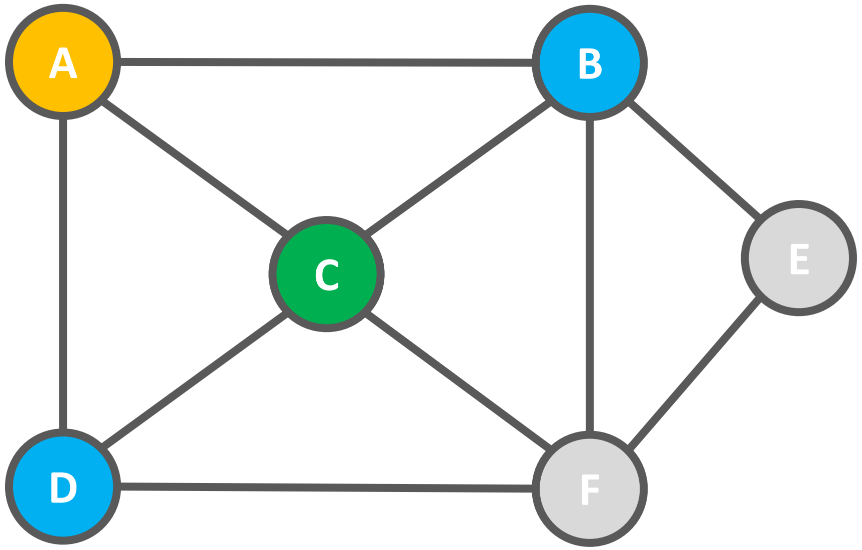

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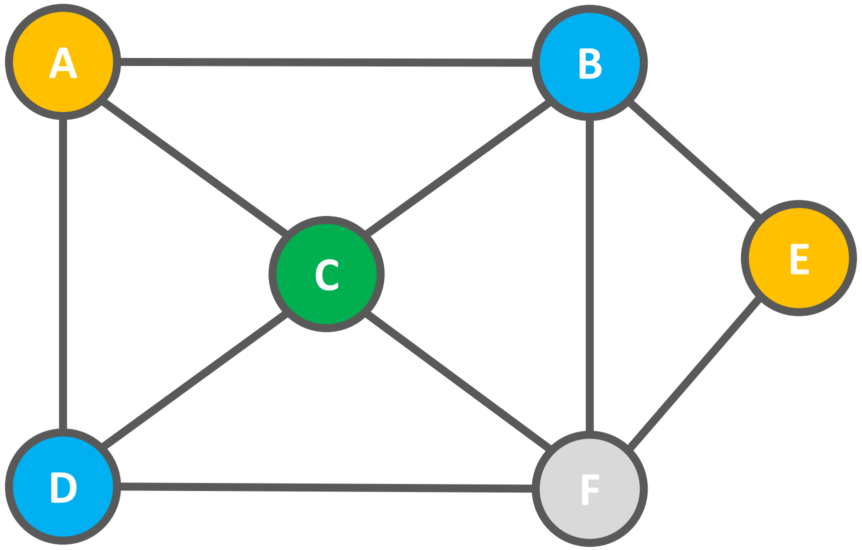

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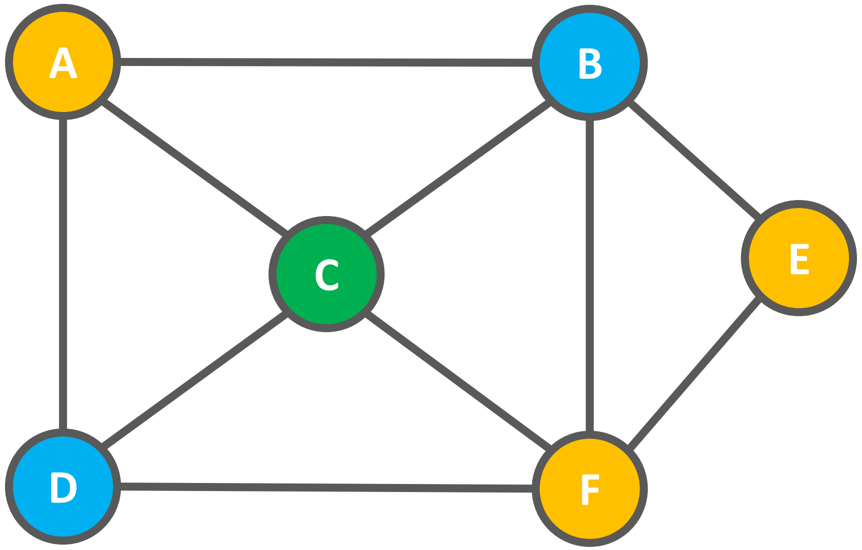

