# Supplementary material: Efficient Computation Discovery for Polynomial Expressions

## 1 RBM Partition Function Approximation

Below we give the full expressions for different terms in the Taylor series approximation of an RBM:

1.1 
$$\mathbf{g}(\mathbf{x} \to \mathbf{x}^3, \mathbf{W})$$

We show below all generated expressions up to degree 3 (for notation clearity, we first define variables A-E).

```
A = sum(W, 2);
B = sum(W, 1);
C = sum(sum(W));
D = repmat(sum(W, 1), [n, 1]);
E = repmat(sum(W, 2), [1, m]);

C, C .^ 2, C .^ 3, sum(B .^ 2),
sum(B .^ 3), sum(A .^ 2), sum(A .^ 3)
sum(sum(B .* E .* W))), sum(sum((W .* W)))
sum(sum(D .^ 2 .* E))
sum(B .* sum(W .* W, 1)))
C .* sum(sum((W .* W), 2))
sum(sum((E .^ 2 .* D))))
sum(sum((E .^ 2 .* D))))
sum(A .* sum((W .* W), 2))
sum(sum((W .* W), 2))
```

Forming a linear system of these expressions and solving, we obtain the following expression for  $g(x \to x^3, W)$ , which is exactly equivalent to the original:

```
(C .^ 3 +
C .* sum(A .^ 2) * 3 +
C .* sum(sum(W .* W, 2)) * 3 +
C .* sum(B .^ 2) * 3 +
sum(A .* sum(W .* D, 2)) * 6)) / 64;
```

