Analysis of the risk of failure of the O-rings on the Challenger shuttle

Arnaud Legrand

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On January 27, 1986, the day before the takeoff of the shuttle *Challenger*, had a three-hour teleconference was held between Morton Thiokol (the manufacturer of one of the engines) and NASA. The discussion focused on the consequences of the temperature at take-off of 31°F (just below 0°C) for the success of the flight and in particular on the performance of the O-rings used in the engines. Indeed, no test had been performed at this temperature.

The following study takes up some of the analyses carried out that night with the objective of assessing the potential influence of the temperature and pressure to which the O-rings are subjected on their probability of malfunction. Our starting point is the results of the experiments carried out by NASA engineers during the six years preceding the launch of the shuttle Challenger.

Loading the data

We start by loading this data:

```r
data = read.csv("shuttle.csv",header=T)
data
```

```
##        Date Count Temperature Pressure Malfunction
## 1   4/12/81     6          66       50           0
## 2  11/12/81     6          70       50           1
## 3   3/22/82     6          69       50           0
## 4  11/11/82     6          68       50           0
## 5   4/04/83     6          67       50           0
## 6   6/18/82     6          72       50           0
## 7   8/30/83     6          73      100           0
## 8  11/28/83     6          70      100           0
## 9   2/03/84     6          57      200           1
## 10  4/06/84     6          63      200           1
## 11  8/30/84     6          70      200           1
## 12 10/05/84     6          78      200           0
## 13 11/08/84     6          67      200           0
## 14  1/24/85     6          53      200           2
## 15  4/12/85     6          67      200           0
## 16  4/29/85     6          75      200           0
## 17  6/17/85     6          70      200           0
## 18  7/29/85     6          81      200           0
## 19  8/27/85     6          76      200           0
## 20 10/03/85     6          79      200           0
## 21 10/30/85     6          75      200           2
## 22 11/26/85     6          76      200           0
## 23  1/12/86     6          58      200           1
```

The data set shows us the date of each test, the number of O-rings (there are 6 on the main launcher), the temperature (in Fahrenheit) and pressure (in psi), and finally the number of identified malfunctions.

Graphical inspection

Flights without incidents do not provide any information on the influence of temperature or pressure on malfunction. We thus focus on the experiments in which at least one O-ring was defective.

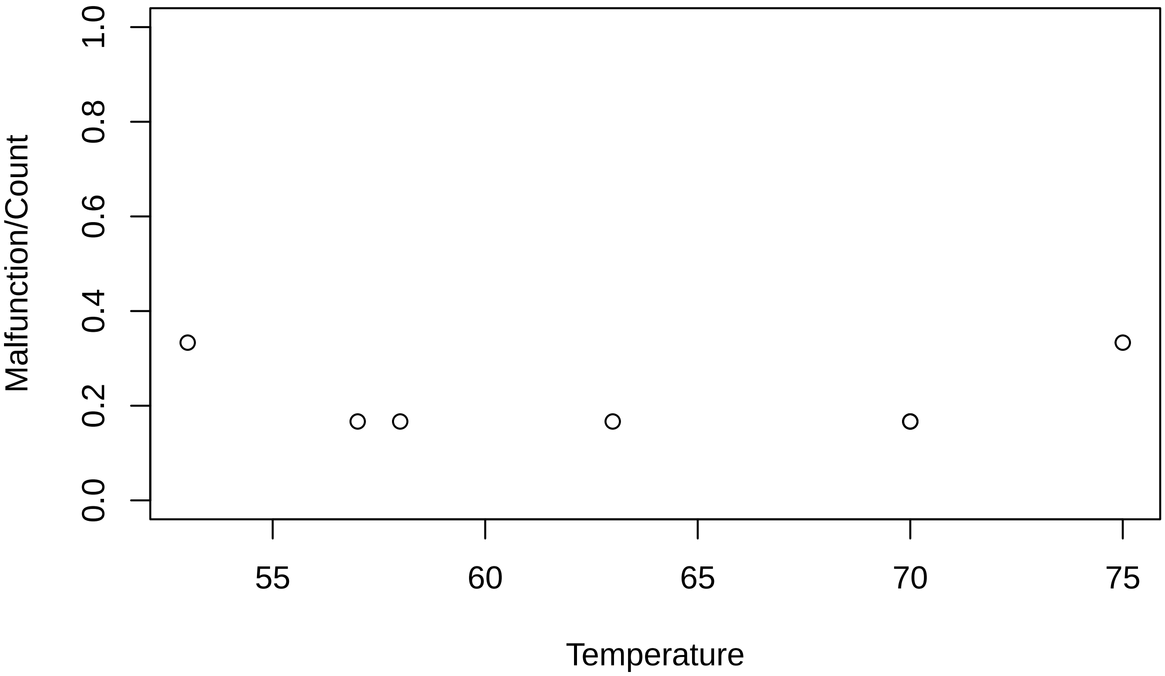
```
data = data[data$Malfunction>0,]
data
```

```
##        Date Count Temperature Pressure Malfunction
## 2  11/12/81     6          70       50           1
## 9   2/03/84     6          57      200           1
## 10  4/06/84     6          63      200           1
## 11  8/30/84     6          70      200           1
## 14  1/24/85     6          53      200           2
## 21 10/30/85     6          75      200           2
## 23  1/12/86     6          58      200           1
```

