Application of Fuzzy Wiener Models in Efficient MPC Algorithms

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Abstract. Efficient Model Predictive Control (MPC) algorithms based on fuzzy Wiener models are proposed in the paper. Thanks to the form of the model the prediction of the control plant output can be easily obtained. It is done in such a way that the MPC algorithm is formulated as a numerically efficient quadratic optimization problem. Moreover, inversion of the static process model, used in other approaches, is avoided. Despite its relative simplicity the algorithm offers practically the same performance as the MPC algorithm in which control signals are generated after solving a nonlinear optimization problem and outperforms the MPC algorithm based on a linear model. The efficacy of the proposed approach is demonstrated in the control system of a nonlinear control plant.

Keywords: fuzzy systems, fuzzy control, predictive control, nonlinear control, constrained control.

1 Introduction

Model predictive control (MPC) algorithms are widely used in practice. It is because they offer very good control performance even for control plants which are difficult to control using other algorithms [4,9,15,18]. The essential feature of these algorithms is to use a control plant model to predict behavior of the control system. Thanks to such an approach, the MPC algorithms are formulated in such a way that constraints existing in the control system can be relatively easily taken into consideration. Moreover, it is possible to use all information about control system operation and on conditions in which it operates to improve prediction and, as a result, operation of an MPC algorithm.

In standard MPC algorithms linear control plant models are used for prediction. Then an algorithm can be formulated as an easy to solve, quadratic optimization problem. Moreover, in the unconstrained case, a control law can be easily obtained. Unfortunately, application of such an MPC algorithm to a nonlinear plant may bring unsatisfactory results or the results can be improved using the algorithm based on a nonlinear model. This problem is especially important if the control system should work in a wide range of set point values.

Direct application of a nonlinear process model to design the MPC algorithm does not solve all issues. It is because it leads to formulation of the algorithm as

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a nonlinear, and in general, non–convex optimization problem. Such a problem is hard to solve and computationally expensive. The approach which does not have these drawbacks consists in obtaining a linear approximation of the nonlinear model at each iteration of the algorithm. It can be done in an efficient way if a control plant is described using a Wiener model.

The Wiener models are composed of a linear dynamic block preceding a nonlinear static block (Fig. 1) [7]. Such a structure of the model simplifies the synthesis of the controllers based on Wiener models. Therefore, Wiener models are often used to model control plants for control purposes; see e.g. [2,10,16].

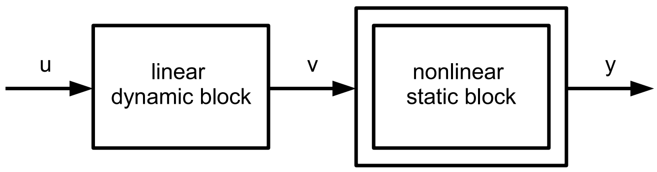

Fig. 1. Structure of the Wiener model; u – input, y – output, v – input of the nonlinear static block

The most popular method of application of Wiener models in the MPC algorithms, in such a way that computationally efficient quadratic optimization problem is solved at each iteration, is to use inverse of the static part of the model; see e.g. [1,16]. On the contrary, in the method proposed in the paper the calculation of the inverse of the static part of the model is avoided. Moreover, the prediction can be performed in a straightforward way what does not influence control performance in a negative way. It is demonstrated in the example control system of a nonlinear plant fuzzy model of which was obtained heuristically.

In the next section the idea of the MPC algorithms is described. Next, the MPC algorithms based on fuzzy Wiener models are proposed. Sect. 4 contains presentation of results obtained in the control system of the nonlinear plant, illustrating excellent performance offered by the proposed approach. The paper is summarized in the last section.