#### 1.2 $\mathbf{g}(\mathbf{x} \to \mathbf{x}^4, \mathbf{W})$

For the 4th order (and subsequent terms), we only show the final expression:

```
2^{(n+m)*}((((sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .* (sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .*
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1) + (sum(((sum(W, 1) .\* sum(W, 1)) .\* (sum(W, 1))))1) \* sum(W, 1)), 2) \* -2) + ( (sum((sum(W, 1) \*sum(W, 1), 2) .\* sum((sum(W, 1) .\* sum(W, 1)), 2))\* 3) + (sum((sum(W, 1) .\* sum(W, 1)) .\* sum((W .\* W), 1)), 2) .\* -12) + ( (sum((sum(W, 1) .\*sum(W, 1), 2) .\* sum(sum((W.\*W), 2), 1)) .\* 6)+ (sum((repmat(sum(sum(W, 2), 1), [n, 1]) .\* sum(((repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, m]))1])) .\* W), 2)), 1) .\* 24) + (( ( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) .\* sum((sum(W, 1)) .\* sum(W, 1)1)), 2)) .\* 6) + ( sum(( ( sum(W, 2) .\* sum(W, 2)) ).\* (sum(W, 2) .\* sum(W, 2)), 1) .\* -2) + ( (sum((sum(W, 2) .\* sum(W, 2), 1) .\* sum(( sum(W, 2) .\*sum(W, 2), 1) .\* 3) + ( (sum(W, 2)) \* ( (W \* (W')) \* sum(W, 2))) .\* 12) + ( ( sum(W, 1) \* (W')) \* (W \* (sum(W, 1)))) \* 12) + (((sum(sum(W, 2), 1))))\* sum(sum(W, 2), 1)) \* sum((sum(W, 2)) \* sum(W, 2)2)), 1)) .\*6) + (sum(((sum(W, 2) .\* sum(W, 2)) .\*sum((W.\*W), 2), 1).\*-12) + ((sum((sum(W, ...))))1) \* sum(W, 1)), 2) \* sum(( sum(W, 2) \* sum(W, (2), (4),  $(2), (1) \cdot *4) + ((sum(sum(W, 2), 1) \cdot *sum(sum(W, 2), 1)) \cdot *sum(sum(W, 2), 1)) \cdot *sum(sum(W, 2), 1) \cdot *sum(sum(W, 2), 1)) \cdot *sum(sum(W, 2), 1) \cdot *sum(su$ (2), (1)) \* sum(sum(( W .\* W), 2), 1)) .\* 6) + ( ( sum(( sum(W, 2) .\* sum(W, 2)), 1) .\* sum(sum(( W .\* W), 2), 1)) .\* 6) + ( ( sum(sum((W.\*W), 2), 1) .\* sum(sum((W.\*W), 2), 1)).\*3) + (sum((sum((W.\*W), 2), 1))).\*3).\* W), 1) .\* sum((W.\*W), 1)), 2) .\* -6) + (sum((sum(( W .\* W), 2) .\* sum(( W .\* W), 2)), 1) .\* -6) + (sum(sum(((W\*(W')).\*(W\*(W'))), 1), 2).\* 6))) / 256;

#### 1.3 $g(x \rightarrow x^5, W)$

-60) + (sum((repmat(sum(sum(W, 2), 1), [n, 1])))\* (repmat(sum(sum(W, 2), 1), [n, 1]) \* sum(( ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* W), 2))), 1) .\* 60) + (sum((sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) .\* sum((repmat(sum(W, 1), [n, 1])), 2)1), [n, 1]) .\* ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])), 2), 1) .\* 60) + (sum(sum(((repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1),[n, 1])) .\* ( W .\* ( W .\* W))), 2), 1) .\* 80) +  $(\operatorname{sum}((\operatorname{sum}(W, 2) .* (\operatorname{sum}(W, 2) .* \operatorname{sum}((\operatorname{rep-}$ mat(sum(W, 2), [1, m]) \* repmat(sum(W, 1), [n, 1])) .\* W), 2))), 1) .\* -40) + (sum((repmat(sum)(sum(W, 2) .\* sum(W, 2)), 1), [n, 1]) .\* sum(( ( repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [n, m])1)) .\*W, 2)), 1) .\*60) + ( ( sum(W, 1) \*(W')) \* ( (W \* (W')) \* sum(W, 2)) .\* 120) + (sum(((sum(W, 2)))) .\* 120) + (sum(((sum(W, 2))))) .\* 120) + (sum(W, 2)) + (sum(W, 2))) .\* 120) + (sum(W, 2)) + (sum(W, 2))) .\* 120) + (sum(W, 2)) + (sum(W, 2)) + (sum(W, 2))) .\* 120) + (sum(W, 2)) + (sum(W,2) .\* sum(W, 2)) .\* sum((repmat(sum(W, 2), [1, m]) .