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Advantages of an easy to design fuzzy predictive algorithm in control systems of nonlinear chemical reactors

Piotr M. Marusak *

Institute of Control and Computation Engineering, Warsaw University of Technology, Nowowiejska 15/19, 00–665 Warsaw, Poland

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ABSTRACT

Advantages of a fuzzy predictive control algorithm are discussed in the paper. The fuzzy predictive algorithm is a combination of a DMC (Dynamic Matrix Control) algorithm and Takagi–Sugeno fuzzy modeling, thus it inherits advantages of both techniques. The algorithm is numerically effective. It is in fact generalization of the standard DMC algorithm widely used in the industry, thus the existing implementations of the DMC algorithm can be extended using the presented fuzzy approach. A simple and easy to apply method of fuzzy predictive control algorithms synthesis is presented in the paper. It can be easy applied also in the case of Multiple Input Multiple Output (MIMO) control plants. Moreover, information about measured disturbance can be included in the algorithms in an easy way. The advantages of the fuzzy predictive control algorithm are demonstrated in the example control systems of two nonlinear chemical reactors: the first one—with inverse response and the second one—a MIMO plant with time delay.

1. Introduction

Model predictive control (MPC) algorithms are widely used in practice due to high-quality control performance they offer. They are usually used in an advanced control layer of the multilayer control system structure; see e.g. [4,27,33]. Basic formulations of the MPC algorithms are based on linear control plant models. The advantage of these algorithms is that they are formulated as easy to solve, linear–quadratic optimization problems. However, application of the MPC algorithm based on a linear plant model to a nonlinear plant, if control in a wide range of set-point values is needed, may bring unsatisfactory results or the results can be improved using the algorithm based on a nonlinear model.

Various kinds of nonlinear models can be used in such algorithms. However, Takagi–Sugeno (TS) fuzzy models [32], thanks to their properties, are particularly suitable for application in MPC algorithms. These models can be utilized in different MPC algorithms. In the first group of MPC algorithms the fuzzy models are directly used to formulate the optimization problem. Unfortunately, a direct usage of a nonlinear model to design the MPC algorithm leads to its formulation as a nonlinear, and in general, non-convex optimization problem that is hard to solve and computationally demanding. Fuzzy predictive algorithms often use nonlinear optimization, see e.g. [10–12,29,34], however such an approach needs application of complex optimization routines and may result in typical problems brought by non-convex optimization, i.e. problem with local minima. Therefore branch and bound method is often used for optimization in these cases, see e.g. [3,25,28]. In this method, however, computational effort is huge, especially in the case of MIMO control systems [25], and possible control actions are discretized, thus the obtained solution is in fact only an approximate one (though it is a global optimum of the discretized problem). The denser the discretization the better approximation is obtained but at the cost of increase of computational effort needed to find the solution. Thus, in practice, suboptimal algorithms that use approximate process models, generated from the nonlinear model at each algorithm iteration, are used; see e.g. [33]. In these algorithms the successive linearization of the process model is used in such a way that the standard linear–quadratic optimization problem is formulated and solved at each algorithm iteration.

In the next group of fuzzy predictive control algorithms are those which use linearization. Approach to the constraints divides this group into two smaller sets. The first set of the algorithms neglects existence of the constraints in the control system during manipulated variable calculation (though constraints can be taken into consideration by clipping, see e.g. [33]). These, analytical fuzzy predictive algorithms, are formulated in such a way that their main advantage is simplicity. It is because in order to obtain manipulated variables only algebraic equation must be solved at

* Tel.: +48 22 234 76 73; fax: +48 22 825 37 19.
E-mail address: P.Marusak@ia.pw.edu.pl.

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each iteration; see e.g. [1,14,18,20,35]. One can use also a PDC approach to design the fuzzy predictive controller, see e.g. [24,33] and obtain the control law with variable parameters. To the group of analytical predictive algorithms belong also algorithms designed for SISO systems in which the control law is very simple thanks to some simplification assumptions. Such a simple control law is obtained in [2,16,30,31] after usage of coincidence points mechanism and assumption than only one such point is taken into consideration at the end of the prediction horizon. Similar effect can be obtained using control horizon equal to one [22].

The second set of fuzzy predictive algorithms which use linearization contains algorithms which are formulated as linear–quadratic optimization problems. They are particularly efficient when constraints must be taken into consideration by the control algorithm. It is because thanks to the formulation as the optimization problem the constraint handling can be done in a systematic and relatively easy way. Different kinds of models can be used as the local models but most often state space equations like in [15,26] or difference equations, like e.g. in [20,23,28,33] are used. In the case of DMC-type algorithms in order to apply a DMC technique like e.g. in [14,20,23,33], at each iteration step responses are obtained from the TS model with difference equations in consequents. Moreover, it can be shown that in the case of all MPC algorithms based on linear (or linearized model) the dynamic matrix, composed of step response coefficients, is present in their formulations [33]. Therefore, in order to simplify the MPC algorithms with fuzzy models even more, the method discussed in the paper was proposed.

The DMC algorithm is practically a standard in the industrial applications; see e.g. [4,5,7,27,33]. It can be relatively easily designed, because it is based on an easy to obtain control plant model in the form of step responses of the process. Moreover, it is claimed in [4] that "Most MPC implementations to date use step response models proven in DMC applications." It is so despite the step response models can be applied only to open-loop stable plants. However, it should be emphasized that MPC algorithms (those formulated as optimization problems, in particular) are usually applied in the multilayer control structure, in the constraint control layer being above direct control layer which is responsible for stabilization of the control plant, see e.g. [33]. Thus, MPC algorithms are, in most cases, applied to stable or pre-stabilized control plants and among them, according to [27], DMC is the most popular algorithm in industrial applications.

The fuzzy DMC (FDMC) algorithm discussed in the paper is easy to design, for it is based on DMC technique and Takagi–Sugeno fuzzy models with step responses used as the local models. Such Takagi–Sugeno fuzzy models can be obtained relatively easy using the expert knowledge and/or simulation experiments. Moreover, the existing control systems with DMC algorithm implementation can be modified in an easy way using the fuzzy approach presented in the paper. Such a modification can lead to significant improvement of the control system performance thanks to the usage of the nonlinear (fuzzy) process model. (The FDMC algorithm is based on the nonlinear control plant model, therefore it usually works well in a wide range of operating point changes, offering better control performance than the algorithms based on linear process models.) The feature of the proposed FDMC algorithm, as well as the DMC algorithm, which may be considered their disadvantage, is relatively big number of parameters of the model. However, time needed to calculate the manipulated variables using modern computers is very short even if long horizons are used (as it was the case in the examples studied in the article). It should be also stressed that the proposed FDMC algorithms are formulated in such a way that the main part of the algorithms (linear–quadratic optimization problem solved at each iteration) is as much complicated as the one for the DMC algorithm with the same horizons. Moreover, at each iteration the linear approximation of the process model in the form of the step response is obtained directly after defuzzification, therefore recalculation of the model is not needed, and dynamic matrix can be directly created using coefficients of the step responses.

The significant advantage of the predictive control algorithms is relative easiness of their application to MIMO control plants, for which standard control techniques are hard to apply and usually do not offer as good control performance as the predictive algorithms; see e.g. [33]. The FDMC algorithm can be applied in control systems of MIMO plants especially easy thanks to the form of the model it is based on.

The FDMC algorithm was tested in control systems of two nonlinear plants. The first one is a chemical plant—an isothermal CSTR in which a van de Vusse reaction takes place. The control plant is a difficult process because except it is nonlinear, it has also an inverse response. The second control plant is the highly nonlinear MIMO pH reactor with time delay. In both cases the experiments were performed in two control systems: the first one—with DMC algorithm based on a linear process model and the second one—with FDMC algorithm. The obtained results demonstrate the superiority of the fuzzy algorithm over the one based on the linear model.

The organization of the paper is as follows. In Section 2 the idea of predictive control algorithms is discussed, the DMC and easy to design FDMC algorithms are described. In Section 3 the chemical reactor in which the van de Vusse reaction takes place is described, algorithms used during the experiments are detailed and discussion of obtained results is presented. In Section 4 the MIMO pH reactor and results of experiments performed in the control system of this plant are described. The paper is summarized in Section 5.

2. Predictive controllers

In the MPC algorithms future behavior of the control plant is predicted many time instants ahead (number of these instants is called the prediction horizon), using a dynamic control plant model and all available knowledge about conditions of control system operation. The future control values are then derived in such a way that the future behavior of the control system fulfills assumed criteria. The idea of predictive control is presented in Fig. 1.

Typically, the minimization of a performance function is demanded subject to the constraints put on manipulated and controlled variables; in the MPC algorithms based on input–output models usually the following optimization problem is solved at

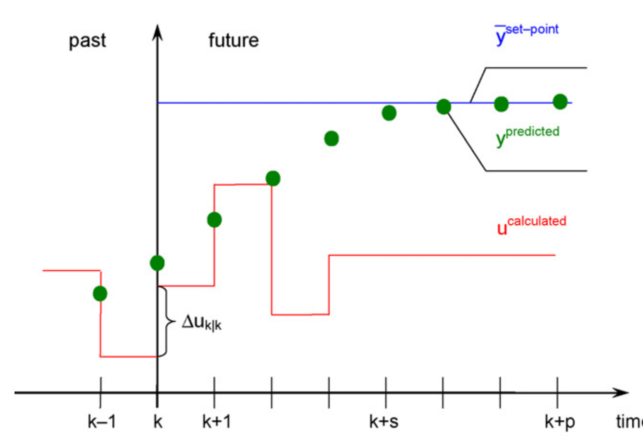

Fig. 1. Idea of predictive control; p – prediction horizon; s – control horizon, $\Delta u_{k|k}$ – control signal change at current iteration.

each iteration:

$$\min_{\Delta u} \sum_{j=1}^{n_y}\sum_{i=1}^{p}\kappa_j \cdot \left(\bar{y}_k^j - y_{k+i|k}^j\right)^2 + \sum_{m=1}^{n_u}\sum_{i=0}^{s-1}\lambda_m \cdot \left(\Delta u_{k+i|k}^m\right)^2 \tag{1}$$

subject to:

$$\Delta \boldsymbol{u}_{\min} \le \Delta \boldsymbol{u} \le \Delta \boldsymbol{u}_{\max}, \tag{2}$$

$$\boldsymbol{u}_{\min} \le \boldsymbol{u} \le \boldsymbol{u}_{\max}, \tag{3}$$

$$\boldsymbol{y}_{\min} \le \boldsymbol{y} \le \boldsymbol{y}_{\max}, \tag{4}$$

where $\bar{y}_k^j$ is a set-point value for jth output, $y_{k+i|k}^j$ is jth output value for $(k+i)$th sampling instant predicted at kth sampling instant using the control plant model, $u_{k+i|k}^m$ are future values of the manipulated variables, $\Delta u_{k+i|k}^m$ are future increments of the manipulated variables, $\kappa_j \ge 0$ and $\lambda_m \ge 0$ are weighting coefficients, p and s denote prediction and control horizons, respectively, and n_y, n_u denote number of outputs and inputs, respectively. The definitions of vectors are as follows:

$$\Delta \boldsymbol{u} = \left[\Delta \boldsymbol{u}_k^1, \Delta \boldsymbol{u}_k^2, \ldots, \Delta \boldsymbol{u}_k^{n_u}\right]^T, \;\; \Delta \boldsymbol{u}_k^m = \left[\Delta u_{k|k}^m, \ldots, \Delta u_{k+s-1|k}^m\right], \tag{5}$$

$$\boldsymbol{u} = \left[\boldsymbol{u}_k^1, \boldsymbol{u}_k^2, \ldots, \boldsymbol{u}_k^{n_u}\right]^T, \;\; \boldsymbol{u}_k^m = \left[u_{k|k}^m, \ldots, u_{k+s-1|k}^m\right], \tag{6}$$

$$\boldsymbol{y} = \left[\boldsymbol{y}_k^1, \boldsymbol{y}_k^2, \ldots, \boldsymbol{y}_k^{n_y}\right]^T, \;\; \boldsymbol{y}_k^j = \left[y_{k+1|k}^j, \ldots, y_{k+p|k}^j\right], \tag{7}$$

$\Delta \boldsymbol{u}_{\min}$, $\Delta \boldsymbol{u}_{\max}$, $\boldsymbol{u}_{\min}$, $\boldsymbol{u}_{\max}$, $\boldsymbol{y}_{\min}$, $\boldsymbol{y}_{\max}$ are vectors of lower and upper bounds of increments and values of the manipulated variables and of the values of output variables, respectively. As a solution to the optimization problem (1)–(4) the optimal vector of increments of the manipulated variables is obtained. From this vector, the $\Delta u_{k|k}^m$ elements are taken and applied in the control system. Then optimization is repeated at the next sampling instant.

The way the predicted output values $y_{k+i|k}^j$ are derived depends on the dynamic control plant model exploited by the algorithm. If the linear model is used then the optimization problem (1)–(4) is a standard linear–quadratic programming problem.