2 MPC Algorithms – Basic Information

The Model Predictive Control (MPC), during control signal generation, predict future behavior of the control plant many sampling instants ahead using a process model. The control signal is derived in such a way that the prediction fulfills assumed criteria. These criteria are, usually, formulated as the following optimization problem [4,9,15,18]:

$$\min_{\Delta \boldsymbol{u}} \left\{ J_{\mathrm{MPC}} = \sum_{i=1}^{p} \left(\overline{y}_k - y_{k+i|k} \right)^2 + \sum_{i=0}^{s-1} \lambda \left(\Delta u_{k+i|k} \right)^2 \right\} \tag{1}$$

subject to:

$$\Delta \boldsymbol{u}_{\min} \leq \Delta \boldsymbol{u} \leq \Delta \boldsymbol{u}_{\max} \ , \tag{2}$$

$$\boldsymbol{u}_{\min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{\max} \ , \tag{3}$$

$$\boldsymbol{y}_{\min} \leq \boldsymbol{y} \leq \boldsymbol{y}_{\max} \ , \tag{4}$$

where $\overline{y}_k$ is a set–point value, $y_{k+i|k}$ is a value of the output for the $(k+i)^{\text{th}}$ sampling instant, predicted at the k^{th} sampling instant, $\Delta u_{k+i|k}$ are future changes of the control signal, $\lambda \geq 0$ is a tuning parameter, p and s denote prediction and control horizons, respectively; $\Delta \boldsymbol{u} = \left[\Delta u_{k+1|k}, \ldots, \Delta u_{k+s-1|k}\right]$, $\boldsymbol{u} = \left[u_{k+1|k}, \ldots, u_{k+s-1|k}\right]$, $\boldsymbol{y} = \left[y_{k+1|k}, \ldots, y_{k+p|k}\right]$; $\Delta \boldsymbol{u}_{\min}$, $\Delta \boldsymbol{u}_{\max}$, $\boldsymbol{u}_{\min}$, $\boldsymbol{u}_{\max}$, $\boldsymbol{y}_{\min}$, $\boldsymbol{y}_{\max}$ are vectors of lower and upper limits of changes and values of the control signal and of the values of the output signal, respectively. The optimization problem (1–4) is solved at each iteration of the algorithm. Its solution is the optimal vector of changes of the control signal. From this vector, the first element is applied to the control plant and then the optimization problem is solved again in the next iteration of the MPC algorithm.

The predicted output variables $y_{k+j|k}$ are derived using a dynamic control plant model. If this model is nonlinear then the optimization problem (1–4) is nonlinear and, in general, non–convex and hard to solve. Examples of this kind of algorithms utilizing fuzzy models one can find e.g. in [3,5] and those utilizing Wiener models – e.g. in [2,10].

If the model used in the MPC algorithm is linear then the optimization problem (1–4) is a standard quadratic programming problem [4,9,15,18]. It is because the superposition principle can be applied and the vector of predicted output values $\boldsymbol{y}$ is given by the following formula:

$$\boldsymbol{y} = \widetilde{\boldsymbol{y}} + \boldsymbol{A} \cdot \Delta \boldsymbol{u} \ , \tag{5}$$

where $\widetilde{\boldsymbol{y}} = \left[\widetilde{y}_{k+1|k}, \ldots, \widetilde{y}_{k+p|k}\right]$ is a free response (contains future values of the output signal calculated assuming that the control signal does not change in the prediction horizon); $\boldsymbol{A} \cdot \Delta \boldsymbol{u}$ is the forced response (depends only on future changes of the control signal (decision variables));

$$\boldsymbol{A} = \begin{bmatrix} a_1 & 0 & \ldots & 0 & 0 \\ a_2 & a_1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_p & a_{p-1} & \ldots & a_{p-s+2} & a_{p-s+1} \end{bmatrix} \tag{6}$$

is the dynamic matrix composed of coefficients of the control plant step response a_i; see e.g. [4,9,15,18].

Let us introduce the vector $\overline{\boldsymbol{y}} = [\overline{y}_k, \ldots, \overline{y}_k]$ of length p. The performance function from (1), after application of the prediction (5), can be rewritten in the matrix–vector form:

$$J_{\text{LMPC}} = (\overline{\boldsymbol{y}} - \widetilde{\boldsymbol{y}} - \boldsymbol{A} \cdot \Delta \boldsymbol{u})^T \cdot (\overline{\boldsymbol{y}} - \widetilde{\boldsymbol{y}} - \boldsymbol{A} \cdot \Delta \boldsymbol{u}) + \Delta \boldsymbol{u}^T \cdot \boldsymbol{\Lambda} \cdot \Delta \boldsymbol{u} \ , \tag{7}$$

where $\boldsymbol{\Lambda} = \lambda \cdot \boldsymbol{I}$ is the $s \times s$ matrix. The performance function (7) depends quadratically on decision variables $\Delta\boldsymbol{u}$. Thus, the optimization problem is in this case a quadratic one. Moreover, if the constraints need not be taken into consideration, the vector minimizing this performance function is given by the following formula:

$$\Delta\boldsymbol{u} = \left(\boldsymbol{A}^T \cdot \boldsymbol{A} + \boldsymbol{\Lambda}\right)^{-1} \cdot \boldsymbol{A}^T \cdot (\overline{\boldsymbol{y}} - \widetilde{\boldsymbol{y}}) \ . \tag{8}$$

