# Supplementary material: Efficient Computation Discovery for Polynomial Expressions

#### 1. Computations

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In the following sections we present computation for various expressions. These results are automatically generated by our software. As it is really hard to format it into human readable form, so we have left them in native Matlab format.

### 1.1. $\sum ABC$

sum(A, 1) \* (B \* sum(C, 2))

## 1.2. $\sum AA^T$

sum(sum(A .\* repmat(sum(A, 2), [1, m]))))

#### 1.3. $\sum AA^TA$

sum(sum(((repmat(sum(A, 2), [1, m]) .\* repmat(sum(A, 1), [n, 1])) .\* A), 2), 1)

## 1.4. $\mathbf{g}(\mathbf{x} \rightarrow \mathbf{x}^4, \mathbf{W})$

 $2^{(n+m)}*((((sum(sum(W, 2), 1) .* sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)))))$ 1)) \* (sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1))) .\*1) + (sum(((sum(W, 1) .\* sum(W, 1)) .\* (sum(W, 1))))1) \* sum(W, 1))), 2) \* -2) + ( (sum((sum(W, 1) \*sum(W, 1), 2) .\* sum((sum(W, 1) .\* sum(W, 1)), 2)).\*3) + (sum((sum(W, 1) .\* sum(W, 1)) .\* sum((W .\* W), 1)), 2) .\* -12) + ( (sum(( sum(W, 1) .\*sum(W, 1), 2) \* sum(sum((W.\*W), 2), 1)) \* 6)+  $(\operatorname{sum}((\operatorname{repmat}(\operatorname{sum}(\operatorname{Sum}(\operatorname{W}, 2), 1), [n, 1])).* \operatorname{sum}(($ (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, m]))1])) .\* W), 2)), 1) .\* 24) + (( ( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) .\* sum((sum(W, 1) .\* sum(W, 1))1)), 2)) .\* 6) + ( sum(( ( sum(W, 2) .\* sum(W, 2)) .\* (sum(W, 2) .\* sum(W, 2)), 1) .\* -2) + ( (sum((sum(W, 2) .\* sum(W, 2), 1) .\* sum(( sum(W, 2) .\*sum(W, 2), 1) .\* 3) + ( (sum(W, 2)) \* ( (W \* (W') \* sum(W, 2)) .\* 12) + ( ( sum(W, 1) \* (W')) \* (W \* (sum(W, 1)))) \* 12) + (((sum(sum(W, 2), 1)))\* sum(sum(W, 2), 1)) \* sum((sum(W, 2) \* sum(W, 2))(2), (3), (3), (4), sum((W.\*W), 2)), 1).\*-12) + ((sum((sum(W,1) \* sum(W, 1)), 2) \* sum(( sum(W, 2) \* sum(W,  $\begin{array}{l} 2)),\,1))\,.^*\,6) + (\,\,\mathrm{sum}(\mathrm{sum}((\,\,(\,\,\mathrm{W}\,.^*\,\,\mathrm{W}\,)\,.^*\,(\,\,\mathrm{W}\,.^*\,\,\mathrm{W}\,)),\\ 2),\,1)\,.^*\,4) + (\,\,(\,\,(\,\,\mathrm{sum}(\mathrm{sum}(\mathrm{W},\,2),\,1)\,.^*\,\,\mathrm{sum}(\mathrm{sum}(\mathrm{W},\,2),\,1))\,.^*\,\,\mathrm{sum}(\mathrm{sum}(\mathrm{W},\,2),\,1))\,.^*\,6) + (\,\,(\,\,\mathrm{sum}((\,\,\mathrm{sum}(\mathrm{W},\,2)\,.^*\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2),\,1))\,.^*\,\,6) + (\,\,(\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2),\,1)\,.^*\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2),\,1)\,.^*\,\,3) + (\,\,\mathrm{sum}((\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,1)\,.^*\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,1)),\,2)\,.^*\,\,-6) + (\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2)\,.^*\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2)),\,1)\,.^*\,\,-6) \\ + (\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2)\,.^*\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2)),\,1)\,.^2\,.^*\,\,6))) / \,\,256; \end{array}$ 

