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# Supplementary material: Efficient Computation Discovery for Polynomial Expressions

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## 1 RBM Partition Function Approximation

Below we give the full expressions for different terms in the Taylor series approximation of an RBM:

### 1.1 $g(\mathbf{x} \rightarrow \mathbf{x}^3, \mathbf{W})$

We show below all generated expressions up to degree 3 (for notation clarity, we first define variables  $A$ - $E$ ).

```
A = sum(W, 2);
B = sum(W, 1);
C = sum(sum(W));
D = repmat(sum(W, 1), [n, 1]);
E = repmat(sum(W, 2), [1, m]);

C, C.^2, C.^3, sum(B.^2),
sum(B.^3), sum(A.^2), sum(A.^3)
sum(sum(B.*E.*W)), sum(sum((W.*W)))
sum(sum(D.^2.*E))
sum(B.*sum(W.*W, 1))
C.*sum(sum((W.*W), 2))
sum(sum((E.^2.*D)))
sum(A.*sum((W.*W), 2))
sum(sum(W.*W.*W))
```

Forming a linear system of these expressions and solving, we obtain the following expression for  $g(x \rightarrow x^3, W)$ , which is exactly equivalent to the original:

```
(C.^3 +
C.*sum(A.^2).*3 +
C.*sum(sum(W.*W, 2)).*3 +
C.*sum(B.^2).*3 +
sum(A.*sum(W.*D, 2)).*6) / 64;
```

### 1.2 $g(\mathbf{x} \rightarrow \mathbf{x}^4, \mathbf{W})$

For the 4th order (and subsequent terms), we only show the final expression:

```
2^(n+m)*((((sum(sum(W, 2), 1).*sum(sum(W, 2),
1)).*(sum(sum(W, 2), 1).*sum(sum(W, 2), 1)))
```

```
1) + (sum((sum(W, 1).*sum(W, 1)).*(sum(W,
1).*sum(W, 1))), 2).*-2) + ((sum((sum(W, 1).*
sum(W, 1)), 2).*sum((sum(W, 1).*sum(W, 1)), 2))
.*3) + (sum((sum(W, 1).*sum(W, 1)).*sum((
W.*W), 1)), 2).*-12) + ((sum((sum(W, 1).*
sum(W, 1)), 2).*sum(sum((W.*W), 2), 1)).*6)
+ (sum(repmat(sum(sum(W, 2), 1), [n, 1]).*sum((
repmat(sum(W, 2), [1, m]).*repmat(sum(W, 1), [n,
1])).*W, 2)), 1).*24) + ((sum(sum(W, 2), 1).*
sum(sum(W, 2), 1)).*sum((sum(W, 1).*sum(W,
1)), 2)).*6) + (sum((sum(W, 2).*sum(W, 2))
.*(sum(W, 2).*sum(W, 2))), 1).*-2) + ((sum((
sum(W, 2).*sum(W, 2)), 1).*sum((sum(W, 2).*
sum(W, 2)), 1)).*3) + ((sum(W, 2)).*(W.*
(W'))).*sum(W, 2)).*12) + ((sum(W, 1).*(W'))
*(W.*(sum(W, 1')))).*12) + ((sum(sum(W, 2), 1)
.*sum(sum(W, 2), 1)).*sum((sum(W, 2).*sum(W,
2)), 1)).*6) + (sum((sum(W, 2).*sum(W, 2)).*
sum((W.*W), 2)), 1).*-12) + ((sum((sum(W,
1).*sum(W, 1)), 2).*sum((sum(W, 2).*sum(W,
2)), 1)).*6) + (sum(sum((W.*W).*(W.*W)),
2), 1).*4) + ((sum(sum(W, 2), 1).*sum(sum(W,
2), 1)).*sum(sum((W.*W), 2), 1)).*6) + ((
sum((sum(W, 2).*sum(W, 2)), 1).*sum(sum((W
.*W), 2), 1)).*6) + ((sum(sum((W.*W), 2), 1)
.*sum(sum((W.*W), 2), 1)).*3) + (sum((sum((W
.*W), 1).*sum((W.*W), 1)), 2).*-6) + (sum((
sum((W.*W), 2).*sum((W.*W), 2)), 1).*-6)
+ (sum(sum((W.*(W')).*(W.*(W'))), 1), 2)
.*6))) / 256;
```

### 1.3 $g(\mathbf{x} \rightarrow \mathbf{x}^5, \mathbf{W})$

```
2^(n+m)*(((sum(sum(repmat((sum(W, 1)
sum(W, 1)), [n, 1]).*(repmat(sum(W, 2), [1,
m]).*repmat(sum(W, 1), [n, 1])).*W), 2), 1)
.*-40) + (sum(((sum(W, 1).*sum(W, 1)).*
sum(repmat(sum(W, 1), [n, 1]).*(repmat(sum(W,
2), [1, m]).*repmat(sum(W, 1), [n, 1]))), 1)), 2)
.*-10) + ((sum(sum(W, 2), 1).*sum((sum(W,
2).*sum(W, 2)).*sum((W.*W), 2)), 1)).*

```





