homework2

March 11, 2024

1 Homework 2:

1.1 Task 1: Linear model

1.1.1 Dataset

	no2	cars	temp	windspeed	tempdiff	winddir	hour	day
0	22.199949	2206.008383	6.4	3.5	-0.3	56	14	196
1	80.500227	1044.997738	-7.2	1.7	1.2	74	23	143
2	77.200042	1840.992302	-1.3	2.6	-0.1	65	11	115
3	46.200009	333.999668	-3.1	1.8	0.3	78	2	55
4	88.399826	3323.986186	1.0	1.2	1.5	215	18	47

First we analyze the distributions and pairplots of features present in the dataset. For example we can notice that the `NO2` distribution has heavy tail. Pairplots for features (excluding target variable) do not show any particular strong correlation. Another note is that treating day as numerical predictor might not be that sensible, however any one hot encoding for this variable makes even less sense.


```
              no2         cars        temp   windspeed    tempdiff  \
count  251.000000   251.000000  251.000000  251.000000  251.000000
mean    50.510360  1598.581544    0.707171    3.008765    0.333068
std     39.490877  1158.345338    5.626803    1.733783    0.877805
min      3.900013    75.000141  -18.600000    0.300000   -3.500000
25%     24.799909   456.498585   -2.800000    1.700000   -0.100000
50%     45.399904  1444.993352    1.500000    2.800000    0.200000
75%     64.950166  2442.996788    4.800000    4.150000    0.800000
max    324.099316  4224.009186   12.200000    9.900000    4.300000

          winddir        hour         day
count  251.000000  251.000000  251.000000
mean   151.633466   12.139442  119.358566
```

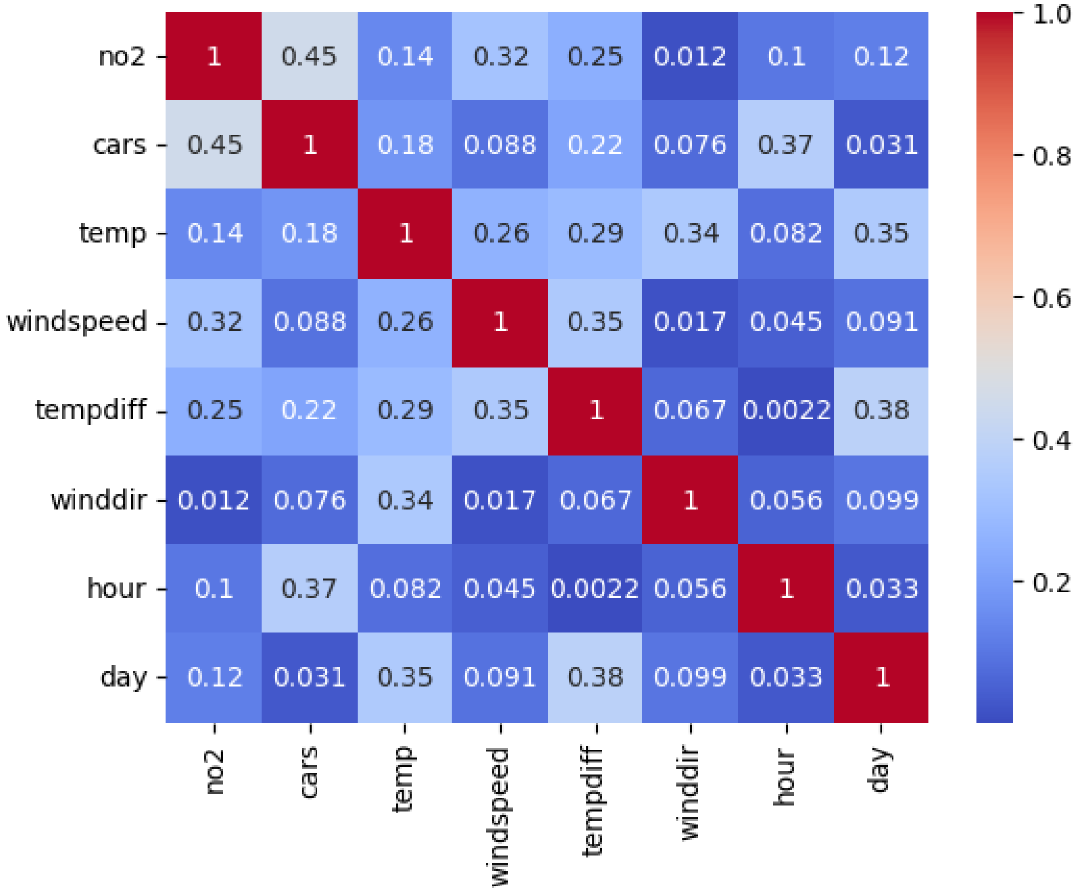
std	87.117949	6.965664	52.133664
min	2.000000	1.000000	32.000000
25%	77.000000	6.000000	78.000000
50%	135.000000	12.000000	119.000000
75%	224.000000	18.000000	165.000000
max	359.000000	24.000000	212.000000

1.1.2 Feature collinearity

We verify if there are any collinearities between predictors and thus if any of the predictors should be removed from our training set. We use Variance Inflation Factor (VIF) and correlation matrix for validation of our findings.