2.1. Standard DMC algorithm

The standard, classical (non-fuzzy) DMC algorithm uses the control plant model in the form of the set of step responses; see e.g. [5,7,19,27,33]:

$$\breve{y}_k^j = \sum_{m=1}^{n_u}\sum_{i=1}^{p_d-1} a_i^{j,m} \cdot \Delta u_{k-i}^m + a_{p_d}^{j,m} \cdot u_{k-p_d}^m, \tag{8}$$

where $\breve{y}_k^j$ is jth output of the control plant model at kth sampling instant, Δu_k^m is a change in mth manipulated variable at kth sampling instant, $a_i^{j,m}$ $(i = 1, \ldots, p_d)$ are step response coefficients of the control plant describing influence of mth input on jth output, $p_d \ge p$ is equal to the number of sampling instants after which the coefficients of the step responses can be assumed as settled, $u_{k-p_d}^m$ is a value of mth manipulated variable at $(k-p_d)$th sampling instant.

The predicted output values are then calculated using the following formula:

$$y_{k+i|k}^j = \sum_{m=1}^{n_u}\left(\sum_{n=1}^{i} a_n^{j,m} \cdot \Delta u_{k-n+i}^m + \sum_{n=i+1}^{p_d-1} a_n^{j,m} \cdot \Delta u_{k-n+i}^m + a_{p_d}^{j,m} \cdot u_{k-p_d+i}^m\right) + d_k^j, \tag{9}$$

where $d_k^j = y_k^j - \breve{y}_{k-1}^j$ are assumed to be the same at each sampling instant in the prediction horizon (it is a DMC-type model of an unmeasured disturbance). Thus (9) can be transformed into the following form:

$$y_{k+i|k}^j = y_k^j + \sum_{m=1}^{n_u}\left(\sum_{n=i+1}^{p_d-1} a_n^{j,m} \cdot \Delta u_{k-n+i}^m + a_{p_d}^{j,m} \cdot \sum_{n=p_d}^{p_d+i-1} \Delta u_{k-n+i}^m - \sum_{n=1}^{p_d-1} a_n^{j,m} \cdot \Delta u_{k-n}^m\right) + \sum_{m=1}^{n_u}\sum_{n=1}^{i} a_n^{j,m} \cdot \Delta u_{k-n+i|k}^m. \tag{10}$$

In (10) only the last component depends on future manipulated variable changes. Thus the vector of predicted output values $\boldsymbol{y}$ can be decomposed into the following components:

$$\boldsymbol{y} = \tilde{\boldsymbol{y}} + \boldsymbol{A} \cdot \Delta \boldsymbol{u}, \tag{11}$$

where $\boldsymbol{A}$ is a matrix of dimensionality $(p \cdot n_y) \times (s \cdot n_u)$, called the dynamic matrix and is composed of the coefficients of the step responses of the control plant:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \cdots & \boldsymbol{A}_{1n_u} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} & \cdots & \boldsymbol{A}_{2n_u} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{A}_{n_y1} & \boldsymbol{A}_{n_y2} & \cdots & \boldsymbol{A}_{n_yn_u} \end{bmatrix}, \tag{12}$$

$$\boldsymbol{A}_{jm} = \begin{bmatrix} a_1^{j,m} & 0 & \cdots & 0 & 0 \\ a_2^{j,m} & a_1^{j,m} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_p^{j,m} & a_{p-1}^{j,m} & \cdots & a_{p-s+2}^{j,m} & a_{p-s+1}^{j,m} \end{bmatrix}, \tag{13}$$

$$\tilde{\boldsymbol{y}} = \left[\tilde{\boldsymbol{y}}_k^1, \tilde{\boldsymbol{y}}_k^2, \ldots, \tilde{\boldsymbol{y}}_k^{n_y}\right]^{\mathrm{T}}, \;\; \tilde{\boldsymbol{y}}_k^j = \left[\tilde{y}_{k+1|k}^j, \ldots, \tilde{y}_{k+p|k}^j\right], \tag{14}$$

$$\tilde{y}_{k+i|k}^j = y_k^j + \sum_{m=1}^{n_u}\left(\sum_{n=i+1}^{p_d-1} a_n^{j,m} \cdot \Delta u_{k-n+i}^m + a_{p_d}^{j,m} \cdot \sum_{n=p_d}^{p_d+i-1} \Delta u_{k-n+i}^m - \sum_{n=1}^{p_d-1} a_n^{j,m} \cdot \Delta u_{k-n}^m\right); \tag{15}$$

$\tilde{\boldsymbol{y}}$ is called the free response of the control plant, because it contains future output values calculated assuming that the control signal does not change in the prediction horizon (describes influence of the manipulated variable values applied to the control plant in previous iterations):

$$\tilde{\boldsymbol{y}} = \overleftarrow{\boldsymbol{y}} + \overleftarrow{\boldsymbol{A}} \cdot \Delta \overleftarrow{\boldsymbol{u}}, \tag{16}$$

where

$$\Delta \overleftarrow{\boldsymbol{u}} = \left[\Delta \overleftarrow{\boldsymbol{u}}_k^1, \Delta \overleftarrow{\boldsymbol{u}}_k^2, \ldots, \Delta \overleftarrow{\boldsymbol{u}}_k^{n_u}\right]^{\mathrm{T}}, \;\; \Delta \overleftarrow{\boldsymbol{u}}_k^m = \left[\Delta u_{k-1}^m, \ldots, \Delta u_{k-p_d+1}^m\right], \tag{17}$$

$$\overleftarrow{\boldsymbol{y}} = \left[\overleftarrow{\boldsymbol{y}}_k^1, \overleftarrow{\boldsymbol{y}}_k^2, \ldots, \overleftarrow{\boldsymbol{y}}_k^{n_y}\right]^T, \;\; \overleftarrow{\boldsymbol{y}}_k^j = \left[y_k^j, \ldots, y_k^j\right], \tag{18}$$

where vectors $\overleftarrow{\boldsymbol{y}}_k^j$ are all of length p;

$$\overleftarrow{\boldsymbol{A}} = \begin{bmatrix} \overleftarrow{\boldsymbol{A}}_{11} & \overleftarrow{\boldsymbol{A}}_{12} & \cdots & \overleftarrow{\boldsymbol{A}}_{1n_u} \\ \overleftarrow{\boldsymbol{A}}_{21} & \overleftarrow{\boldsymbol{A}}_{22} & \cdots & \overleftarrow{\boldsymbol{A}}_{2n_u} \\ \vdots & \vdots & \ddots & \vdots \\ \overleftarrow{\boldsymbol{A}}_{n_y1} & \overleftarrow{\boldsymbol{A}}_{n_y2} & \cdots & \overleftarrow{\boldsymbol{A}}_{n_yn_u} \end{bmatrix}, \tag{19}$$

$$\overleftarrow{\boldsymbol{A}}_{jm} = \begin{bmatrix} a_2^{j,m} - a_1^{j,m} & a_3^{j,m} - a_2^{j,m} & \cdots & a_{p_d-1}^{j,m} - a_{p_d-2}^{j,m} & a_{p_d}^{j,m} - a_{p_d-1}^{j,m} \\ a_3^{j,m} - a_1^{j,m} & a_4^{j,m} - a_2^{j,m} & \cdots & a_{p_d}^{j,m} - a_{p_d-2}^{j,m} & a_{p_d}^{j,m} - a_{p_d-1}^{j,m} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{p+1}^{j,m} - a_1^{j,m} & a_{p+2}^{j,m} - a_2^{j,m} & \cdots & a_{p_d}^{j,m} - a_{p_d-2}^{j,m} & a_{p_d}^{j,m} - a_{p_d-1}^{j,m} \end{bmatrix}; \tag{20}$$

Using the prediction (11) one can formulate the optimization problem (1)–(4), solution of which contains the future increments of the manipulated variables. The details concerning formulation of the DMC algorithm can be found, e.g. in [5,7,19,27,33].

Remark. The step responses can be obtained either directly from the control plant or using the process model. In the latter case one can simulate the process and easily collect step responses starting simulation from an assumed operating point; it was done so during preparation of examples discussed later. During collection of the step responses one should remember to apply a step change at only one of the process inputs at the same time. Then the step responses should be normalized, i.e. all coefficients should be recalculated according to the formula:

$$a_i^{j,m} = \frac{y_i^{j,m} - y_0^j}{\Delta u_{appl}^m}, \tag{21}$$

where $y_i^{j,m}$ are step response coefficients of the control plant describing influence of mth input on jth output before normalization, y_0^j is the value of jth output at the beginning of an experiment, Δu_{appl}^m is the value of change applied to mth input of the process.

Disturbance measurement can be taken into consideration in an easy way. Despite that, it can offer significant control system performance improvement. In the DMC algorithm the control plant model (8) should be supplemented by terms describing influence of the disturbance on the output variable:

$$\breve{y}_k^j = \sum_{m=1}^{n_u} \sum_{i=1}^{p_d-1} a_i^{j,m} \cdot \Delta u_{k-i}^m + a_{p_d}^{j,m} \cdot u_{k-p_d}^m + \sum_{n=1}^{n_z} \sum_{i=1}^{p_z-1} b_i^{j,n} \cdot \Delta z_{k-i}^n + b_{p_z}^{j,n} \cdot z_{k-p_z}^n, \tag{22}$$

where Δz_k^n is a change in nth disturbance variable at kth sampling instant, $b_i^{j,n}$ $(i = 1, \ldots, p_z)$ are disturbance step response coefficients, p_z is equal to the number of sampling instants after which the coefficients of the disturbance step responses can be assumed as settled, $z_{k-p_z}^n$ is a value of nth disturbance variable at $(k - p_z)$th sampling instant.

If there is no information available about future disturbance, the future changes of disturbance are assumed equal to 0. The influence of past changes of the disturbance should be included in the free response. Thus, this time, the free response is described by the following formula:

$$\tilde{\boldsymbol{y}} = \overleftarrow{\boldsymbol{y}} + \overleftarrow{\boldsymbol{A}} \cdot \Delta \overleftarrow{\boldsymbol{u}} + \overleftarrow{\boldsymbol{A}}_z \cdot \Delta \overleftarrow{\boldsymbol{z}}, \tag{23}$$

where

$$\Delta \overleftarrow{\boldsymbol{z}} = \left[\Delta \overleftarrow{\boldsymbol{z}}_k^1, \Delta \overleftarrow{\boldsymbol{z}}_k^2, \ldots, \Delta \overleftarrow{\boldsymbol{z}}_k^{n_z}\right]^T, \quad \Delta \overleftarrow{\boldsymbol{z}}_k^n = \left[\Delta z_{k-1}^n, \ldots, \Delta z_{k-p_z+1}^n\right], \tag{24}$$

$$\overleftarrow{\boldsymbol{A}}_z = \begin{bmatrix} \overleftarrow{\boldsymbol{A}}_{11}^z & \overleftarrow{\boldsymbol{A}}_{12}^z & \cdots & \overleftarrow{\boldsymbol{A}}_{1n_z}^z \\ \overleftarrow{\boldsymbol{A}}_{21}^z & \overleftarrow{\boldsymbol{A}}_{22}^z & \cdots & \overleftarrow{\boldsymbol{A}}_{2n_z}^z \\ \vdots & \vdots & \ddots & \vdots \\ \overleftarrow{\boldsymbol{A}}_{n_y1}^z & \overleftarrow{\boldsymbol{A}}_{n_y2}^z & \cdots & \overleftarrow{\boldsymbol{A}}_{n_yn_z}^z \end{bmatrix}, \tag{25}$$

$$\overleftarrow{\boldsymbol{A}}_{jn}^z = \begin{bmatrix} b_2^{j,n} - b_1^{j,n} & b_3^{j,n} - b_2^{j,n} & \cdots & b_{p_z-1}^{j,n} - b_{p_z-2}^{j,n} & b_{p_z}^{j,n} - b_{p_z-1}^{j,n} \\ b_3^{j,n} - b_1^{j,n} & b_4^{j,n} - b_2^{j,n} & \cdots & b_{p_z}^{j,n} - b_{p_z-2}^{j,n} & b_{p_z}^{j,n} - b_{p_z-1}^{j,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ b_{p+1}^{j,n} - b_1^{j,n} & b_{p+2}^{j,n} - b_2^{j,n} & \cdots & b_{p_z}^{j,n} - b_{p_z-2}^{j,n} & b_{p_z}^{j,n} - b_{p_z-1}^{j,n} \end{bmatrix}. \tag{26}$$

2.2. Easy to design fuzzy DMC algorithms

The FDMC algorithms used in the paper exploit Takagi–Sugeno (TS) fuzzy models with local models in the form of step responses. Thanks to such an approach each iteration of the algorithm is simple. Moreover, the local models are easy to obtain. The TS fuzzy models, the FDMC algorithms are based on, are described by the sets of the following rules:

Rule f: if

$$\underbrace{\begin{aligned} &y_k^j \text{ is } B_1^{f,j} \text{ and } \ldots \text{ and } y_{k-n_P+1}^j \text{ is } B_{n_P}^{f,j} \text{ and } \ldots \text{ and } u_k^m \text{ is } C_1^{f,m} \\ &\quad \text{and } \ldots \text{ and } u_{k-m_P+1}^m \text{ is } C_{m_P}^{f,m} \end{aligned}}_{\text{antecedent}}$$

then

$$\underbrace{\tilde{y}_{k+1}^{j,f} = \sum_{m=1}^{n_u} \sum_{i=1}^{p_d-1} a_i^{j,m,f} \cdot \Delta u_{k-i}^m + a_{p_d}^{j,m,f} \cdot u_{k-p_d}^m}_{\text{consequent}} \tag{27}$$

where y_k^j is a value of jth output variable at kth sampling instant, u_k^m is a value of mth manipulated variable at kth sampling instant, $B_1^{f,j}, \ldots, B_{n_P}^{f,j}, C_1^{f,m}, \ldots C_{m_P}^{f,m}$ are fuzzy sets, $a_i^{j,m,f}$ are the coefficients of step responses in fth local model, $j = 1, \ldots, n_y$, $m = 1, \ldots, n_u$, $f = 1, \ldots, l$, l – number of rules.

