**Wzory Wallisa i Stirlinga**

$$\binom{2n}{n} \sim \frac{4^n}{\sqrt{\pi n}} \qquad n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

**Podstawienie z tangensem**

$$t = \tan\left(\frac{x}{2}\right) \qquad \sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad dx = \frac{2}{1+t^2}\,dt$$

**Całkowanie ułamków prostych**

1. Wyrażenia postaci $(x-a)^{-k}$:

$$\int \frac{1}{x-a}\,dx = \ln|x-a|$$

$$\int \frac{1}{(x-a)^k}\,dx = \frac{1}{1-k}\cdot\frac{1}{(x-a)^{k-1}},\ k \geq 2$$

2. Wyrażenia postaci $\frac{Bx+C}{(x^2+bx+c)^k}$ dla $\Delta < 0$:

$$\frac{Bx+C}{((x-\beta)^2+\gamma)^k}$$

$$x-\beta = \sqrt{\gamma}u : \frac{\widetilde{B}u+\widetilde{C}}{(u^2+1)^k}$$

3. Wyrażenia postaci $\frac{u}{(u^2+1)^k}$:

$$\int \frac{u}{u^2+1}\,du = \frac{1}{2}\ln(u^2+1)$$

$$\int \frac{u}{(u^2+1)^k}\,du = \frac{1}{1-k}\cdot\frac{1}{(u^2+1)^{k-1}},\ k \geq 2$$

4. Wyrażenia postaci $\frac{1}{(u^2+1)^k}$:

$$I_k = \int \frac{1}{(u^2+1)^k}\,du,\ k \geq 1$$

$$I_1 = \arctan u$$

$$I_{k+1} = \frac{1}{2k}\cdot\frac{u}{(u^2+1)^k} + \frac{2k-1}{2k}\cdot I_k,\ k \geq 1$$

**Trygonometria**

$$\sin\alpha\sin\beta = -\frac{1}{2}(\cos(\alpha+\beta)-\cos(\alpha-\beta))$$

$$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))$$

$$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$$

$$\sin^2 x + \cos^2 x = 1$$

$$1+\tan^2 x = \sec^2 x$$

$$1+\cot^2 x = \csc^2 x$$

$$\sin(x+y) = \sin x\cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x\tan y}$$

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

| Kąt | sin | cos | tan |
|---|---|---|---|
| $0°$ | 0 | 1 | 0 |
| $15°$ | $\frac{\sqrt{6}-\sqrt{2}}{4}$ | $\frac{\sqrt{6}+\sqrt{2}}{4}$ | $2-\sqrt{3}$ |
| $30°$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ |
| $45°$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 |
| $60°$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| $90°$ | 1 | 0 | – |

wypukła :) $f''(x) \geqslant 0$ | wklęsła :( $f''(x) \leqslant 0$

**Funkcje hiperboliczne**

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

$$\sinh(x+y) = \sinh x\cosh y + \cosh x\sinh y$$

$$\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$$

$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1+\tanh x\tanh y}$$

$$\sinh 2x = 2\sinh x\cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$

**Fajne całki**

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$$

$$\int \frac{dx}{\sqrt{x^2+1}} = \operatorname{arsinh} x$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arcosh} x$$

$$\int \frac{dx}{x^2+1} = \arctan x$$

$$\int \sqrt{1-e^x}\,dx = 2\sqrt{1-e^x} + x - 2\ln(1+\sqrt{1-e^x})$$

$$\int \sqrt{x^2+1}\,dx = \frac{1}{2}\ln(x+\sqrt{x^2+1}) + \frac{1}{2}x\sqrt{x^2+1}$$

$$\int \sqrt{x^2-1}\,dx = -\frac{1}{2}\ln(x+\sqrt{x^2-1}) + \frac{1}{2}x\sqrt{x^2-1}$$

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin x$$

$$\operatorname{arsinh} x = \ln(x+\sqrt{x^2+1}),\ \mathbb{R}\to\mathbb{R}$$

$$\operatorname{arcosh} x = \ln(x+\sqrt{x^2-1}),\ [1,\infty)\to[0,\infty)$$