	Feature	VIF
0	const	24.241779
1	cars	1.243074
2	temp	1.512238
3	windspeed	1.192568
4	tempdiff	1.418973
5	winddir	1.229814
6	hour	1.167743
7	day	1.351740

All of the features have VIF ~ 1, suggesting no major collinearities. The variance of the regression coefficients is not inflated. We validate that observation by looking at the correlation matrix. As we can see, the predictors are not highly correlated, which is a good sign. The biggest observable correlation is between `no2` and `cars` (~ 0.45), which might be a suggestion that the number of cars is a valuable predictor.



1.1.3 Linear regression fit

Dep. Variable:	no2	**R-squared:**	0.411
Model:	OLS	**Adj. R-squared:**	0.394
Method:	Least Squares	**F-statistic:**	24.21
Date:	Mon, 11 Mar 2024	**Prob (F-statistic):**	6.77e-25
Time:	18:00:57	**Log-Likelihood:**	-1211.9
No. Observations:	251	**AIC:**	2440.
Df Residuals:	243	**BIC:**	2468.
Df Model:	7		
Covariance Type:	nonrobust		

	coef	std err	t	P> |t|	[0.025	0.975]
Intercept	18.6681	4.777	3.908	0.000	9.258	28.078
cars	0.0198	0.002	10.603	0.000	0.016	0.024
const	18.6681	4.777	3.908	0.000	9.258	28.078
day	0.0131	0.043	0.303	0.762	-0.072	0.099
hour	-0.5144	0.302	-1.705	0.089	-1.109	0.080
temp	-0.6995	0.425	-1.646	0.101	-1.537	0.138
tempdiff	12.0540	2.639	4.568	0.000	6.857	17.252
winddir	-0.0025	0.025	-0.100	0.920	-0.051	0.046
windspeed	-5.6553	1.225	-4.618	0.000	-8.068	-3.243

Omnibus:	210.841	**Durbin-Watson:**	2.228
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4141.571
Skew:	3.223	**Prob(JB):**	0.00
Kurtosis:	21.827	**Cond. No.**	1.11e+18

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 7.93e-28. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

The R^2 and adjusted R^2 indexes are relatively small and might signal not great fit of linear regression model. P-value for F-statistic is small and we can assume that not all of the predictors are insignificant. However, looking at the t-tests for predictor significance, we can observe high p-value for `winddir` and `day` columns, suggesting the removal of these predictors from model. Of course, removing variables can only lead to increase of SSE on the training data, but can improve generalisation on test data and overall interpretability of model.

Also p-values for `hour` and `temp` are higher than 0.05 which could indicate that these variables are also insignificant and removing these variables should at least be considered.

1.1.4 MSE for full model

```
MSE: 915.0655311951564
```

1.1.5 Model after excluding `winddir` and `day`

Dep. Variable:	no2	**R-squared:**	0.411
Model:	OLS	**Adj. R-squared:**	0.399
Method:	Least Squares	**F-statistic:**	34.14
Date:	Mon, 11 Mar 2024	**Prob (F-statistic):**	2.08e-26
Time:	04:22:17	**Log-Likelihood:**	-1212.0
No. Observations:	251	**AIC:**	2436.
Df Residuals:	245	**BIC:**	2457.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P> |t|	[0.025	0.975]
Intercept	19.4244	2.872	6.763	0.000	13.768	25.081
cars	0.0198	0.002	10.654	0.000	0.016	0.023
const	19.4244	2.872	6.763	0.000	13.768	25.081
hour	-0.5170	0.300	-1.721	0.086	-1.109	0.075
temp	-0.6792	0.368	-1.844	0.066	-1.405	0.046
tempdiff	11.7343	2.461	4.768	0.000	6.887	16.582
windspeed	-5.6911	1.212	-4.696	0.000	-8.078	-3.304

Omnibus:	210.498	**Durbin-Watson:**	2.225
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	4120.157
Skew:	3.217	**Prob(JB):**	0.00
Kurtosis:	21.777	**Cond. No.**	9.70e+18

