Solutions to exercises from Leonard Susskinds "Quantum Mechanics: The Theoretical Minimum"

Contents

1 Chapter 1

Exercise 1.1

1. $(\langle A| + \langle B|)\,|C\rangle = [\langle C|\,(|A\rangle + |B\rangle)]^* = \langle C|A\rangle^* + \langle C|B\rangle^* = \langle A|C\rangle + \langle B|C\rangle$
2. $\langle A|A\rangle^* = \langle A|A\rangle$

Exercise 1.2 It's obvious, that this definition is linear and antisimmetric function – thus it is scalar product.

2 Chapter 2

Exercise 2.1 $\langle p|l\rangle = \frac{1}{\sqrt{2}}(\langle g| + \langle d|)\frac{1}{\sqrt{2}}(|g\rangle - |d\rangle) = \frac{1}{2}(\langle g| + \langle d|)(|g\rangle - |d\rangle) = \frac{1}{2}(\langle g|g\rangle - \langle g|d\rangle + \langle d|g\rangle - \langle d|d\rangle) = \frac{1}{2}(1 - 0 + 0 - 1) = 0$

Exercise 2.2

$$|w\rangle = \frac{1}{\sqrt{2}}(|g\rangle + i\,|d\rangle)$$

$$|z\rangle = \frac{1}{\sqrt{2}}(|g\rangle - i\,|d\rangle)$$

1. Condition (2.7)

$$\langle w|z\rangle = \frac{1}{2}(\langle g| - i\,\langle d|)(|g\rangle - i\,|d\rangle) = 0$$

2. Condition (2.8)

$$\langle z|g\rangle = \frac{1}{\sqrt{2}} \Rightarrow |\,\langle z|g\rangle\,|^2 = \frac{1}{2}$$

$$\langle z|d\rangle = \frac{i}{\sqrt{2}} \Rightarrow |\,\langle z|d\rangle\,|^2 = \frac{1}{2}$$

$$\langle w|g\rangle = \frac{1}{\sqrt{2}} \Rightarrow |\,\langle w|g\rangle\,|^2 = \frac{1}{2}$$

$$\langle w|d\rangle = -\frac{i}{\sqrt{2}} \Rightarrow |\,\langle w|d\rangle\,|^2 = \frac{1}{2}$$

3. Condition (2.9)

$$|p\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |d\rangle)$$

$$|l\rangle = \frac{1}{\sqrt{2}}(|g\rangle - |d\rangle)$$

$$\langle z|p\rangle = \frac{1}{2} + \frac{i}{2} \Rightarrow |\,\langle z|p\rangle\,|^2 = \frac{1}{2}$$

$$\langle z|l\rangle = \frac{1}{2} - \frac{i}{2} \Rightarrow |\,\langle z|p\rangle\,|^2 = \frac{1}{2}$$

$$\langle w|p\rangle = \frac{1}{2} - \frac{i}{2} \Rightarrow |\,\langle z|p\rangle\,|^2 = \frac{1}{2}$$

$$\langle w|l\rangle = \frac{1}{2} + \frac{i}{2} \Rightarrow |\,\langle z|p\rangle\,|^2 = \frac{1}{2}$$

These vectors are not unique, because they can be multiplied by phase factor.

Exercise 2.3

1. Equation (2.8) tells precisely, that square of modulus of vectors' components is equal to $\frac{1}{2}$

2.
$$\langle p|w\rangle = \frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}}$$
$$(\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}})(\frac{\alpha^*}{\sqrt{2}} + \frac{\beta^*}{\sqrt{2}}) = \frac{1}{2}$$
$$(\alpha + \beta)(\alpha^* + \beta^*) = 1$$
$$|\alpha|^2 + \alpha\beta^* + \alpha^*\beta + |\beta|^2 = 1$$
$$\alpha\beta^* + \alpha^*\beta = 0$$
Analogously $\gamma^*\delta + \gamma\delta^* = 0$

3.
$$x = a + ib$$
$$x^* + x = 0$$
$$a + ib + a - ib = 0$$
$$a = 0$$

Exercise 2.3 TODO

3 Chapter 3

Exercise 3.2 Obvious.