If a measured disturbance is taken into consideration, the local model has the following form:

$$\breve{y}_{k+1}^{j,f} = \sum_{m=1}^{n_u} \sum_{i=1}^{p_d-1} a_i^{j,m,f} \cdot \Delta u_{k-i}^m + a_{p_d}^{j,m,f} \cdot u_{k-p_d}^m + \sum_{n=1}^{n_z} \sum_{i=1}^{p_z-1} b_i^{j,n,f} \cdot \Delta z_{k-i}^n + b_{p_z}^{j,n,f} \cdot z_{k-p_z}^n, \tag{28}$$

where $b_i^{j,n,f}$ are the coefficients of the disturbance step responses in fth local model.

The design process of the TS model (27) can be simplified to a large extent. It is because instead of obtaining a complicated local models (as e.g. difference equations) it is sufficient to obtain a few step responses of the control plant (from the environs of a few operating points). The membership functions can be chosen using expert knowledge, simulation experiments, fuzzy neural networks or all these techniques combined. Using such an approach it is relatively easy to extend the existing DMC algorithm to an FDMC one.

The FDMC algorithm uses the TS fuzzy model (27), at each iteration, in the following way:

1. A linear model for current sampling instant is derived using current values of process variables, the TS fuzzy model (27) and fuzzy reasoning. The output value of the model is calculated using the following formula (in the case when a measured disturbance is taken into consideration):

$$\breve{y}_{k+1}^j = \sum_{m=1}^{n_u} \sum_{i=1}^{p_d-1} \tilde{a}_i^{j,m} \cdot \Delta u_{k-i}^m + \tilde{a}_{p_d}^{j,m} \cdot u_{k-p_d}^m + \sum_{n=1}^{n_z} \sum_{i=1}^{p_z-1} \tilde{b}_i^{j,n} \cdot \Delta z_{k-i}^n + \tilde{b}_{p_z}^{j,n} \cdot z_{k-p_z}^n, \tag{29}$$

where $\tilde{a}_i^{j,m} = \sum_{f=1}^{l} \tilde{w}_f \cdot a_i^{j,m,f}$, $\tilde{b}_i^{j,m} = \sum_{f=1}^{l} \tilde{w}_f \cdot b_i^{j,m,f}$ and $\tilde{w}_f$ are the normalized weights calculated using standard fuzzy reasoning, see e.g. [32].

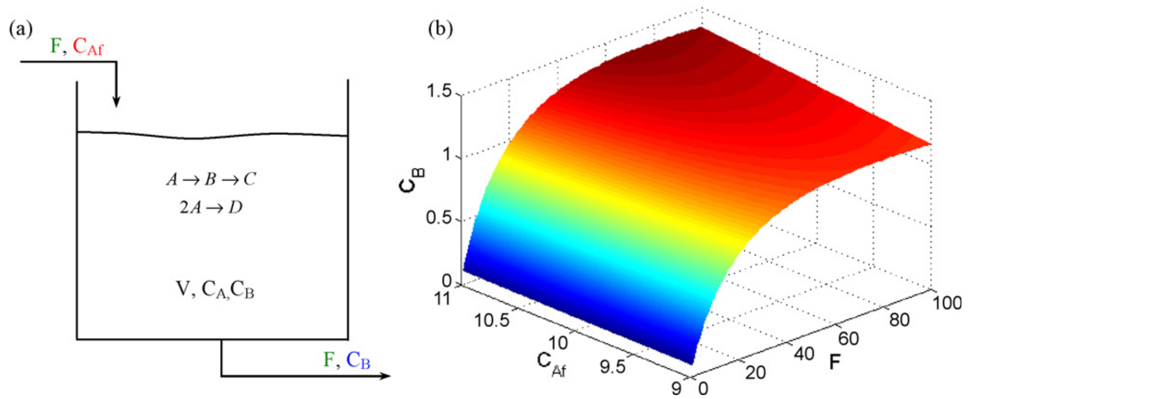

Fig. 2. Isothermal CSTR with van de Vusse reaction; (a) diagram of the plant; (b) steady-state characteristic of the plant; F – manipulated variable; C_B – output variable; C_{Af} – disturbance.

In fact (29) is the step response control plant model valid for the current values of process variables. Thus, next steps of the FDMC algorithm are the same as in the standard DMC algorithm (Section 2.1), i.e.

2. The obtained step response coefficients $\tilde{a}_i^{j,m}$ are used to generate the dynamic matrix (12).
3. The step response model (29) is used to generate the elements of the control plant free response (15).
4. The free response and the dynamic matrix are used to formulate the quadratic optimization problem (1)–(4).
5. The optimization problem is solved and, using the obtained solution, the values of the manipulated variables are generated. Then the controller passes to the next iteration.

Remark. The formulation of the FDMC algorithm given above has an advantage that the optimization problem which must be solved is exactly as complicated as in the case of the DMC algorithm, i.e. respective matrices are of the same dimensionality in DMC and FDMC algorithms. Moreover the same tuning parameters are used and their number depends on the problem not on the algorithm (that was the case in the examples studied in the next sections). It means that the already applied DMC algorithm can be very easily extended to FDMC algorithm and in consequence the performance of the control system may be improved with relatively little effort.

3. Control system of a chemical reactor with van de Vusse reaction

3.1. Control plant

The first of considered control plants is an isothermal CSTR in which a van de Vusse reaction carries out (Fig. 2a) [9]. Its steady-state characteristic is shown in Fig. 2b.

The process model contains composition balance equations:

$$\begin{aligned}\frac{dC_A}{dt} &= -k_1 C_A - k_3 C_A^2 + \frac{F}{V}(C_{Af} - C_A),\\ \frac{dC_B}{dt} &= k_1 C_A - k_2 C_B - \frac{F}{V} C_B.\end{aligned} \tag{30}$$

where C_A, C_B are the concentrations of components A and B, respectively, F is the inlet flow rate (equal to the outlet flow rate), V is the volume in which the reaction is carried out (it is assumed constant and $V = 1$ l), C_{Af} is the concentration of component A in the inlet flow stream (if it is not stated differently it is assumed that $C_{Af0} = 10$ mol/l). The values of the kinetic parameters are: $k_1 = 50$ l/h, $k_2 = 100$ l/h, k_3=10 l/h mol.

The output variable is the concentration C_B of substance B, the manipulated variable is the inlet flow rate F of the raw substance, the disturbance variable is the concentration of component A in the inlet flow stream C_{Af}.

It was assumed that the manipulated variable is constrained

$$F_{\min} \le F \le F_{\max} \tag{31}$$

where $F_{\min} = 0$ l/h, $F_{\max} = 60$ l/h.

For the presented control plant two controllers were designed: the first one – a DMC controller based on a linear model and the second one – an FDMC controller based on a Takagi–Sugeno fuzzy model. The sampling period of both controllers was assumed equal to 3.6 s. The following tuning parameters of both algorithms were assumed: $p_d = p_z = p = 70$, $s = 35$, $\lambda = 0.001$; parameter p_z is present because disturbance measurement is used in the algorithm.

The DMC algorithm is based on a step response obtained in environs of the operating point [9]:

$$P1: C_{B0} = 1.12\,\text{mol/l},\ \ C_{A0} = 3\,\text{mol/l},\ \ F = 34.3\,\text{l/h}.$$

In the case of the FDMC algorithm, two additional step responses are used (the conventional DMC algorithm is in fact extended). The step responses were obtained in environs of the following points:

$$P2: C_{B0} = 0.91\,\text{mol/l},\ \ C_{A0} = 2.18\,\text{mol/l},\ \ F = 20\,\text{l/h, and}$$

$$P3: C_{B0} = 1.22\,\text{mol/l},\ \ C_{A0} = 3.66\,\text{mol/l},\ \ F = 50\,\text{l/h}.$$

Thus, the TS control plant model used by the FDMC controller is composed of three rules [21]. The assumed membership functions are shown in Fig. 3. The step responses were obtained using the model (30) by simulating behavior of the process after step changes of the manipulated variable and of the disturbance. The normalized responses are shown in Fig. 4. They are good illustration of nonlinear properties of the process. The numerical

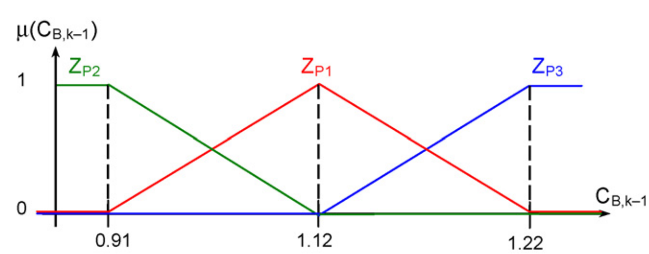

Fig. 3. Membership functions of the model used by the FDMC controller activating step responses obtained in environs of operating points P1, P2 and P3 (listed in the text).

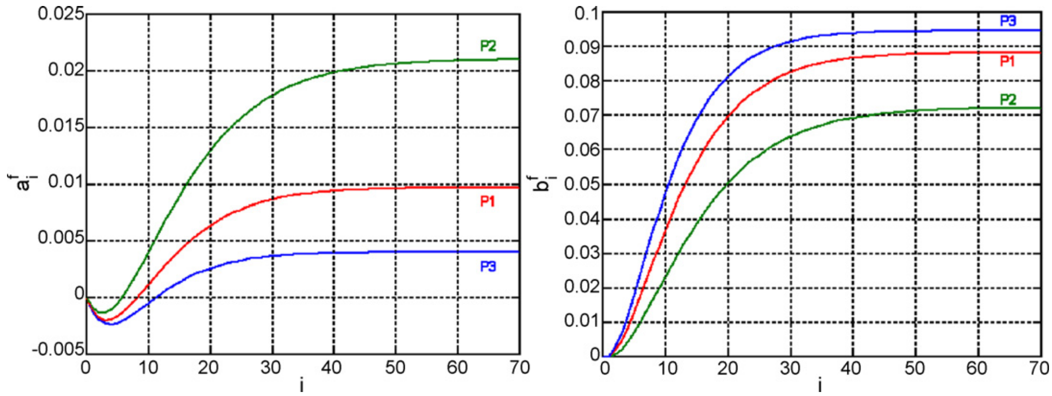

Fig. 4. Normalized step responses of the control plant to changes of the manipulated variable (left) and to changes of the disturbance variable (right); a_i^f, b_i^f – coefficients of the step responses to the changes of manipulated and disturbance variable, respectively; i – index of the coefficients of the step response; f – index of the local models.

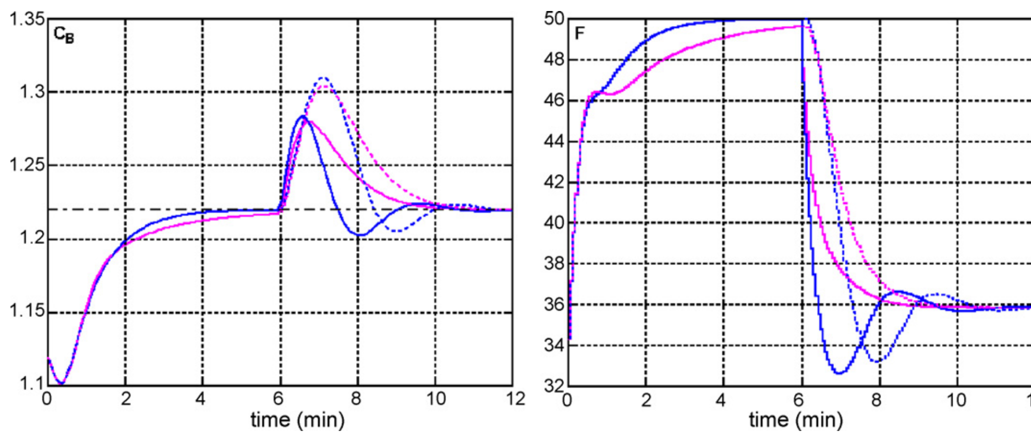

Fig. 5. Responses of the control system with: DMC and FDMC algorithms to the change of C_B set-point to $\bar{C}_B = 1.22$ and the change of the disturbance C_{Af} by 1 mol/l at the 6th minute; disturbance measurement: used – solid lines, not used – dotted lines; left – output variable C_B; right – manipulated variable F.

values of the step response coefficients are listed in the Appendix A.