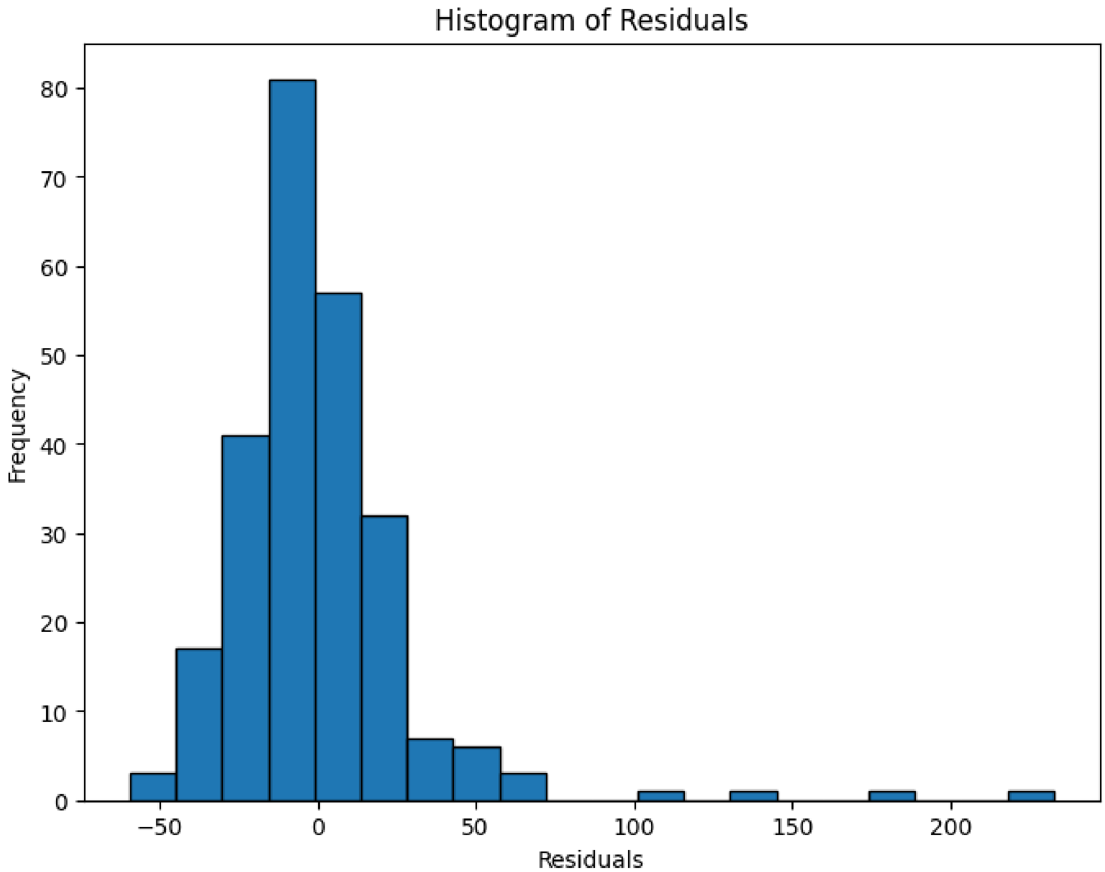
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 1.04e-29. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

```
915.5139090872674
```

After excluding 2 `day` and `winddir` variables from the set of predictors we barely increase MSE on training, while reduce complexity of model and potentially improve generalization.

