Cooperation of model predictive control with steady-state economic optimisation

Article *in* Control and Cybernetics · January 2008

Maciej Ławryńczuk
Warsaw University of Technology

Piotr M. Marusak
Warsaw University of Technology

Control and Cybernetics

vol. **37** (2008) No. 1

Cooperation of model predictive control with steady-state economic optimisation*

by

Maciej Ławryńczuk, Piotr M. Marusak and Piotr Tatjewski

Institute of Control and Computation Engineering,
Faculty of Electronics and Information Technology,
Warsaw University of Technology
ul. Nowowiejska 15/19, 00-665 Warszawa, Poland
e-mail: M.Lawrynczuk@ia.pw.edu.pl, P.Marusak@ia.pw.edu.pl,
P.Tatjewski@ia.pw.edu.pl

Abstract: The problem of cooperation of Model Predictive Control (MPC) algorithms with steady-state economic optimisation is investigated in this paper. It is particularly important when the dynamics of disturbances is comparable with the dynamics of the process, since in such a case the classical hierarchical multilayer structure is likely to be not efficient and give the economic yield smaller than expected. This is because the economic nonlinear optimisation problem cannot be then solved on-line to update the optimal operating point as frequently as needed. On the other hand, simple target set-point optimisation based on linear models can be also insufficiently accurate. This paper introduces approximate formulations of the target set-point optimisation problem which tightly cooperates with the MPC and is solved as frequently as the MPC controller executes. Linear, linear-quadratic and piecewise-linear formulations are discussed, tuning guidelines are also given.

Keywords: predictive control, optimal control, optimisation, economic steady-state optimisation, nonlinear control systems, constrained control.

1. Introduction

The technique of process automation has been based on the hierarchical (multilayer) approach for years. In general, from a functional point of view, the main control layers are: the regulatory (feedback) control layers, which keep process at given operating points and the optimisation layer, which calculates these set-points (Brdys and Tatjewski, 2005; Findeisen et al., 1980). The optimal set-points can be either optimal dynamic trajectories or optimal constant

*Submitted: October 2006; Accepted: February 2008

(i.e. steady-state) values of the set-points, which result in maximising the economic yield. Moreover, the optimal trajectories or the set-points must satisfy the constraints, which determine safety and quality of production.

As far as the contemporary, advanced control systems are concerned, the regulatory control layer consists of basic (direct) dynamic control layer, which is usually comprised of PID controllers, and a higher constraint control layer, called also advanced control layer or Model Predictive Control (MPC) control layer (Blevins et al., 2003; Brdys and Tatjewski, 2005; Kassmann et al., 2000; Qin and Badgwell, 2003; Tatjewski et al., 2006). Model Predictive Control is recognised as the only advanced control technique (i.e. more advanced than the well known PID approach) which has been very successful in practical applications. MPC has influenced not only the directions of development of industrial control systems but also research in this area (Brdys and Tatjewski, 2005; Henson, 1998; Maciejowski, 2002; Morari and Lee, 1999; Qin and Badgwell, 2003; Rossiter, 2003; Tatjewski 2007). The idea of MPC consists in predicting at each sampling instant behavior of the controlled plant for a predefined future time horizon under assumed sequence of control input values and solving a dynamic optimisation problem. As a result, an optimal sequence of control values is obtained, but only the first value from this sequence is used. Having updated the measurement of the process output (or state) variables, the prediction is shifted one step forward and the whole procedure is repeated at the next sampling instant. The optimisation problem with control-type performance index that minimises, over a predefined prediction horizon, the values of the control error is usually used. The most important advantage of the MPC algorithms is the fact that they have the unique ability to take into account constraints imposed on process inputs (manipulated variables) and outputs (controlled variables) or state variables, which usually decide on quality, economic efficiency and safety of production. Furthermore, MPC techniques are very efficient in multivariable process control. Finally, the underlying idea of MPC is relatively easy to explain to engineering and operating staff, which is of fundamental importance when it comes to introducing new techniques into industrial practice.

The values of measured or estimated disturbances have to be taken into account in the economic optimisation problem since they determine the optimal operating point. When the classical multilayer control system structure is used, it is usually assumed that the disturbances are slowly-varying when compared to the dynamics of the process. Hence, the steady-state economic optimisation problem can be solved reasonably less frequently than the MPC controllers executes. Provided that the dynamics of disturbances is much slower than the dynamics of the plant, such an approach performs well. Unfortunately, much more typical of industrial practice are the cases when the dynamics of the disturbances is not slow, in the worst case it may be comparable with the process dynamics. Very often the disturbances, for example flow rates, properties of feed and energetic streams etc., vary significantly and not much slower than the dynamics of the controlled process. In such cases operation in the classical hierarchical structure with infrequent economic optimisation can result in a significant loss of economic effectiveness. Hence, the optimal set-points have to be computed more often. This can be done efficiently in the control systems which use MPC algorithms.

The paper is concerned with the problem of cooperation of MPC algorithms with steady-state optimisation when the dynamics of disturbances is not much slower than that of the process. Ideally, it would be best to perform full nonlinear optimisation but with increased frequency. Unfortunately, such an approach has limited applicability and is rarely implementable on-line. To circumvent this drawback, in practice the MPC algorithm is supplemented with an additional simple steady-state target optimisation. Since it usually employs a linear model corresponding to the dynamic one used in the MPC algorithm, it may be not effective. Hence, more advanced target recalculation is necessary.

The contribution of this paper is the introduction of approximate formulations of the target set-point optimisation which uses nonlinear steady-state model approximation. It tightly cooperates with the MPC and is performed at every sampling instant. Linear, linear-quadratic and piecewise-linear approaches to this approximation problem are proposed. The method giving the best results for a particular process actually depends on its nature, precisely on the nonlinearity of its steady-state characteristics.

The outline of the paper is as follows. First, in Section 2, the multilayer, hierarchical control system structure with model predictive control algorithm and steady-state economic optimisation is presented. Section 3 describes MPC dynamic optimisation problems. The main part of the article, Section 4, details the MPC target calculation problem with steady-state model approximation. Linear and linear-quadratic approximations and resulting optimisation problems are thoroughly discussed, piecewise-linear approach is also briefly described. Simulation results of the described algorithms applied to two nonlinear benchmark processes are presented in Section 5, and the paper is summarised in Section 6.

2. Model predictive control with steady-state economic optimisation

Fig. 1 depicts the structure of the hierarchical control system. In general, each layer operates with different frequency of intervention. A supervisory global plant-wide optimisation layer aims at maximising the economic yield obtained from many technological processes. On the contrary, the Local Steady-State Optimisation (LSSO) layer is used only so as to maximise the yields from one single plant. Typically, the plant-wide optimisation period is one day, the LSSO problem is repeatedly solved every hour, the MPC layer executes every minute and the basic controller layer is activated as frequently as every second (Qin and Badgwell, 2003).

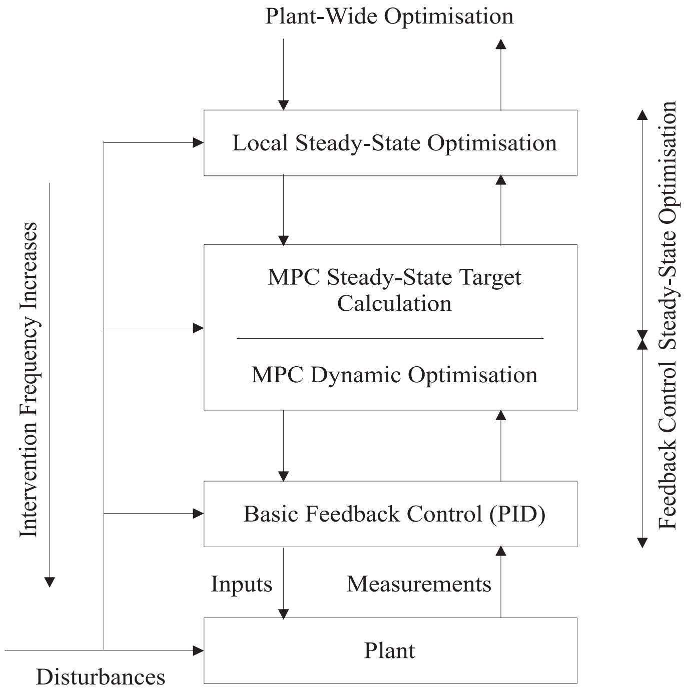

Figure 1. Hierarchical control system structure with MPC advanced control (set-point control) layer with steady-state target calculation

The LSSO layer uses a comprehensive nonlinear steady-state model of the process, the resulting economic optimisation problem is usually difficult and time consuming, with constraints which significantly decrease the set of possible solutions. The LSSO layer computes the optimal set-point values for the MPC layer taking into account current values of disturbances (measured or estimated) affecting the plant, the information from the plant-wide optimisation layer and from the operator. As mentioned in the introduction, if variability of disturbances is not significant in comparison with the dynamic properties of the process, such a structure works well, because the steady-state operating points are close to optimal over long time periods. Unfortunately, when the disturbances vary faster, i.e. with the dynamics comparable with the dynamics of the process, the economic performance is likely to be below expectations.