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## 1.5. $\mathbf{g}(\mathbf{x} \to \mathbf{x^5}, \mathbf{W})$

 $2^{n+m}*(((sum(sum((repmat((sum(W,$ 1) sum(W, 1)), [n, 1]) \* ( ( repmat(sum(W, 2), [1,m]) .\* repmat(sum(W, 1), [n, 1])) .\* W)), 2), 1) .\* -40) + (sum(((sum(W, 1) .\* sum(W, 1)) .\*sum((repmat(sum(W, 1), [n, 1]).\* (repmat(sum(W, (2), [1, m]) \* repmat(sum(W, 1), (n, 1])), (1), (2).\* -10) + (( sum(sum(W, 2), 1) .\* sum(( ( sum(W, 2) .\* sum(W, 2)) .\* sum(( W .\* W), 2)), 1)) .\* -60) + (sum((repmat(sum(sum(W, 2), 1), [n, 1]))).\* (repmat(sum(sum(W, 2), 1), [n, 1]) .\* sum((repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* W), 2))), 1) .\* 60) + (sum((sum((W.\* repmat(sum(W, 1), [n, 1])), 2) .\* sum((repmat(sum(W, 1), [n, 1]) .\* ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])), 2), 1).\*60) + (sum(sum(((repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1),[n, 1])) .\* ( W .\* ( W .\* W))), 2), 1) .\* 80) + (sum((sum(W, 2) .\* (sum(W, 2) .\* sum(( repmat(sum(W, 2), [1, m]) \* repmat(sum(W, 1), [n, 1])) .\* W), 2))), 1) .\* -40) + (sum((repmat(sum)(sum(W, 2) .\* sum(W, 2)), 1), [n, 1]) .\* sum(( (repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [n, m])1])) .\* W), 2)), 1) .\* 60) + ( ( sum(W, 1) \* (W')) \* ((W \* (W')) \* sum(W, 2)). \* 120) + (sum(((sum(W, 2)))) \* (sum(W, 2))). 2) .\* sum(W, 2)) .\* sum((repmat(sum(W, 2), [1, m]) .\* ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1]), 2), 1) .\* -10) + (( sum(sum(W, 2), 1) .\* sum(( ( sum(W, 1) .\* sum(W, 1)) .\* sum(( W .\* W),1)), 2)) .\* -60) + (sum((sum(repmat(( sum(W, 2) .\*sum(W, 2)), [1, m]), 1).\* sum((repmat(sum(W, 2), [1, [m]) .\* (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])), 1)), 2) .\* 15) + ( ( sum(sum(W, 2), 1)\* (sum(W, 2)')) \* ((W \* (W')) \* sum(W, 2))) .\* 60) + (((sum(sum(W, 2), 1) \* (sum(W, 1) \* (W'))) \* (W \* (sum(W, 1)')) .\* 60) + ((sum(sum(W, 2), 1) .\* (sum((sum(W, 1).\* sum(W, 1)), 2).\* sum((sum(W, 1) \* sum(W, 1)), 2))) \* 15) + ((sum(sum(W, 2), 1) \* (( sum(sum(W, 2), 1) \* sum(sum(W, 2), 1)1)) \* ( sum(sum(W, 2), 1) \* sum(sum(W, 2),1)))) .\* 1) + ((sum((sum(W, 1) .\* sum(W, 1)),2) \* ( sum(sum(W, 2), 1) \* ( sum(sum(W,