3.2. Results of the experiments

During the experiments the operation of both control systems (the first one—with the DMC and the second one—with the FDMC algorithms) was compared. First, the set-point value $\bar{C}_B$ was changed towards high values of the concentration C_B (to 1.22 mol/l). Moreover, at the 6th minute the disturbance C_{Af} was changed from 10 mol/l to 11 mol/l. The response obtained in the control system with the FDMC algorithm (blue lines in Fig. 5) is better than the one acquired by the standard DMC algorithm (magenta lines in Fig. 5) – the output variable much faster reaches the demanded set-point value. The response to the change of the C_{Af} disturbance is much better when the measurement of the disturbance is used (solid lines

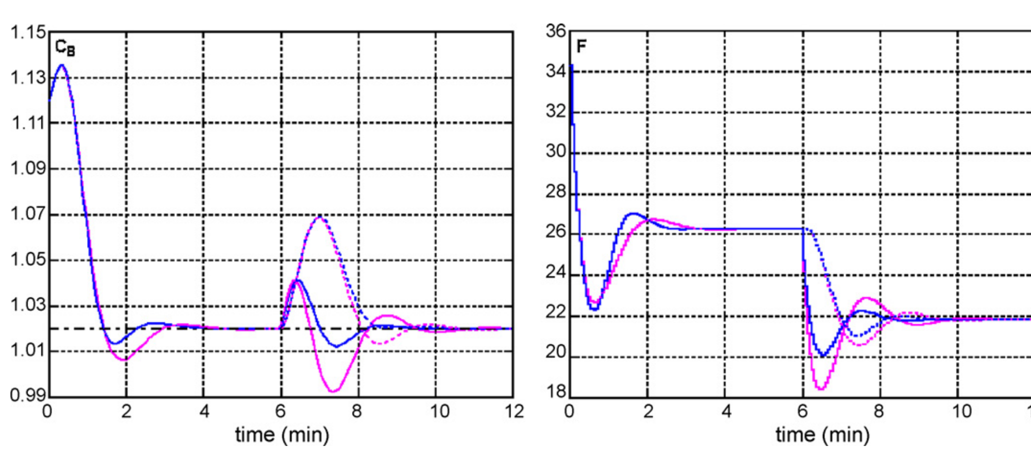

Fig. 6. Responses of the control system with: DMC and FDMC algorithms to the change of C_B set-point to $\bar{C}_B = 1.02$ and the change of the disturbance C_{Af} by 1 mol/l at the 6th minute; disturbance measurement: used – solid lines; not used – dotted lines; left – output variable C_B; right – manipulated variable F.

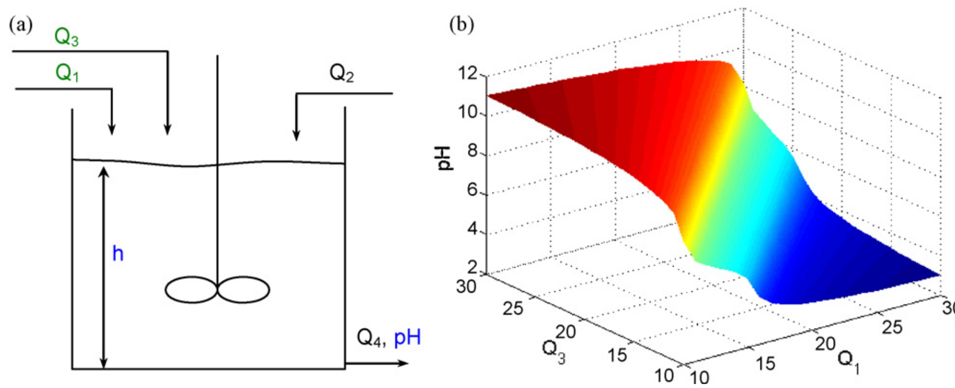

Fig. 7. pH reactor; (a) diagram of the plant; (b) steady-state characteristic pH (Q_1, Q_3); manipulated variables: Q1, Q3; controlled variables: h, pH.

in Fig. 5). If the measurement is not used then the overshoot is significantly bigger and the control time is longer (dotted lines in Fig. 5).

It can be noticed that both responses at the beginning of control action (for C_B values close to the 1.12 mol/l) are practically the same. It is caused by the fact that the responses, the standard DMC controller is based on, were used as a local model in one of the rules of the TS model exploited by the FDMC controller. This fact illustrates well that the FDMC controller can be designed by simply extending the existing DMC controller (usually well tuned to work near the operating point it was designed for) if needed.

In the second experiment, the set-point value $\bar{C}_B$ was changed towards low values of the concentration C_B (to 1.02 mol/l). Moreover, as in the previous experiment, at the 6th minute the disturbance C_{Af} was increased by 1 mol/l. The response obtained in the control system with the FDMC algorithm (blue lines in Fig. 6) is again better than the one generated by the control system with the standard DMC algorithm (magenta lines in Fig. 6)—the output variable also reaches the set-point value faster in the case of the control system with the fuzzy algorithm. Moreover, the overshoot is significantly lower in the control system with the FDMC algorithm.

The response to the change of disturbance C_{Af} is also better in the system with the FDMC controller comparing to the case when the DMC controller was used. In both cases – when the disturbance measurement is used (solid lines) and when it is not used (dotted lines) – the disturbance influence is compensated faster in the control system with the fuzzy controller. The obtained results clearly show the profits gained thanks to the usage of the fuzzy approach. The best response was obtained in the control system

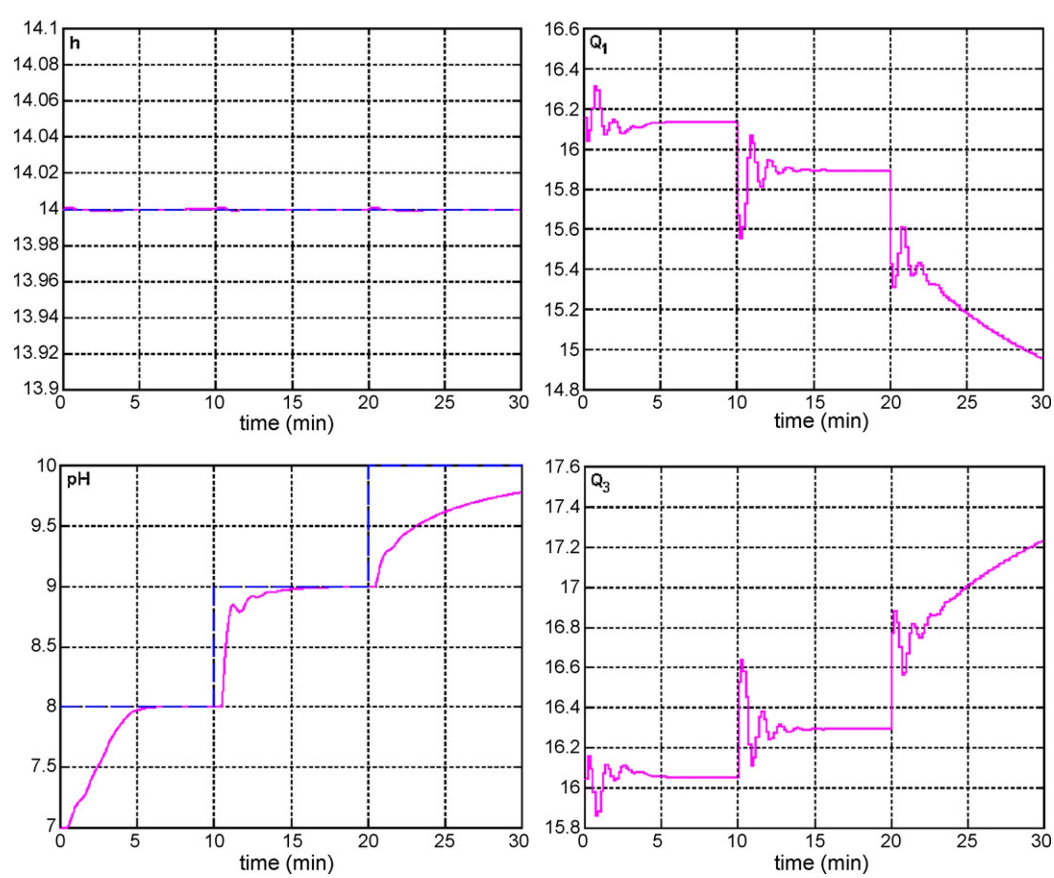

Fig. 8. Responses of the control system of the pH reactor with the DMC predictive control algorithm to the sequence of changes of pH set-points from 7 to 8, 9 and 10; dashed lines – set-points; left – output variables h and pH; right – manipulated variables Q_1 and Q_3.

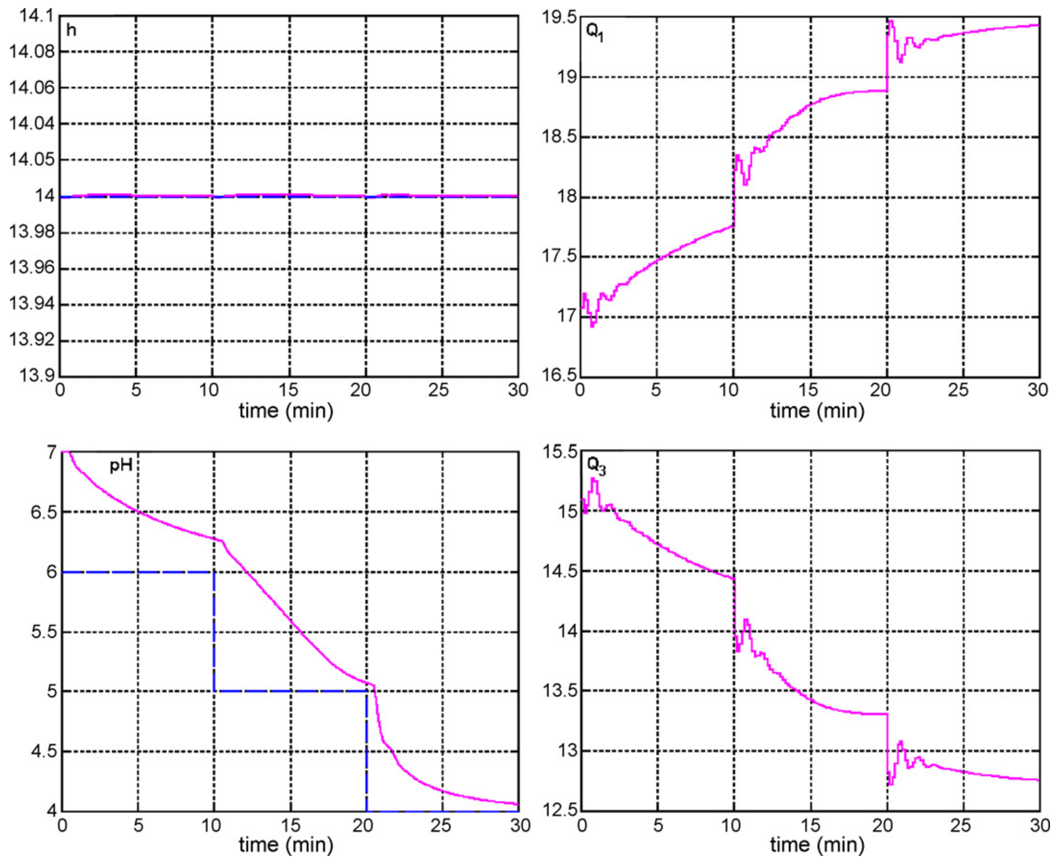

Fig. 9. Responses of the control system of the pH reactor with the DMC predictive control algorithm to the sequence of changes of pH set-points from 7 to 6, 5 and 4; dashed lines – set-points; left – output variables h and pH, right – manipulated variables Q_1 and Q_3.

with the FDMC controller and the disturbance measurement (solid blue lines in Fig. 6)—the settling time and the overshoot are much smaller than in the case without disturbance measurement (dotted blue lines in Fig. 6).

4. Control system of a pH reactor

4.1. Control plant

The second of the considered control plants is a pH reactor (Fig. 7a) it is often used for benchmarks; see e.g. [6,8,13]. In the paper the configuration of manipulated variables and controlled variables from [17] is assumed. The control plant is highly nonlinear which is illustrated well by the steady-state characteristic pH(Q_1, Q_3) shown in Fig. 7b.