As increasing the frequency of the LSSO layer is limited in practice because of its high computational burden, the MPC layer is supplemented with an additional Steady-State Target Optimisation (SSTO) layer, as it is shown in Fig. 1 (Blevins et al., 2003; Kassmann et al., 2000; Qin and Badgwell, 2003; Tatjewski, 2007). The SSTO closely cooperates with the MPC layer, the steady-state operating-point determined by the LSSO layer activated less often is recalculated as frequently as the MPC executes. Because of that, the SSTO layer uses a simplified steady-state model, rather than the comprehensive model used in the LSSO layer. In practice, a linear steady-state model resulting from the dynamic model used in the MPC algorithm is typically employed (Blevins et al., 2003; Kassmann et al., 2000; Qin and Badgwell, 2003). Such an approach leads to a linear programming SSTO problem assuming the objective function is a linear one. Unfortunately, it may lead to substantial loss of economic optimality, since this model is only approximate and may be significantly different from the comprehensive one used at the LSSO layer, for most of the operating points. The problem has been recognised in Kassmann et al., (2000), treated explicitly using uncertainty estimation in the steady-state gain matrix, in the framework of robust target steady-state calculation. Linearisation of the comprehensive nonlinear model, which leads to a quadratic programming SSTO problem has been reported (Qin and Badgwell, 2003). Recently, the on-line adaptation of the linear, linear-quadratic or piecewise-linear approximate models has been proposed (Ławryńczuk et al., 2006; Tatjewski et al., 2006). The idea behind the latter approach is that the steady-state model used at the SSTO layer should be consistent with the one used at the LSSO layer rather than with the linear dynamic model used at the MPC layer. The best solution would be, certainly, to repeat the nonlinear LSSO every time the MPC controller executes, thus eliminating the need for the MPC-SSTO task. This approach may be nowadays possible, but still only for rather limited cases of slow process and relatively simple nonlinear steady-state models.

As far as economic optimisation performed at the LSSO layer is concerned, one usually aims at maximising the production profit. Typically, linear dependence of costs of individual materials on their prices is assumed. Thus, in the multilayer control system structure the economic optimisation layer has to solve usually the following problem

$$\begin{aligned} &\min_{u^{ss}} \left\{ J_E(k) = c_u^T u^{ss} - c_y^T y^{ss} \right\} \\ &\text{subject to:} \\ &\qquad u_{\min} \le u^{ss} \le u_{\max} \\ &\qquad y_{\min} \le y^{ss} \le y_{\max} \\ &\qquad y^{ss} = F(u^{ss}, \tilde{w}) \end{aligned} \tag{1}$$

where a differentiable function $F : \Re^{n_u} \times \Re^{n_w} \to \Re^{n_y}$ denotes a comprehensive steady-state process model, usually a nonlinear mapping, which can be often given in an implicit numerical form, n_u, n_w, n_y are the numbers of manipulated variables, disturbances affecting the plant and controlled variables, respectively, $\tilde{w}$ is the current estimate or short-term prediction or measurement of disturbances. The vectors $u^{ss} \in \Re^{n_u}$ and $y^{ss} \in \Re^{n_y}$ are related by the steady-state process model $y^{ss} = F(u^{ss}, \tilde{w})$. The vectors $c_u \in \Re^{n_u}$, $c_y \in \Re^{n_y}$ represent the prices resulting from economic considerations, $u_{\min}$, $u_{\max}$, $y_{\min}$, $y_{\max}$ are vectors of constraints imposed on input and output variables, respectively.

The disturbances affecting the process can be regarded as the parameters of the optimisation problem (1). In order to obtain both reliable and economically viable solution two requirements have to be met. Firstly, the values of these disturbances (measured or estimated) must be known. Secondly, the steady-state model F of the process should be accurate enough and take into account all the significant disturbances. It should be remembered, however, that in practice, because of inaccuracies and uncertainties, the obtained solution to the economic optimisation problem may be far from the real optimal operating point. If the disturbance measurements and the model do not correspond to the reality, the optimisation layer is likely to give not adequate set-points. Even if significant uncertainty is encountered, but the disturbances can be assumed to be constant over long time periods, using in an approximate way additional measurements from the plant can lead to improved optimality of the determined set-point values (Brdys and Tatjewski, 2005; Tatjewski 2007).

3. MPC dynamic optimisation problems

At first, the most typical situation is considered when the numbers of controlled variables, y, and manipulated variables, u, are equal, i.e. $n_u = n_y$. The MPC dynamic optimisation problem is as follows

$$\min_{\Delta \boldsymbol{u}(k)} \left\{ J_{MPC}(k) = \sum_{p=1}^{N} \| y^{sp}(k+p|k) - y(k+p|k) \|^2_{\boldsymbol{M}_p} + \sum_{p=0}^{N_u-1} \| \Delta \boldsymbol{u}(k+p|k) \|^2_{\boldsymbol{\Lambda}_p} \right\}$$

subject to:

$$\begin{aligned} & u_{\min} \leq u(k+p|k) \leq u_{\max}, \qquad p = 0, \ldots, N_u - 1 \\ & -\Delta u_{\max} \leq \Delta u(k+p|k) \leq \Delta u_{\max}, \qquad p = 0, \ldots, N_u - 1 \\ & y_{\min} \leq y(k+p|k) \leq y_{\max}, \qquad p = 1, \ldots, N \end{aligned} \tag{2}$$

where

$$\Delta \boldsymbol{u}(k) = \begin{bmatrix} \Delta u(k|k) \\ \vdots \\ \Delta u(k + N_u - 1|k) \end{bmatrix} \tag{3}$$

is the vector of future control increments (i.e. decision variables of the algorithm), N and N_u denote prediction and control horizons, respectively, $\boldsymbol{M}_p \geq \boldsymbol{0}$ and $\boldsymbol{\Lambda}_p > \boldsymbol{0}$ are diagonal weighting matrices of dimension $n_y \times n_y$ and $n_u \times n_u$, $y(k+p|k)$ denotes the output prediction of the outputs for a future sampling instant $k+p$, calculated at current sampling instant k using a dynamic model of the process. The set-point trajectory is typically assumed to be constant over the prediction horizon and equal to the desired set-point

$$y^{sp}(k+p|k) = y^{ss}, \qquad p = 1, \ldots, N \tag{4}$$

or, alternatively, a reference trajectory can be used

$$y^{sp}(k+p|k) = \gamma y^{sp}(k+p-1|k) + (1-\gamma)y^{ss}, \qquad p = 1,\ldots,N. \tag{5}$$

In the latter case $y^{sp}(k|k) = y(k)$, γ is a design parameter satisfying $0 < \gamma < 1$. It means that a prescribed continuous trajectory approaching the required steady-state, defined by a first-order filter, is applied as the set-point trajectory in the MPC algorithm.

For future considerations, without loss of generality, the constant set-point trajectory is assumed. Thus, the problem (2) can be rewritten as

$$\begin{aligned}
&\min_{\Delta \boldsymbol{u}(k)} \left\{ J_{MPC}(k) = \|\boldsymbol{y}^{ss} - \boldsymbol{y}(k)\|^2_{\boldsymbol{M}} + \|\Delta \boldsymbol{u}(k)\|^2_{\boldsymbol{\Lambda}} \right\} \\
&\text{subject to:} \\
&\qquad \boldsymbol{u}_{\min} \le \boldsymbol{J}\Delta \boldsymbol{u}(k) + \boldsymbol{u}^{k-1} \le \boldsymbol{u}_{\max} \\
&\qquad -\Delta \boldsymbol{u}_{\max} \le \Delta \boldsymbol{u}(k) \le \Delta \boldsymbol{u}_{\max} \\
&\qquad \boldsymbol{y}_{\min} \le \boldsymbol{y}(k) \le \boldsymbol{y}_{\max}
\end{aligned} \tag{6}$$

where

$$\boldsymbol{u}_{\min} = \begin{bmatrix} u_{\min} \\ \vdots \\ u_{\min} \end{bmatrix}, \boldsymbol{u}_{\max} = \begin{bmatrix} u_{\max} \\ \vdots \\ u_{\max} \end{bmatrix}, \boldsymbol{u}^{k-1} = \begin{bmatrix} u(k-1) \\ \vdots \\ u(k-1) \end{bmatrix}, \Delta \boldsymbol{u}_{\max} = \begin{bmatrix} \Delta u_{\max} \\ \vdots \\ \Delta u_{\max} \end{bmatrix} \tag{7}$$

are vectors of length $n_u N_u$,

$$\boldsymbol{y}(k) = \begin{bmatrix} y(k+1|k) \\ \vdots \\ y(k+N|k) \end{bmatrix}, \boldsymbol{y}_{\min} = \begin{bmatrix} y_{\min} \\ \vdots \\ y_{\min} \end{bmatrix}, \boldsymbol{y}_{\max} = \begin{bmatrix} y_{\max} \\ \vdots \\ y_{\max} \end{bmatrix}, \boldsymbol{y}^{ss} = \begin{bmatrix} y^{ss} \\ \vdots \\ y^{ss} \end{bmatrix} \tag{8}$$

are vectors of length $n_y N$,

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{0}_{n_u \times n_u} & \boldsymbol{0}_{n_u \times n_u} & \cdots & \boldsymbol{0}_{n_u \times n_u} \\ \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{0}_{n_u \times n_u} & \cdots & \boldsymbol{0}_{n_u \times n_u} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{I}_{n_u \times n_u} & \boldsymbol{I}_{n_u \times n_u} & \cdots & \boldsymbol{I}_{n_u \times n_u} \end{bmatrix} \tag{9}$$