1) \* sum(sum(W, 2), 1)))) \* 10) + (sum((sum(( (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [1, m]))1), [n, 1])) .\* W), 2) .\* repmat(sum(sum(( W .\* 113 W), 2), 1), [n, 1])), 1) .\* 60) + ((sum(( sum(W, 2) .\* sum(W, 2)), 1) .\* ( sum(sum(W, 2), 1) .\* ( sum(sum(W, 2), 1) \* sum(sum(W, 2), 1))) \* 10) +((sum(sum((W.\*W), 2), 1).\*(sum(sum(W, 2),1) \* ( sum(sum(W, 2), 1) \* sum(sum(W, 2), 1))))118 .\* 10) +  $(\operatorname{sum}((\operatorname{sum}(\operatorname{repmat}((\operatorname{sum}(W, 1) .* \operatorname{sum}(W,$ 1)), [n, 1]), 2) .\* sum((repmat(sum(W, 2), [1, m]) .\* ( 119 120 repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, (1)), (2)), (1), (3)) + (sum(sum)(((((repmat(sum)(W, 1)))))2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* W) .\* repmat(sum(( W .\* W), 1), [n, 1])), 2), 1) .\* -120) 124 + ((sum(sum(W, 2), 1) .\* (sum((sum(W, 1) .\*125 sum(W, 1)), 2) .\* sum(sum(( W .\* W), 2), 1))) .\* 126 30) + (sum(sum(((repmat(sum(W, 2), [1, m]).\*)127 repmat(sum(( W .\* W), 1), [n, 1])) .\* repmat(sum(( 128 2), 1) .\* ( sum(( sum(W, 2) .\* sum(W, 2)), 1) 129 130 .\* sum(sum((W.\*W), 2), 1))) .\* 30) + (sum((repmat(sum(sum(W, 2), 1), [n, 1]) .\* ( sum(( W .\* W), 2) .\* sum(( W .\* W), 2))), 1) .\* -30) + (sum((  $\operatorname{repmat}(\operatorname{sum}(\operatorname{sum}(W,\ 2),\ 1),\ [n,\ 1])$  .\*  $\operatorname{sum}((\ (\ W\ .*$ W) .\* (W.\*W), 2), 1) .\* 20) + ((sum(sum(W, 2),1) .\* ( sum(sum(( W .\* W), 2), 1) .\* sum(sum(( W 136 .\* W), 2), 1))) .\* 15) + (sum((sum((W.\*W), 2).\*sum(( ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1]) .\* W), 2)), 1) .\* -120) + ( sum(( ( W \* 138 139 (W')) .\* (W \* (W'))) \* repmat(sum(sum(W, 2), 1), [n, 1])), 1) .\* 30))) / 1024; 141

#### 1.6. $g(x \to x^6, W)$

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 $2^{(n+m)}*(((sum(sum(((W.**W)..*W)..*)((rep$ mat(sum(W, 2), [1, m]) \* repmat(sum(W, 1), [n, 1])) .\* ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1]))), 2), 1) .\* 360) + (sum(sum((( W .\* repmat(sum(W, 2), [1, m])).\* ( (W.\* repmat(sum(W, 2), [1, m]) \* repmat(sum((W.\*W), 1), [n, 1])), 2), 1).\* 360) + (sum(((sum(W, 1) .\* sum(W, 1)) .\* ((sum(W, 1) \* sum(W, 1)) \* (sum(W, 1) \* sum(W, 1))), 2) \*16) + ((sum((sum(W, 1) .\* sum(W, 1)), 2) .\* sum(((sum(W, 1) .\* sum(W, 1)) .\* (sum(W, 1) .\* sum(W, 1))1))), 2)) .\* -30) + ( ( sum(W, 1) \* ( (W') \* sum(W, 1) \* (W') \* (W') \* sum(W, 1) \* (W') \* (W') \* (W') \* sum(W, 1) \* (W') \*2))) \* (sum(W, 1) \* ((W') \* sum(W, 2)))) .\* 360) + (((sum(W, 1) \* (W')) \* ((W.\* (W.\* W)) \* (sum(W, 1))) \* (sum(W, 1)) \* (s1)'))) .\*480) + (sum((( sum(W, 2) .\* sum(W, 2)) .\* ( (sum(W, 2) .\* sum(W, 2)) .\* (sum(W, 2) .\* sum(W, 2))(2))), 1) .\* 16) + (sum(sum(((W.\*repmat(sum(W,1), [n, 1])) .\* ( ( W .\* repmat(sum(W, 1), [n, 1])) .\* repmat(sum(( W .\* W), 2), [1, m])), 2), 1) .\* 360) + ((sum((sum(W, 1) .\* sum(W, 1)), 2) .\* sum(((sum(W, 1) .\* sum(W, 1)), 3) .\* sum(((sum(W, 1) \* sum(W, 1) \* sum((W.\*W), 1), 2)

