

Problem Description

The problem considered in this project involves a set of nodes, each represented by three integer values: two coordinates (x,y) that define the node's position in a two-dimensional plane, and a cost value associated with the node. The objective is to select exactly 50% of all available nodes (if the total number of nodes is odd, the number of selected nodes is rounded up) and construct a Hamiltonian cycle — that is, a closed path visiting each selected node exactly once and returning to the starting node. The goal is to minimize the sum of two components: 1. The total length of the constructed cycle, and 2. The total cost of all selected nodes. The distances between nodes are calculated using the Euclidean metric, rounded to the nearest integer.

Implemented Algorithms

Greedy Heuristics with a Weighted Sum Criterion

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Greedy heuristics with a weighted sum criterion

Algorithm GreedyWeighted(D, nodes, k, start,  $\alpha$ ,  $\beta$ )
Input: D, nodes[], k, start, weights  $\alpha$  and  $\beta$ 
Output: Best tour and objective

1. selected[start]  $\leftarrow$  true
2. Choose j  $\neq$  start minimizing D[start][j] + nodes[j].cost
3. selected[j]  $\leftarrow$  true
4. tour  $\leftarrow$  [start, j]

5. While count(selected) < k:
6.     bestNode  $\leftarrow$  None
7.     bestScore  $\leftarrow$   $-\infty$ 
8.     For each node v not in selected:
9.         (bestInc, bestPos)  $\leftarrow$  BestInsertion(v, tour, D)
10.        secondInc  $\leftarrow$  2nd best insertion increase
11.        bestTotal  $\leftarrow$  bestInc + nodes[v].cost
12.        secondTotal  $\leftarrow$  secondInc + nodes[v].cost
13.        regret  $\leftarrow$  secondTotal - bestTotal
14.        score  $\leftarrow$   $\alpha$  * regret -  $\beta$  * bestTotal
15.        If score > bestScore:
16.            bestScore  $\leftarrow$  score
17.            bestNode  $\leftarrow$  v
18.            bestPos  $\leftarrow$  bestPos
19. Insert bestNode into tour at bestPos
20. selected[bestNode]  $\leftarrow$  true

21. length  $\leftarrow$  TourLength(tour, D)
22. costSum  $\leftarrow$  sum(nodes[i].cost for i in selected)
23. objective  $\leftarrow$  length + costSum
24. return (tour, objective)
```

2-Regret Insertion Heuristic



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Algorithm Greedy2Regret(D, nodes, k, start)
Input: D, nodes[], k, start
Output: Best tour and objective

1. selected[start] ← true
2. Choose j ≠ start minimizing D[start][j] + nodes[j].cost
3. selected[j] ← true
4. tour ← [start, j]

5. While count(selected) < k:
6.     bestNode ← None
7.     bestScore ← -∞
8.     For each node v not in selected:
9.         Find (bestInc, bestPos) ← BestInsertion(v, tour, D)
10.        Find (secondInc) ← 2nd smallest insertion increase for v
11.        bestTotal ← bestInc + nodes[v].cost
12.        secondTotal ← secondInc + nodes[v].cost
13.        regret ← secondTotal - bestTotal
14.        If (regret > bestScore) OR (regret = bestScore and bestTotal <
currentBestTotal):
15.            bestNode ← v
16.            bestPos ← bestPos
17.            bestScore ← regret
18. Insert bestNode into tour at bestPos
19. selected[bestNode] ← true

20. length ← TourLength(tour, D)
21. costSum ← sum(nodes[i].cost for i in selected)
22. objective ← length + costSum
23. return (tour, objective)
```

Results

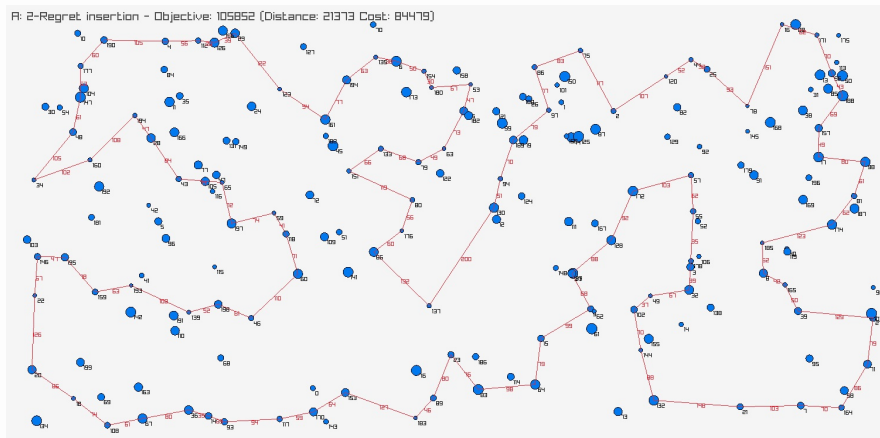
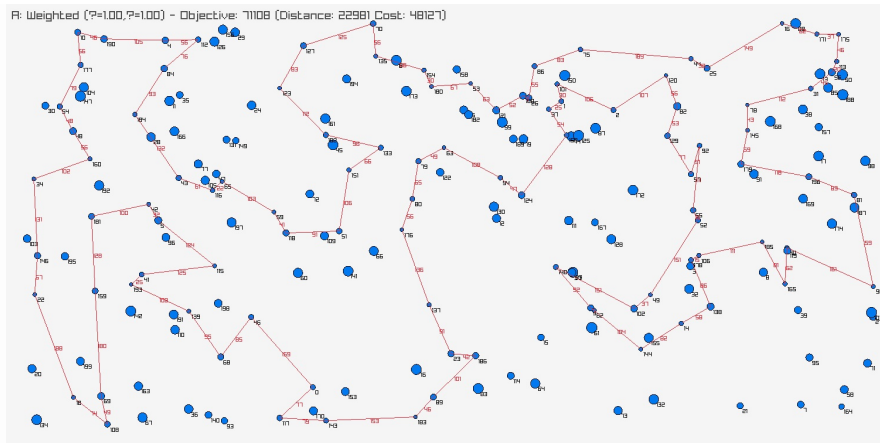
Instance A