We have a high temperature variability but the pressure is almost always 200, which should simplify the analysis.

How does the frequency of failure vary with temperature?

```
plot(data=data, Malfunction/Count ~ Temperature, ylim=c(0,1))
```



At first glance, the dependence does not look very important, but let's try to estimate the impact of temperature t on the probability of O-ring malfunction.

Estimation of the temperature influence

Suppose that each of the six O-rings is damaged with the same probability and independently of the others and that this probability depends only on the temperature. If $p(t)$ is this probability, the number D of malfunctioning O-rings during a flight at temperature t follows a binomial law with parameters $n = 6$ and $p = p(t)$. To link $p(t)$ to t, we will therefore perform a logistic regression.

```r
logistic_reg = glm(data=data, Malfunction/Count ~ Temperature, weights=Count,
                   family=binomial(link='logit'))
summary(logistic_reg)
```

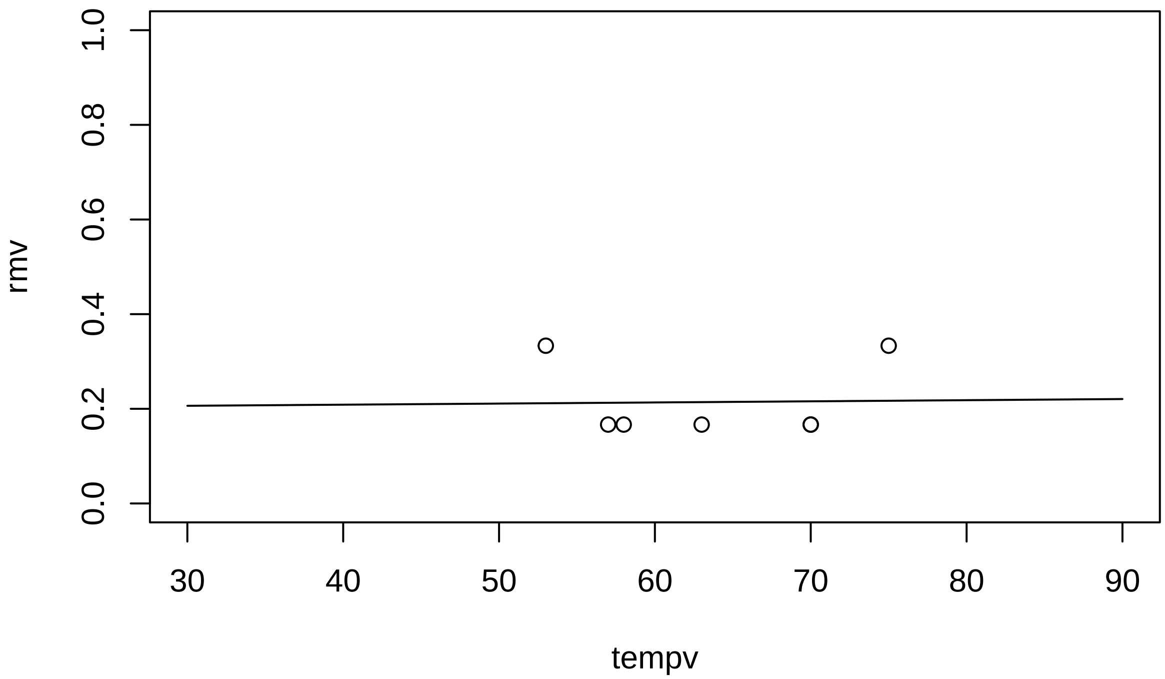
```
##
## Call:
## glm(formula = Malfunction/Count ~ Temperature, family = binomial(link = "logit"),
##     data = data, weights = Count)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.389528   3.195752  -0.435    0.664
## Temperature  0.001416   0.049773   0.028    0.977
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1.3347  on 6  degrees of freedom
## Residual deviance: 1.3339  on 5  degrees of freedom
## AIC: 18.894
##
## Number of Fisher Scoring iterations: 4
```