\*-180) + (sum((repmat(sum(sum(W, 2), 1), [n, 1]).\* sum(((repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [1, m]))))1), [n, 1]) .\* ( W .\* ( W .\* W)), 2)), 1) .\* 480) +  $(\operatorname{sum}((\operatorname{repmat}(\operatorname{sum}(\operatorname{Sum}(\operatorname{W}, 2), 1), [n, 1]))) * (\operatorname{sum}(\operatorname{W}, 2), 1))$ 2) .\* (sum(W, 2) .\* sum(( ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* W), 2))), 1) .\* -240) + (sum((sum((W.\*repmat(sum(W, 1), [n, 1])), 2))).\* (sum(W, 2) .\* sum(repmat(sum((repmat(sum(W, 2), [1, m]) .\* ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])), 1), [n, 1], 2))), 1).\*360) +((sum(sum(W, 2), 1) \* (sum(W, 1) \* (W'))) \* ((W \* (W')) \* sum(W, 2))) .\* 720) + ((( sum(sum(W, 2))) .\* 720) + (( sum(Sum(W, 2))) .\* ((2), (1) \* (1) 1) \* sum(sum(W, 2), 1)) \* (sum(sum(W, 2), 1) 1) .\* sum(W, 1)), 2) .\* (( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) .\* ( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1))) .\* 15) + (sum((sum((W.\*)repmat(sum(W, 1), [n, 1])), 2) .\* (sum(W, 2) .\* repmat((sum(sum(W, 2), 1) .\* (sum(sum(W, 2), 1) .\*sum(sum(W, 2), 1)), [n, 1])), 1) \* 120) + ((sum((sum(W, 2) .\* sum(W, 2), 1) .\* (( sum(sum(W, 2), ...)) .\* (( sum(sum(W, 2), ...)) ...) ...)1) \* sum(sum(W, 2), 1)) \* (sum(sum(W, 2), 1) \*sum(sum(W, 2), 1))).\* 15) + ((sum(sum((W.\*W),2), 1) .\* (( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) \* ( sum(sum(W, 2), 1) \* sum(sum(W, 2), 1)))) .\* 15) + (sum(sum((repmat((sum(W, 2) .\* sum(W,2)), [1, m]) .\* (( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* ( repmat(sum(W, 2), [1, [m]) .\* repmat(sum(W, 1), [n, 1]))), 1), 2) .\* -30) + (sum((sum((repmat(sum(W, 2), [1, m])).\* (repmat(sum(W, 2), [1, m]) \* repmat(sum(W, 1), [n, 1])), 1) .\* sum((repmat(sum(W, 2), [1, m]) .\* ( repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [n, 1]))), 1)), 2) .\* 45) + (sum((sum(( W .\* repmat(sum(W, 2), [1, m])), 1) .\* (sum(( W .\* repmat(sum(W, 2), [1, m])), 1) .\* repmat(sum(( sum(W, 1) .\* sum(W, 1)), 2), [1, m])), 2).\* 180) + (sum((sum((W.\*repmat(sum(W, 2), [1, m])), 1) .\* (sum(W, 1) .\* sum(( (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1),[n, 1]) .\* W), 1))), 2) .\* -360) + ((sum(( sum(W, 1) .\* sum(W, 1), 2) .\* ( sum((sum(W, 1) .\* sum(W, 1) .\*1)), 2) \* sum(( sum(W, 1) \* sum(W, 1)), 2))) \*15) + (sum((sum((repmat(sum(W, 1), [n, 1]) .\* (repmat(sum(W, 2), [1, m]) \* repmat(sum(W, 1), [n, 1]))), 1) .\* sum((repmat(sum(W, 1), [n, 1]) .\* ( repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [n, 1]))), 1)), 2).\*-30) + ((sum(sum(W, 2), 1).\* (sum(sum(W, 2), 1)). (2), (1) \* (sum((sum(W, 1) .\* sum(W, 1)), (2) .\* sum((sum(W, 1) \* sum(W, 1), 2))) \* 45) + ((( sum(W, 1), 2))) \* 45) + ((( sum(W, 1), 2), 2))) \* 45) + (( sum(W, 1), 2))) \* (( sum(W, 1), 2)) \* (( sum(W, 1), 2))) \* (( sum(W, 1), 2))) \* (( sum(W, 1), 2)) \* (( sum(W, 1), 2))) \* (( sum(W, 1), 2))) \* (( sum(W, 1), 2))) \* (( sum(W, 1), 2)) \* (( sum(W, 1), 2))) \* (( s1) .\* sum(W, 1)) \* ( ( repmat(sum(W, 1), [m, 1]) \* (W')) \* (W \* (sum(W, 1)'))) .\* 180) + (sum((sum((W.\* repmat(sum(W, 1), [n, 1])), 2).\* ((sum(W, 2).\*)repmat(sum(sum(W, 2), 1), [n, 1])) .\* sum(repmat((