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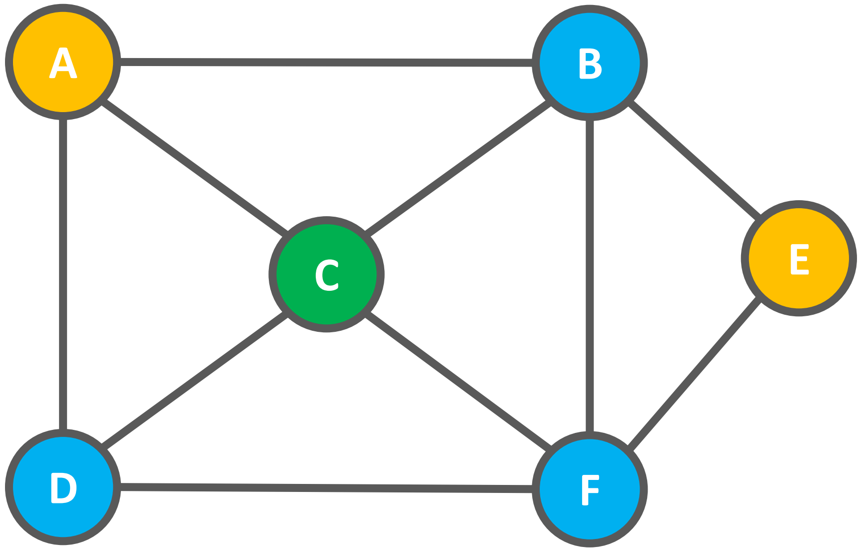

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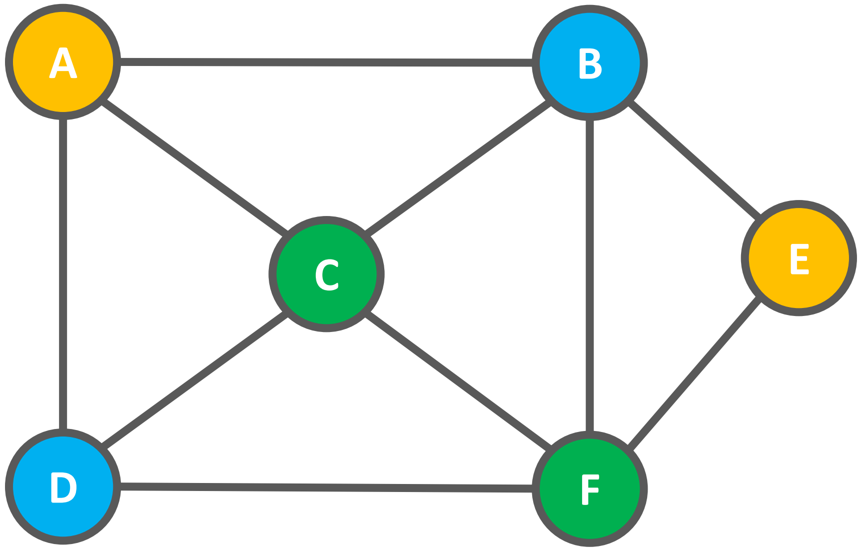

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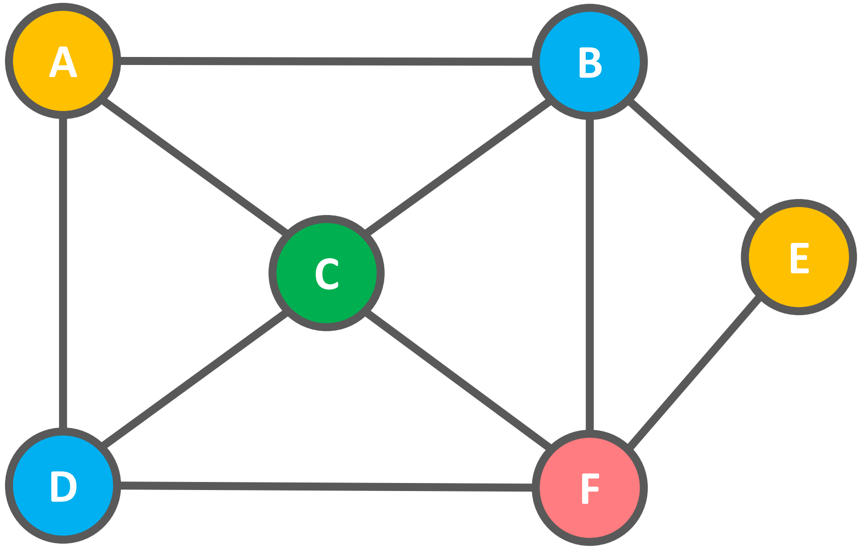
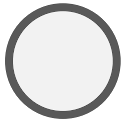


