$$\begin{bmatrix} 4 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 12 \end{bmatrix} \longrightarrow A\bar{x} = \bar{b}$$

$$m_{21} = \frac{a_{21}}{a_{11}} = \frac{2}{4} = 0,5 \qquad L_2 = L_2 - m_{21} \cdot L_1$$

$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{1}{4} = 0,25 \qquad L_3 = L_3 - m_{31} \cdot L_1$$

$$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 0.5 \\ 0 & 1.5 & 1,75 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 11 \end{bmatrix}$$

$$m_{32} = \frac{a_{32}}{a_{22}} = \frac{1,5}{1} = 1,5 \qquad L_3 = L_3 - m_{32} \cdot L_2$$

$$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 0,5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 0,5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix} \Rightarrow U\bar{x} = \bar{d}$$

substituições regressivas

$L_3 \to \boxed{x_3} = 2$

$L_2 \to \boxed{x_2} - 0,5\,x_3 = 6$

$L_1 \to 4x_1 + 2x_2 + x_3 = 4$

$$x_2 = \frac{6 - 0,5x_3}{1} = 5$$

$$x_1 = \frac{-2x_2 - x_3 + 4}{4}$$

$$x_1 = \frac{-2 \cdot 5 - 2 + 4}{4} = -2$$

3.2) Decomposição LU

$$\mathbb{A}\bar{x} = \vec{b} \xrightarrow[\text{de Gauss}]{\text{eliminação}} \mathbb{U}\bar{x} = \bar{d}$$

$$\mathbb{A}\bar{x} - \bar{b} = 0 \qquad\qquad \underbrace{\mathbb{U}x - \bar{d} = 0}_{\times \mathbb{L}}$$

$$\mathbb{A} = \mathbb{L} \cdot \mathbb{U} \qquad\qquad \mathbb{L}\mathbb{U}\bar{x} - \mathbb{L}\bar{d} = 0$$

$$\mathbb{L}\bar{d} = \bar{b} \qquad\qquad \mathbb{L} = \begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 12 \end{bmatrix} \longrightarrow \mathbb{A}\bar{x} = \bar{b}$$

Eliminação de Gauss

$$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 0{,}5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix} \longrightarrow \mathbb{U}x = \bar{d} \uparrow \longrightarrow \bar{x} = \begin{bmatrix} -2 \\ 5 \\ 2 \end{bmatrix}$$

Encontre a solução p/ o sistema abaixo usando a decomposição LU.

$$\begin{bmatrix} 4 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} \longrightarrow \mathbb{A}\bar{x} = \bar{b}_n$$

$$\mathbb{A} \nearrow \mathbb{L} = \begin{bmatrix} 1 & 0 & 0 \\ 0{,}5 & 1 & 0 \\ 0{,}25 & 1{,}5 & 1 \end{bmatrix}$$

$$\mathbb{A} \searrow \mathbb{U} = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 0{,}5 \\ 0 & 0 & 1 \end{bmatrix}$$

p/ testar se tá certo só multiplica $\mathbb{L} \times \mathbb{U}$ que voltará p/ matriz original

$$\mathbb{A} \to \mathbb{L} = \begin{bmatrix} 1 & 0 & 0 \\ 0{,}5 & 1 & 0 \\ 0{,}25 & 1{,}5 & 1 \end{bmatrix}$$

$$\mathbb{A} \to \mathbb{U} = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 0{,}5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 4 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}}_{\mathbb{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} \to \mathbb{A}\bar{x} = \bar{b}_m$$

$$\mathbb{U}\bar{x}_m = \bar{d}_m$$

$$\mathbb{L}\,\bar{d}_m = \bar{b}_m$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0{,}5 & 1 & 0 \\ 0{,}25 & 1{,}5 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$$

$L_1 \to d_1 = 0$

$L_2 \to 0{,}5 \cdot d_1 + d_2 = 2$

$L_3 \to 0{,}25 d_1 + 1{,}5 d_2 + d_3 = 4$

$d_3 = 4 - 1{,}5 \cdot 2 = 1$

$$\bar{d}_m = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 0{,}5 \\ 0 & 0 & 1 \end{bmatrix} \qquad \bar{d}_m = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$U \bar{x}_m = \bar{d}_m \quad \underbrace{\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 0{,}5 \\ 0 & 0 & 1 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\bar{x}_m} = \underbrace{\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}}_{\bar{d}_m}$$

$$L_3 \rightarrow x_3 = \boxed{1}$$

$$L_2 \rightarrow x_2 + 0{,}5x_3 = 2 \Rightarrow x_2 = \boxed{1{,}5}$$

$$L_1 \rightarrow 4x_1 + 2x_2 + x_3 = 0 \Rightarrow 4x_1 = -3 - 1$$

$$x_1 = \frac{-4}{4} = \boxed{1}$$

$$\boxed{\bar{x}_m = \begin{bmatrix} -1 \\ 1{,}5 \\ 1 \end{bmatrix}}$$