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sum(W, 1) .\* sum(W, 1)), [n, 1]), (2))), (1) .\* 360) +( ( sum(( W .\* W), 1) \* ( ( (W') \* sum(W, 2)) .\* ( (W') \* sum(W, 2))) .\* -360) + (sum(( sum(( W .\*223 W), 1) .\* ( (sum(W, 1) .\* sum(W, 1)) .\* (sum(W, 1) 224 .\* sum(W, 1))), 2) .\* 240) + ((sum(sum((W.\*W),(2), 1) \* sum(((sum(W, 1) \* sum(W, 1)) \* (sum(W, 1)) \* (226 1) \* sum(W, 1)), 2)) \* -30) + ((sum(( sum(W, 1) \*227 sum(W, 1), 2).\* (sum((sum(W, 1).\*sum(W, 1)), 2)228 \* sum(sum((W.\*W), 2), 1))) .\* 45) + ((sum(sum(W, 45), 45)) + ((sum(sum(W, 45), 45)))) .\* 45) + ((sum(sum(W, 45), 45))) .\* 45) + ((sum(sum(W, 45), 45)) + ((sum(sum(W, 45), 45))) .\* 45) + ((sum(Sum(W, 45), 45)) + ((sum(Sum(W, 45), 45))) .\* 45) + ((sum(Sum(W, 45), 45)) + ((sum(Sum(W, 45), 45)) + ((sum(Sum(W, 45), 45)) + ((sum(Sum(W, 45), 45))) + ((sum(Sum(W, 45), 45)) + ((sum(Sum(W229 2), 1) .\* ( sum(sum(W, 2), 1) .\* sum(( ( sum(W, 1) .\* 230 sum(W, 1)) \*\* sum((W.\*W), 1), 2))) \*\* -180) + (((sum(sum(W, 2), 1) \* sum(sum(W, 2), 1)) \* (sum((231 232 sum(W, 1) .\* sum(W, 1)), 2) .\* sum(sum((W.\*W),233 (2), (1)) .\* 90) + (sum(sum(((repmat(sum((W.\*W), 234 2), [1, m]) .\* repmat(sum(( W .\* W), 1), [n, 1])) .\* ( 235 W \* W), 2, 1 \* 360 + (sum((sum(( W \* W), 1)236 .\* ( sum(( W .\* W), 1) .\* repmat(sum(( sum(W, 1) 237  $* \operatorname{sum}(W, 1), 2, [1, m]), 2 .* -90 + ((\operatorname{sum}(\operatorname{sum}(($ 238 W \* W), 2), 1) \* sum(( ( sum(W, 2) \* sum(W, 2)))239 .\* (sum(W, 2) .\* sum(W, 2)), 1)) .\* -30) + ((sum((sum(W, 2) .\* sum(W, 2)), 1) .\* ( sum(( sum(W, 2) .\*241 sum(W, 2), 1) \* sum(sum((W.\*W), 2), 1))) \* 45)+ (sum((sum(( W .\* repmat(sum(W, 2), [1, m])), 1) 243 .\* (sum(( W .\* repmat(sum(W, 2), [1, m])), 1) .