The process model is given by a set of the following equations [6,8,13,17]:

$$\frac{dh}{dt} = \frac{Q_1 + Q_2 + Q_3 - Q_4}{A}, \quad Q_4 = C_V\sqrt{h}, \tag{32}$$

$$\frac{dW_{a4}}{dt} = \frac{(W_{a1} - W_{a4}) \cdot Q_1 + (W_{a2} - W_{a4}) \cdot Q_2 + (W_{a3} - W_{a4}) \cdot Q_3}{A \cdot h}, \tag{33}$$

$$\frac{dW_{b4}}{dt} = \frac{(W_{b1} - W_{b4}) \cdot Q_1 + (W_{b2} - W_{b4}) \cdot Q_2 + (W_{b3} - W_{b4}) \cdot Q_3}{A \cdot h}, \tag{34}$$

$$W_{a4} + 10^{\mathrm{pH}-14} + W_{b4}\frac{1 + 2 \cdot 10^{\mathrm{pH}-\mathrm{p}K_2}}{1 + 10^{\mathrm{p}K_1-\mathrm{pH}} + 10^{\mathrm{p}K_2-\mathrm{pH}}} - 10^{-\mathrm{pH}} = 0, \tag{35}$$

where Q_1 is the flow rate of acid (HNO_3), Q_2 is the flow rate of buffer ($NaHCO_3$), Q_3 is a flow rate of base (NaOH), Q_4 is the flow of the product (it is a gravitational outflow), h is the level of the liquid in the reactor, pH indicates the composition of the product. The values of parameters in the pH reactor model are following:

$$\begin{aligned}&W_{a1} = 3 \times 10^{-3}\,\mathrm{M},\ W_{a2} = -3 \times 10^{-2}\,\mathrm{M},\ W_{a3} = -3.05 \times 10^{-3}\,\mathrm{M},\\ &W_{b1} = 0\,\mathrm{M},\ W_{b2} = 3 \times 10^{-2}\,\mathrm{M},\ W_{b3} = 5 \times 10^{-5}\,\mathrm{M},\\ &A = 207\,\mathrm{cm}^2,\ C_V = 8.75\,\mathrm{ml/(cm\,s)},\ \mathrm{p}K_1 = 6.35,\ \mathrm{p}K_2 = 10.25.\end{aligned} \tag{36}$$

Moreover, there is a delay of pH measurement, which is equal to T_d = 30 s. Thus, the control task is difficult because of the delay and high nonlinearity of the control plant.

4.2. Results of the experiments

First, for the presented control plant, a DMC algorithm was designed. The output variables are: the liquid level h and pH value

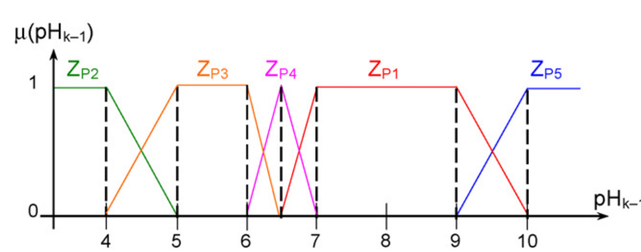

Fig. 10. Membership functions of the model used by the FDMC controller activating step responses obtained in environs of operating points P1, P2, P3, P4 and P5 (listed in the text).

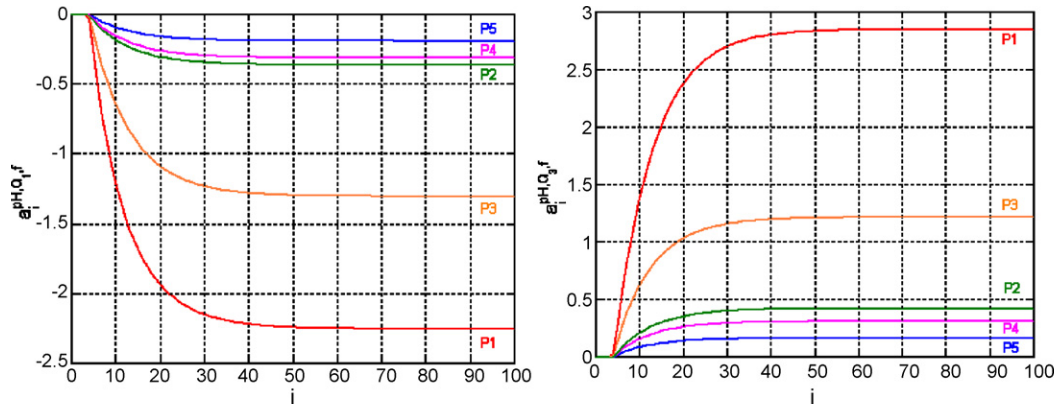

Fig. 11. Normalized step responses of pH to changes of the manipulated variable Q_1 (left) and to changes of the manipulated variable Q_3 (right); $a_i^{pH,Q_1,f}$, $a_i^{pH,Q_3,f}$ – coefficients of the step responses, i – index of the coefficients of the step responses; f – index of the local models.

(pH), the manipulated variables are: acid flow rate Q_1 and base flow rate Q_3. The sampling period was assumed equal to $T_s = 10$ s. The values of the tuning parameters were assumed as follows: $p_d = p = 100$, $s = 50$, $\lambda_1 = \lambda_2 = 1$, $\kappa_1 = \kappa_2 = 1$. There is no p_z parameter because the disturbance measurement is not used. There are two λ_i and two κ_i parameters (more than in the previous example) because the control plant has two inputs and two outputs.

The DMC algorithm is based on a set of step responses obtained in environs of the operating point:

$$P1: Q_{10} = 16.14\,\text{ml/s},\;\; Q_{30} = 16.05\,\text{ml/s},\;\; h_0 = 14\,\text{cm},\;\; \text{pH}_0 = 8.$$

It is assumed that the liquid level should be stabilized on the fixed value $h_0 = 14$ cm. It is a reasonable assumption because if maximum production rate should be obtained then all available volume of the reactor should be used (see [17]).

During the experiments operation in a wide range of pH values, as in [8], was considered. The values of process variables at the beginning of experiment are as follows:

$$Q_{10} = 16.6\,\text{ml/s},\;\; Q_{20} = 0.55\,\text{ml/s},\;\; Q_{30} = 15.6\,\text{ml/s},$$
$$h_0 = 14\,\text{cm},\;\; \text{pH}_0 = 7.$$

The experiments performed in the control system with DMC algorithm clearly demonstrate nonlinear character of the plant. The DMC algorithm must be tuned to work in acceptable way for all range of set-point values. Therefore, it operates well for values between 7 and 9, but very slowly for other pH set-points (10, 6, 5,

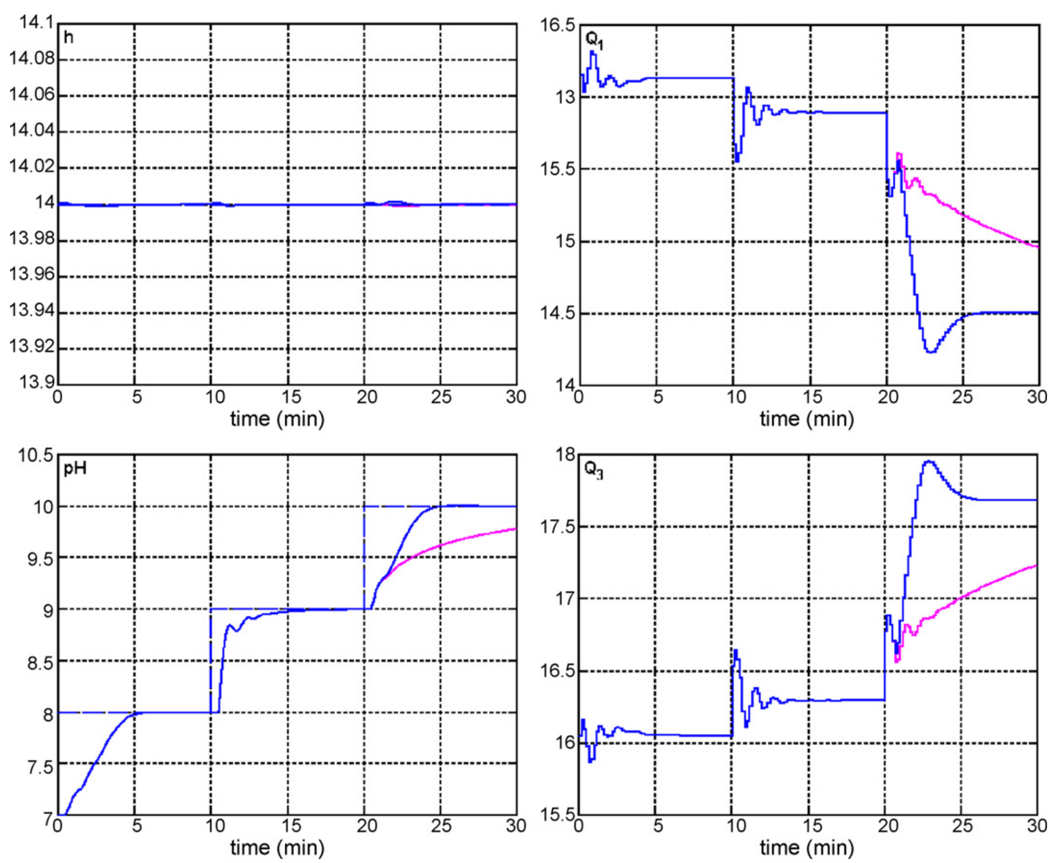

Fig. 12. Responses of the control system of the pH reactor with: DMC and FDMC predictive control algorithms to the sequence of changes of pH set-points from 7 to 8, 9 and 10; dashed lines – set-points; left – output variables h and pH; right – manipulated variables Q_1 and Q_3.

4) (Figs. 8 and 9). It is good to notice that the level h of the liquid in the reactor is stabilized very well. The controller changes the flow rates Q_1 and Q_3 in such a way that influence of these changes on the liquid level h is very low.

In order to improve operation of the control system, the DMC algorithm was extended using the fuzzy approach described in the article. The resulting FDMC controller has the same parameters as the DMC algorithm but is based on a few sets of step responses. Except the one used by the DMC algorithm, additional four sets were obtained from environs of the following points:

$$P2: Q_{10} = 19.48\,\text{ml/s},\;\; Q_{30} = 12.71\,\text{ml/s},\;\; h_0 = 14\,\text{cm},\;\; \text{pH}_0 = 4,$$

$$P3: Q_{10} = 18.89\,\text{ml/s},\;\; Q_{30} = 13.30\,\text{ml/s},\;\; h_0 = 14\text{cm},\;\; \text{pH}_0 = 5,$$

$$P4: Q_{10} = 17.29\,\text{ml/s},\;\; Q_{30} = 14.90\,\text{ml/s},\;\; h_0 = 14\,\text{cm},\;\; \text{pH}_0 = 6.5,$$

$$P5: Q_{10} = 14.51\,\text{ml/s},\;\; Q_{30} = 17.68\,\text{ml/s},\;\; h_0 = 14\,\text{cm},\;\; \text{pH}_0 = 10.$$

The TS control plant model used by the FDMC controller is, thus, composed of five rules; the assumed membership functions are shown in Fig. 10.

The normalized step responses of *pH* to changes of manipulated variables Q_1 and Q_3 are shown in Fig. 11. The numerical values of this responses and the response of liquid level h to the changes of both manipulated variables are listed in Appendix B. Only one set of data is given there for the liquid level h because due to the assumption on level stabilization on the fixed value h_0 = 14 cm and properties of the process, the same responses are obtained for both manipulated variables and all operating points (all local models).

The responses obtained in the control system with the FDMC algorithm are shown in Figs. 12 and 13. The operation of the control system for set points between 7 and 9 is the same as in the case of the control system with the DMC algorithm. It is natural, because in this range of pH values the same set of the step responses is active as the one that is used by the DMC algorithm.

In the case of other pH set-point values, the FDMC algorithm offers much better control performance. The pH output variable reaches the set-point values 10, 6, 5 and 4 much faster in the control system with the fuzzy algorithm. Moreover, in the case of all set-point values, they are achieved without the overshoot. The obtained responses are similar to each other in the case of set-point changes to 10, 8, 5 and 4. Thus, the effect of the control plant nonlinearities on control quality is reduced too a large extent by the application of the fuzzy controller.

It is also good to notice that the liquid level in the reactor h, as in the control system with the DMC algorithm, changes only a little in the control system with the FDMC algorithm. Moreover despite relatively long horizons were used in the examples, in both cases calculation of values of the manipulated variables took fractions of a second thanks to the numerically efficient formulation of the FDMC algorithm.

5. Summary

The Fuzzy DMC (FDMC) algorithm and its advantages are discussed in the paper. The formulation of the FDMC algorithm is presented for Multiple Input Multiple Output (MIMO) control plants. The mechanism of taking measured disturbances into consideration in the algorithm is also discussed. The FDMC algorithm under consideration is based on Takagi–Sugeno fuzzy models with step responses used as local models. Therefore a simple method of algorithm synthesis, described in the paper, can be used.