is the matrix of dimension $n_u N_u \times n_u N_u$, $\boldsymbol{M}$ and $\boldsymbol{\Lambda}$ are diagonal matrices of dimension $n_y N \times n_y N$ and $n_u N_u \times n_u N_u$, comprised of matrices $\boldsymbol{M}$ and $\boldsymbol{\Lambda}$, respectively. If for prediction purposes a linear dynamic model of the plant is used, as it is e.g. in the case of DMC (Cutler and Ramaker, 1979) or GPC (Clarke et al., 1987) algorithms, the output prediction can be expressed as the sum of the forced trajectory, which depends only on the future, i.e. on the input moves $\Delta \boldsymbol{u}(k)$ and the free trajectory $\boldsymbol{y}^0(k)$, which depends only on the past

$$\boldsymbol{y}(k) = \boldsymbol{G}\Delta \boldsymbol{u}(k) + \boldsymbol{y}^0(k) \tag{10}$$

where

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{S}_1 & \mathbf{0}_{n_y \times n_u} & \dots & \mathbf{0}_{n_y \times n_u} \\ \boldsymbol{S}_2 & \boldsymbol{S}_1 & \dots & \mathbf{0}_{n_y \times n_u} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{S}_N & \boldsymbol{S}_{N-1} & \dots & \boldsymbol{S}_{N-N_u+1} \end{bmatrix} \tag{11}$$

is the dynamic matrix of the dimension $n_y N \times n_u N_u$, which is composed of step response coefficients.

If the output constraints are present in the economic optimisation task (1) and, consequently, in the MPC optimisation task (2) or (6), the MPC may be affected by the infeasibility problem. To cope with such a situation, the well known approach is to soften the output constraints by using slack variables (Maciejowski, 2002). Using a quadratic penalty for constraint violations, assuming that the set-point trajectory is constant over the prediction horizon (4) and using for prediction a linear model of the plant (10), the MPC optimisation problem would then be as follows

$$\min_{\Delta \boldsymbol{u}(k),\, \boldsymbol{\varepsilon}_{\min},\, \boldsymbol{\varepsilon}_{\max}} \left\{ \begin{array}{c} J_{MPC}(k) = \left\| \boldsymbol{y}^{ss} - \boldsymbol{G}\Delta\boldsymbol{u}(k) - \boldsymbol{y}^0(k) \right\|^2_{\boldsymbol{M}} + \left\| \Delta\boldsymbol{u}(k) \right\|^2_{\boldsymbol{\Lambda}} + \\ +\rho_{\min} \left\| \boldsymbol{\varepsilon}_{\min} \right\|^2 + \rho_{\max} \left\| \boldsymbol{\varepsilon}_{\max} \right\|^2 \end{array} \right\}$$

subject to:

$$\begin{aligned} &\boldsymbol{u}_{\min} \le \boldsymbol{J}\Delta\boldsymbol{u}(k) + \boldsymbol{u}^{k-1} \le \boldsymbol{u}_{\max} \\ &-\Delta\boldsymbol{u}_{\max} \le \Delta\boldsymbol{u}(k) \le \Delta\boldsymbol{u}_{\max} \\ &\boldsymbol{y}_{\min} - \boldsymbol{\varepsilon}_{\min} \le \boldsymbol{G}\Delta\boldsymbol{u}(k) + \boldsymbol{y}^0(k) \le \boldsymbol{y}_{\max} + \boldsymbol{\varepsilon}_{\max} \\ &\boldsymbol{\varepsilon}_{\min} \ge 0 \\ &\boldsymbol{\varepsilon}_{\max} \ge 0 \end{aligned} \tag{12}$$

where $\boldsymbol{\varepsilon}_{\min}$, $\boldsymbol{\varepsilon}_{\max}$ are vectors of length $n_y N$, and $\rho_{\min}$, $\rho_{\max}$ are positive weights.

A disadvantage of the quadratic penalty is the fact that if the constraints are active, for all finite values of $\rho_{\min}$ and $\rho_{\max}$ this approach results in them being violated to some extent, even if the violation is not necessary (Maciejowski, 2002). As an alternative, the 1-norm (sum of violations) of the constraint violations may be considered. The MPC optimisation problem would then be as follows

$$\begin{aligned} &\min_{\Delta \boldsymbol{u}(k),\, \varepsilon} \left\{ \begin{array}{c} J_{MPC}(k) = \left\| \boldsymbol{y}^{ss} - \boldsymbol{G}\Delta\boldsymbol{u}(k) - \boldsymbol{y}^0(k) \right\|^2_{\boldsymbol{M}} + \left\| \Delta\boldsymbol{u}(k) \right\|^2_{\boldsymbol{\Lambda}} + \\ +\rho_{\min} \left\| \boldsymbol{\varepsilon}_{\min} \right\|_1 + \rho_{\max} \left\| \boldsymbol{\varepsilon}_{\max} \right\|_1 \end{array} \right\} \\ &\text{subject to:} \\ &\quad \boldsymbol{u}_{\min} \le \boldsymbol{J}\Delta\boldsymbol{u}(k) + \boldsymbol{u}^{k-1} \le \boldsymbol{u}_{\max} \\ &\quad -\Delta\boldsymbol{u}_{\max} \le \Delta\boldsymbol{u}(k) \le \Delta\boldsymbol{u}_{\max} \\ &\quad \boldsymbol{y}_{\min} - \varepsilon_{\min} \le \boldsymbol{G}\Delta\boldsymbol{u}(k) + \boldsymbol{y}^0(k) \le \boldsymbol{y}_{\max} + \varepsilon_{\max} \\ &\quad \boldsymbol{\varepsilon}_{\min} \ge 0 \\ &\quad \boldsymbol{\varepsilon}_{\max} \ge 0. \end{aligned} \tag{13}$$

4. MPC target calculation with steady-state model approximation

The best solution would be to always solve the nonlinear optimisation problem (1) as frequently as needed. This frequency should be high enough to make it possible to follow the changes of the disturbances. In the ideal case it should be solved as frequently as the MPC controller executes. Unfortunately, mainly because of computational complexity, it is usually impossible. That is why, instead of the LSSO problem, the MPC SSTO problem is solved at each sampling instant. In order to reduce the computation requirements, a simplified steady-sate model of the process is used and this optimisation problem is posed in the form of linear or quadratic programming.

4.1. Linear MPC steady-state target optimisation

In the simplest, standard approach to the MPC SSTO problem which is widely applied in industrial practice (Blevins et al., 2003; Kassmann et al., 2000), it is based on a constant linear steady-state model of the process derived off-line from the dynamic (linear) one used in the MPC algorithm. Let $\boldsymbol{H}$ denote the gain matrix of dimension $n_y \times n_u$ corresponding to this model. Taking into account the nonlinear LSSO problem (1), the equivalent MPC SSTO problem is cast in the following linear programming form

$$\begin{aligned}
&\min_{u^{ss}} \left\{ J_E(k) = c_u^T \Delta u^{ss} - c_y^T \Delta y^{ss} \right\} \\
&\text{subject to:} \\
&\qquad u_{\min} \leq u^{ss} \leq u_{\max} \\
&\qquad y_{\min} \leq y^{ss} \leq y_{\max} \\
&\qquad \Delta y^{ss} = \boldsymbol{H} \Delta u^{ss} \\
&\qquad y^{ss} = y^0(k+N|k) + \Delta y^{ss} \\
&\qquad u^{ss} = u(k-1) + \Delta u^{ss}
\end{aligned} \tag{14}$$

where $y^0(k+N|k)$ is the value of predicted free output trajectory at the end of the prediction horizon. In this formulation the free output trajectory should be calculated first, then the LP SSTO problem (14) is solved to determine the optimal steady-state u^{ss} and corresponding optimal y^{ss}, which is finally used in the MPC optimisation problem (12) or (13) to calculate $\Delta \boldsymbol{u}(k)$. In the above SSTO problem formulation the free response $y^0(k+N|k)$ is used since it comprises the information about the disturbances. More specifically, it is influenced by disturbances affecting the output measurements or the process itself.

In general, the SSTO optimisation task (14) may be affected by the infeasibility problem, i.e. for the current operating point determined by $u(k-1)$ and $y^0(k-1)$ it may be infeasible. Since such a situation is unacceptable in on-line control, one has to soften the steady-state output constraints $y_{\min} \leq y^{ss} \leq y_{\max}$.

The SSTO problem becomes

$$\min_{u^{ss},\, v_{\min},\, v_{\max}} \left\{ J_E(k) = c_u^T \Delta u^{ss} - c_y^T \Delta y^{ss} + \rho_{\min} \left\| v_{\min} \right\|_1 + \rho_{\max} \left\| v_{\max} \right\|_1 \right\}$$

subject to:

$$\begin{aligned}
& u_{\min} \leq u^{ss} \leq u_{\max} \\
& y_{\min} - v_{\min} \leq y^{ss} \leq y_{\max} + v_{\max} \\
& \Delta y^{ss} = \boldsymbol{H} \Delta u^{ss} \\
& y^{ss} = y^0(k+N|k) + \Delta y^{ss} \\
& u^{ss} = u(k-1) + \Delta u^{ss} \\
& v_{\min} \geq 0 \\
& v_{\max} \geq 0
\end{aligned} \tag{15}$$

where $v_{\min}$, $v_{\max}$ are vectors of length n_y, and $\rho_{\min}$, $\rho_{\max}$ are positive weights.