Coloring Problem

ROOT

this is the empty state

(starting state)



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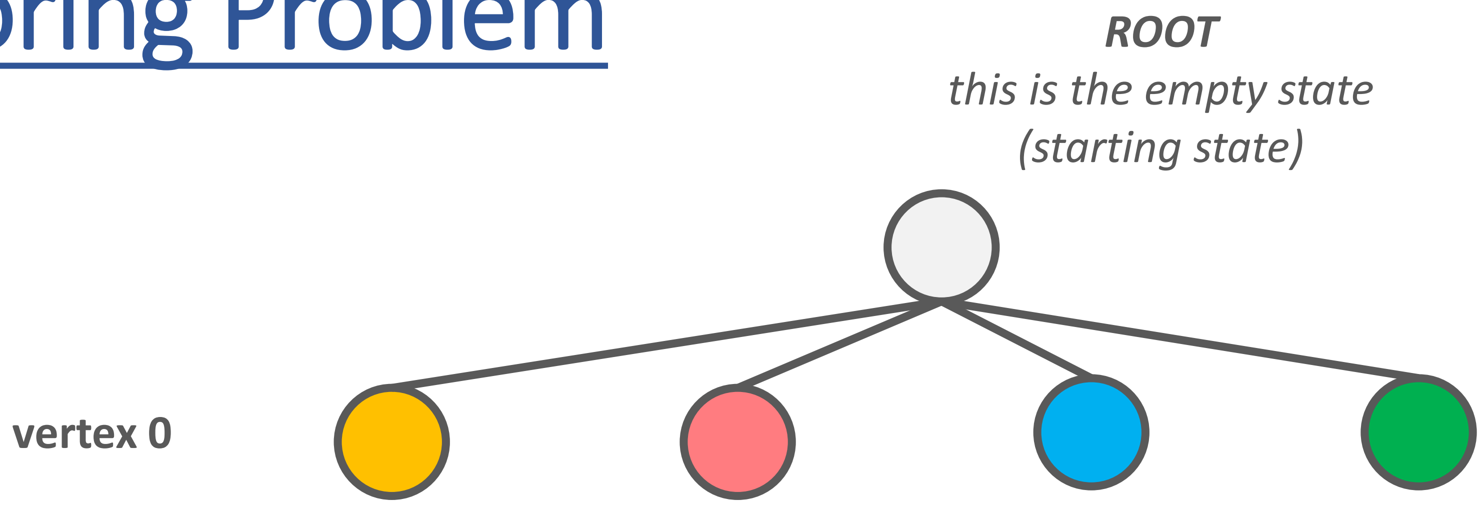