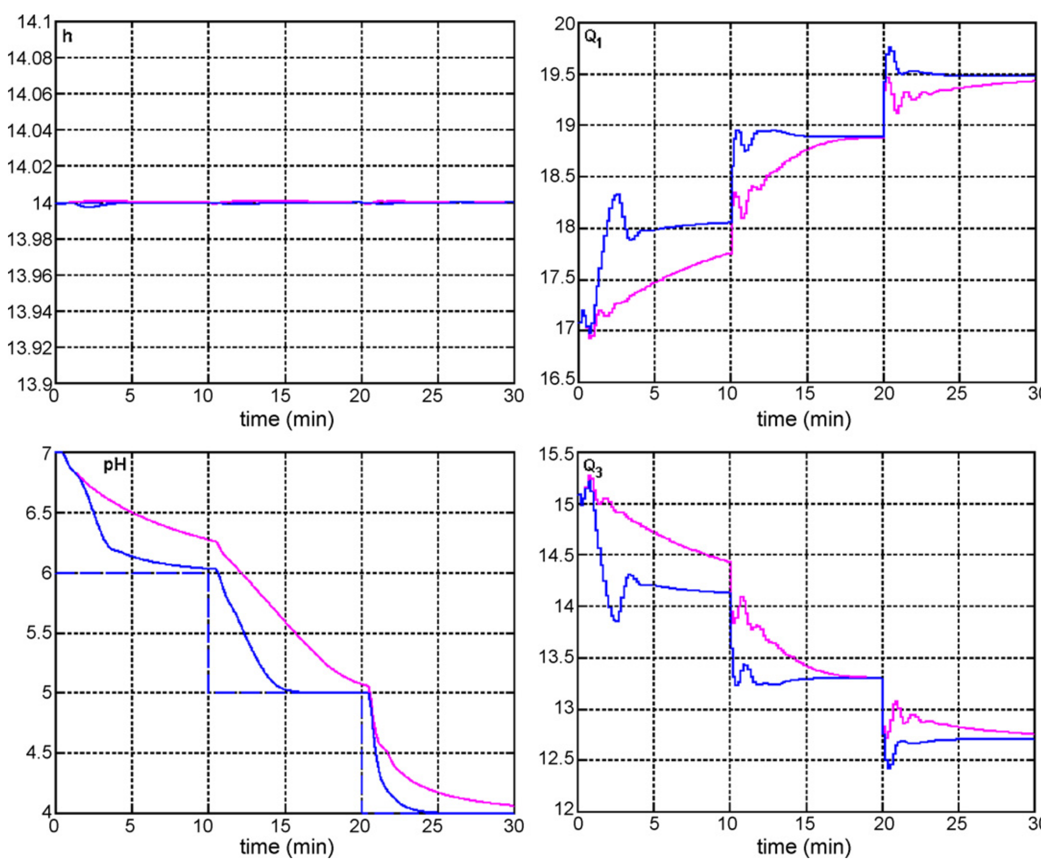

Fig. 13. Responses of the control system of the pH reactor with: DMC and FDMC predictive control algorithms to the sequence of changes of pH set-points from 7 to 6, 5 and 4; dashed lines – set-points; left – output variables h and pH; right – manipulated variables Q_1 and Q_3.

The DMC algorithm (based on only one set of step responses) is very common in practical applications. The presented method of design of the FDMC algorithm can be applied to extend controllers working in existing control systems in a fast and easy way. It is sufficient to obtain a few additional sets of step responses from environs of a few operating points, and using these responses synthesize the FDMC algorithm being in fact the extension of the DMC algorithm.

Application of the FDMC algorithm to both example nonlinear control plants with difficult dynamics (inverse response in the case of the first one and time delay in the case of the second one) brought improvement of the control systems operation comparing to the case when the standard DMC algorithm (based on the linear model – only one set of step responses) was used. The improvement of control system operation was achieved despite relatively little effort needed to design the fuzzy algorithms. In the case of the controllers for both control plants, the parameters of the FDMC algorithm were chosen exactly the same as in the case of the DMC algorithm. Despite that the control systems with the FDMC controllers offer better control performance and the further the original operating point the system operates the bigger difference of operation between DMC and FDMC algorithm is. It should be also stressed, that in the case of the FDMC algorithm it is possible to use different values of the tuning parameters for different rules of the control plant model. Thus, it is possible to tune the fuzzy algorithm even better, however, an additional effort and time is needed to do it.

Appendix A

Table A.1 Step response coefficients obtained for the chemical reactor described in Section 3.

i	a_i^{P1}	a_i^{P2}	a_i^{P3}	b_i^{P1}	b_i^{P2}	b_i^{P3}
1	−1.1180e−003	−9.0788e−004	−1.2201e−003	−8.8000e−007	−2.9100e−004	2.4400e−005
2	−1.7356e−003	−1.3157e−003	−1.9400e−003	1.7136e−003	4.9085e−004	2.5400e−003
3	−1.9706e−003	−1.3279e−003	−2.2899e−003	4.6654e−003	2.0985e−003	6.7399e−003
4	−1.9177e−003	−1.0314e−003	−2.3706e−003	8.4765e−003	4.3276e−003	1.2011e−002
5	−1.6526e−003	−4.9835e−004	−2.2601e−003	1.2850e−002	7.0101e−003	1.7896e−002
6	−1.2354e−003	2.1202e−004	−2.0182e−003	1.7555e−002	1.0009e−002	2.4054e−002
7	−7.1387e−004	1.0509e−003	−1.6902e−003	2.2415e−002	1.3212e−002	3.0240e−002
8	−1.2506e−004	1.9786e−003	−1.3102e−003	2.7294e−002	1.6530e−002	3.6283e−002
9	5.0211e−004	2.9628e−003	−9.0369e−004	3.2093e−002	1.9891e−002	4.2064e−002
10	1.1455e−003	3.9777e−003	−4.8905e−004	3.6740e−002	2.3240e−002	4.7509e−002
11	1.7884e−003	5.0026e−003	−7.9498e−005	4.1184e−002	2.6531e−002	5.2573e−002
12	2.4185e−003	6.0214e−003	3.1581e−004	4.5391e−002	2.9732e−002	5.7236e−002
13	3.0268e−003	7.0214e−003	6.9083e−004	4.9340e−002	3.2818e−002	6.1495e−002
14	3.6072e−003	7.9931e−003	1.0418e−003	5.3023e−002	3.5772e−002	6.5356e−002
15	4.1557e−003	8.9295e−003	1.3668e−003	5.6436e−002	3.8582e−002	6.8838e−002
16	4.6698e−003	9.8255e−003	1.6651e−003	5.9584e−002	4.1240e−002	7.1961e−002
17	5.1486e−003	1.0678e−002	1.9369e−003	6.2473e−002	4.3743e−002	7.4750e−002
18	5.5919e−003	1.1485e−002	2.1829e−003	6.5116e−002	4.6090e−002	7.7230e−002
19	6.0002e−003	1.2244e−002	2.4044e−003	6.7524e−002	4.8284e−002	7.9430e−002
20	6.3748e−003	1.2958e−002	2.6030e−003	6.9711e−002	5.0328e−002	8.1374e−002
21	6.7171e−003	1.3624e−002	2.7803e−003	7.1693e−002	5.2226e−002	8.3087e−002
22	7.0289e−003	1.4246e−002	2.9381e−003	7.3485e−002	5.3984e−002	8.4594e−002
23	7.3119e−003	1.4824e−002	3.0779e−003	7.5100e−002	5.5609e−002	8.5916e−002
24	7.5683e−003	1.5360e−002	3.2016e−003	7.6553e−002	5.7107e−002	8.7073e−002
25	7.7998e−003	1.5856e−002	3.3107e−003	7.7858e−002	5.8486e−002	8.8085e−002
26	8.0085e−003	1.6314e−002	3.4067e−003	7.9029e−002	5.9754e−002	8.8968e−002
27	8.1962e−003	1.6736e−002	3.4910e−003	8.0076e−002	6.0916e−002	8.9737e−002
28	8.3648e−003	1.7125e−002	3.5649e−003	8.1013e−002	6.1981e−002	9.0406e−002
29	8.5159e−003	1.7481e−002	3.6296e−003	8.1849e−002	6.2954e−002	9.0987e−002
30	8.6511e−003	1.7808e−002	3.6860e−003	8.2594e−002	6.3844e−002	9.1492e−002
31	8.7719e−003	1.8108e−002	3.7353e−003	8.3257e−002	6.4655e−002	9.1929e−002
32	8.8797e−003	1.8382e−002	3.7783e−003	8.3848e−002	6.5394e−002	9.2308e−002
33	8.9759e−003	1.8632e−002	3.8156e−003	8.4372e−002	6.6067e−002	9.2636e−002
34	9.0614e−003	1.8861e−002	3.8481e−003	8.4838e−002	6.6679e−002	9.2919e−002
35	9.1375e−003	1.9069e−002	3.8763e−003	8.5251e−002	6.7234e−002	9.3164e−002
36	9.2051e−003	1.9259e−002	3.9007e−003	8.5617e−002	6.7739e−002	9.3375e−002
37	9.2652e−003	1.9432e−002	3.9219e−003	8.5941e−002	6.8196e−002	9.3557e−002
38	9.3184e−003	1.9589e−002	3.9402e−003	8.6227e−002	6.8611e−002	9.3714e−002
39	9.3655e−003	1.9731e−002	3.9561e−003	8.6481e−002	6.8986e−002	9.3849e−002
40	9.4073e−003	1.9861e−002	3.9698e−003	8.6705e−002	6.9326e−002	9.3966e−002
41	9.4442e−003	1.9978e−002	3.9816e−003	8.6902e−002	6.9633e−002	9.4066e−002
42	9.4769e−003	2.0084e−002	3.9918e−003	8.7077e−002	6.9911e−002	9.4152e−002
43	9.5057e−003	2.0181e−002	4.0006e−003	8.7230e−002	7.0161e−002	9.4226e−002
44	9.5312e−003	2.0268e−002	4.0082e−003	8.7366e−002	7.0387e−002	9.4289e−002
45	9.5536e−003	2.0347e−002	4.0148e−003	8.7486e−002	7.0592e−002	9.4344e−002
46	9.5734e−003	2.0418e−002	4.0204e−003	8.7591e−002	7.0776e−002	9.4391e−002
47	9.5909e−003	2.0483e−002	4.0253e−003	8.7683e−002	7.0941e−002	9.4431e−002
48	9.6063e−003	2.0541e−002	4.0294e−003	8.7765e−002	7.1091e−002	9.4465e−002
49	9.6198e−003	2.0594e−002	4.0330e−003	8.7836e−002	7.1225e−002	9.4495e−002
50	9.6318e−003	2.0641e−002	4.0361e−003	8.7899e−002	7.1346e−002	9.4520e−002
51	9.6422e−003	2.0684e−002	4.0387e−003	8.7954e−002	7.1455e−002	9.4542e−002
52	9.6515e−003	2.0723e−002	4.0410e−003	8.8003e−002	7.1553e−002	9.4560e−002
53	9.6596e−003	2.0758e−002	4.0430e−003	8.8046e−002	7.1641e−002	9.4576e−002

Appendix A (*Continued*)

i	a_i^{P1}	a_i^{P2}	a_i^{P3}	b_i^{P1}	b_i^{P2}	b_i^{P3}
54	9.6667e−003	2.0789e−002	4.0446e−003	8.8083e−002	7.1720e−002	9.4590e−002
55	9.6729e−003	2.0817e−002	4.0461e−003	8.8116e−002	7.1791e−002	9.4601e−002
56	9.6784e−003	2.0843e−002	4.0473e−003	8.8145e−002	7.1854e−002	9.4611e−002
57	9.6832e−003	2.0866e−002	4.0484e−003	8.8170e−002	7.1912e−002	9.4620e−002
58	9.6874e−003	2.0886e−002	4.0493e−003	8.8192e−002	7.1963e−002	9.4627e−002
59	9.6912e−003	2.0905e−002	4.0501e−003	8.8211e−002	7.2009e−002	9.4633e−002
60	9.6944e−003	2.0921e−002	4.0507e−003	8.8228e−002	7.2050e−002	9.4639e−002
61	9.6972e−003	2.0936e−002	4.0513e−003	8.8243e−002	7.2087e−002	9.4643e−002
62	9.6997e−003	2.0950e−002	4.0518e−003	8.8256e−002	7.2120e−002	9.4647e−002
63	9.7019e−003	2.0962e−002	4.0522e−003	8.8268e−002	7.2150e−002	9.4650e−002
64	9.7038e−003	2.0973e−002	4.0525e−003	8.8278e−002	7.2177e−002	9.4653e−002
65	9.7055e−003	2.0982e−002	4.0528e−003	8.8286e−002	7.2201e−002	9.4656e−002
66	9.7070e−003	2.0991e−002	4.0531e−003	8.8294e−002	7.2222e−002	9.4658e−002
67	9.7083e−003	2.0999e−002	4.0533e−003	8.8301e−002	7.2241e−002	9.4659e−002
68	9.7094e−003	2.1006e−002	4.0535e−003	8.8306e−002	7.2258e−002	9.4661e−002
69	9.7104e−003	2.1012e−002	4.0537e−003	8.8311e−002	7.2273e−002	9.4662e−002
70	9.7112e−003	2.1018e−002	4.0538e−003	8.8316e−002	7.2287e−002	9.4663e−002

a_i^f, b_i^f – coefficients of the step responses to the changes of manipulated and disturbance variable, respectively; i – index of the coefficients of the step responses; f – index of the local models.