4.2. Linear MPC steady-state target optimisation with successive linearisation

In linear MPC steady-state target optimisation problems, (14) or (15), the steady-state properties of the process are characterised by the constant matrix $\boldsymbol{H}$ corresponding to the dynamic model used in the MPC optimisation problem (12) or (13). Such a method is likely to fail if the process is significantly nonlinear, because real steady-state nature of the process, which is dependent on the current operating point, differs significantly from the steady-state description $\Delta y^{ss} = \boldsymbol{H} \Delta u^{ss}$. An appropriate alternative is then to use a successive linearisation approach, i.e. to use in the SSTO problem the gains matrix $\boldsymbol{H}(k)$ calculated from the comprehensive nonlinear model of the process, $F(\cdot,\cdot)$, which is also used in the local steady-state optimisation (LSSO) problem (1). The matrix

$$\boldsymbol{H}(k) = \begin{bmatrix} \dfrac{\partial F(u(k-1),\tilde{w})}{\partial u_1} & \cdots & \dfrac{\partial F(u(k-1),\tilde{w})}{\partial u_{n_u}} \end{bmatrix} \tag{16}$$

$$= \begin{bmatrix} \dfrac{\partial f_1(u(k-1),\tilde{w})}{\partial u_1} & \cdots & \dfrac{\partial f_1(u(k-1),\tilde{w})}{\partial u_{n_u}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_{n_y}(u(k-1),\tilde{w})}{\partial u_1} & \cdots & \dfrac{\partial f_{n_y}(u(k-1),\tilde{w})}{\partial u_{n_u}} \end{bmatrix} \tag{17}$$

may be updated as frequently as the MPC controller executes (i.e. at every sampling instant), or, alternatively, less frequently. In the latter case the SSTO problems solved at a few consecutive sampling instants use the same matrix $\boldsymbol{H}(k)$. The derivatives comprising the matrix $\boldsymbol{H}(k)$ are usually computed numerically using finite difference approach. Analogously to (14), the SSTO problem is then as follows

$$\begin{aligned}
&\min_{u^{ss}} \left\{ J_E(k) = c_u^T \Delta u^{ss} - c_y^T \Delta y^{ss} \right\} \\
&\text{subject to:} \\
&\qquad u_{\min} \leq u^{ss} \leq u_{\max} \\
&\qquad y_{\min} \leq y^{ss} \leq y_{\max} \\
&\qquad \Delta y^{ss} = \boldsymbol{H}(k)\Delta u^{ss} + b \\
&\qquad y^{ss} = F(u^{ss}, \tilde{w}) + \Delta y^{ss} \\
&\qquad u^{ss} = u(k-1) + \Delta u^{ss}
\end{aligned} \tag{18}$$

where b is the bias corresponding to unmeasured disturbances based on a comparison of measured and predicted outputs. To avoid the infeasibility problems, the SSTO task with soft output constraints can be formulated, analogously to (15).

Since the comprehensive steady-state nonlinear model of the process is used to calculate on-line, taking into account current state of the plant, the gains matrix $\boldsymbol{H}(k)$, it would be also reasonable to obtain successively a local linearisation of the nonlinear dynamic model and employ it in the MPC algorithm. Of course, this is possible, provided that such a model is available. If only the steady-state nonlinear model is used and the MPC algorithm uses a linear dynamic model it is then logical to update the gains matrix of this model each time a new matrix $\boldsymbol{H}(k)$ is determined.

The MPC optimisation problems (2), (6), (12) and (13) are well posed provided that $n_u = n_y$. However, the cases when the process has more manipulated variables than controlled ones, i.e. $n_u > n_y$, are encountered in practice. Because the solution to the MPC dynamic optimisation problem is then not unique, the control algorithm has to take advantage of additional degrees of freedom to enforce economically better solution. This could be achieved by imposing an additional constraint

$$u(k + N_u - 1|k) = u^{ss} \tag{19}$$

and use it in the MPC optimisation problem. Alternatively, the same can be achieved by adding to the performance function an additional penalty term. Considering hard output constraints for short, analogously to (6) and using (10), one has

$$\begin{aligned}
&\min_{\Delta \boldsymbol{u}(k)} \Big\{ J_{MPC}(k) = \left\| \boldsymbol{y}^{ss} - \boldsymbol{G}\Delta\boldsymbol{u}(k) - \boldsymbol{y}^0(k) \right\|_{\boldsymbol{M}}^2 + \left\| \Delta\boldsymbol{u}(k) \right\|_{\boldsymbol{\Lambda}}^2 \\
&\qquad\qquad + \left\| u^{ss} - u(k + N_u - 1|k) \right\|_{\boldsymbol{R}}^2 \Big\} \\
&\text{subject to:} \\
&\qquad \boldsymbol{u}_{\min} \leq \boldsymbol{J}\Delta\boldsymbol{u}(k) + \boldsymbol{u}^{k-1} \leq \boldsymbol{u}_{\max} \\
&\qquad -\Delta\boldsymbol{u}_{\max} \leq \Delta\boldsymbol{u}(k) \leq \Delta\boldsymbol{u}_{\max} \\
&\qquad \boldsymbol{y}_{\min} \leq \boldsymbol{G}\Delta\boldsymbol{u}(k) + \boldsymbol{y}^0(k) \leq \boldsymbol{y}_{\max}
\end{aligned} \tag{20}$$

where the matrix $\boldsymbol{R}$ is of dimension $n_u \times n_u$. The purpose of the last term in the performance function is to enforce the future controls $u(k+N_u-1|k)$ to be as close as possible to the optimal steady-state u^{ss}. Remembering that $n_u > n_y$, it is obvious that the controller has enough freedom to calculate such values of the decision variables since it is not in contradiction to the requirement that the steady-state values y^{ss} should be achieved at the end of the prediction horizon, i.e. $y(k+N|k) = y^{ss}$. It is possible because the steady state values u^{ss} and y^{ss} are consistent, provided that the same gains matrices are used in both MPC and SSTO problems. Of course, the above formulation can also use soft output constraints, similarly to (12) or (13).

4.3. Quadratic MPC steady-state target optimisation with successive linear-quadratic approximation

For some nonlinear plants linear approximation of the steady-state characteristic may be not sufficiently accurate. In such cases the quadratic approximation can be employed provided that the economic optimisation performance index $J_E(k)$ is a linear function as it is assumed in the LSSO problem formulation (1). Taking into account the current operating point determined by $u(k-1)$, at each sampling instant one has

$$y^{ss} = F(u(k-1), \tilde{w}) + \boldsymbol{H}(k)\Delta u^{ss} + 0.5 \begin{bmatrix} (\Delta u^{ss})^T & \boldsymbol{B}_1(k) & \Delta u^{ss} \\ & \vdots & \\ (\Delta u^{ss})^T & \boldsymbol{B}_{n_y}(k) & \Delta u^{ss} \end{bmatrix} \tag{21}$$

where the matrices $\boldsymbol{B}_n(k)$, $n = 1, \ldots, n_y$ have the following structure

$$\boldsymbol{B}_n(k) = \begin{bmatrix} \dfrac{\partial^2 f_n(u(k-1), \tilde{w})}{\partial u_1 \partial u_1} & \cdots & \dfrac{\partial^2 f_n(u(k-1), \tilde{w})}{\partial u_1 \partial u_{n_u}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial^2 f_n(u(k-1), \tilde{w})}{\partial u_{n_u} \partial u_1} & \cdots & \dfrac{\partial^2 f_n(u(k-1), \tilde{w})}{\partial u_{n_u} \partial u_{n_u}} \end{bmatrix}. \tag{22}$$

The quadratic approximation given by (21) can be rewritten in a more compact form as

$$y^{ss} = F(u(k-1), \tilde{w}) + \boldsymbol{H}(k)\Delta u^{ss} + 0.5(\boldsymbol{I}_{n_y \times n_y} \otimes (\Delta u^{ss})^T)\boldsymbol{B}(k)\Delta u^{ss} \tag{23}$$

where

$$\boldsymbol{B}(k) = \begin{bmatrix} \boldsymbol{B}_1(k) \\ \vdots \\ \boldsymbol{B}_{n_y}(k) \end{bmatrix} \tag{24}$$

is a matrix of dimension $n_y n_u \times n_u$ and $\otimes$ denotes the Kronecker tensor product. Analogously to (14) and (18) the SSTO problem is then as follows

$$\min_{u^{ss}} \left\{ \begin{array}{c} J_E(k) = c_u^T \Delta u^{ss} - c_y^T \Delta y^{ss} \\ = c_u^T \Delta u^{ss} - c_y^T (\boldsymbol{H}(k)\Delta u^{ss} + 0.5(\boldsymbol{I}_{n_y \times n_y} \otimes (\Delta u^{ss})^T)\boldsymbol{B}(k)\Delta u^{ss}) \end{array} \right\}$$

subject to:

$$\begin{aligned} & u_{\min} \leq u^{ss} \leq u_{\max} \\ & y_{\min} \leq y^{ss} \leq y_{\max} \\ & \Delta y^{ss} = \boldsymbol{H}(k)\Delta u^{ss} + b \\ & y^{ss} = F(u^{ss}, \tilde{w}) + \Delta y^{ss} \\ & u^{ss} = u(k-1) + \Delta u^{ss} \end{aligned} \tag{25}$$

which is a standard quadratic programming problem. It is worth emphasising that the quadratic approximation (21) is used in the above SSTO problem only in the objective function whereas the linear approximation is used in the constraints, otherwise it would not be possible to keep all the constraints linear. Unfortunately, it may be the main disadvantage of the linear-quadratic approximate formulation (25), since in many applications at the optimal steady-state operating point the constraints are active. It may be then necessary to use a more accurate approximation of the steady-state characteristic of the process in the constraints than in the objective function $J_E(k)$. Nevertheless, in general, the linear-quadratic approximation is expected to be more accurate than the successive linearization only, especially, if nonlinearity of the steady-state model corresponding to actively constrained outputs is not strong or only process inputs are constrained.

