8 (done)

10 January, 2024 17:05

L12.8. $\boxed{\text{1 punkt}}$ Jak należy dobrać n, aby stosując złożony wzór Simpsona S_n obliczyć przybliżoną wartość całki $\int_{-3\pi}^{\pi/6} \sin(5x - \pi/3)\,dx$ z błędem względnym $\leq 10^{-10}$?

$$R_n(f) = (a-b)\frac{h_n^4}{180} \cdot f^{(4)}(\alpha) = \frac{(a-b)\cdot(b-a)^4}{n^4 \cdot 180} f^{(4)}(\alpha)$$

$\text{względny} = \left|\frac{R}{c}\right|$ c - rzeczywista wartość całki

$$-\frac{1}{5}\cos\left(5x-\frac{\pi}{3}\right)\Big|_{-3\pi}^{\frac{\pi}{6}}$$

$$\int_{-3\pi}^{\frac{\pi}{6}} \sin\left(5x-\frac{\pi}{3}\right)dx = -\frac{1}{5}\cos\left(5\cdot\frac{\pi}{6}-\frac{\pi}{3}\right) + \frac{1}{5}\cos\left(5\cdot 3\pi - \frac{\pi}{3}\right) =$$

$$-\frac{1}{5}\cos\frac{\pi}{2} + \frac{1}{5}\cos\underset{14\frac{2\pi}{3}}{\frac{44\pi}{3}} = 0 + \frac{1}{5}\cdot\left(-\frac{1}{2}\right) = -\frac{1}{10}$$

$$\int \sin\left(5x-\frac{\pi}{3}\right)dx = \left\{\begin{matrix} 5x-\frac{\pi}{3}=t \\ 5dx = dt \end{matrix}\right\} =$$

$$\int \sin(t)\frac{1}{5}dt = \frac{1}{5}(-\cos(t)) = -\frac{1}{5}\cos\left(5x-\frac{\pi}{3}\right)$$

$$f'(x) = 5\sin\left(5x+\frac{\pi}{6}\right)$$

$$f''(x) = 25\cos\left(5x+\frac{\pi}{6}\right)$$

$$f'''(x) = -125\sin\left(5x+\frac{\pi}{6}\right)$$

$$f''''(x) = -625\cos\left(5x+\frac{\pi}{6}\right)$$

$$\max\left|f^{(4)}\right| = 625$$

wyjątkiem $-$ z $(b-a)$ i z $-\frac{1}{10}$

$$\left|\frac{R}{c}\right| \leq 10^{-10}$$

$$\frac{(b-a)^5}{\frac{1}{10}\cdot n^4 \cdot 180}\left|f^{(4)}(\alpha)\right| \leq 10^{-10}$$

$$n^4 \geq \frac{(b-a)^5}{\frac{1}{10}\cdot 180 \cdot 10^{-10}} \cdot 625$$

$$b-a = \frac{19\pi}{6}$$

$$n^4 \geq \frac{19^5 \cdot 625}{18\cdot 6^5} \cdot 10^{10}$$

kalkulator

$$n \geq \sim 3242{,}685$$