4 (done)

25 October, 2023 16:47

L5.4. $\boxed{\text{1 punkt}}$ Które z ciągów: $\frac{1}{n^2}, \frac{1}{2^{2^n}}, \frac{1}{\sqrt{n}}, \frac{1}{e^n}, \frac{1}{n^n}$ są zbieżne kwadratowo? Odpowiedź uzasadnij.

Warunek: zbieżny, oraz:

$$\underbrace{\frac{|x_{n+1}-\alpha|}{|x_n-\alpha|^p} = C \quad C>0}_{\text{(kwadratowy dla } p=2)}$$

a)

$$\lim_{n\to\infty} \frac{1}{n^2} = 0$$

dla $p=1$, $C\in(0,1)$

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)^2}}{\left(\frac{1}{n^2}\right)^2} = \lim_{n\to\infty} \frac{n^4}{n^2+2n+1} = \lim_{n\to\infty} n^2 = +\infty$$

b)

$$\lim_{n\to\infty} \frac{1}{2^{2^n}} = 0$$

$$\lim_{n\to\infty} \frac{\frac{1}{2^{2^{n+1}}}}{\left(\frac{1}{2^{2^n}}\right)^2} = \lim_{n\to\infty} \frac{(2^{2^n})^2}{2^{2^{n+1}}} = \lim_{n\to\infty} \frac{2^{2^n\cdot 2}}{2^{2^n\cdot 2}} = 1$$

c)

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$$

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+1}}}{\left(\frac{1}{\sqrt{n}}\right)^2} = \lim_{n\to\infty} \frac{(\sqrt{n})^2}{\sqrt{n+1}} = \lim_{n\to\infty} \frac{n}{\sqrt{n+1}} = +\infty$$

d)

$$\lim_{n\to\infty} \frac{1}{e^n} = 0$$

$$\lim_{n\to\infty} \frac{\frac{1}{e^{n+1}}}{\left(\frac{1}{e^n}\right)^2} = \lim_{n\to\infty} \frac{(e^n)^2}{e^{n+1}} = \lim_{n\to\infty} e^{2n-n-1} = \lim_{n\to\infty} e^{n-1} = \infty$$

e)

$$\lim_{n\to\infty} \frac{1}{n^n} = 0$$

$$\lim_{n\to\infty} \frac{n^{2n}}{(n+1)^{n+1}} = \lim_{n\to\infty} \frac{n^{2n}}{n^{n}(1+\frac{1}{n})^n(n+1)} = \lim_{n\to\infty} \frac{n^{n}}{e(n+1)} =$$

$$\lim_{n\to\infty} \frac{n^n}{e\, n(1+\frac{1}{n})} = \lim_{n\to\infty} \frac{n^{n-1}}{e} = +\infty$$

n^{n-1} rośnie szybciej niż 2^n