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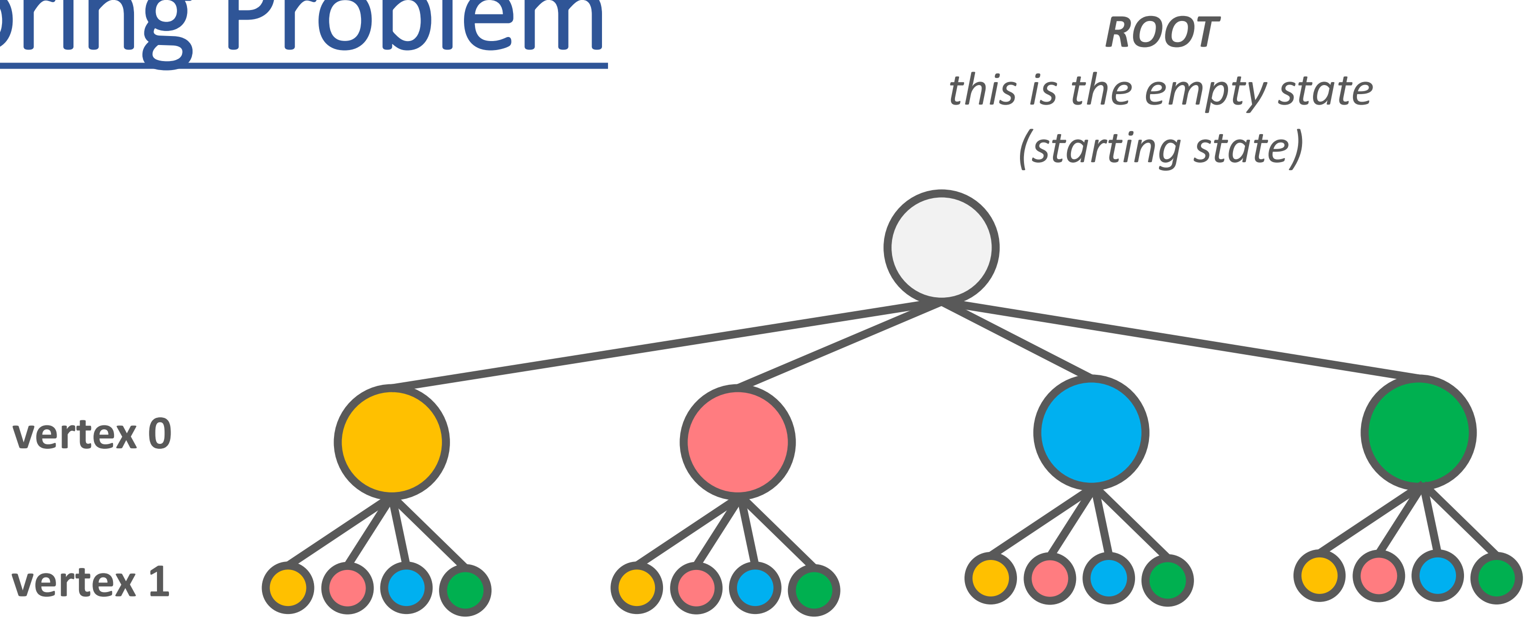

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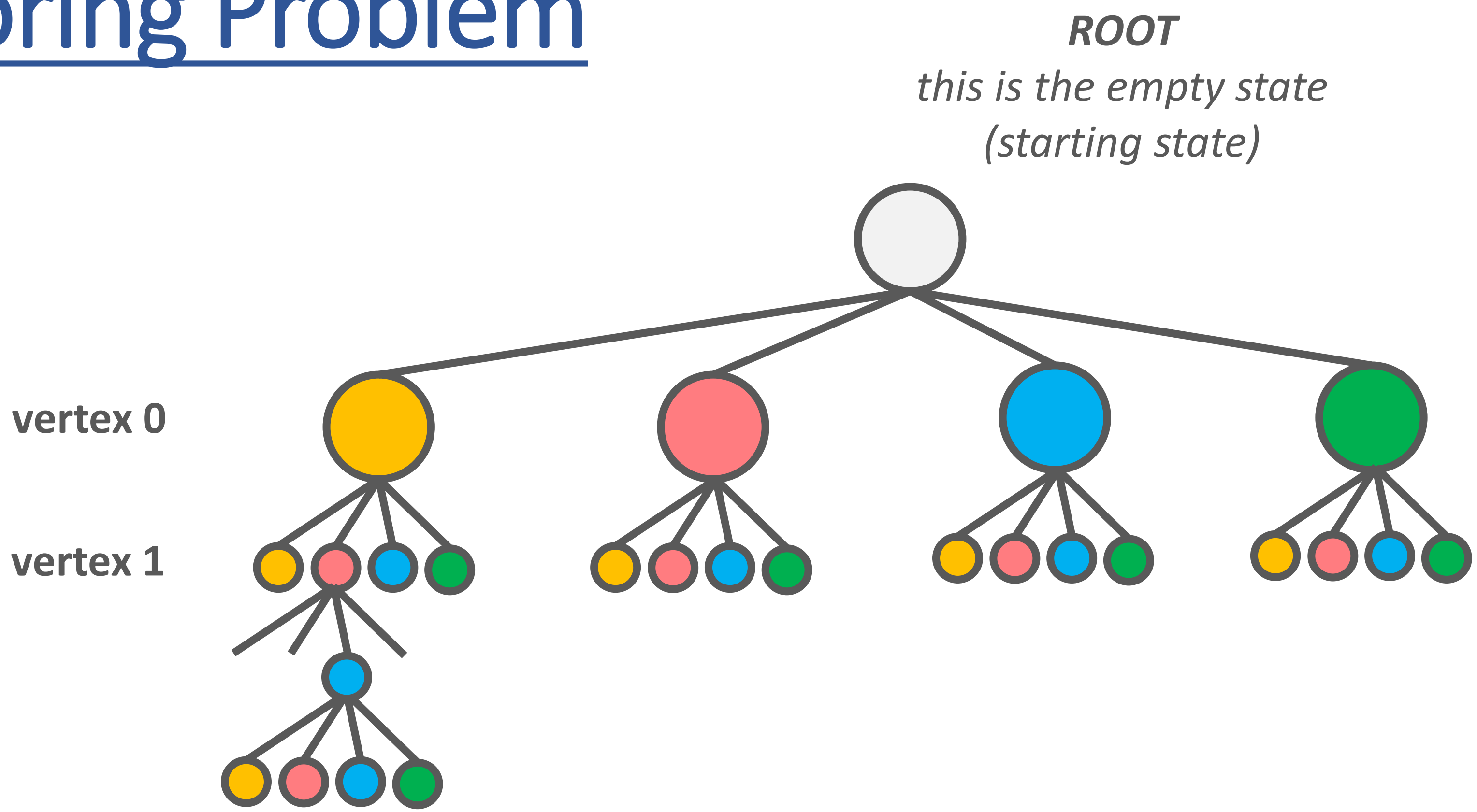