1.1.6 Diagnostics

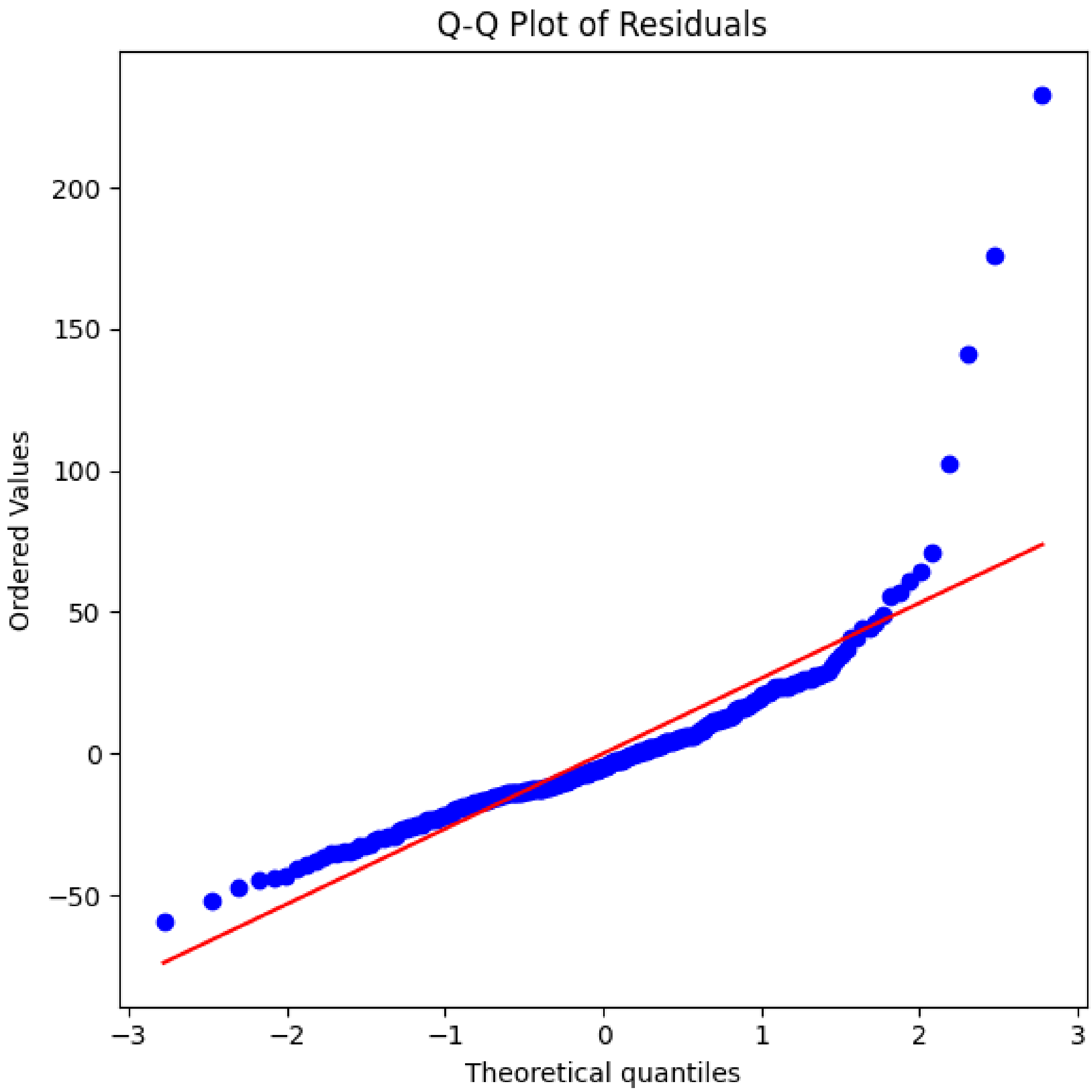
Distribution of residuals



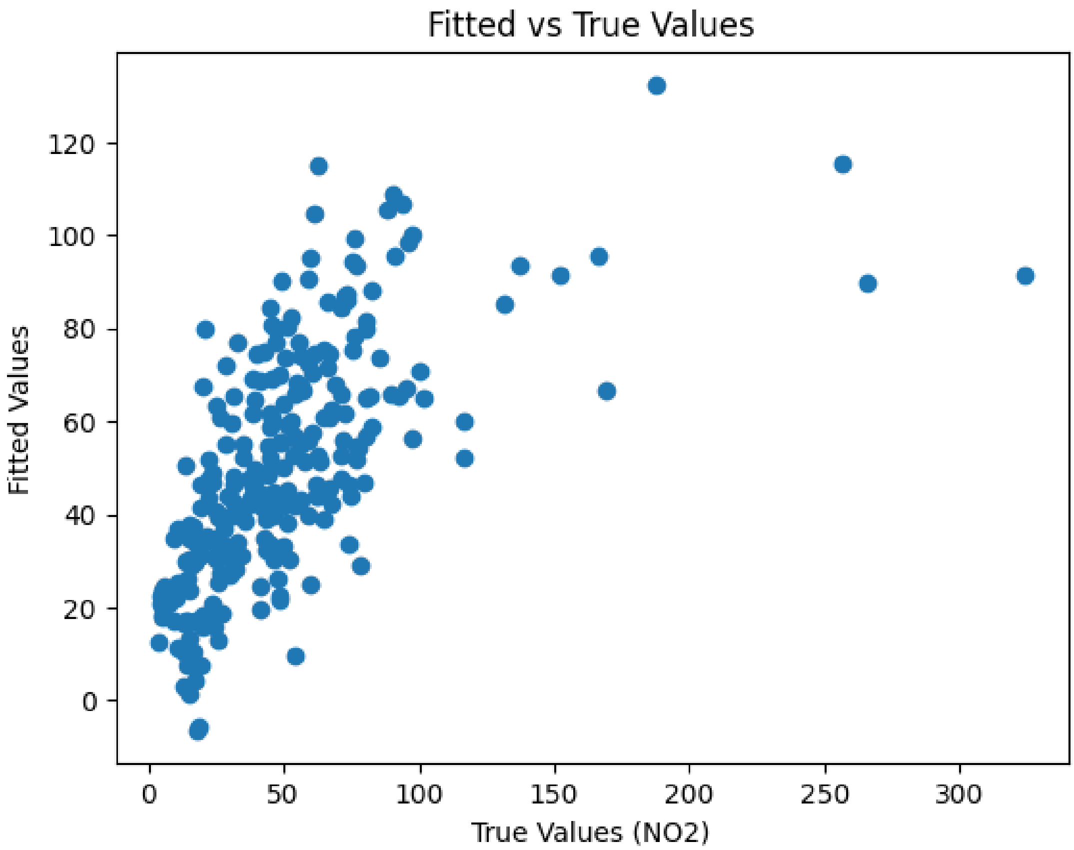

```
Shapiro-Wilk Test Statistic: 0.7688661343173065
Shapiro-Wilk p-value: 1.6377012765726963e-18
```

The residuals of the linear regression model should optimally have normal distribution. The histogram shows some concern and Shapiro-Wilk test was conducted to statistically verify the hypothesis of normality of residuals. The p-value is very low, suggesting to discard the null hypothesis about the normality of the residuals.

