

L7.2. 1 punkt Używając postaci Newtona, podaj wielomian interpolacyjny dla następujących danych:

a)  $\begin{array}{c|ccc} x_k & -3 & 0 & 3 & 4 \\ y_k & 2 & 2 & 38 & 142 \end{array}$ , b)  $\begin{array}{c|ccc} x_k & 3 & 4 & -3 & -1 & 0 \\ y_k & 38 & 142 & 2 & 62 & 2 \end{array}$

c)  $\begin{array}{c|ccc} x_k & -4 & -3 & -2 & 2 & 3 & 4 \\ y_k & 9 & 7 & 5 & -3 & -5 & 1977 \end{array}$

Uwaga. Na pewno zauważysz, że rozwiązując podpunkty b) oraz c) nie musisz wykonywać wielu obliczeń.

Wzór:

$$w(x) = a_0 + \sum_{i=1}^n a_i \prod_{j=0}^{i-1} (x - x_j) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_{n-1}) \dots (x - x_1)(x - x_0)$$

a)  $\begin{array}{c|ccc} x_k & -3 & 0 & 3 & 4 \\ y_k & 2 & 2 & 38 & 142 \end{array}$

$x_0 = -3, y_0 = 2$   
 $x_1 = 0, y_1 = 2$   
 $x_2 = 3, y_2 = 38$   
 $x_3 = 4, y_3 = 142$

$f[x_1, x_0] = \frac{2-2}{0-(-3)} = 0 = a_1$   
 $f[x_2, x_1, x_0] = \frac{38-2}{3-0} = 12$   
 $f[x_3, x_2, x_1, x_0] = \frac{142-38}{4-3} = 104$   
 $x_0 = -3, y_0 = 2$   
 $x_1 = 0, y_1 = 2$   
 $x_2 = 3, y_2 = 38$   
 $x_3 = 4, y_3 = 142$

$w(x) = 2 + 2x(x+3) + 3x(x-3)(x+3) = 3x^3 + 2x^2 - 21x + 2$

b)  $\begin{array}{c|ccc} x_k & 3 & 4 & -3 & -1 & 0 \\ y_k & 38 & 142 & 2 & 62 & 2 \end{array}$

$x_0 = 3, y_0 = 38$   
 $x_1 = 4, y_1 = 142$   
 $x_2 = -3, y_2 = 2$   
 $x_3 = -1, y_3 = 62$   
 $x_4 = 0, y_4 = 2$

$x_4 = 0, y_4 = 2$

Wzór:

$$w(x) = a_0 + \sum_{i=1}^n a_i \prod_{j=0}^{i-1} (x - x_j) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_{n-1}) \dots (x - x_1)(x - x_0)$$

$w(x) = 38 + 104(x-3) + 14(x-4)(x-3) + \frac{7}{2}(x+3)(x-4)(x-3) - \frac{19}{4}(x+1)(x+3)(x-4)(x-3) = -\frac{19}{4}x^4 + \frac{71}{4}x^3 + \frac{247}{4}x^2 - \frac{615}{4}x - 154$

c)  $\begin{array}{c|ccc} x_k & -4 & -3 & -2 & 2 & 3 & 4 \\ y_k & 9 & 7 & 5 & -3 & -5 & 1977 \end{array}$

$x_0 = -4, y_0 = 9$   
 $x_1 = -3, y_1 = 7$   
 $x_2 = -2, y_2 = 5$   
 $x_3 = 2, y_3 = -3$   
 $x_4 = 3, y_4 = -5$   
 $x_5 = 4, y_5 = 1977$

$w(x) = 1984 - \frac{19}{4}x^4 + \frac{71}{4}x^3 - \frac{247}{4}x^2 + \frac{730}{7}x + \frac{2983}{7}$

$$X_4 = 3 \quad y_4 = - \quad FL(x_5/x_4) = \frac{-1}{4-3} = 1 \cdot x_4$$

$$X_5 = 4 \quad y_5 = 197$$

$$w(x) = 9 - (x+4) + \frac{62}{21}(x+4)(x+3)(x+2)(x-2)(x-3) = \frac{62}{21}x^5 + \frac{248}{21}x^4 - \frac{806}{21}x^3 - \frac{3224}{21}x^2 + \frac{730}{7}x + \frac{2983}{7}$$