Although in this paper the economic objective function $J_E(k)$ minimised at the LSSO and SSTO layers is a linear combination of input and output values, it is also possible, and in some cases natural for certain processes, that the objective function is nonlinear, for example a quadratic one. In such cases the linear-quadratic approximation is more appropriate than the linearisation.

The linear-quadratic approximation (21) and (23) depends on the current state of the process. If the steady-state characteristic is of appropriate nature, one can imagine that the approximation is a global one, calculated off-line, i.e. the approximation is then not updated on-line since it does not depend on the current operating point. It means that, instead of (23), one has

$$y^{ss} = F(u(k-1), \tilde{w}) + \boldsymbol{H}\Delta u^{ss} + 0.5(\boldsymbol{I}_{n_y \times n_y} \otimes (\Delta u^{ss})^T)\boldsymbol{B}\Delta u^{ss} \tag{26}$$

where the matrices $\boldsymbol{H}$ and $\boldsymbol{B}$ are constant. As far as the adaptive version of the approximation is concerned, in some applications, it may be sufficient to update the matrices $\boldsymbol{H}(k)$ and $\boldsymbol{B}(k)$ not at each sampling instant, but less frequently.

4.4. Linear MPC steady-state target optimisation with piecewise-linear approximations

If the process under consideration has significantly nonlinear steady-state properties, either linear MPC steady-state target optimisation based on successive linearisation or MPC steady-state target optimisation based on successive linear-quadratic approximation may give economic yields still worse than expected (Tatjewski et al., 2006). In most cases the reason is that the linear approximation may simply be too inaccurate. Even if the linear-quadratic approximation is used in the objective function, still the linear approximation must be used for the constraints to keep them linear in the quadratic-programming formulation. The significance of accurate satisfaction of these constraints, which are usually active at the optimum steady-state operating point, is crucial, since they decide of the quality and safety of production. It is then necessary to introduce yet another, more accurate and efficient, approach to nonlinear function approximation to be used in the SSTO problem. A piecewise-linear approximation can be used as a method having the required properties. It is a well known concept in mathematical programming, especially in separable programming. It leads to mixed linear programming problems, or linear programming problems if the solver used allows Special Ordered Sets (SOS) to be declared (Williams, 1995). Similarly to the previously described approximations, the piecewise-linear approximation should also be performed locally at each sampling instant or every few instants, depending on the degree of nonlinearity and the dimensionality of the problem. The approximation can be also done globally, analogously to the linear-quadratic approximation. As far as the local approximation is concerned, it is based on a limited number of points in the neighbourhood of the current operating point, whereas in the global approach the whole region $u_{\min} \leq u^{ss} \leq u_{\max}$, $y_{\min} \leq y^{ss} \leq y_{\max}$ must be considered.

As an alternative to the exact piecewise-linear approximation, a conceptually very similar but reasonably simpler and slightly more approximate approach is to simply check the values of the objective function $J_E(k)$ and satisfaction of the nonlinear inequality constraints imposed on the output variables only for a defined mesh of grid points from the input domain $u_{\min} \leq u^{ss} \leq u_{\max}$. As a result, the solution is found which both satisfies the nonlinear constraints and yields good economic performance. If the obtained solution is too rough, a finer grid mesh around the found solution should be used, i.e. the mesh of points can be selected once again so as to be close to the current region of operation (Tatjewski et al., 2006; Tatjewski 2007). Apparently, the piecewise-linear approximation to be used in the SSTO problem can also be considered as a less complicated, approximate version of the comprehensive nonlinear LSSO problem, but the approximate-based optimisation problem is solved more frequently and it is expected to give improved results only in the vicinity of the current point of approximation.

In case of some processes the introduction of SSTO problem with a linear

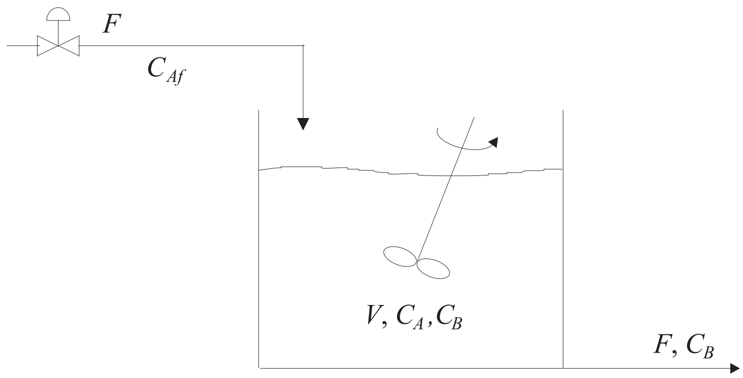

Figure 2. Van de Vusse reactor

constant model or successive linearisations may not give expected set-points leading to economic optimality, whereas the SSTO with quadratic approximation could not be used because of the nonlinear nature of the constraints. Thus, for example, the distillation process has these properties, but the steady-state target optimisation with piecewise-linear approximations was found to be successful (Tatjewski et al., 2006; Tatjewski 2007).

5. Simulation results

5.1. Van de Vusse reactor economic optimisation and control

The considered control plant is an isothermal Continuous Stirred Tank Reactor (CSTR) with van de Vusse reaction depicted in Fig. 2 (Maner et al., 1996). It is often used as a benchmark plant for nonlinear control systems evaluation. Its distinct feature is the shape of static characteristic shown in Fig. 3. The reaction scheme is as follows

$$\begin{aligned} A &\longrightarrow B \longrightarrow C \\ 2A &\longrightarrow D\,. \end{aligned} \tag{27}$$

The process model contains composition balance equations for components A and B

$$\begin{aligned} \frac{dC_A}{dt} &= -k_1 C_A - k_3 C_A^2 + \frac{F}{V}(C_{Af} - C_A) \\ \frac{dC_B}{dt} &= k_1 C_A - k_2 C_B - \frac{F}{V} C_B \end{aligned} \tag{28}$$

where C_A, C_B are the concentrations of components A and B in the reactor, respectively, F is the inlet flow rate (equal to the outlet flow rate), V is the

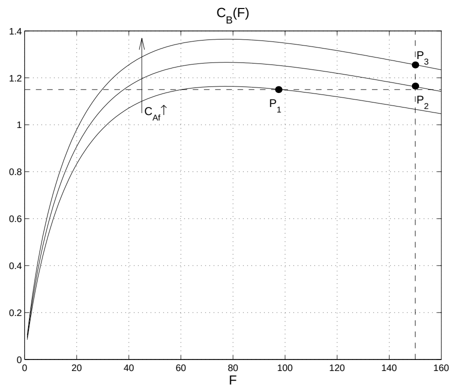

Figure 3. Steady-state characteristic $C_B(F)$ of the van de Vusse reactor, P_i – optimal operating points for different values of disturbance C_{Af}

volume in which the reaction is carried out (it is assumed constant and $V = 1$ l), C_{Af} is the concentration of component A in the inlet flow stream (if it is not stated differently it is assumed that $C_{Af0} = 10\ mol/l$). The values of the kinetic parameters are: $k_1 = 50\ 1/h$, $k_2 = 100\ 1/h$, $k_3 = 10\ l/(h \cdot mol)$. The output variable is C_B, the manipulated variable is F, it is assumed that the disturbance C_{Af} is changing according to the equation

$$C_{Af} = C_{Af0} - \sin\left(\frac{2\pi t}{100}\right). \tag{29}$$

Since maximum production rate is required, the following performance function at the economic optimisation layer is minimised

$$J_E(k) = -F^{ss}. \tag{30}$$

Manipulated variable is constrained

$$F_{\min} \leq F \leq F_{\max} \tag{31}$$

where $F_{\min} = 0\ l/h$, $F_{\max} = 150\ l/h$. It is also assumed that the product should fulfill some purity criteria, i.e. the output variable is constrained

$$C_{B\min} \leq C_B \tag{32}$$

where $C_{B\min} = 1.15\ mol/l$.

Optimal operating points for different values of disturbance $9\ mol/l \leq C_{Af} \leq 11\ mol/l$ are shown in Fig. 3. Particularly, for small concentration of substance A the optimal steady-state operating points P_1 is on the output constraint $C_{B\min}$. For bigger values of C_{Af} the input constraint $F_{\max}$ becomes active (optimal operating points P_2, P_3).