Method	Best	Worst	Average
2-Regret Insertion	105852	123428	115474.93
Weighted Sum ($\alpha=1.00$, $\beta=1.00$)	71108	73438	72130.85

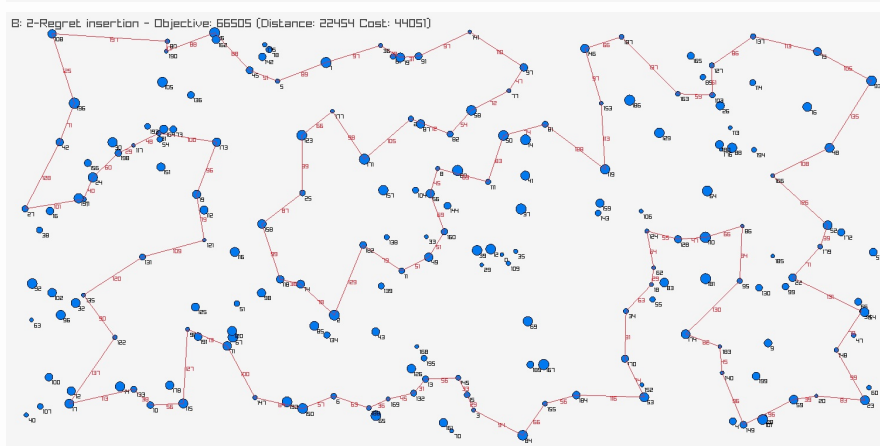
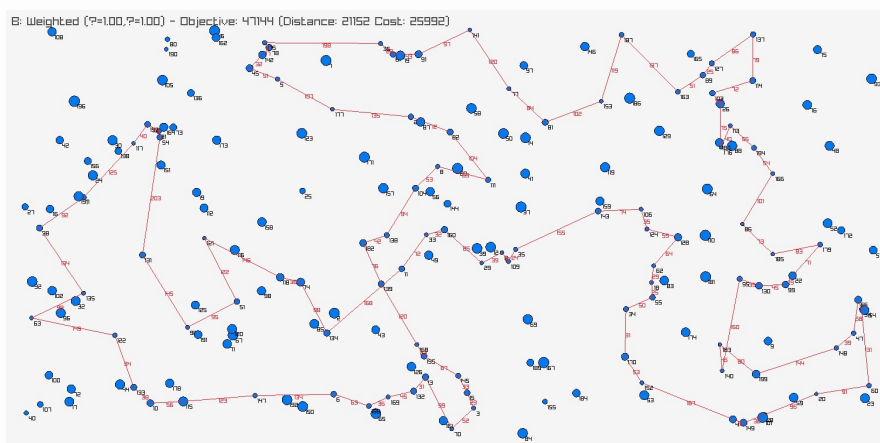
Instance B

Method	Best	Worst	Average
2-Regret Insertion	66505	77072	72454.77
Weighted Sum ($\alpha=1.00$, $\beta=1.00$)	47144	55700	50918.82

Best path for the instance A



Best path for the instance B



Comparison

Instance A

Algorithm	Total Distance	Total Cost	Objective Value
2-Regret Insertion	21373	84479	105852
Weighted Sum ($\alpha=1.00$, $\beta=1.00$)	22981	48127	71108
NN - End	-	-	89198
NN - Anywhere	-	-	71488
Greedy Cycle	-	-	72639

Instance B

Algorithm	Total Distance	Total Cost	Objective Value
2-Regret Insertion	22454	44051	66505
Weighted Sum ($\alpha=1.00$, $\beta=1.00$)	21152	25992	47144
NN - End	-	-	62606
NN - Anywhere	-	-	49001
Greedy Cycle	-	-	50243

Conclusions

Among the four implemented heuristics, the Weighted Greedy (2-Regret + Cost) method achieved the best overall performance, producing the lowest objective values and the most consistent results across runs. The 2-Regret heuristic also performed well, showing a good balance between exploration and exploitation, while the simpler Nearest and Best Insertion methods were faster and often yielded beter solutions. Overall, incorporating regret and weighted selection significantly improved solution quality and stability compared to purely local greedy approaches.