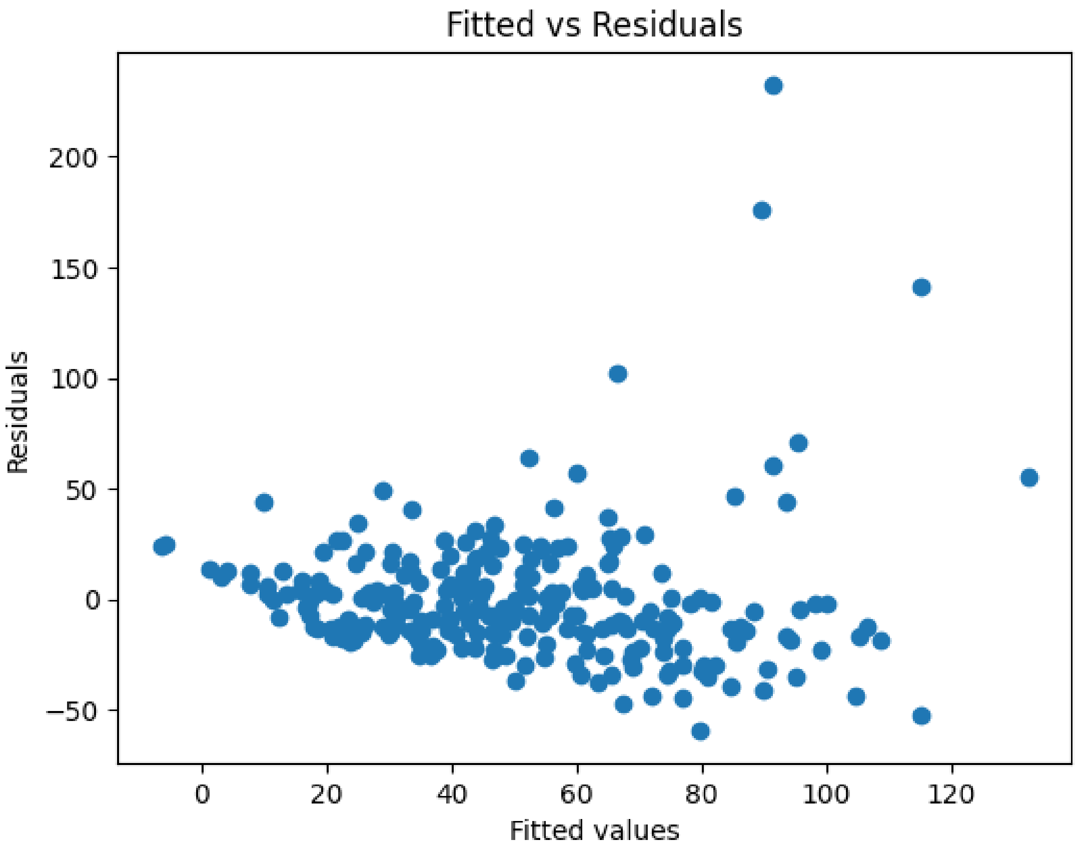
QQ-plot



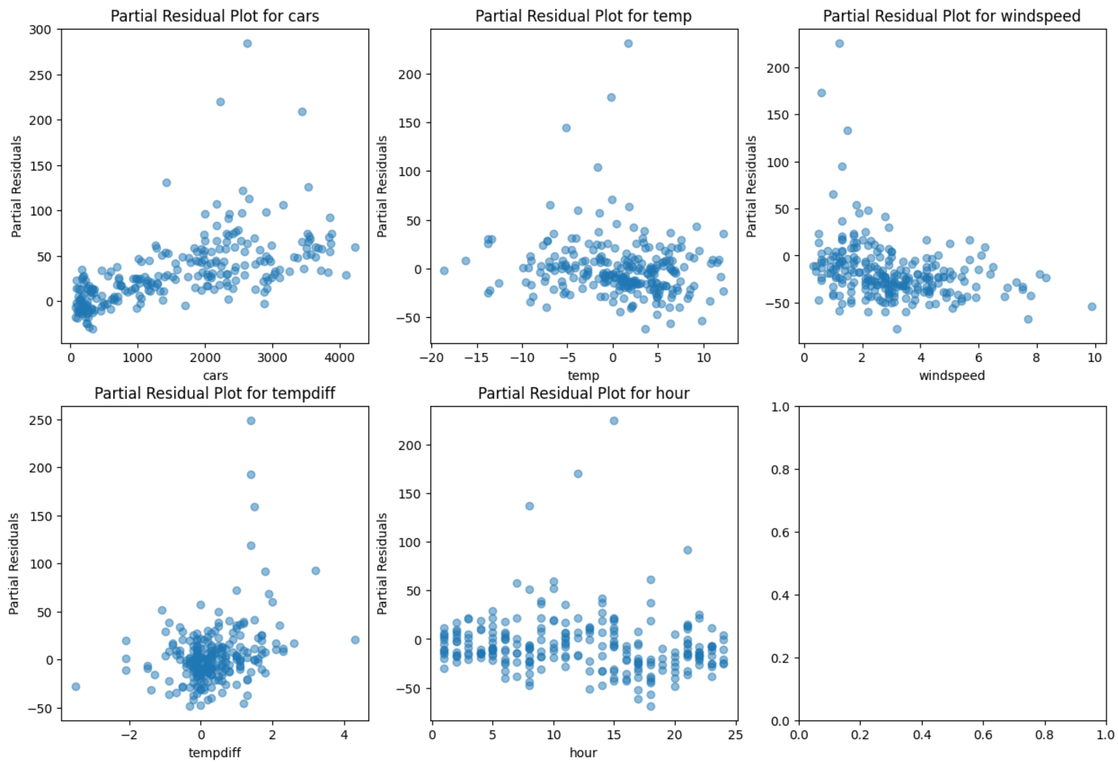
The QQ-plot reinforces the hypotheses that the residuals do not fulfill normality assumption.



The desirable shape for the points from this plot would be a straight line. Unfortunately we can see some anomalies for larger values of the NO2.



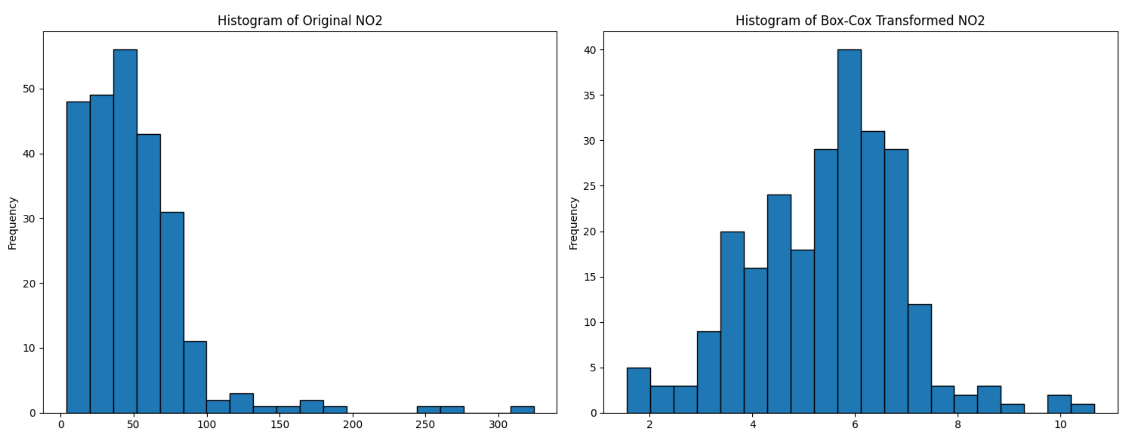
Fitted vs Residuals
Residuals
Fitted values
200
150
100
50
0
−50
0
20
40
60
80
100
120



1.1.7 Box-cox transformation of response variable

One possibility to stabilize variance and normalize distributions of `NO2` and `cars` variables is to apply Box-Cox transformation. After the transformation, I verify if it gives some improvement for the model.

```
Lambda value used for transformation: 0.1939379324703674
```



As we can see, box-cox transformation normalized values of NO2 variable. We do the same for `cars` variable and fit a model with transformed ones.

Dep. Variable:	no2_boxcox	**R-squared:**	0.553
Model:	OLS	**Adj. R-squared:**	0.543
Method:	Least Squares	**F-statistic:**	60.52
Date:	Mon, 11 Mar 2024	**Prob (F-statistic):**	6.96e-41
Time:	18:07:30	**Log-Likelihood:**	-355.77
No. Observations:	251	**AIC:**	723.5
Df Residuals:	245	**BIC:**	744.7
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
Intercept	1.9732	0.110	17.930	0.000	1.756	2.190
cars_boxcox	0.0467	0.003	14.171	0.000	0.040	0.053
const	1.9732	0.110	17.930	0.000	1.756	2.190
hour	-0.0308	0.011	-2.922	0.004	-0.052	-0.010
temp	-0.0457	0.012	-3.756	0.000	-0.070	-0.022
tempdiff	0.4094	0.082	4.991	0.000	0.248	0.571
windspeed	-0.2521	0.040	-6.299	0.000	-0.331	-0.173