\* rep-244 mat(sum(sum(( W .\* W), 2), 1), [1, m]))), 2) .\* 180) 245 + (sum((sum((W.\*repmat(sum(W, 1), [n, 1])), 2))).\* (sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) .\* repmat(sum(sum(( W .\* W), 2), 1), [n, 1]))), 1) .\* 180) 248 + (sum(((sum(W, 2).\* repmat(sum(sum(W, 2), 1),249 [n, 1])) .\* (sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) 250 .\* repmat(sum(sum(( W .\* W), 2), 1), [n, 1]))), 1) .\* 360) + (sum((sum((W.\*repmat(sum(W, 1), [n, 1])),252 2) \* sum(( repmat(( sum(W, 1) \* sum(W, 1)), [n, 1]) 253 .\* (W.\* repmat(sum(W, 1), [n, 1])), 2)), 1).\* -240) 254 + (sum(sum((W.\* repmat((repmat(sum(sum(W, 2), 1), [1, m]) .\* (sum(W, 1) .\* sum(( ( repmat(sum(W, 256 2), [1, m]) \* repmat(sum(W, 1), [n, 1])) \* W), 257 1))), [n, 1])), 2), 1) .\* -240) + ((sum(( sum(W, 2) 258 .\* sum(W, 2), 1) .\* sum(( ( sum(W, 1) .\* sum(W, 1) .\*1)) .\* (sum(W, 1) .\* sum(W, 1)), 2)) .\* -30) + (((260 sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) .\* sum((261 (sum(W, 2) .\* sum(W, 2)) .\* sum((W.\*W), 2)),1)) \* -180) + ((( sum(sum(W, 2), 1) \* sum(sum(W, 263 2), 1)) .\* sum(sum(( ( W .\* W) .\* ( W .\* W)), 2), 264 1)) .\* 60) + (sum((sum((repmat(sum(W, 2), [1, m]) 265 .\* (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 266 1), [n, 1]))), 2) .\* sum((repmat(sum(W, 2), [1, m]) 267 .\* (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 268 1), [n, 1])), 2)), 1) .\* -30) + ((sum(sum(W, 2), 1) 269 .\* sum((sum(repmat(( sum(W, 2) .\* sum(W, 2)), [1, 270 m]), 1) .\* sum((repmat(sum(W, 2), [1, m]) .\* ( rep-271 mat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [n, 1]))), 272 1)), 2)) .\*45) + (sum((sum((W.\*repmat(sum(W, 2), 273 [1, m]), 1) .\* (sum(( W .\* repmat(sum(W, 2), [1, m])), 274