The most likely estimator of the temperature parameter is 0.001416 and the standard error of this estimator is 0.049, in other words we cannot distinguish any particular impact and we must take our estimates with caution.

Estimation of the probability of O-ring malfunction

The expected temperature on the take-off day is 31°F. Let's try to estimate the probability of O-ring malfunction at this temperature from the model we just built:

```r
# shuttle=shuttle[shuttle$r!=0,]
tempv = seq(from=30, to=90, by = .5)
rmv <- predict(logistic_reg,list(Temperature=tempv),type="response")
plot(tempv,rmv,type="l",ylim=c(0,1))
points(data=data, Malfunction/Count ~ Temperature)
```



As expected from the initial data, the temperature has no significant impact on the probability of failure of the O-rings. It will be about 0.2, as in the tests where we had a failure of at least one joint. Let's get back to the initial dataset to estimate the probability of failure:

```r
data_full = read.csv("shuttle.csv",header=T)
sum(data_full$Malfunction)/sum(data_full$Count)
```

```
## [1] 0.06521739
```

This probability is thus about $p = 0.065$. Knowing that there is a primary and a secondary O-ring on each of the three parts of the launcher, the probability of failure of both joints of a launcher is $p^2 \approx 0.00425$. The probability of failure of any one of the launchers is $1 - (1 - p^2)^3 \approx 1.2$. That would really be bad luck.... Everything is under control, so the takeoff can happen tomorrow as planned.

But the next day, the Challenger shuttle exploded and took away with her the seven crew members. The public was shocked and in the subsequent investigation, the reliability of the O-rings was questioned. Beyond the internal communication problems of NASA, which have a lot to do with this fiasco, the previous analysis includes (at least) a small problem.... Can you find it? You are free to modify this analysis and to look at this dataset from all angles in order to to explain what's wrong.