The advantages offered by the quadratic optimization led to design of MPC algorithms based on linear approximations of the nonlinear process models obtained at each iteration; see e.g. [8,18]. The algorithms of this type based on fuzzy process models one can find e.g. in [11,12,13].

3 Efficient MPC Algorithms Based on Fuzzy Wiener Models

The Wiener process model (Fig. 1) with fuzzy static block is considered. It is assumed that the static part of the model is a fuzzy Takagi–Sugeno model which consists of the following rules:

Rule j: if v_k is M_j, then

$$y_k^j = g_j \cdot v_k + h_j, \tag{9}$$

where g_j, h_j are coefficients of the model, M_j are fuzzy sets, $j = 1, \ldots, l$, l is the number of fuzzy rules (local models).

The output of the static part of the model is described by the following formula:

$$\widehat{y}_k = \sum_{j=1}^{l} w_j(v_k) \cdot y_k^j \ , \tag{10}$$

where $\widehat{y}_k$ is the output of the static block (and the output of the Wiener model), v_k is the input to the static block and the output of the dynamic block, $w_j(v_k)$ are weights obtained using fuzzy reasoning (see e.g. [14,17]). Therefore, the output of the Wiener model can be described by:

$$\widehat{y}_k = \widetilde{g}_k \cdot v_k + \widetilde{h}_k \ , \tag{11}$$

where $\widetilde{g}_k = \sum_{j=1}^{l} w_j(v_k) \cdot g_j$, $\widetilde{h}_k = \sum_{j=1}^{l} w_j(v_k) \cdot h_j$ It is assumed that the dynamic part of the model is a difference equation (a model often used in linear dynamic block of the Wiener models):

$$v_k = b_1 \cdot v_{k-1} + \ldots + b_n \cdot v_{k-n} + c_1 \cdot u_{k-1} + \ldots + c_m \cdot u_{k-m} \ , \tag{12}$$

where b_j, c_j are parameters of the linear model.

Thus, the output of the Wiener model is given by the following formula:

$$\widehat{y}_k = \widetilde{g}_k \cdot \left(\sum_{j=1}^{n} b_j \cdot v_{k-j} + \sum_{j=1}^{m} c_j \cdot u_{k-j} \right) + \widetilde{h}_k \ , \tag{13}$$

In the proposed approach the fuzzy (nonlinear) Wiener model is used to obtain the free response of the plant. Thanks to the structure of the Wiener model it can be obtained in a straightforward way.

The output of the linear part of the model in the $(k+i)^{\mathrm{th}}$ sampling instant calculated after assumption of constant control values $(u_k = u_{k+1} = \ldots = u_{k+p})$ is described by the following formula:

$$\widehat{v}_{k+i} = \sum_{j=1}^{n} b_j \cdot \widehat{v}_{k-j+i} + \sum_{j=1}^{i} c_j \cdot u_k + \sum_{j=i+1}^{m} c_j \cdot u_{k-j+i} \ , \tag{14}$$

where $\widehat{v}_{k+i}$ are values of the internal signal of the model obtained after assumption of constant control values. The free response is calculated taking into consideration also the estimated disturbances and modeling errors:

$$d_k = y_k - \widehat{y}_k \ . \tag{15}$$

The final formula describing the elements of the free response is, thus, as follows:

$$\widetilde{y}_{k+i|k} = \widetilde{g}_k \cdot \left(\sum_{j=1}^{n} b_j \cdot \widehat{v}_{k-j+i} + \sum_{j=1}^{i} c_j \cdot u_k + \sum_{j=i+1}^{m} c_j \cdot u_{k-j+i} \right) + \widetilde{h}_k + d_k \ , \tag{16}$$

where d_k is the DMC–type disturbance model, i.e. it is assumed the same on the whole prediction horizon.