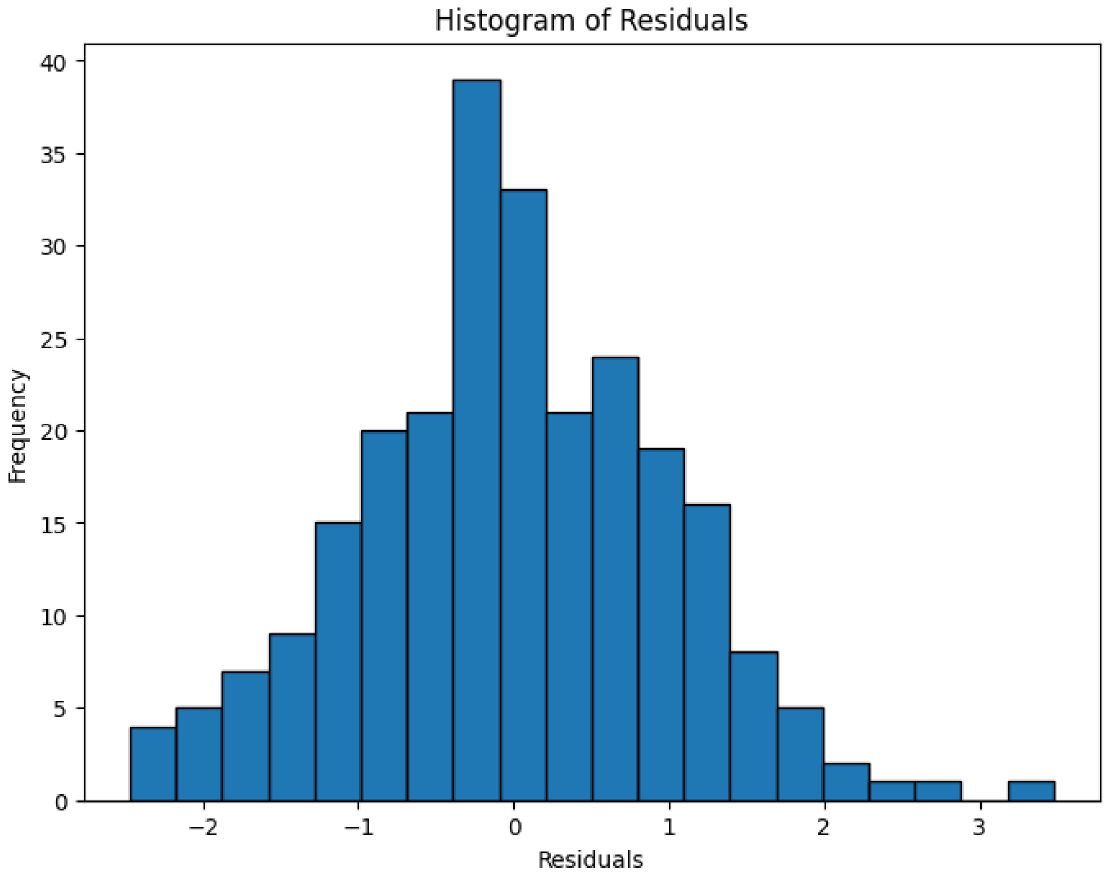
Omnibus:	1.314	**Durbin-Watson:**	2.188
Prob(Omnibus):	0.518	**Jarque-Bera (JB):**	1.010
Skew:	0.119	**Prob(JB):**	0.604
Kurtosis:	3.200	**Cond. No.**	3.03e+17

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 1e-29. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

