$$
\begin{array}{rcl}
Var \ni x & ::= & x_1 \mid x_2 \mid ... \\
Const \ni n & ::= & ... \mid \texttt{-1} \mid \texttt{0} \mid \texttt{1} \mid ... \\
Exp \ni e & ::= & n \mid x \mid e_1 + e_2 \mid e_1 - e_2 \\
Stm \ni i & ::= & x := e \mid i_1; i_2 \mid \texttt{skip} \mid \texttt{if } e_1 = e_2 \texttt{ then } i_1 \texttt{ else } i_2 \mid \\
 & & \texttt{guard } e \texttt{ in } i \texttt{ end} \mid \texttt{loop } i \texttt{ end} \\
Prg \ni p & ::= & i
\end{array}
$$

$$
\begin{array}{rcl}
s \in State & = & Var \to \mathbf{Z}_\perp \\
g \in Guard & = & State \to \mathbf{Z}_\perp \\
N & : & Const \to \mathbf{Z}_\perp \\
\mathbb{E} & : & Exp \to State \to \mathbf{Z}_\perp \\
\mathbb{I} & : & Stm \to State \to Guard \to State \\
\mathbb{P} & : & Stm \to State
\end{array}
$$

Założenia:

1. Funkcja N w sposób klasyczny konwertuje stałe na liczby.
2. Operatory działań ($+$, $-$) są różne w zależności od kontekstu (definiują drzewo programu, lub operację na liczbach).

Wyrażenia:

$$
\begin{array}{rcl}
\mathbb{E}[\![n]\!]\ s & = & N(n) \\
\mathbb{E}[\![x]\!]\ s & = & s(x) \\
\mathbb{E}[\![e_1 + e_2]\!]\ s & = & \mathbb{E}[\![e_1]\!]s + \mathbb{E}[\![e_2]\!]s \\
\mathbb{E}[\![e_1 - e_2]\!]\ s & = & \mathbb{E}[\![e_1]\!]s - \mathbb{E}[\![e_2]\!]s
\end{array}
$$

Instrukcje:

$$
\begin{array}{rcl}
\mathbb{I}[\![x := e]\!]\ s\ g & = & s[x \to \mathbb{E}[\![e]\!]\ s] \\
\mathbb{I}[\![i_1; i_2]\!]\ s\ g & = & \mathbb{I}[\![i_2]\!]\ (\mathbb{I}[\![i_1]\!]\ s\ g)\ g \\
\mathbb{I}[\![\texttt{skip}]\!]\ s\ g & = & s \\
\mathbb{I}[\![\texttt{if } e_1 = e_2 \texttt{ then } i_1 \texttt{ else } i_2]\!]\ s\ g & = & \mathsf{if}\ \mathbb{E}[\![e_1]\!]\ s = \mathbb{E}[\![e_2]\!]\ s\ \mathsf{then}\ \mathbb{I}[\![i_1]\!]\ s\ g\ \mathsf{else}\ \mathbb{I}[\![i_2]\!]\ s\ g \\
\mathbb{I}[\![\texttt{guard } e \texttt{ in } i \texttt{ end}]\!]\ s\ g & = & \mathsf{let}\ g' = (\lambda s.\mathsf{if}\ \mathbb{E}[\![e]\!]\ s = 0\ \mathsf{then}\ 0\ \mathsf{else}\ g(s)) \\
 & & \mathsf{in}\ \mathbb{I}[\![i]\!]\ s\ g' \\
\mathbb{I}[\![\texttt{loop } i \texttt{ end}]\!] & = & Fix(\lambda f.\ \lambda s.\ \lambda g.\ \mathsf{if}\ g(s) = 0\ \mathsf{then}\ s\ \mathsf{else}\ f(\mathbb{I}[\![i]\!]\ s\ g)\ g)
\end{array}
$$

Programy:

$$
\mathbb{P}[\![i]\!] \ = \ \mathbb{I}[\![i]\!](\lambda x.0)(\lambda s.1)
$$