Next, the dynamic matrix, needed to predict the influence of the future control changes should be derived. It can be done in a straightforward way. First, one should obtain the step response coefficients of the dynamic part of the Wiener model a_n ($n = 1, \ldots, p_d$; p_d is the dynamics horizon equal to the number of sampling instants after which the step response can be assumed as settled). Then, the proper value of gain must be derived. It can be noticed that it can be approximated by:

$$dy = \frac{\left(\left(\sum_{j=1}^{l} w_j(v_k) \cdot (g_j \cdot v_k + h_j) \right) - \left(\sum_{j=1}^{l} w_j(v_{k-}) \cdot (g_j \cdot (v_{k-}) + h_j) \right) \right)}{dv} \ , \tag{17}$$

where $v_{k-} = v_k - dv$, dv is a small number. Thus, at each iteration of the algorithm the following linear approximation of the fuzzy Wiener model (13) is used:

$$\widehat{y}_k = dy \cdot \left(\sum_{n=1}^{p_d-1} a_n \cdot \Delta u_{k-n} + a_{p_d} \cdot u_{k-p_d} \right) \ . \tag{18}$$

The dynamic matrix will be therefore described by the following formula:

$$\boldsymbol{A}_k = dy \cdot \begin{bmatrix} a_1 & 0 & \dots & 0 & 0 \\ a_2 & a_1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_p & a_{p-1} & \dots & a_{p-s+2} & a_{p-s+1} \end{bmatrix} . \tag{19}$$

The free response (16) and the dynamic matrix (19) are used to obtain the prediction:

$$\boldsymbol{y} = \widetilde{\boldsymbol{y}} + \boldsymbol{A}_k \cdot \Delta\boldsymbol{u} \ . \tag{20}$$

After application of prediction (20) to the performance function from (1), one obtains:

$$J_{\mathrm{FMPC}} = (\overline{\boldsymbol{y}} - \widetilde{\boldsymbol{y}} - \boldsymbol{A}_k \cdot \Delta\boldsymbol{u})^T \cdot (\overline{\boldsymbol{y}} - \widetilde{\boldsymbol{y}} - \boldsymbol{A}_k \cdot \Delta\boldsymbol{u}) + \Delta\boldsymbol{u}^T \cdot \boldsymbol{\Lambda} \cdot \Delta\boldsymbol{u} \ . \tag{21}$$

Thus, as in the case of the MPC algorithm based on a linear model, a quadratic optimization problem is obtained.

4 Testing of the Proposed Approach

4.1 Control Plant – Description and Fuzzy Modeling

The control plant under consideration is a valve for control of fluid flow. It is described by the following Wiener model [1,6]:

$$v_k = 1.4138 \cdot v_{k-1} - 0.6065 \cdot v_{k-2} + 0.1044 \cdot u_{k-1} + 0.0883 \cdot u_{k-2} \ , \tag{22}$$

$$y_k = \frac{0.3163 \cdot v_k}{\sqrt{0.1 + 0.9 \cdot (v_k)^2}} \ , \tag{23}$$

where u_k is the pneumatic control signal applied to the stem, v_k is the stem position (it is the output signal of the linear dynamic block and the input signal of the nonlinear static block), y_k is flow through the valve (it is the output of the plant). The static part of the model was approximated using the fuzzy model. It was done heuristically because the nonlinear function in the control plant model (23) resembles the sigmoid function; see Fig. 2.

As a result of a few experiments the simple fuzzy model of the statics of the control plant was obtained. It consists of two rules:

Rule 1: if v_k is M_1, then

$$y^1_{k+1} = -0.3289, \tag{24}$$

Rule 2: if v_k is M_2, then

$$y^2_{k+1} = 0.3289. \tag{25}$$

The assumed membership functions are shown in Fig. 3. Fuzzy approximation of the static nonlinearity is presented as the dashed line in Fig. 2.