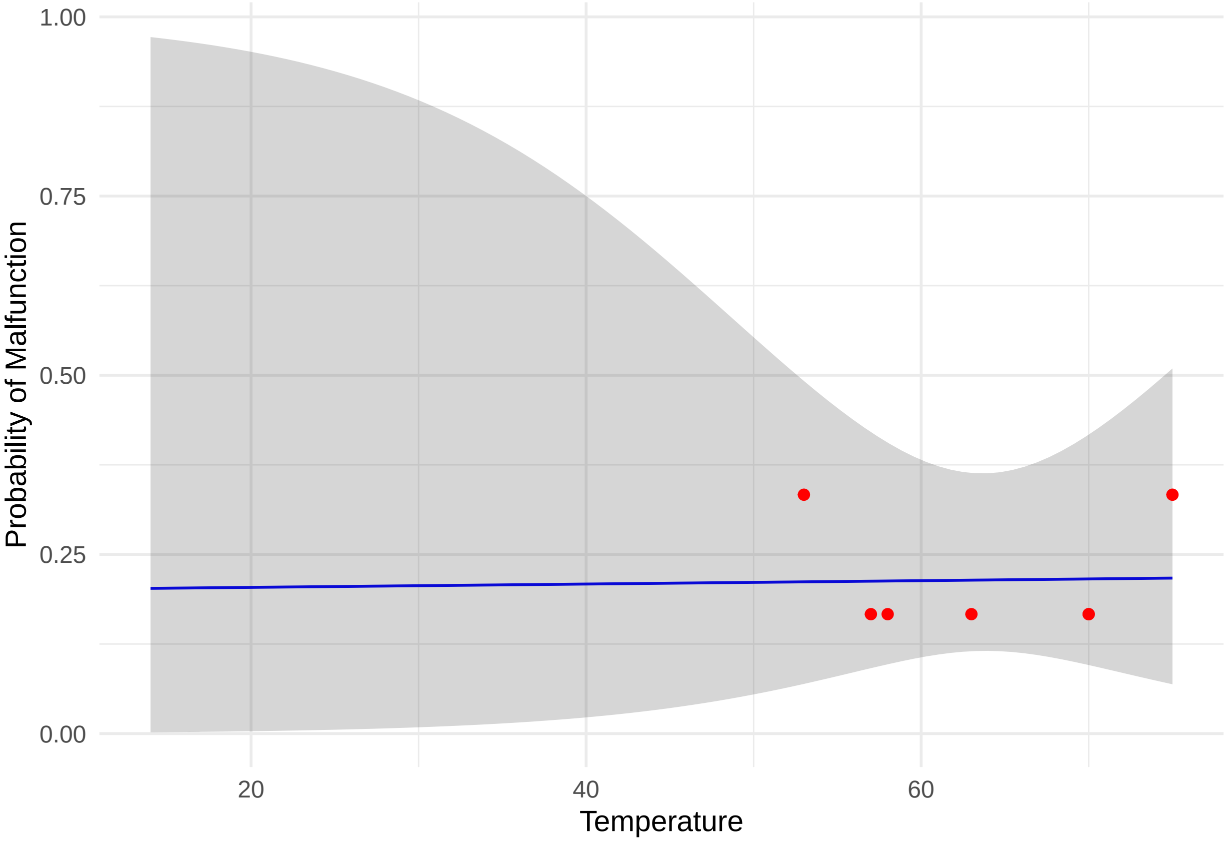
Finding error

in the provided data from tests, the range of temperatures is small, all of them vary between 60-70, based on this data we cannot reason about what will happen in the temperatures around 30. To show this we can visualize confidence intervals for out logistic regression

```r
# Create a sequence of Temperature values for plotting
newdata <- data.frame(Temperature = seq(14,
                                        max(data$Temperature),
                                        length.out = 84))

# Predict on the link (logit) scale with standard errors
pred <- predict(logistic_reg, newdata, type = "link", se.fit = TRUE)

# Compute 95% CI on the link scale
```

```
newdata$fit <- pred$fit
newdata$lower <- pred$fit - 1.96 * pred$se.fit
newdata$upper <- pred$fit + 1.96 * pred$se.fit

# Transform back to probability scale
newdata$fit_prob <- plogis(newdata$fit)
newdata$lower_prob <- plogis(newdata$lower)
newdata$upper_prob <- plogis(newdata$upper)
```

```
library(ggplot2)

ggplot(newdata, aes(x = Temperature, y = fit_prob)) +
  geom_line(color = "blue") +                              # Predicted probability line
  geom_ribbon(aes(ymin = lower_prob, ymax = upper_prob), alpha = 0.2) + # 95% CI
  geom_point(data = data, aes(x = Temperature, y = Malfunction/Count), color = "red") + # observed prop
  labs(y = "Probability of Malfunction",
       x = "Temperature") +
  theme_minimal()
```



Model was to simple and took into account only, temperature not pressure etc, we should do the same based on pressure, temperature, malfucntion types