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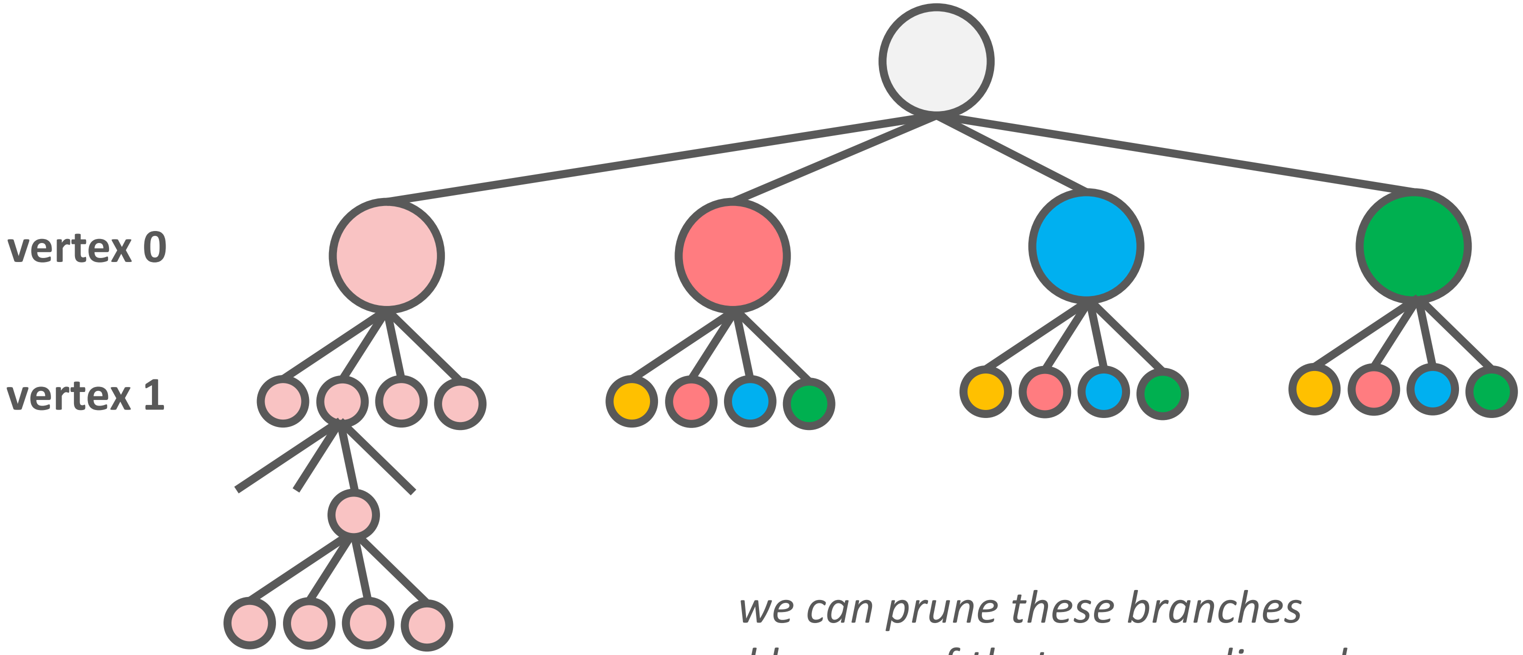

we can prune these branches
and beause of that we can discard
mutiple bad states (subtrees) in the same iteration
PRUNING BOOSTS THE RUNNING TIME OF BACKTRACKING