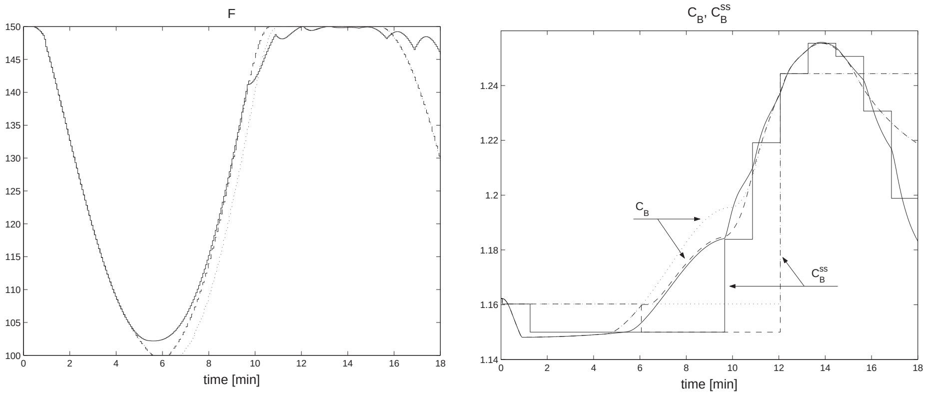

Figure 4. Simulation results of the control system of the van de Vusse reactor in the LSSO+MPC structure with economic optimisation repeated every 72 *s* (*solid line*), 6 *min* (*dashed line*), 12 *min* (*dashdot line*)

The same constraints imposed on manipulated and controlled variables are taken into account in the LSSO, SSTO and MPC optimisation problems. The infeasibility problem, resulting from output constraints in the MPC optimisation problem, was not encountered during the simulations, hence the output constraint could be treated as hard.

In order to design a DMC predictive controller the step response was obtained with sampling time 3.6 *s*. Tuning parameters of the controller are: $N = 30$, $N_u = 15$, $\boldsymbol{M} = \boldsymbol{I}$, $\boldsymbol{\Lambda} = \lambda \boldsymbol{I}$, $\lambda = 0.001$.

Let T_E denote the intervention period of the economic LSSO layer, when it is activated as often as the MPC controller $T_E = 1$. At first, the hierarchical LSSO+MPC structure was considered, intervention frequency of the optimisation layer was changed. The set-point optimisation was repeated every 72 *s* ($T_E = 20$), 6 *min* ($T_E = 100$) and 12 *min* ($T_E = 200$), respectively (Fig. 4). For comparison, temporary values of the economic performance index summed and divided by the number of iterations were calculated. The higher the frequency the better responses are obtained. In case of the optimisation repeated every 72 s (solid line in Fig. 4) performance was $J_E = -133.2657$, if the optimisation was repeated every 6 min (dashed line in Fig. 4) $J_E = -132.2807$, and in the case when the optimisation was repeated every 12 min (dotted line in Fig. 4) $J_E = -131.1948$. It should be noticed that the frequency of the set-point optimisation should be chosen in relation to the dynamics of the disturbances.

In the second experiment, the SSTO layer was introduced into the control system. Simulation results of the LSSO+MPC structure versus the LSSO+MPC +SSTO structure with single linear steady-state model are depicted in Fig. 5, economic optimisation at the LSSO layer was repeated every 6 *min*. Fig. 6 com-

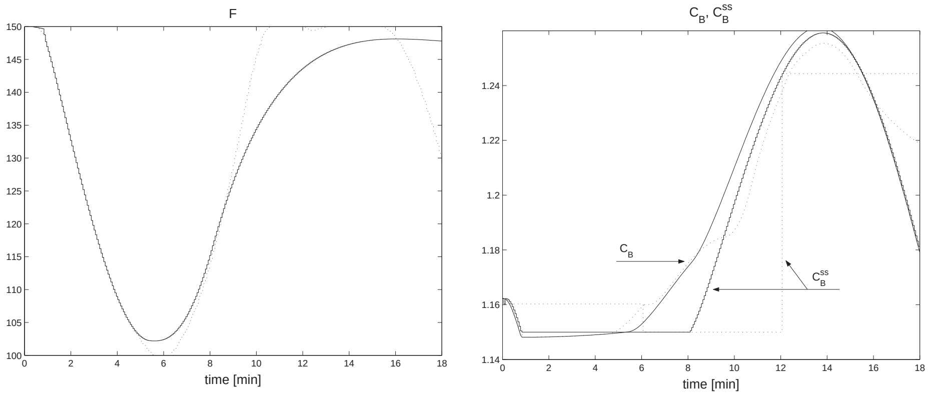

Figure 5. Simulation results of the control system of the van de Vusse reactor in the LSSO+MPC structure (*dotted line*) and in the LSSO+MPC+SSTO structure with single linear steady-state model (*solid line*), economic optimisation in the LSSO layer repeated every 6 *min*

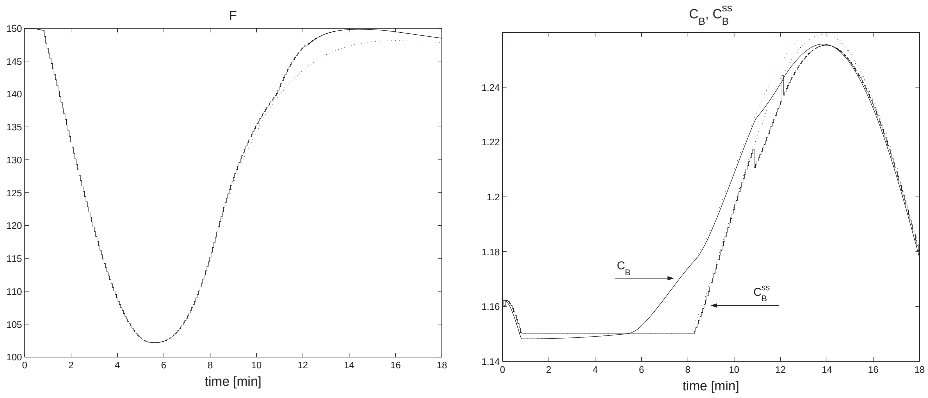

Figure 6. Simulation results of the control system of the van de Vusse reactor in the LSSO+MPC+SSTO structure with single linear steady-state model (*dotted line*) and in the LSSO+MPC+SSTO structure with the linear approximate model updated iteratively (*solid line*), economic optimisation in the LSSO layer repeated every 6 *min*

pares the LSSO+MPC+SSTO structure with single linear steady-state model and the LSSO+MPC+SSTO structure with the linear approximate model updated iteratively. As previously, the LSSO problem was solved every 6 *min*. In the LSSO+MPC+SSTO structure with single linear steady-state model the average performance index $J_E = -131.3750$. It was worse than in the case

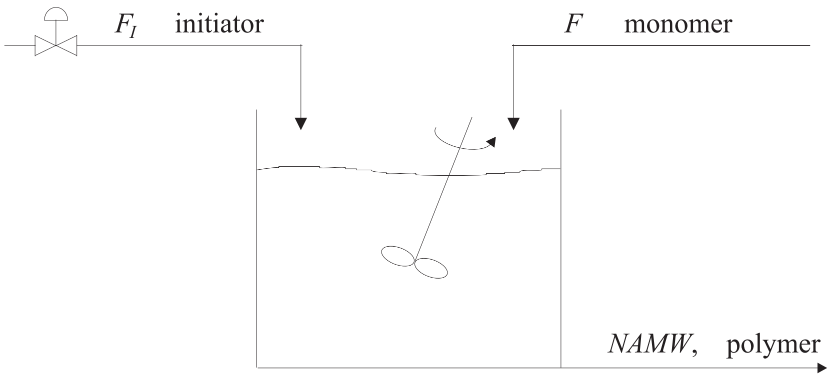

Figure 7. Polymerisation reactor

without the SSTO layer. However, the application of the proposed SSTO with iteratively updated linear model (Fig. 6) brought an improvement of the performance index, which was equal $J_E = -132.2875$.

Experiment in the LSSO+MPC+SSTO structure with iteratively updated quadratic model were also performed. The obtained responses are almost the same as in the case with iteratively updated linear model and shown in Fig. 6, so they are not presented here. The obtained average performance index was slightly better, $J_E = -132.2876$.

5.2. Economic optimisation and control of a polymerisation reactor

The control process is a polymerization reaction taking place in a jacketed continuous stirred tank reactor depicted in Fig. 7 (Maner et al., 1996). The reaction under consideration is the free-radical polymerization of methyl methacrylate with azo-bis-isobutyronitrile as initiator and toluene as solvent. The output $NAMW$ (Number Average Molecular Weight) is controlled by manipulating the inlet initiator flow rate F_I. The main disturbance is the feed flow of the monomer and the solvent stream F.

The fundamental (i.e. first-principle) model is as follows

$$
\begin{aligned}
\frac{dx_1}{dt} &= 10(6 - x_1) - 2.4568 x_1 \sqrt{x_2} \qquad (33)\\
\frac{dx_2}{dt} &= 80F_I - (0.10225 + 10F)x_2\\
\frac{dx_3}{dt} &= 0.0024121 x_1 \sqrt{x_2} + 0.112191 x_2 - 10x_3\\
\frac{dx_4}{dt} &= 245.978 x_1 \sqrt{x_2} - 10x_4\\
NAMW &= \frac{x_4}{x_3}
\end{aligned}
$$

where x_1, x_2, x_3, x_4 are state variables. Empirical dynamic linear model used in the MPC algorithm of the GPC type (Clarke et al., 1987) has the form

$$y(k) = b_2 u(k-2) - a_1 y(k-1) - a_2 y(k-2) \tag{34}$$

where $u(k) = 100(F_I(k) - F_{I0})$, $y(k) = 0.0001(NAMW(k) - NAMW_0)$, $F_{I0} = 0.033566\ m^3/h$, $NAMW_0 = 2.494749 \cdot 10^4\ kg/kmol$. The horizons were set to: $N = 10$ and $N_u = 3$, the weighting matrices to $\boldsymbol{M} = \boldsymbol{I}$, $\boldsymbol{\Lambda} = \lambda \boldsymbol{I}$, $\lambda = 0.2$, the sampling time was set to 1.8 min.