\* (repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1),[n, 1])), 2), 1) \* -10) + ((sum(sum(W, 2), 1) \*sum(((sum(W, 1) .\* sum(W, 1)) .\* sum((W .\* W),1)), 2)) .\* -60) + (sum((sum(repmat(( sum(W, 2) .\*sum(W, 2), [1, m], 1).\* sum((repmat(sum(W, 2), [1, m]), 1). m]) .\* (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])), 1)), 2) .\* 15) + ( ( sum(sum(W, 2), 1)\* (sum(W, 2)')) \* ((W \* (W')) \* sum(W, 2))) .\* 60) + ((sum(sum(W, 2), 1) \* (sum(W, 1) \* (W'))) \* (W \* (sum(W, 1)'))) .\* 60) + ((sum(sum(W, 2), 1) .\* (sum((sum(W, 1) .\* sum(W, 1)), 2) .\* sum((sum(W, 1), 2) .\* sum((sum(W, 1), 2), 2)))1) \* sum(W, 1), 2))) \* 15) + ((sum(sum(W, 2), 1) \* (( sum(sum(W, 2), 1) \* sum(sum(W, 2), 1)1)) .\* ( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 2)1)))) .\* 1) + ((sum((sum(W, 1) .\* sum(W, 1)), 2) \* ( sum(sum(W, 2), 1) \* ( sum(sum(W, 2), 1)1) \* sum(sum(W, 2), 1)))) \* 10) + (sum((sum(( (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [1, m]))1), [n, 1])) .\* W), 2) .\* repmat(sum(sum(( W .\* (W), (2), (1), (n, 1)), (1), (3) (1) (3) (4) (4)2) \* sum(W, 2)), 1) \* ( sum(sum(W, 2), 1) \* ( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1))) .\* 10) +((sum(sum((W.\*W), 2), 1).\*(sum(sum(W, 2),1) \* ( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)))).\* 10) +  $(\operatorname{sum}((\operatorname{sum}(\operatorname{repmat}((\operatorname{sum}(W, 1)).* \operatorname{sum}(W,$ 1)), [n, 1]), 2) .\* sum((repmat(sum(W, 2), [1, m]) .\* ( repmat(sum(W, 2), [1, m]) \* repmat(sum(W, 1), [n, m])1))), 2)), 1) .\* 30) + (sum(sum((( ( repmat(sum(W,2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* W) .\* repmat(sum(( W .\* W), 1), [n, 1])), 2), 1) .\* -120) + ((sum(sum(W, 2), 1) .\* (sum((sum(W, 1) .\*sum(W, 1)), 2) .\* sum(sum(( W .\* W), 2), 1))) .\* 30) + (sum(sum(((repmat(sum(W, 2), [1, m]).\*)repmat(sum(( W .\* W), 1), [n, 1])) .\* repmat(sum(( (2), (3) (3) (3) (4)\* sum(sum(( W .\* W), 2), 1))) .\* 30) + (sum((  $\operatorname{repmat}(\operatorname{sum}(\operatorname{sum}(W, 2), 1), [n, 1]) .* (\operatorname{sum}((W.*)))$ 

### $\textbf{1.4} \quad g(x \rightarrow x^6, W)$

 $2^{(n+m)}*(((sum(sum(((W .* W) .* W) .* ((rep$ mat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\*  $(\operatorname{repmat}(\operatorname{sum}(W, 2), [1, m])$ .\*  $\operatorname{repmat}(\operatorname{sum}(W, 1),$ [n, 1]))), 2), 1) \* 360) + (sum(sum((( W .\* repmat(sum(W, 2), [1, m])).\* ( (W.\* repmat(sum(W, 2), [1, m]).\* repmat(sum((W.\*W), 1), [n, 1])), 2), 1).\* 360) + (sum(((sum(W, 1) .\* sum(W, 1)) .\* ((sum(W, 1) \* sum(W, 1)) \* (sum(W, 1) \* sum(W, 1))), 2) \*16) + ((sum((sum(W, 1) .\* sum(W, 1)), 2) .\* sum(((sum(W, 1) \* sum(W, 1)) \* (sum(W, 1) \* sum(W, 1))1))), 2)) .\* -30) + ( ( sum(W, 1) \* ((W') \* sum(W, 1))2))) \* (sum(W, 1) \* ((W') \* sum(W, 2)))) .\* 360) + (((sum(W, 1) \* (W')) \* ((W.\* (W.\* W)) \* (sum(W, 1))) \* (sum(W, 1)) \* (s1)'))) .\*480) + (sum((( sum(W, 2) .\* sum(W, 2)) .\* ( (sum(W, 2) .\* sum(W, 2)) .\* (sum(W, 2) .\* sum(W, 2))(2))), 1) .\* 16) + (sum(sum(((W.\*repmat(sum(W,1), [n, 1])) .\* ( ( W .\* repmat(sum(W, 1), [n, 1])) .\* repmat(sum(( W .