Solutions to exercises from Ramamurti Shankar "Quantum Mechanics"

Contents

1 Chapter 1

Exercise 1.1.1

1. There is exactly one null vector.

   Let $|0\rangle, |0'\rangle$ be null vectors against addition operation. Then for any $|v\rangle$ we have
   $$|v\rangle + |0\rangle = |0\rangle + |v\rangle = |0\rangle\,,$$
   $$|v\rangle + |0'\rangle = |0'\rangle + |v\rangle = |0'\rangle\,.$$
   For that reason $|0\rangle + |0'\rangle = |0\rangle$. Similarly $|0\rangle + |0'\rangle = |0'\rangle$. Eventually, $|0\rangle = |0'\rangle$.

2. $0\,|v\rangle = |0\rangle$.

   We have $|0\rangle = |v\rangle + |-v\rangle = (0+1)\,|v\rangle + |-v\rangle = 0\,|v\rangle + |v\rangle + |-v\rangle = 0\,|v\rangle + |0\rangle = |0\rangle$.

3. $|-v\rangle = -\,|v\rangle$.

   We have $|v\rangle + |-v\rangle = |0\rangle =$ (point 2) $= 0\,|v\rangle = (1+(-1))\,|v\rangle = |v\rangle + (-\,|v\rangle)$

   For that reason $|-v\rangle = -\,|v\rangle$

4. for any $|v\rangle$ there is exactly one $|-v\rangle$.

   For any $|v\rangle$ let $|w\rangle$ it's inversion. There is only one null vector $|0\rangle$ (according to point 1). For that reason $|v\rangle + |w\rangle = |v\rangle + (-\,|v\rangle)$. We have then $|w\rangle = -\,|v\rangle$, because there is exactly one inversion.

Exercise 1.1.2

$$(a,b,c) + (d,e,f) = (a+d, b+e, c+f)$$
$$\alpha(a,b,c) = (\alpha a, \alpha b, \alpha c)$$
$$\text{null vector: } |0\rangle = (0,0,0)$$
$$\text{inverse vector:} (-a,-b,-c)$$

Vectors $(a,b,1)$ don't create linear space, because their set don't have null vector.

Exercise 1.1.3

1. Yes
2. No, because there is not null vector
3. No, because there is not null vector

Exercise 1.1.4 Space of these vectors isn't linearly independent, because

$$-1\,|1\rangle + 2\,|2\rangle + 1\,|3\rangle = |0\rangle\,.$$

Exercise 1.1.5

1. This set of vectors isn't linearly independent, because we have $|1\rangle = \frac{1}{2}(|3\rangle - |2\rangle)$
2. This set of vectors is linearly independent, because only solution of $a_1\,|1\rangle + a_2\,|2\rangle + a_3\,|3\rangle = |0\rangle$ is $a_1 = a_2 = a_3 = 0$:

$$a_1(1,1,0) + a_2(1,0,1) + a_3(0,1,1) = (0,0,0)$$

$$\begin{cases} a_1 + a_2 = 0 \\ a_1 + a_3 = 0 \\ a_2 + a_3 = 0 \end{cases}$$