Analogously to the van de Vusse reactor, maximum production rate is required, and the following performance function at the economic optimisation layer is used

$$J_E(k) = -F_I^{ss}. \tag{35}$$

The following constraints are imposed on the manipulated variable

$$F_{I\min} \leq F_I \leq F_{I\max} \tag{36}$$

where $F_{I\min} = 0.0035\ m^3/h$, $F_{I\max} = 0.033566\ m^3/h$. In addition to that, the product should satisfy some purity criteria, i.e. the output variable is constrained

$$NAMW_{\min} \leq NAMW \tag{37}$$

where $NAMW_{\min} = 20000\ kg/kmol$. The same constraints imposed on manipulated and controlled variables are taken into account in the LSSO, SSTO and MPC optimisation problems. Output constraints in the MPC optimisation problem were implemented as soft ones, 2-norm formulation (12) was used. It is assumed that the changes in the disturbance signal can be described by the equation

$$F(k) = F_0 - 1.6(\sin(0.008k) - \sin(0.08)) \tag{38}$$

where $F_0 = 2\ m^3/h$.

Taking into account the constraints imposed on input and output variables, the steady-state characteristic of the polymerisation reactor is depicted in Fig. 8. Optimal operating points for different values of disturbance $0.5\ m^3/h \leq F \leq 2\ m^3/h$ are also shown. In particular, for $F = 2\ m^3/h$ the optimal steady-state operating point P_1 is on the input constraint $F_{I\max}$. Initially, as the value of F decreases, this constraint is also active (optimal operating points P_2, P_3, P_4). Finally, for small values of the disturbance F, this constraint is not active, but the output constraint $NAMW = NAMW_{\min}$ becomes active (optimal operating points P_5, P_6, P_7).

Simulation results of the polymerisation reactor in the LSSO+MPC structure with $T_E = 1$ and with single economic optimisation (performed for sampling instant $k = 3$) are depicted in Fig. 9. Economic optimisation repeated

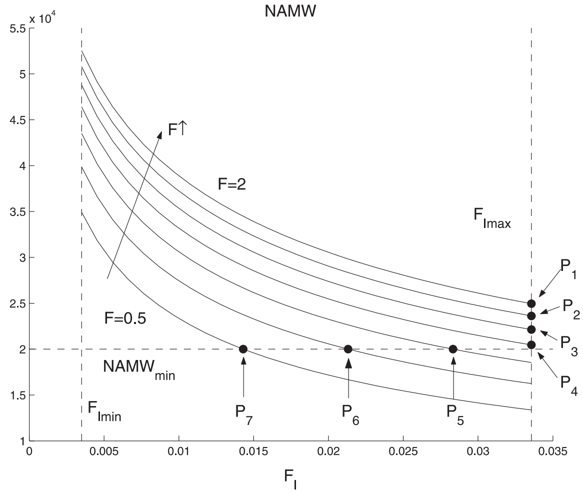

Figure 8. Steady-state characteristic $NAMW(F_I, F)$ of the polymerisation reactor, P_i – optimal operating points for different values of disturbance F

as frequently as the MPC controller executes takes into account changes in the disturbance F, new optimal steady-state operating point is calculated for each sampling instant. Apparently, this structure gives the best economic objective function value (calculated for the whole simulation horizon after completing the simulations), $J_E = -5.2459$. On the contrary, if the nonlinear economic optimisation is performed only once, the set-point value is constant, the economic objective function deteriorates to $J_E = -3.7188$. Figs. 10 and 11 show how the obtained responses of the system with the LSSO+MPC structure change according to different values of T_E, the results for $T_E = 1$ are shown as the reference. Table 1 compares the obtained economic objective function values (calculated for the whole simulation horizon after completing the simulations) for LSSO+MPC structure for different values of the intervention period T_E. In general, analogously as in the case of the van de Vusse reactor considered in the previous subsection, the higher the period, the worse the economic results obtained.

Finally, the LSSO+MPC+SSTO structure was studied. Figs. 12 and 13 depict simulation results obtained in both LSSO+MPC and LSSO+MPC+SSTO structures for different values of T_E. Corresponding values of the economic performance function values J_E are compared in Table 1. In the case of the considered polymerisation reactor, introduction of the steady-state target optimisation into the LSSO+MPC structure with nonlinear economic optimisation layer activated infrequently leads to the same economic performance as in the reference control system with $T_E = 1$. At the SSTO layer an iteratively updated approximate linear steady-state model was used.

Table 1. Comparison of the economic performance function values J_e obtained in LSSO+MPC and LSSO+MPC+SSTO control structures of the polymerisation reactor

T_E	LSSO+MPC	LSSO+MPC+SSTO
1	-5.2459	$-$
2	-5.2459	-5.2459
5	-5.2441	-5.2459
10	-5.2134	-5.2459
15	-5.1763	-5.2459
20	-5.1323	-5.2459
25	-5.1011	-5.2459
30	-5.0426	-5.2459

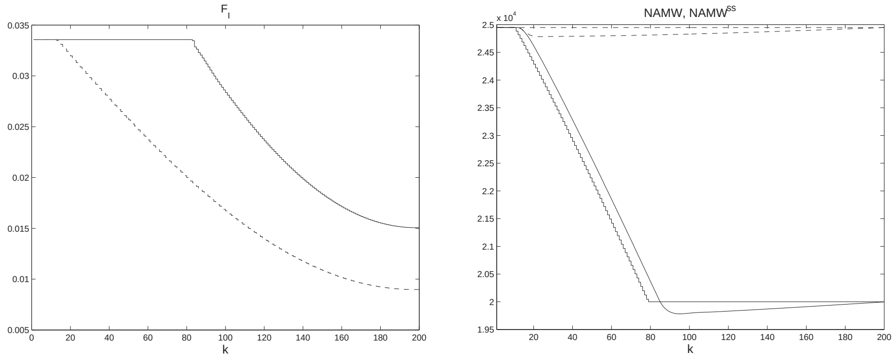

Figure 9. Simulation results of the control system of the polymerisation reactor in the LSSO+MPC structure with $T_E = 1$ (*solid line*), single economic optimisation (*dashed line*)

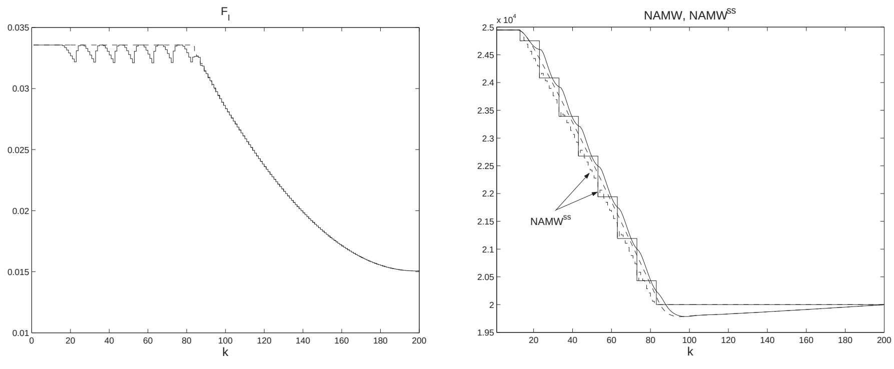

Figure 10. Simulation results of the control system of the polymerisation reactor in the LSSO+MPC structure with $T_E = 1$ (*solid line*), $T_E = 10$ (*dashed line*)

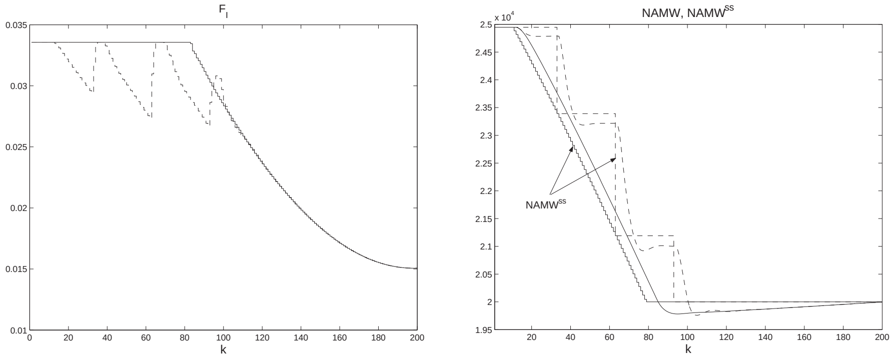

Figure 11. Simulation results of the control system of the polymerisation reactor in the LSSO+MPC structure with $T_E = 1$ (*solid line*), $T_E = 30$ (*dashed line*)