Appendix B

Table B.1 Step response coefficients obtained for the pH reactor described in Section 4; part 1/2.

i	$a_i^{\mathrm{pH},Q_1,P1}$	$a_i^{\mathrm{pH},Q_1,P2}$	$a_i^{\mathrm{pH},Q_1,P3}$	$a_i^{\mathrm{pH},Q_1,P4}$	$a_i^{\mathrm{pH},Q_1,P5}$	a_i^h
1	0	0	0	0	0	4.8309e−003
2	0	0	0	0	0	5.1664e−002
3	0	0	0	0	0	9.5919e−002
4	−4.4664e−003	−1.4088e−002	−3.5127e−003	−2.9426e−002	−2.0732e−003	1.3774e−001
5	−4.4142e−002	−1.4866e−001	−3.6430e−002	−3.0151e−001	−2.1735e−002	1.7725e−001
6	−7.9244e−002	−2.6974e−001	−6.5775e−002	−5.3739e−001	−3.9325e−002	2.1460e−001
7	−1.1033e−001	−3.7854e−001	−9.1940e−002	−7.4231e−001	−5.5058e−002	2.4988e−001
8	−1.3789e−001	−4.7619e−001	−1.1527e−001	−9.2076e−001	−6.9127e−002	2.8323e−001
9	−1.6233e−001	−5.6375e−001	−1.3608e−001	−1.0765e+000	−8.1705e−002	3.1474e−001
10	−1.8403e−001	−6.4219e−001	−1.5464e−001	−1.2127e+000	−9.2949e−002	3.4451e−001
11	−2.0330e−001	−7.1243e−001	−1.7120e−001	−1.3321e+000	−1.0300e−001	3.7265e−001
12	−2.2043e−001	−7.7528e−001	−1.8596e−001	−1.4370e+000	−1.1198e−001	3.9924e−001
13	−2.3566e−001	−8.3150e−001	−1.9914e−001	−1.5292e+000	−1.2000e−001	4.2437e−001
14	−2.4921e−001	−8.8177e−001	−2.1090e−001	−1.6105e+000	−1.2717e−001	4.4812e−001
15	−2.6126e−001	−9.2670e−001	−2.2138e−001	−1.6822e+000	−1.3357e−001	4.7056e−001
16	−2.7199e−001	−9.6686e−001	−2.3074e−001	−1.7456e+000	−1.3929e−001	4.9177e−001
17	−2.8155e−001	−1.0027e+000	−2.3909e−001	−1.8016e+000	−1.4440e−001	5.1181e−001
18	−2.9006e−001	−1.0348e+000	−2.4655e−001	−1.8512e+000	−1.4897e−001	5.3075e−001
19	−2.9764e−001	−1.0634e+000	−2.5320e−001	−1.8951e+000	−1.5304e−001	5.4865e−001
20	−3.0440e−001	−1.0890e+000	−2.5913e−001	−1.9341e+000	−1.5668e−001	5.6556e−001
21	−3.1042e−001	−1.1118e+000	−2.6443e−001	−1.9687e+000	−1.5993e−001	5.8155e−001
22	−3.1579e−001	−1.1322e+000	−2.6915e−001	−1.9994e+000	−1.6284e−001	5.9666e−001
23	−3.2057e−001	−1.1505e+000	−2.7337e−001	−2.0267e+000	−1.6543e−001	6.1093e−001
24	−3.2484e−001	−1.1667e+000	−2.7714e−001	−2.0510e+000	−1.6774e−001	6.2442e−001
25	−3.2865e−001	−1.1812e+000	−2.8050e−001	−2.0726e+000	−1.6981e−001	6.3717e−001
26	−3.3204e−001	−1.1942e+000	−2.8350e−001	−2.0918e+000	−1.7165e−001	6.4922e−001
27	−3.3507e−001	−1.2058e+000	−2.8618e−001	−2.1088e+000	−1.7330e−001	6.6061e−001
28	−3.3777e−001	−1.2161e+000	−2.8857e−001	−2.1241e+000	−1.7477e−001	6.7137e−001
29	−3.4018e−001	−1.2254e+000	−2.9070e−001	−2.1376e+000	−1.7609e−001	6.8154e−001
30	−3.4233e−001	−1.2336e+000	−2.9261e−001	−2.1497e+000	−1.7726e−001	6.9115e−001
31	−3.4425e−001	−1.2409e+000	−2.9431e−001	−2.1605e+000	−1.7831e−001	7.0024e−001
32	−3.4596e−001	−1.2475e+000	−2.9583e−001	−2.1700e+000	−1.7924e−001	7.0882e−001
33	−3.4749e−001	−1.2534e+000	−2.9718e−001	−2.1786e+000	−1.8008e−001	7.1694e−001
34	−3.4886e−001	−1.2586e+000	−2.9839e−001	−2.1862e+000	−1.8082e−001	7.2460e−001
35	−3.5007e−001	−1.2633e+000	−2.9947e−001	−2.1930e+000	−1.8149e−001	7.3185e−001
36	−3.5116e−001	−1.2675e+000	−3.0044e−001	−2.1991e+000	−1.8208e−001	7.3870e−001
37	−3.5213e−001	−1.2712e+000	−3.0130e−001	−2.2045e+000	−1.8261e−001	7.4517e−001
38	−3.5299e−001	−1.2745e+000	−3.0207e−001	−2.2093e+000	−1.8309e−001	7.5129e−001
39	−3.5377e−001	−1.2775e+000	−3.0275e−001	−2.2136e+000	−1.8351e−001	7.5707e−001
40	−3.5446e−001	−1.2801e+000	−3.0336e−001	−2.2174e+000	−1.8389e−001	7.6253e−001
41	−3.5507e−001	−1.2825e+000	−3.0391e−001	−2.2209e+000	−1.8422e−001	7.6769e−001
42	−3.5562e−001	−1.2846e+000	−3.0440e−001	−2.2239e+000	−1.8452e−001	7.7257e−001
43	−3.5611e−001	−1.2865e+000	−3.0483e−001	−2.2266e+000	−1.8479e−001	7.7718e−001
44	−3.5655e−001	−1.2882e+000	−3.0522e−001	−2.2291e+000	−1.8503e−001	7.8154e−001
45	−3.5694e−001	−1.2897e+000	−3.0557e−001	−2.2313e+000	−1.8525e−001	7.8566e−001
46	−3.5729e−001	−1.2911e+000	−3.0588e−001	−2.2332e+000	−1.8544e−001	7.8955e−001
47	−3.5760e−001	−1.2922e+000	−3.0616e−001	−2.2349e+000	−1.8561e−001	7.9323e−001
48	−3.5788e−001	−1.2933e+000	−3.0641e−001	−2.2365e+000	−1.8576e−001	7.9671e−001

Appendix B (*Continued*)

i	$a_i^{pH,Q_1,P1}$	$a_i^{pH,Q_1,P2}$	$a_i^{pH,Q_1,P3}$	$a_i^{pH,Q_1,P4}$	$a_i^{pH,Q_1,P5}$	a_i^h
49	−3.5813e−001	−1.2943e+000	−3.0663e−001	−2.2378e+000	−1.8590e−001	7.9999e−001
50	−3.5835e−001	−1.2951e+000	−3.0682e−001	−2.2391e+000	−1.8602e−001	8.0310e−001
51	−3.5855e−001	−1.2959e+000	−3.0700e−001	−2.2402e+000	−1.8613e−001	8.0603e−001
52	−3.5872e−001	−1.2966e+000	−3.0716e−001	−2.2412e+000	−1.8623e−001	8.0881e−001
53	−3.5888e−001	−1.2972e+000	−3.0730e−001	−2.2420e+000	−1.8631e−001	8.1143e−001
54	−3.5902e−001	−1.2977e+000	−3.0742e−001	−2.2428e+000	−1.8639e−001	8.1391e−001
55	−3.5915e−001	−1.2982e+000	−3.0753e−001	−2.2435e+000	−1.8646e−001	8.1625e−001
56	−3.5926e−001	−1.2986e+000	−3.0763e−001	−2.2441e+000	−1.8652e−001	8.1846e−001
57	−3.5936e−001	−1.2990e+000	−3.0772e−001	−2.2447e+000	−1.8657e−001	8.2055e−001
58	−3.5945e−001	−1.2994e+000	−3.0780e−001	−2.2452e+000	−1.8662e−001	8.2253e−001
59	−3.5953e−001	−1.2997e+000	−3.0787e−001	−2.2456e+000	−1.8667e−001	8.2440e−001
60	−3.5960e−001	−1.3000e+000	−3.0794e−001	−2.2460e+000	−1.8671e−001	8.2616e−001
61	−3.5966e−001	−1.3002e+000	−3.0799e−001	−2.2464e+000	−1.8674e−001	8.2783e−001
62	−3.5972e−001	−1.3004e+000	−3.0804e−001	−2.2467e+000	−1.8677e−001	8.2941e−001
63	−3.5977e−001	−1.3006e+000	−3.0809e−001	−2.2470e+000	−1.8680e−001	8.3090e−001
64	−3.5982e−001	−1.3008e+000	−3.0813e−001	−2.2472e+000	−1.8683e−001	8.3230e−001
65	−3.5986e−001	−1.3009e+000	−3.0817e−001	−2.2475e+000	−1.8685e−001	8.3363e−001
66	−3.5989e−001	−1.3011e+000	−3.0820e−001	−2.2477e+000	−1.8687e−001	8.3489e−001
67	−3.5993e−001	−1.3012e+000	−3.0823e−001	−2.2478e+000	−1.8689e−001	8.3608e−001
68	−3.5996e−001	−1.3013e+000	−3.0825e−001	−2.2480e+000	−1.8690e−001	8.3720e−001
69	−3.5998e−001	−1.3014e+000	−3.0828e−001	−2.2481e+000	−1.8691e−001	8.3827e−001
70	−3.6000e−001	−1.3015e+000	−3.0830e−001	−2.2483e+000	−1.8693e−001	8.3927e−001
71	−3.6002e−001	−1.3016e+000	−3.0831e−001	−2.2484e+000	−1.8694e−001	8.4022e−001
72	−3.6004e−001	−1.3017e+000	−3.0833e−001	−2.2485e+000	−1.8695e−001	8.4111e−001
73	−3.6006e−001	−1.3017e+000	−3.0834e−001	−2.2486e+000	−1.8696e−001	8.4196e−001
74	−3.6007e−001	−1.3018e+000	−3.0836e−001	−2.2486e+000	−1.8697e−001	8.4276e−001
75	−3.6009e−001	−1.3018e+000	−3.0837e−001	−2.2487e+000	−1.8697e−001	8.4352e−001
76	−3.6010e−001	−1.3019e+000	−3.0838e−001	−2.2488e+000	−1.8698e−001	8.4423e−001
77	−3.6011e−001	−1.3019e+000	−3.0839e−001	−2.2488e+000	−1.8699e−001	8.4491e−001
78	−3.6012e−001	−1.3019e+000	−3.0840e−001	−2.2489e+000	−1.8699e−001	8.4555e−001
79	−3.6013e−001	−1.3020e+000	−3.0840e−001	−2.2489e+000	−1.8699e−001	8.4615e−001
80	−3.6013e−001	−1.3020e+000	−3.0841e−001	−2.2490e+000	−1.8700e−001	8.4672e−001
81	−3.6014e−001	−1.3020e+000	−3.0842e−001	−2.2490e+000	−1.8700e−001	8.4726e−001
82	−3.6015e−001	−1.3021e+000	−3.0842e−001	−2.2491e+000	−1.8701e−001	8.4777e−001
83	−3.6015e−001	−1.3021e+000	−3.0843e−001	−2.2491e+000	−1.8701e−001	8.4825e−001
84	−3.6016e−001	−1.3021e+000	−3.0843e−001	−2.2491e+000	−1.8701e−001	8.4871e−001
85	−3.6016e−001	−1.3021e+000	−3.0843e−001	−2.2491e+000	−1.8701e−001	8.4914e−001
86	−3.6016e−001	−1.3021e+000	−3.0844e−001	−2.2492e+000	−1.8702e−001	8.4955e−001
87	−3.6017e−001	−1.3021e+000	−3.0844e−001	−2.2492e+000	−1.8702e−001	8.4993e−001
88	−3.6017e−001	−1.3022e+000	−3.0844e−001	−2.2492e+000	−1.8702e−001	8.5029e−001
89	−3.6017e−001	−1.3022e+000	−3.0845e−001	−2.2492e+000	−1.8702e−001	8.5064e−001
90	−3.6018e−001	−1.3022e+000	−3.0845e−001	−2.2492e+000	−1.8702e−001	8.5096e−001
91	−3.6018e−001	−1.3022e+000	−3.0845e−001	−2.2492e+000	−1.8702e−001	8.5127e−001
92	−3.6018e−001	−1.3022e+000	−3.0845e−001	−2.2492e+000	−1.8702e−001	8.5156e−001
93	−3.6018e−001	−1.3022e+000	−3.0845e−001	−2.2492e+000	−1.8702e−001	8.5183e−001
94	−3.6018e−001	−1.3022e+000	−3.0845e−001	−2.2493e+000	−1.8703e−001	8.5209e−001
95	−3.6018e−001	−1.3022e+000	−3.0846e−001	−2.2493e+000	−1.8703e−001	8.5233e−001
96	−3.6019e−001	−1.3022e+000	−3.0846e−001	−2.2493e+000	−1.8703e−001	8.5256e−001
97	−3.6019e−001	−1.3022e+000	−3.0846e−001	−2.2493e+000	−1.8703e−001	8.5278e−001
98	−3.6019e−001	−1.3022e+000	−3.0846e−001	−2.2493e+000	−1.8703e−001	8.5299e−001
99	−3.6019e−001	−1.3022e+000	−3.0846e−001	−2.2493e+000	−1.8703e−001	8.5318e−001
100	−3.6019e−001	−1.3022e+000	−3.0846e−001	−2.2493e+000	−1.8703e−001	8.5337e−001

$a_i^{pH,Q_1,f}$ – coefficients of the step responses of pH to change of manipulated variable Q_1; i – index of the coefficients of the step responses; f – index of the local models; a_i^h – coefficients of the step responses of liquid level h to change of both manipulated variables (due to the assumptions and properties of the process the same for both manipulated variables and all operating points).