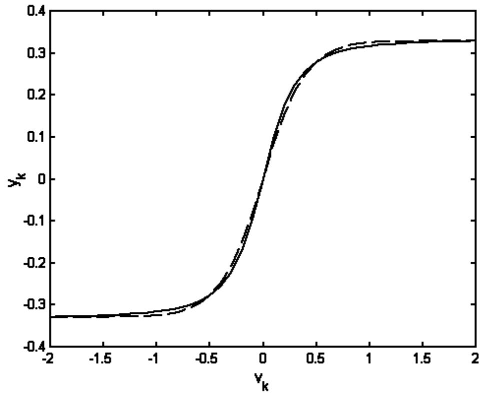

Fig. 2. Static characteristic of the valve; solid line – original model, dashed line – fuzzy approximation

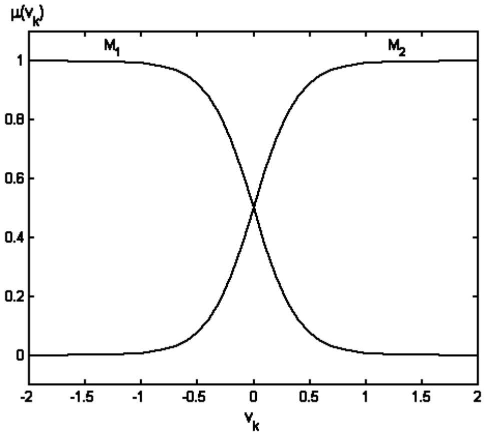

Fig. 3. Membership functions in the fuzzy model of the static characteristic of the valve

4.2 Simulation Experiments

The operation of the proposed MPC algorithm was compared with other approaches. Thus, three MPC algorithms were designed for the considered control plant:

1. LMPC – with a linear model,
2. NMPC – with nonlinear optimization,
3. FMPC – with prediction based on fuzzy Wiener model.

Tuning parameters of all three algorithms were assumed the same and: prediction horizon $p = 30$, control horizon $s = 15$, weighting coefficient $\lambda = 4$.

Performance of control systems with LMPC, NMPC and FMPC algorithms was compared. The example responses obtained after changes of the set–point value from 0 to 0.3 at the beginning of the experiment and then back from 0.3 to 0 in the half of the experiment are shown in Fig. 4. The LMPC algorithm gives the worst responses (dashed lines in Fig. 4). They are much slower than those obtained using other MPC algorithms. The responses obtained in the control systems with the FMPC (solid lines in Fig. 4) and NMPC algorithms (dotted lines in Fig. 4) are very similar. However, in the FMPC algorithm the control signal is generated much faster as a solution of the quadratic programming problem.

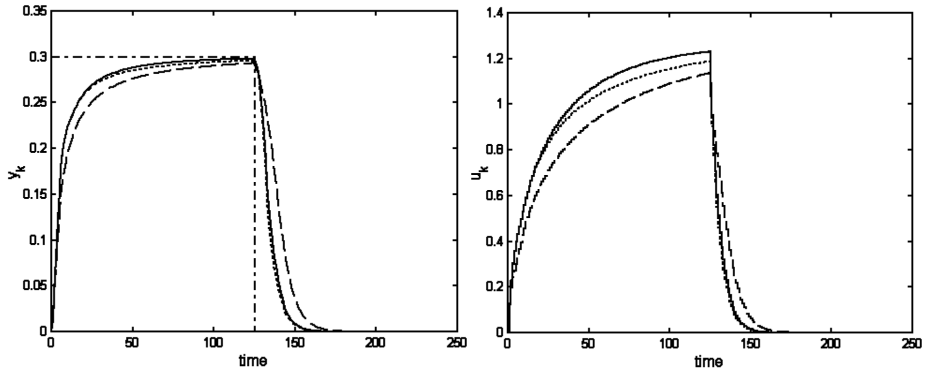

Fig. 4. Responses of the control systems to the changes of the set–point value to $\overline{y}_1 = 0.3$ and $\overline{y}_2 = 0$; FMPC (solid lines), NMPC (dotted lines), LMPC (dashed lines); dash–dotted line – set–point signal; right – output signal, left – control signal

5 Summary

The MPC algorithms proposed in the paper are based on fuzzy Wiener models. They use the nonlinear process model to derive the free response and its linear approximation to derive the forced response. Thanks to the form of the control plant model the prediction is easy to derive. The proposed algorithms are formulated as the efficient linear–quadratic optimization problems but they offer practically the same performance as the algorithms consisting in nonlinear optimization outperforming their counterparts based on linear process models.

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