Exercise 3.3 TODO

$$\sigma_n = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$$
$$(\cos\theta - \lambda)(-\cos\theta - \lambda) - \sin^2\theta = 0$$
$$-(\cos^2\theta - \lambda^2) - \sin^2\theta = 0$$
$$\lambda^2 = 1$$
$$\lambda_1 = 1, \lambda_2 = -1$$

- $|\lambda_1\rangle$

$$\begin{cases} x(\cos\theta - 1) + y\sin\theta = 0 \\ x\sin\theta + y(-\cos\theta - 1) = 0 \end{cases}$$

$$\begin{cases} y = \frac{-\cos\theta+1}{\sin\theta}x \\ y = \frac{\sin\theta}{\cos\theta+1}x \end{cases}$$

Let $x = 1$. Then

- $|\lambda_2\rangle$

Exercise 3.4

$$\sigma_n = \begin{bmatrix} \cos\theta & \sin\theta\cos\phi - i\sin\theta\sin\phi \\ \sin\theta\cos\phi + i\sin\theta\sin\phi & -\cos\theta \end{bmatrix}$$

$$(\cos\theta - \lambda)(-\cos\theta - \lambda) - (\sin\theta\cos\phi + i\sin\theta\sin\phi)(\sin\theta\cos\phi - i\sin\theta\sin\phi) = 0$$

$$-(\cos^2\theta - \lambda^2) - (\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi) = 0$$

$$-\cos^2\theta + \lambda^2 - \sin^2\theta = 0$$

$$\lambda_1 = 1, \lambda_2 = -1$$

- $|\lambda_1\rangle$

4 Chapter 4

Exercise 4.1

$$\langle UA|UB\rangle = \langle A|\, U^\dagger U\, |B\rangle = \langle A|B\rangle$$

Exercise 4.2 $(i[M,L])^\dagger = (i(ML - LM))^\dagger = (iML - iLM)^\dagger = -iL^\dagger M^\dagger + iM^\dagger L^\dagger = -iLM + iML = i(ML - LM) = i[M,L]$

5 Chapter 7

Exercise 7.1

$$I \otimes \tau_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$(I \otimes \tau_x)\,|gg\rangle = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = |gd\rangle$$

$$(I \otimes \tau_x)\left|gd\right\rangle = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \left|gg\right\rangle$$

$$(I \otimes \tau_x)\left|dg\right\rangle = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \left|dd\right\rangle$$

$$(I \otimes \tau_x)\left|dd\right\rangle = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \left|dg\right\rangle$$

Exercise 7.2

$$\sigma_z \otimes \tau_x = \begin{bmatrix} \langle gg|\,\sigma_z\tau_x\,|gg\rangle & \langle gg|\,\sigma_z\tau_x\,|gd\rangle & \langle gg|\,\sigma_z\tau_x\,|dg\rangle & \langle gg|\,\sigma_z\tau_x\,|dd\rangle \\ \langle gd|\,\sigma_z\tau_x\,|gg\rangle & \langle gd|\,\sigma_z\tau_x\,|gd\rangle & \langle gd|\,\sigma_z\tau_x\,|dg\rangle & \langle gd|\,\sigma_z\tau_x\,|dd\rangle \\ \langle dg|\,\sigma_z\tau_x\,|gg\rangle & \langle dg|\,\sigma_z\tau_x\,|gd\rangle & \langle dg|\,\sigma_z\tau_x\,|dg\rangle & \langle dg|\,\sigma_z\tau_x\,|dd\rangle \\ \langle dd|\,\sigma_z\tau_x\,|gg\rangle & \langle dd|\,\sigma_z\tau_x\,|gd\rangle & \langle dd|\,\sigma_z\tau_x\,|dg\rangle & \langle dd|\,\sigma_z\tau_x\,|dd\rangle \end{bmatrix}$$

6 Chapter 8

Exercise 8.1 Obvious.

7 Chapter 9

Exercise 9.1

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}e^{\frac{ipx}{\hbar}} = Ee^{\frac{ipx}{\hbar}}$$

$$-\frac{\hbar^2}{2m}(\frac{ip}{\hbar})^2 e^{\frac{ipx}{\hbar}} = Ee^{\frac{ipx}{\hbar}}$$

$$E = -\frac{\hbar^2}{2m}(\frac{ip}{\hbar})^2 = \frac{p^2}{2m}$$

Exercise 9.2 $P[P,X] + [P,X]P = P(PX - XP) + (PX - XP)P = P^2X - PXP + PXP - XP^2 = P^2X - XP^2 = [P^2, X]$

Exercise 9.3 $-i\hbar V(x)\frac{d\Psi}{dx} + i\hbar(\frac{dV}{dx} + V\frac{d\Psi}{dx}) = i\hbar\frac{dV}{dx}\Psi = [V(x), P]\Psi(x)$

8 Chapter 10

Exercise 10.1

$$\ddot{x} = \omega^2(-A\cos(\omega t) - B\sin(\omega t)) = -\omega^2 x$$