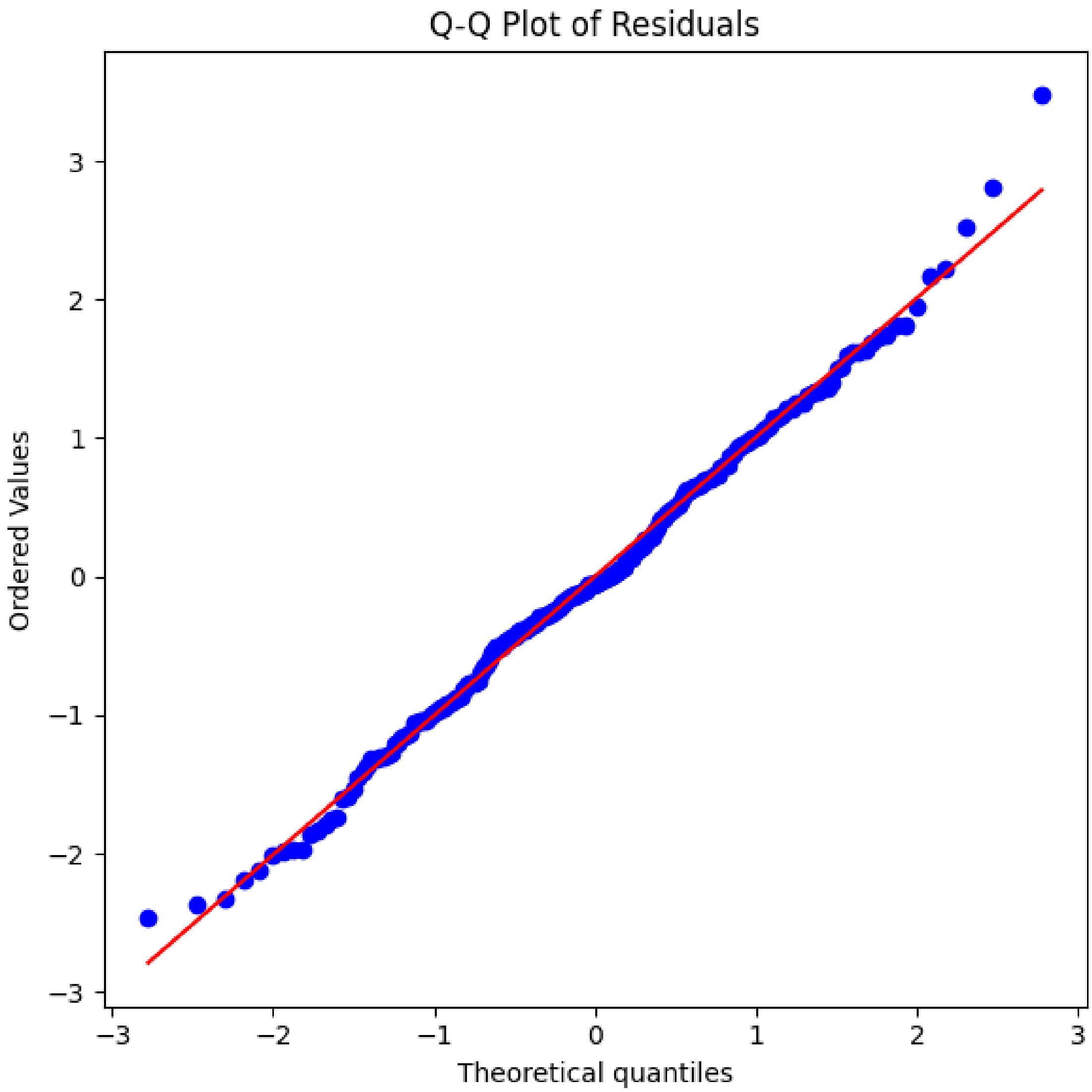
In order to calculate MSE, we need to find the inverse function of our Box-Cox transformation.

```
MSE: 862.3729217742655
```

The MSE is significantly reduced and R^2 index increased by over 0.1, which is a solid improvement.




```
Shapiro-Wilk Test Statistic: 0.9948681045940815
Shapiro-Wilk p-value: 0.5631110423306341
```



Also the residuals improved and they pass Shapiro-Wilk test for normality of distribution. Quantiles on QQ-plot almost perfectly resemble the normal distribution.

1.1.8 Outlier detection

At last we analyse the influence of outlier removal on the results. We use Cook's distance for outlier detection and set threshold based on sample size (typically $\frac{4}{N}$ is considered a reasonable threshold)

Dep. Variable:	no2_boxcox	**R-squared:**	0.565
Model:	OLS	**Adj. R-squared:**	0.556
Method:	Least Squares	**F-statistic:**	61.32
Date:	Mon, 11 Mar 2024	**Prob (F-statistic):**	9.12e-41
Time:	18:10:10	**Log-Likelihood:**	-320.16
No. Observations:	242	**AIC:**	652.3
Df Residuals:	236	**BIC:**	673.2
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
Intercept	1.9831	0.101	19.704	0.000	1.785	2.181
cars_boxcox	0.0445	0.003	14.496	0.000	0.038	0.051
const	1.9831	0.101	19.704	0.000	1.785	2.181
hour	-0.0280	0.010	-2.876	0.004	-0.047	-0.009
temp	-0.0478	0.011	-4.175	0.000	-0.070	-0.025
tempdiff	0.3677	0.077	4.757	0.000	0.215	0.520
windspeed	-0.2364	0.037	-6.364	0.000	-0.310	-0.163

Omnibus:	1.507	**Durbin-Watson:**	2.083
Prob(Omnibus):	0.471	**Jarque-Bera (JB):**	1.578
Skew:	-0.150	**Prob(JB):**	0.454
Kurtosis:	2.742	**Cond. No.**	4.70e+16

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 3.99e-28. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

```
MSE: 890.2473820578655
```

We can notice that the fit of the model is better, but it should be intuitive as we removed the annoying observations. But the MSE calculated on the whole data grows.