

Knight's Tour Problem

(Algorithmic Problems)

Knight's Tour Problem

- the problem is that we have to visit every cell on an **NxN** chessboard such that we visit every cell exactly once
- closed tour - when the knight end point is the same as the starting point
- the knight's tour problem is an instance of the more general **Hamiltonian path** problem
- of course the closed knight's tour is the same as finding the **Hamiltonian cycle** of a **G(V,E)** graph

Knight's Tour Problem

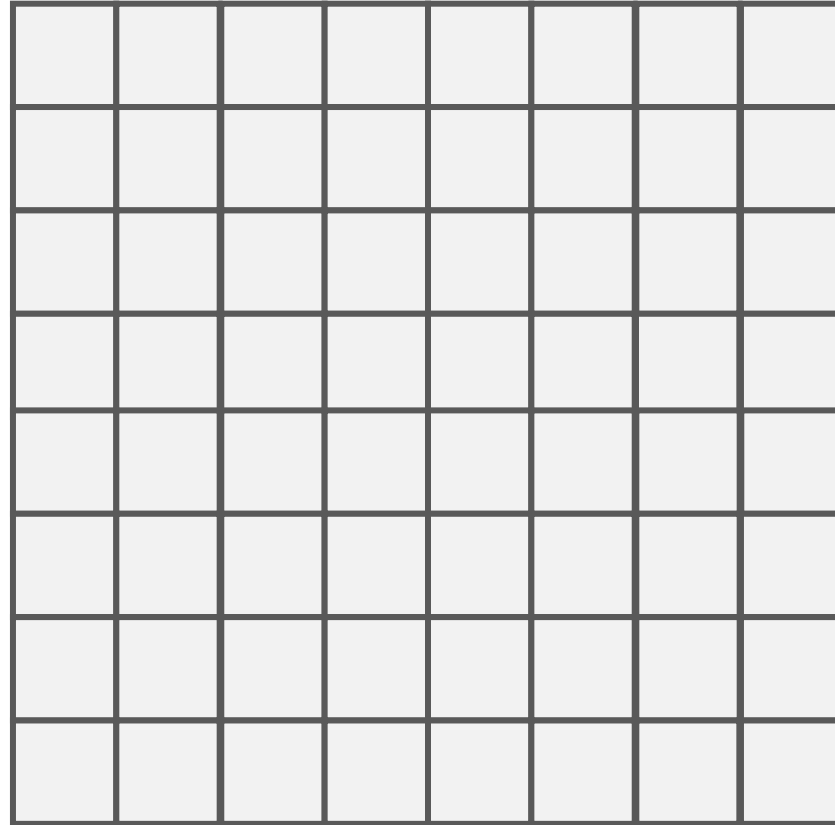
- the problem is that we have to visit every cell on an **NxN** chessboard such that we visit every cell exactly once
- what is the size of the search space?
- we can make **8** possible move with the knight in a single iteration but we have to do it to **NxN** every cell of the board
- it has **$O(2^{3(N \times N)})$ exponential running time** complexity
- unlike Hamiltonian path problem it can be solved in **$O(N)$** linear running time with **divide-and-conquer** approach

Knight's Tour Problem

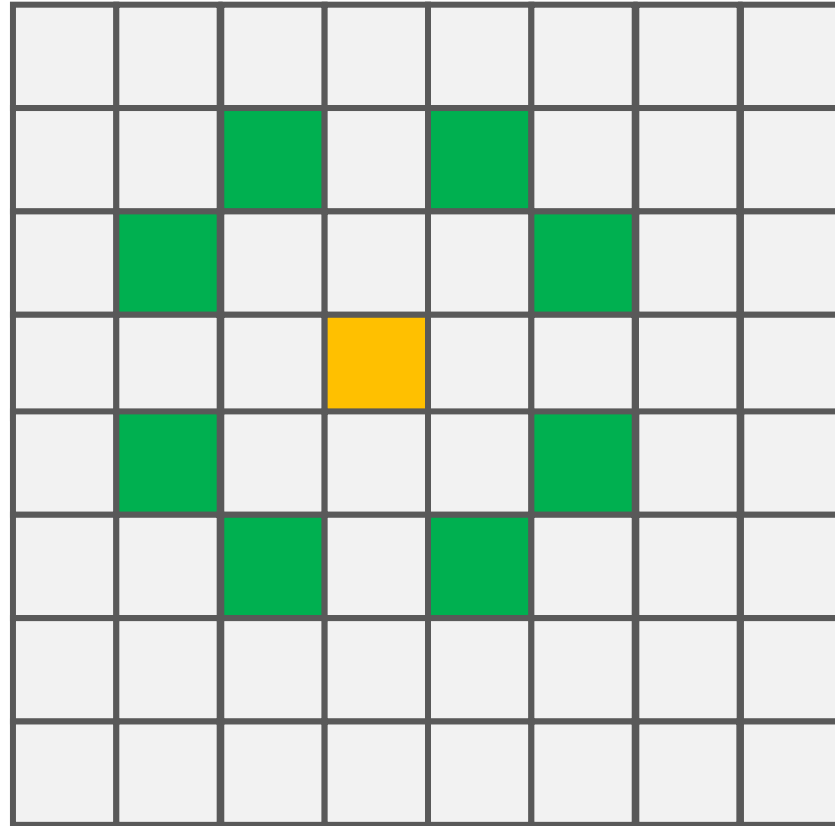
For an **MxN** chessboard the closed knight tour problem is always feasible, unless:

- **M** and **N** are both odds
- **M** = 1, 2 or 4
- **M** = 3 and **N** = 4, 6 or 8

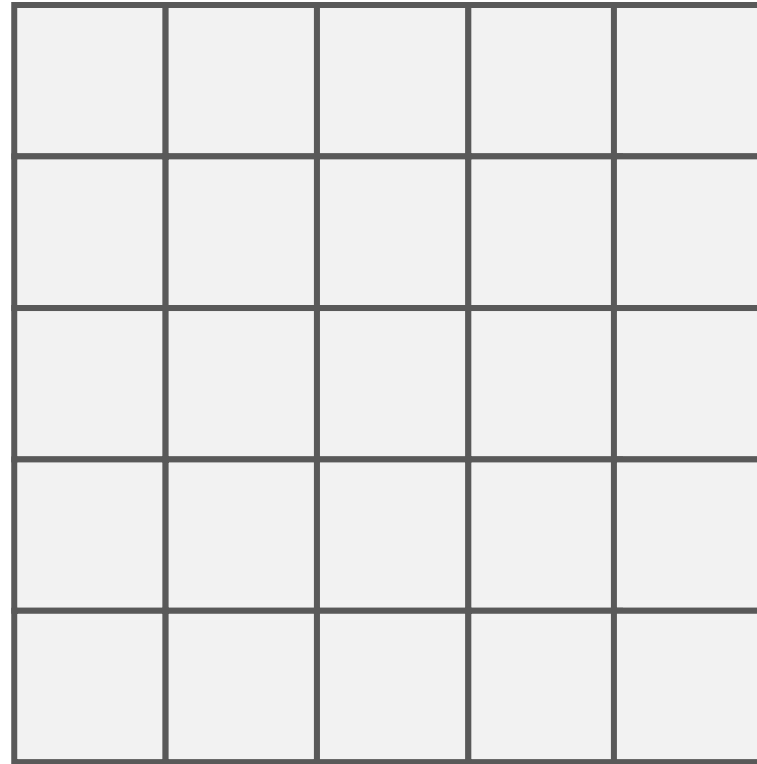
Knight's Tour Problem



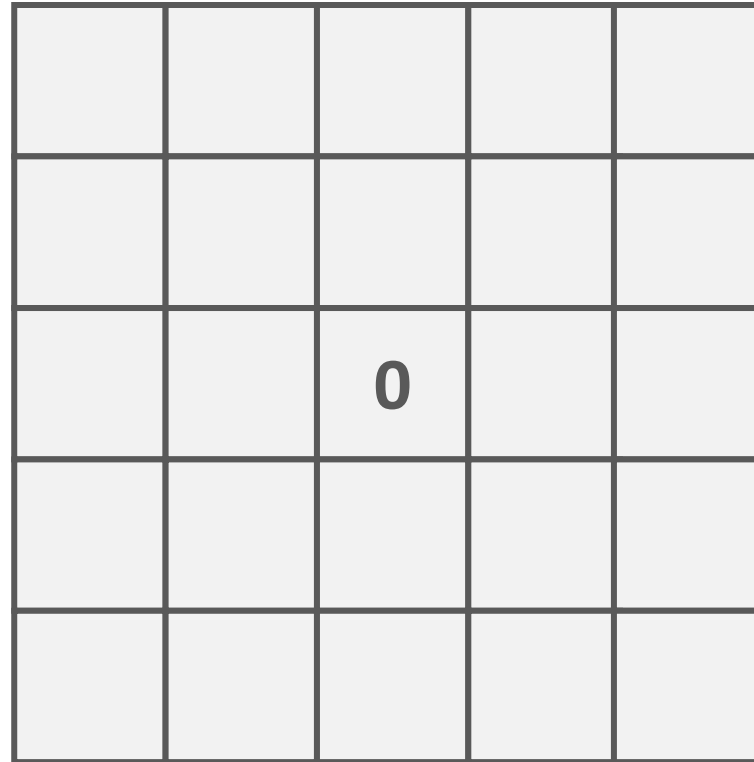
Knight's Tour Problem



Knight's Tour Problem



Knight's Tour Problem



Knight's Tour Problem

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Knight's Tour Problem

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Knight's Tour Problem

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Knight's Tour Problem

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Knight's Tour Problem

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Knight's Tour Problem

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Knight's Tour Problem

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| 13 | 2 | | 6 | 11 |
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Knight's Tour Problem

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| 13 | 2 | | 6 | 11 |
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| 3 | 14 | 17 | 10 | 5 |
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Knight's Tour Problem

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| 13 | 2 | | 6 | 11 |
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Knight's Tour Problem

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| 13 | 2 | 21 | 6 | 11 |
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| 3 | 14 | 17 | 10 | 5 |
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Knight's Tour Problem

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| 3 | 14 | 17 | 10 | 5 |
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Knight's Tour Problem

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| 3 | 14 | 17 | 10 | 5 |
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Knight's Tour Problem

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| 13 | 2 | 21 | 6 | 11 |
| 8 | 23 | 0 | 19 | 16 |
| 3 | 14 | 17 | 10 | 5 |
| 24 | 9 | 4 | 15 | 18 |