Diffusion Processes On Complex Networks

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1 Assignment 1

1. Task 1

    (a) Figure 1 represents of network defined by table where nodes are named as first letters of people names.
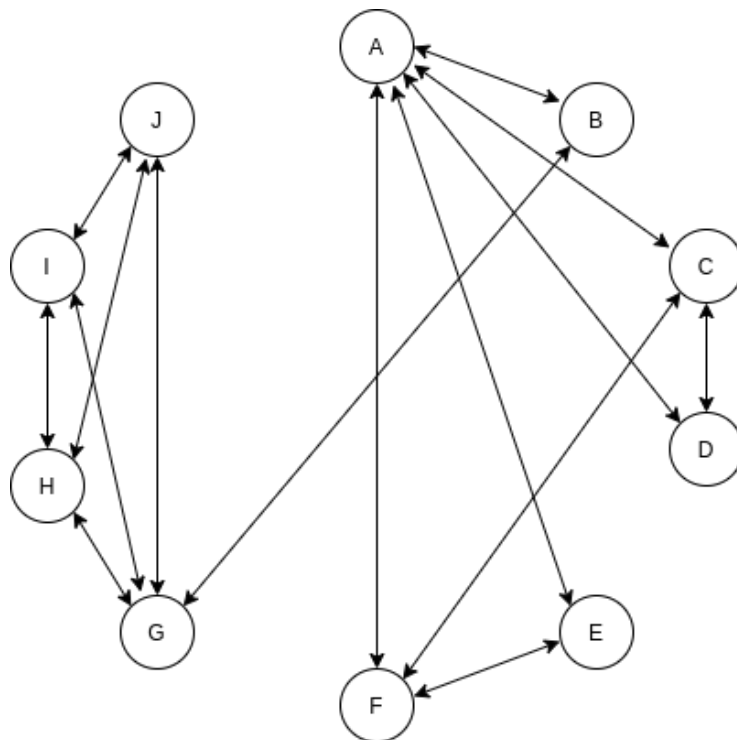

Figure 1: Network

    (b) # of nodes = 10.

(c) Density $= \frac{15}{\frac{n(n-1)}{2}} = \frac{1}{3}$.

(d) Degrees: $K_A = 5$, $K_B = 2$, $K_C = 3$, $K_D = 2$, $K_E = 2$, $K_F = 3$, $K_G = 4$, $K_H = 3$, $K_I = 3$, $K_J = 3$.

In this measure the node A is the most central.

(e) Clustering: $C_A = \frac{3}{10}$, $C_B = 0$, $C_C = \frac{2}{3}$, $C_D = 1$, $C_E = 1$, $C_F = \frac{2}{3}$, $C_G = \frac{1}{2}$, $C_H = C_I = C_J = 1$.

Average clustering: $<C> = \frac{\frac{3}{10} + \frac{2}{3} + 1 + 1 + \frac{2}{3} + \frac{1}{2} \cdot + 1 + 1 + 1}{10} = \frac{107}{150}$.

(f) Closeness centrality:

- $C_C(A) = \frac{1}{1+1+1+1+1+2+3+3+3} = \frac{1}{16}$;
- $C_C(B) = \frac{1}{1+2+2+2+2+1+2+2+2} = \frac{1}{16}$;
- $C_C(C) = \frac{1}{1+2+1+2+1+3+4+4+4} = \frac{1}{22}$;
- $C_C(D) = \frac{1}{1+2+1+2+2+3+4+4+4} = \frac{1}{23}$;
- $C_C(E) = \frac{1}{23}$;
- $C_C(F) = \frac{1}{22}$;
- $C_C(G) = \frac{1}{2+1+3+3+3+3+1+1+1} = \frac{1}{18}$;
- $C_C(H) = C_C(I) = C_C(J) = \frac{1}{3+2+4+4+4+4+1+1+1} = \frac{1}{24}$.

According closeness centrality the most central nodes are A and B.

(g) Betweeness centrality:

- $C_B(A) = 0.61$;
- $C_B(B) = 0.56$;
- $C_B(C) = 0.01$;
- $C_B(D) = 0$;
- $C_B(E) = 0$;
- $C_B(F) = 0.01$;
- $C_B(G) = 0.56$;
- $C_B(H) = C_B(I) = C_B(J) = 0$.

According to betweeness centrality the most central node is A with value 0.61.

2. Task 2

   The point of the task 2 was to realize two things. First, prepare the csv file, which content is presented at Listing 1. Second, use the the Gephi software to visualize the graph and calculate it's properties, what is shown by 2.

Listing 1: network.csv

```
1   Alice ,Bob
2   Carl , Alice
3   Alice ,David
4   Alice ,Ernst
5   Alice ,Frank
6   Bob, Gail
7   Gail ,Harry
8   Harry ,Jen
9   Jen ,Gail
10  Harry ,Irene
11  Irene ,Gail
12  Irene ,Jen
13  Ernst ,Frank
14  David ,Carl
15  Carl ,Frank
```
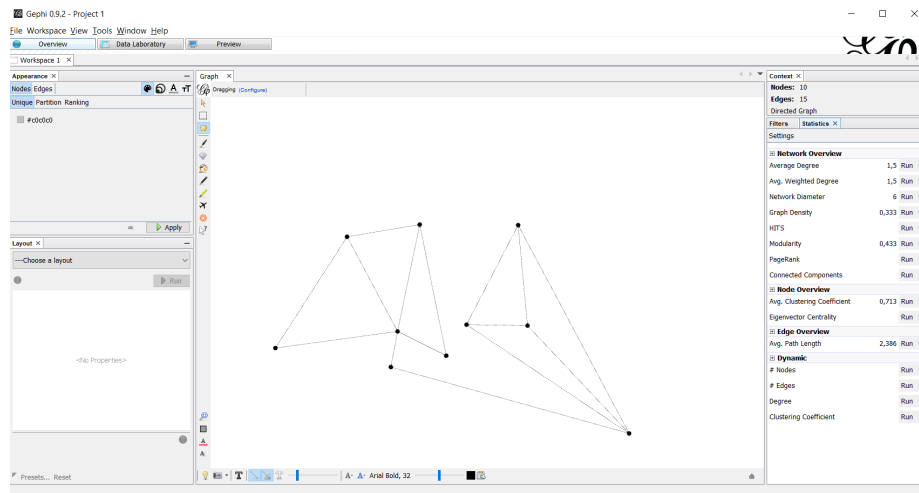

Figure 2: Gephi

3. Task 3

   (a) $K = e^T A$.

   (b) $L = \frac{1}{2} I_1 (e^T A)^T$ where $I_1 = [1]$.

   (c) $N = A^2$.

   (d) -

   (e) Write down number 1 and look at the first row. Write down numbers of columns with value 1. Next, take one written number e.g. i, look at i's row and add to the list numbers of columns with value 1. Repeat last sentense, until you have checked all written numbers.
   If you have written numbers of all rows, then the graph is connected, otherwise the graph is disconnected.