1) .\* repmat(( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)), [1, m])), 2) .\* 180) + ((( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) .\* ( sum((sum(W, 1)) .\* sum(W, 1) .\* 1)), 2) \* sum(( sum(W, 2) \* sum(W, 2)), 1))) \* 90) + (sum((sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) .\* (sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) .\* repmat((sum(sum(W, 2), 1).\*sum(sum(W, 2), 1)), [n,1]))), 1) .\* 180) + ( ( (sum(W, 1) \* ((W') \* W)) \* (((W') \* W) \* (sum(W, 1)'))) .\* 360) + (sum((sum((W')))) .\* 360) + (sum((W'))) - (sum((W'))) -.\* repmat(sum(W, 1), [n, 1])), 2) .\* (sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) \* repmat(sum(( sum(W, 2) \* sum(W, 2)), 1), [n, 1])), 1) \* 180) + ((sum(( sum(W, 1) .\* sum(W, 1)), 2) .\* ( sum(( sum(W, 1) .\*sum(W, 1), 2) \* sum((sum(W, 2) \* sum(W, 2)), 1))).\*45) + (((sum(W, 2)') \* W) \* ((W') \* W) \* ((W') \* sum(W, 2)))) .\* 360) + (sum((sum((W.\* repmat(sum(W, 2), [1, m])), 1).\*sum((repmat((sum(W, 2), [1, m]))))2) .\* sum(W, 2)), [1, m]) .\* (W .\* repmat(sum(W, 2), [1, m])), 1), 2) .\* -240) + ((( (sum(W, 2)') \* W) \* ((W') \* ( repmat(sum(W, 2), [1, n]) \* ( sum(W, 2) .\* sum(W, 2))))) .\* 180) + ( ( (sum(W, 2)') \* W) \* (((W.\*(W.\*W))')\*sum(W, 2))).\*480) + ((sum((sum(W, 2) .\* sum(W, 2)), 1) .\* sum(( ( sum(W, 2)  $* \operatorname{sum}(W, 2)$  .\*  $(\operatorname{sum}(W, 2) .* \operatorname{sum}(W, 2)), 1)$ .\* -30) + ((sum((sum(W, 2) .\* sum(W, 2)), 1) .\* (sum((sum(W, 2) .\* sum(W, 2)), 1) .\* sum((sum(W, 2), 2)), 1)2) \* sum(W, 2)), 1))) \* 15) + ((sum(( sum(W, 2) \*sum(W, 2), 1) .\* sum(( ( sum(W, 2) .\* sum(W, 2)) .\*sum((W.\*W), 2), 1).\* -180) + (sum((Sum((W.\*V), 2)), 1)).\* W), 2).\* sum((W.\* W), 2)).\* repmat(sum((sum(W, 2) \* sum(W, 2)), 1), [n, 1])), 1) \* -90) + ((sum(( sum(W, 1) .\* sum(W, 1)), 2) .\* ( sum(( sum(W, 2) .\*sum(W, 2), 1) \* sum(sum((W.\*W), 2), 1)) \* 90)+ (sum(( sum(W, 1) .\* sum(W, 1)) .\* sum(( W .\*W) .\* (W.\*W), 1), 2) .\* -480) + ((sum((sum(W, W)))1) .\* sum(W, 1)), 2) .\* sum(sum(( ( W .\* W) .\* ( W (\* W), 2), 1)) (\* 60) + (sum((sum((W.\* W), 1).\* sum(( ( W .\* W) .\* ( W .\* W)), 1)), 2) .\* -480) +  $(\operatorname{sum}((\operatorname{sum}(\operatorname{Repmat}((\operatorname{sum}(\operatorname{W}, 2)), *\operatorname{sum}(\operatorname{W}, 2)), [1,$ m]), 1) .\* sum(( ( W .\* W) .\* ( W .\* W)), 1)), 2) .\* 60) + ((sum(W, 1) \* sum((W \* W), 1)) \* (((W') \* W) \* (sum(W, 1)')) .\* -720) + ((sum(sum(W, 1)'))) .\* -720) + ((sum(W, 1)')) - ((sum(W, 1)')2), 1) .\* sum(sum((( ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* W) .\* repmat(sum(( W .\* W), 1), [n, 1]), 2), 1)) .\* -720) + ((sum(( sum(W, 2) \* sum(W, 2)), 1) \* sum(( ( sum(W, 1) \* sum(W, 1)) \* sum(( W \* W), 1)), 2)) \* -180) + ((sum(sum(( W.\*W), 2), 1).\*sum(((sum(W, 1).\*sum(W, 1)).\*sum((W.\*W), 1)), 2)).\*-180) + (sum((sum(W,1) .\* sum(W, 1)) .\* ( sum(( W .\* W), 1) .\* sum(( W .\* W), 1))), 2) .\* 720) + ( sum(( sum(( W .\* W), 1).\* ( sum(( W .\* W), 1) .\* sum(( W .\* W), 1))), 2) .\* 240) + (sum(sum((repmat(sum(( W .\* W), 1), [n, 1)  $\cdot$ \* (repmat((sum(W, 2)  $\cdot$ \* repmat(sum(sum(W, 2),