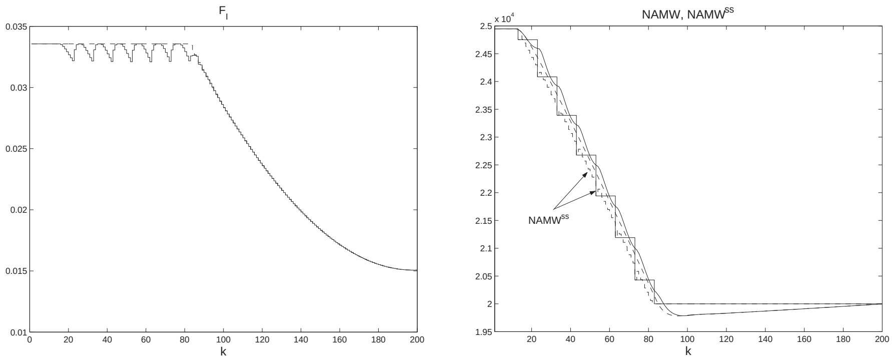

Figure 12. Simulation results of the control system of the polymerisation reactor in the LSSO+MPC structure (*solid line*), the LSSO+MPC+SSTO structure (*dashed line*), $T_E = 10$

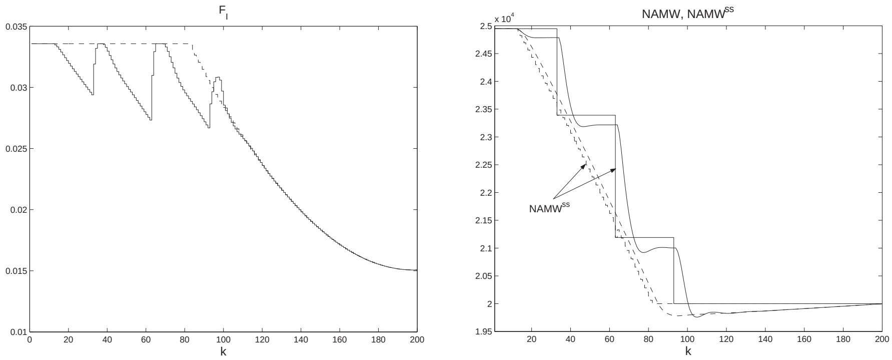

Figure 13. Simulation results of the control system of the polymerisation reactor in the LSSO+MPC structure (*solid line*), and the LSSO+MPC+SSTO structure, (*dashed line*), $T_E = 30$

6. Conclusions

This paper describes the cooperation problem of model predictive control algorithms with steady-state optimisation assuming that the dynamics of disturbances is comparable with that of the process. Since the classical hierarchical multilayer approach is not economically efficient, in such a case an additional steady-state target optimisation (SSTO) layer activated as frequently as the MPC controller executes is introduced. Unlike the typical approach, this paper introduces the SSTO problems with on-line adaptation of approximate models. Specifically, linear, linear-quadratic or piecewise-linear approximate models are considered. It is emphasised that the steady-state model used in the SSTO should be consistent with the comprehensive nonlinear model used at the local steady-state optimisation (LSSO) layer rather than with the linear dynamic one used at the MPC layer.

In spite of the fact that the proposed approaches are straightforward, they have two advantages: *i)* computational efficiency (they result in linear or quadratic programming SSTO problems), *ii)* unlike the standard SSTO with the linear model corresponding to the dynamic one used at the MPC layer, the calculated steady-state values are closer to the optimal ones, which would be calculated by the LSSO using a comprehensive nonlinear model of the plant. The efficiency of the proposed approaches was assessed in control systems of two instances of nonlinear benchmark plants: the van de Vusse reactor and a polymerisation reactor.

For some processes, the cooperation problem of MPC algorithms with steady-state economic optimisation can be also approached in an integrated manner (Ławryńczuk et al., 2006, 2007a, 2007b; Tvrzska and Odolak,1998; Zanin et al., 2000; Zanin et al., 2002). Instead of solving three optimisation problems (i.e. LSSO, SSTO and MPC problems) it is possible to integrate MPC optimisation with economic optimisation, which leads to solving on-line at each sampling instant only one optimisation problem. Using approximate steady-state models, similarly it is done in the SSTO problem, the resulting optimisation task can be transformed into a linear or a quadratic programming one (Ławryńczuk et al., 2006, 2007a, 2007b). Although the integrated approach has more limited applicability than the hierarchical structure described in this paper, it may lead to quite good results.

Acknowledgement

The work presented in this paper was supported by the Polish national budget funds for science in 2005-2007 as a research project.

References

Becerra, V.M., Roberts, P.D. and Griffiths, G.W. (1997) Novel developments in process optimization using predictive control. *Journal of Pro-*

cess Control **8** (2), 117–138.

Blevins, T.L., McMillan, G.K., Wojsznis, W.K. and Brown, M.W. (2003) *Advanced Control Unleashed.* ISA.

Brdys, M.A. and Tatjewski, P. (2005) *Iterative Algorithms for Multilayer Optimizing Control.* Imperial College Press/World Scientific, London.

Clarke, D.W., Mohtadi, C. and Tuffs, P.S. (1987) Generalized predictive control - I. The basic algorithm, II. Extensions and interpretations. *Automatica* **23** (2), 137–160.

Cutler, R. and Ramaker, B.(1979) Dynamic matrix control - a computer control algorithm. *AIChE National Meeting*, Houston.

Doyle, F., Ogunnaike, B.A. and Pearson, R.K. (1995) Nonlinear model-based control using second-order Volterra models. *Automatica* **31** (5), 697–714.

Findeisen, W., Bailey, F.N., Brdys, M., Malinowski, K., Tatjewski, P. and Woniak, A. (1980) *Control and Coordination in Hierarchical Systems.* J.W̃iley & Sons, Chichester, New York, Brisbane, Toronto.

Goodwin, G.C., Graebe, S.F. and Salgado, M.E. (2001) *Control System Design.* Prentice Hall, Upper Saddle River.

Henson, M.A. (1998) Nonlinear model predictive control: current status and future directions. *Computers and Chemical Engineering* **23** (2), 187–202.

Kassmann, D.E., Badgwell, T.A. and Hawkins, R.B. (2000) Robust Steady-State Target Calculation for Model Predictive Control. *AIChE Journal* **46** (5), 1007–1024.

Lautenschlager, M.L.F. and Odloak, D. (1995) Constrained multivariable control of fluid catalytic converters. *Journal of Process Control* **5** (1), 29–39.

Luyben, W.L. (1973) *Process Modeling, Simulation, and Control for Chemical Engineers.* McGraw-Hill.

Ławryńczuk, M., Marusak, P. and Tatjewski, P. (2007a) An efficient MPC algorithm integrated with economic optimisation for MIMO systems. *Proceedings of the 13th IEEE/IFAC International Conference on Methods and Models in Automation and Robotics, MMAR 2007*, Szczecin, Poland, 295–302.

Ławryńczuk, M., Marusak, P. and Tatjewski, P. (2007b) Efficient model predictive control integrated with efficient economic optimisation. *Proceedings of the 15th IEEE Mediterranean Conference on Control and Automation, MED'07*, Athens, Greece, CD-ROM, paper T27-001.

Ławryńczuk, M., Marusak, P. and Tatjewski, P. (2006) Integrating predictive control with steady-state optimisation. *Proceedings of the 12th International Conference on Methods and Models in Automation and Robotics, MMAR 2006*, Międzyzdroje, Poland, 445–452.

Ławryńczuk, M., Marusak, P. and Tatjewski, P. (2005) Optimising predictive range control for a distillation process. *Proceedings of the 11th International Conference on Methods and Models in Automation and Robotics,*

MMAR 2005, Międzyzdroje, Poland, 379–384.
Maciejowski, J.M. (2002) *Predictive Control with Constraints.* Prentice Hall, Harlow.
Maner, B.R., Doyle, F.J., Ogunnaike, B.A. and Pearson, R.K. (1996) Nonlinear model predictive control of a simulated multivariable polymerization reactor using second-order Volterra models. *Automatica* **32** (9), 1285–1301.
Morari, M. and Lee, J. (1999) Model predictive control: past, present and future. *Computers and Chemical Engineering* **23** (4/5), 667–682.
Qin, S.J. and Badgwell, T. (2003) A survey of industrial model predictive control technology. *Control Engineering Practice* **11** (7), 733–764.
Rossiter, J.A. (2003) *Model-Based Predictive Control.* CRC Press, Boca Raton.
Tatjewski, P. (2007) *Advanced Control of Industrial Processes, Structures and Algorithms.* Springer.
Tatjewski, P., Ławryńczuk, M. and Marusak, P. (2006) Linking nonlinear steady-state and target set-point optimisation for model predictive control. *IEE International Conference Control 2006*, Glasgow.
Tvrzska de Gouvea, M. and Odloak, D. (1998) One-layer real time optimization of LPG production in the FCC unit: procedure, advantages and disadvantages. *Computers and Chemical Engineering* **22**, Supplement 1, S191–S198.
Williams, H.P. (1995) *Model Building in Mathematical Programming.* J. Wiley, Chichester, New York.
Zanin, A., Tvrzska de Gouvea, M. and Odloak, D. (2002) Integrating real-time optimization into model predictive controller of the FCC system. *Control Engineering Practice* **10** (8), 819–831.
Zanin, A., Tvrzska de Gouvea, M. and Odloak, D. (2000) Industrial implementation of a real-time optimization strategy for maximizing production of LPG in a FCC unit. *Computers and Chemical Engineering* **24** (2–7), 525–531.