Table B.2 Step response coefficients obtained for the pH reactor described in Section 4; part 2/2.

i	$a_i^{pH,Q_3,P1}$	$a_i^{pH,Q_3,P2}$	$a_i^{pH,Q_3,P3}$	$a_i^{pH,Q_3,P4}$	$a_i^{pH,Q_3,P5}$
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	0
4	4.3943e−003	1.4707e−002	3.5220e−003	2.9505e−002	1.9572e−003
5	4.8011e−002	1.5070e−001	3.6725e−002	3.1117e−001	2.0176e−002
6	8.7275e−002	2.7087e−001	6.6398e−002	5.6892e−001	3.6394e−002
7	1.2259e−001	3.7714e−001	9.2913e−002	8.0368e−001	5.0834e−002
8	1.5433e−001	4.7118e−001	1.1660e−001	1.0166e+000	6.3696e−002
9	1.8283e−001	5.5445e−001	1.3777e−001	1.2091e+000	7.5154e−002
10	2.0841e−001	6.2824e−001	1.5667e−001	1.3826e+000	8.5364e−002
11	2.3136e−001	6.9367e−001	1.7356e−001	1.5387e+000	9.4463e−002
12	2.5193e−001	7.5172e−001	1.8864e−001	1.6787e+000	1.0257e−001
13	2.7037e−001	8.0326e−001	2.0211e−001	1.8043e+000	1.0980e−001
14	2.8689e−001	8.4903e−001	2.1414e−001	1.9167e+000	1.1625e−001
15	3.0167e−001	8.8971e−001	2.2488e−001	2.0172e+000	1.2200e−001
16	3.1491e−001	9.2587e−001	2.3448e−001	2.1071e+000	1.2713e−001

(*Continued*)

i	$a_i^{pH,Q_3,P1}$	$a_i^{pH,Q_3,P2}$	$a_i^{pH,Q_3,P3}$	$a_i^{pH,Q_3,P4}$	$a_i^{pH,Q_3,P5}$
17	3.2676e−001	9.5804e−001	2.4304e−001	2.1873e+000	1.3170e−001
18	3.3736e−001	9.8666e−001	2.5069e−001	2.2590e+000	1.3578e−001
19	3.4684e−001	1.0121e+000	2.5752e−001	2.3230e+000	1.3942e−001
20	3.5532e−001	1.0348e+000	2.6362e−001	2.3802e+000	1.4266e−001
21	3.6290e−001	1.0550e+000	2.6907e−001	2.4312e+000	1.4556e−001
22	3.6967e−001	1.0730e+000	2.7393e−001	2.4767e+000	1.4814e−001
23	3.7573e−001	1.0890e+000	2.7827e−001	2.5173e+000	1.5045e−001
24	3.8114e−001	1.1033e+000	2.8214e−001	2.5535e+000	1.5251e−001
25	3.8598e−001	1.1161e+000	2.8560e−001	2.5858e+000	1.5434e−001
26	3.9030e−001	1.1274e+000	2.8869e−001	2.6147e+000	1.5598e−001
27	3.9416e−001	1.1375e+000	2.9145e−001	2.6404e+000	1.5745e−001
28	3.9761e−001	1.1466e+000	2.9391e−001	2.6634e+000	1.5875e−001
29	4.0069e−001	1.1546e+000	2.9611e−001	2.6839e+000	1.5992e−001
30	4.0345e−001	1.1618e+000	2.9807e−001	2.7022e+000	1.6096e−001
31	4.0590e−001	1.1682e+000	2.9983e−001	2.7185e+000	1.6188e−001
32	4.0810e−001	1.1739e+000	3.0139e−001	2.7331e+000	1.6271e−001
33	4.1006e−001	1.1790e+000	3.0279e−001	2.7461e+000	1.6345e−001
34	4.1181e−001	1.1836e+000	3.0404e−001	2.7577e+000	1.6411e−001
35	4.1338e−001	1.1876e+000	3.0515e−001	2.7680e+000	1.6470e−001
36	4.1477e−001	1.1912e+000	3.0614e−001	2.7773e+000	1.6523e−001
37	4.1602e−001	1.1945e+000	3.0703e−001	2.7855e+000	1.6570e−001
38	4.1714e−001	1.1974e+000	3.0782e−001	2.7929e+000	1.6612e−001
39	4.1813e−001	1.1999e+000	3.0853e−001	2.7995e+000	1.6649e−001
40	4.1902e−001	1.2022e+000	3.0916e−001	2.8053e+000	1.6683e−001
41	4.1981e−001	1.2043e+000	3.0973e−001	2.8106e+000	1.6712e−001
42	4.2052e−001	1.2061e+000	3.1023e−001	2.8152e+000	1.6739e−001
43	4.2115e−001	1.2078e+000	3.1068e−001	2.8194e+000	1.6763e−001
44	4.2172e−001	1.2092e+000	3.1108e−001	2.8231e+000	1.6784e−001
45	4.2222e−001	1.2105e+000	3.1144e−001	2.8265e+000	1.6803e−001
46	4.2267e−001	1.2117e+000	3.1176e−001	2.8294e+000	1.6820e−001
47	4.2307e−001	1.2127e+000	3.1204e−001	2.8321e+000	1.6835e−001
48	4.2343e−001	1.2136e+000	3.1230e−001	2.8345e+000	1.6849e−001
49	4.2375e−001	1.2145e+000	3.1253e−001	2.8366e+000	1.6861e−001
50	4.2404e−001	1.2152e+000	3.1273e−001	2.8385e+000	1.6871e−001
51	4.2430e−001	1.2159e+000	3.1291e−001	2.8401e+000	1.6881e−001
52	4.2452e−001	1.2165e+000	3.1307e−001	2.8417e+000	1.6889e−001
53	4.2473e−001	1.2170e+000	3.1322e−001	2.8430e+000	1.6897e−001
54	4.2491e−001	1.2175e+000	3.1335e−001	2.8442e+000	1.6904e−001
55	4.2507e−001	1.2179e+000	3.1346e−001	2.8453e+000	1.6910e−001
56	4.2522e−001	1.2182e+000	3.1357e−001	2.8462e+000	1.6916e−001
57	4.2535e−001	1.2186e+000	3.1366e−001	2.8471e+000	1.6920e−001
58	4.2546e−001	1.2189e+000	3.1374e−001	2.8478e+000	1.6925e−001
59	4.2557e−001	1.2191e+000	3.1381e−001	2.8485e+000	1.6929e−001
60	4.2566e−001	1.2194e+000	3.1388e−001	2.8491e+000	1.6932e−001
61	4.2574e−001	1.2196e+000	3.1394e−001	2.8497e+000	1.6935e−001
62	4.2581e−001	1.2198e+000	3.1399e−001	2.8502e+000	1.6938e−001
63	4.2588e−001	1.2200e+000	3.1404e−001	2.8506e+000	1.6940e−001
64	4.2594e−001	1.2201e+000	3.1408e−001	2.8510e+000	1.6943e−001
65	4.2599e−001	1.2202e+000	3.1412e−001	2.8513e+000	1.6945e−001
66	4.2604e−001	1.2204e+000	3.1415e−001	2.8516e+000	1.6946e−001
67	4.2608e−001	1.2205e+000	3.1418e−001	2.8519e+000	1.6948e−001
68	4.2612e−001	1.2206e+000	3.1420e−001	2.8521e+000	1.6949e−001
69	4.2615e−001	1.2207e+000	3.1423e−001	2.8524e+000	1.6951e−001
70	4.2618e−001	1.2207e+000	3.1425e−001	2.8526e+000	1.6952e−001
71	4.2621e−001	1.2208e+000	3.1427e−001	2.8527e+000	1.6953e−001
72	4.2623e−001	1.2209e+000	3.1428e−001	2.8529e+000	1.6954e−001
73	4.2625e−001	1.2209e+000	3.1430e−001	2.8530e+000	1.6954e−001
74	4.2627e−001	1.2210e+000	3.1431e−001	2.8532e+000	1.6955e−001
75	4.2629e−001	1.2210e+000	3.1432e−001	2.8533e+000	1.6956e−001
76	4.2630e−001	1.2210e+000	3.1434e−001	2.8534e+000	1.6956e−001
77	4.2631e−001	1.2211e+000	3.1435e−001	2.8535e+000	1.6957e−001
78	4.2633e−001	1.2211e+000	3.1435e−001	2.8535e+000	1.6957e−001
79	4.2634e−001	1.2211e+000	3.1436e−001	2.8536e+000	1.6958e−001
80	4.2635e−001	1.2212e+000	3.1437e−001	2.8537e+000	1.6958e−001
81	4.2636e−001	1.2212e+000	3.1437e−001	2.8537e+000	1.6958e−001
82	4.2636e−001	1.2212e+000	3.1438e−001	2.8538e+000	1.6959e−001
83	4.2637e−001	1.2212e+000	3.1438e−001	2.8538e+000	1.6959e−001
84	4.2638e−001	1.2212e+000	3.1439e−001	2.8539e+000	1.6959e−001
85	4.2638e−001	1.2213e+000	3.1439e−001	2.8539e+000	1.6959e−001
86	4.2639e−001	1.2213e+000	3.1440e−001	2.8539e+000	1.6959e−001
87	4.2639e−001	1.2213e+000	3.1440e−001	2.8540e+000	1.6960e−001
88	4.2639e−001	1.2213e+000	3.1440e−001	2.8540e+000	1.6960e−001
89	4.2640e−001	1.2213e+000	3.1440e−001	2.8540e+000	1.6960e−001
90	4.2640e−001	1.2213e+000	3.1441e−001	2.8540e+000	1.6960e−001
91	4.2640e−001	1.2213e+000	3.1441e−001	2.8540e+000	1.6960e−001
92	4.2641e−001	1.2213e+000	3.1441e−001	2.8541e+000	1.6960e−001
93	4.2641e−001	1.2213e+000	3.1441e−001	2.8541e+000	1.6960e−001

(*Continued*)

i	$a_i^{pH,Q_3,P1}$	$a_i^{pH,Q_3,P2}$	$a_i^{pH,Q_3,P3}$	$a_i^{pH,Q_3,P4}$	$a_i^{pH,Q_3,P5}$
94	4.2641e−001	1.2213e+000	3.1441e−001	2.8541e+000	1.6960e−001
95	4.2641e−001	1.2213e+000	3.1441e−001	2.8541e+000	1.6960e−001
96	4.2641e−001	1.2213e+000	3.1442e−001	2.8541e+000	1.6960e−001
97	4.2642e−001	1.2213e+000	3.1442e−001	2.8541e+000	1.6960e−001
98	4.2642e−001	1.2213e+000	3.1442e−001	2.8541e+000	1.6961e−001
99	4.2642e−001	1.2213e+000	3.1442e−001	2.8541e+000	1.6961e−001
100	4.2642e−001	1.2213e+000	3.1442e−001	2.8541e+000	1.6961e−001

$a_i^{pH,Q_3,f}$ – coefficients of the step responses of pH to change of manipulated variable Q_3; i – index of the coefficients of the step responses; f – index of the local models.

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