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1), [n, 1])), [1, m]) .* repmat(sum(( W .* W), 1), [n,
                      1))), 2), 1) .* -90) + (sum((sum(repmat((sum(W, 2)
                      .* sum(W, 2)), [1, m]), 1) .* ( sum(( W .* W), 1) .*
                      sum((W.*W), 1)), 2).*-90) + (sum((sum((W.*
                      W), 2).* (sum((W.*W), 2).* repmat((sum(sum(W,
                      2), 1) * sum(sum(W, 2), 1)), [n, 1])), 1) * -90) +
                      + (((sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .* (
339
                      sum(sum((W.*W), 2), 1).*sum(sum((W.*W), 2),
340
                      1))) .*45) + ((( sum(sum(W, 2), 1) .* sum(sum(W,
                      2), 1)) .* ( sum(( sum(W, 2) .* sum(W, 2)), 1) .*
                      sum(sum((W.*W), 2), 1))).*90) + (sum((sum((W.*W), 2), 1)))
                      .* W), 2) .* ( (sum(W, 2) .* sum(W, 2)) .* (sum(W, 2))
                     2) * sum(W, 2))), 1) * 240) + (sum(( (sum(W, 2)
                      .* sum(W, 2)) .* sum(( ( W .* W) .* ( W .* W)), 2)),
                      1) .* -480) + (sum(( ( sum(W, 2) .* sum(W, 2)) .* (
                      sum(( W .* W), 2) .* sum(( W .* W), 2))), 1) .* 720)
                      + ((sum(sum(W, 2), 1) .* sum((sum((W.*W), 2) .*
                      sum(( (repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 2), [1, m]) ))))
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350
                      1), [n, 1]) .* W), 2)), 1)) .* -720) + ( ( (sum(W, 2)')
                      * (W * (W')) * (sum(W, 2) .* sum((W .* W), 2))
                      .* -720) + (sum((sum((W.*W), 2).*sum((W.*W), 2)))
                      .* W) .* (W .* W)), 2)), 1) .* -480) + (sum((sum((
                     W.* W), 2).* (sum((W.* W), 2).* sum((W.*
                      (W), (S)), (S), 
                      1)), 2) .* sum(( ( sum(W, 2) .* sum(W, 2)) .* sum((
                      358
                      2), 1) * sum(( ( sum(W, 2) * sum(W, 2)) * sum((
359
                      W.*W), 2)), 1)).*-180) + (sum((sum((W.*rep-
360
                      mat(sum(W, 1), [n, 1])), 2) .* (sum(W, 2) .* sum(( (
361
                      repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n,
                      1])) .* W), 2))), 1) .* -360) + ( ( ( sum(W, 1) *
                      (W')) .* (sum(W, 1) * (W'))) * sum((W.*W), 2))
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364
                      * -360) + (sum((sum(repmat((sum(W, 1) * sum(W,
                      1)), [n, 1]), 2) .* ( sum(( W .* W), 2) .* sum(( W .*
                      (W), (S)), (S), 
367
                      2)), 1) .* ( sum(sum(( W .* W), 2), 1) .* sum(sum((
368
                      W * W), 2), 1))) * 45) + (sum(sum(( ( W * W))))) * 45) + (sum(sum(( ( W * W)))))) * (W * W))) * (W * W)) * (
369
                      .* ( ( W .* W) .* ( W .* W))), 2), 1) .* 256) + ( (
                     sum(sum(( W .* W), 2), 1) .* sum(sum(( ( W .* W)
                     .* (W.*W), 2), 1)) .* 60) + ((sum(sum((W.*W),
                     2), 1) .* ( sum(sum(( W .* W), 2), 1) .* sum(sum((
                      W * W), 2), 1))) * 15) + (sum(sum((repmat(sum((
                      W.* W), 1), [n, 1]) .* ( repmat(sum(( W.* W), 2),
375
                      [1, m]) .* repmat(sum(( W .* W), 1), [n, 1])), 1), 2)
                      .* -90) + ((repmat(sum(sum(W, 2), 1), [1, m]) * (
                      ((W') * W) .* ((W') * W)) * repmat(sum(sum(W, W))) * repmat(sum(W, W))) * repmat(sum(Sum(W, W))) * repmat(sum(W, W))) * repmat(sum(Sum(W, W))) * repmat(sum(Sum(W, W))) * repmat(sum(W, W))) * repmat(sum(W, W)) * repmat(sum(W, W)) * repmat(sum(W, W)) * repmat(sum(W, W))) * repmat(sum(W, W)) * repmat(sum(W
378
                     (2), (1), (m, 1))) * (90) + (sum(sum((repmat(sum((W
379
                      .* W), 2), [1, m]) .* ( repmat(sum(( W .* W), 2), [1,
380
                      m]) .* repmat(sum(( W .* W), 1), [n, 1]))), 2), 1) .*
381
                     -90) + (sum(( ( ( W * (W')) .* ( W * (W'))) * (
382
                     sum(W, 2) .* sum(W, 2)), 1) .* -360) + (sum(( ( (
383
                      W * (W')) .* (W * (W'))) * sum((W .* W), 2)), 1)
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