\* W), 2), [1, m])), 2), 1) .\* 360) + ((sum((sum(W, 1) .\* sum(W, 1)), 2) .\* sum(((sum(W, 1)), 2) .\* sum((sum(W, 1)), 3) .\* sum((sum(W, 1)),sum(W, 1) .\* sum(W, 1)) .\* sum((W.\*W), 1)), 2))\*-180) + (sum((repmat(sum(sum(W, 2), 1), [n, 1]).\* sum(((repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [1, m]))))1), [n, 1])) .\* ( W .\* ( W .\* W))), 2)), 1) .\* 480) +  $(\operatorname{sum}((\operatorname{repmat}(\operatorname{sum}(\operatorname{W}, 2), 1), [n, 1])) * (\operatorname{sum}(\operatorname{W}, 2), 1))$ 2) \* (sum(W, 2) \* sum(( (repmat(sum(W, 2), [1, m])))).\* repmat(sum(W, 1), [n, 1])) .\* W), 2))), 1) .\* -240) + (sum((sum((W, 1), [n, 1])), 2)).\* (sum(W, 2) .\* sum(repmat(sum((repmat(sum(W, 2), [1, m]) .\* ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])), 1), [n, 1], 2)), 1) \* 360) +((sum(sum(W, 2), 1) \* (sum(W, 1) \* (W'))) \* ((W \* (W')) \* sum(W, 2))) .\* 720) + ((( sum(sum(W, 2))) .\* 720) + (( sum(sum(W, 2))) .\* 720) + (( sum(sum(W, 2)) .\* 720) + (( sum(w, 2)) .\* 72(2), (1) \* (1) 1) \* sum(sum(W, 2), 1)) \* (sum(sum(W, 2), 1) 1) .\* sum(W, 1)), 2) .\* (( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) .\* ( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1))) .\* 15) + (sum((sum((W.\*) $\operatorname{repmat}(\operatorname{sum}(W, 1), [n, 1])), 2)$  .\*  $(\operatorname{sum}(W, 2)$  .\*  $\operatorname{rep-}$ mat((sum(sum(W, 2), 1) .\* (sum(sum(W, 2), 1) .\*sum(sum(W, 2), 1)), [n, 1])), 1) .\* 120) + ((sum((sum(W, 2) .\* sum(W, 2), 1) .\* (( sum(sum(W, 2), 2), 3))1) .\* sum(sum(W, 2), 1)) .\* ( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1))) .\* 15) + ((sum(sum((W.\*W),

2), 1) .\* (( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) \* ( sum(sum(W, 2), 1) \* sum(sum(W, 2), 1)))) .\* 15) + (sum(sum((repmat((sum(W, 2) .\* sum(W, 2) .\* su2)), [1, m]) .\* (( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* ( repmat(sum(W, 2), [1,[m]) .\* repmat(sum(W, 1), [n, 1]))), 1), 2) .\* -30) + (sum((sum((repmat(sum(W, 2), [1, m])).\* (repmat(sum(W, 2), [1, m]) \* repmat(sum(W, 1), [n, 1]))), 1) .\* sum((repmat(sum(W, 2), [1, m]) .\* ( repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [n, 1]))), 1)), 2) .\* 45) + (sum((sum(( W .\* repmat(sum(W, 2), [1, m])), 1) .\* (sum(( W .\* repmat(sum(W, 2), [1, m]), 1) .\* repmat(sum(( sum(W, 1) .\* sum(W, 1)), 2), [1, m])), 2) .\* 180) + (sum((sum((W.\* repmat(sum(W, 2), [1, m])), 1) .\* (sum(W, 1) .\* sum(((repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1),[n, 1]) .\* W), 1))), 2) .\* -360) + ((sum(( sum(W, 1) \* sum(W, 1), 2) \* (sum((sum(W, 1)) \* sum(W, 1))1)), 2) \* sum(( sum(W, 1) \* sum(W, 1)), 2))) \* $15) + (sum((sum((repmat(sum(W, 1), [n, 1])))^* (rep$ mat(sum(W, 2), [1, m]) \* repmat(sum(W, 1), [n, 1]))), 1) .\* sum((repmat(sum(W, 1), [n, 1]) .\* ( repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [n, 1]))), 1)), 2).\* -30) + ((sum(sum(W, 2), 1).\* (sum(sum(W, 2), 1)). (2), 1) \* (sum((sum(W, 1) \* sum(W, 1)), 2) \* sum((sum(W, 1) \* sum(W, 1), 2))) \* 45) + ((( sum(W, 1), 2))) \* 45) + ((( sum(W, 1), 2), 2))) \* 45) + (( sum(W, 1), 2)) \* (( sum(W, 1), 2))) \* (( s1) .\* sum(W, 1)) \* ( ( repmat(sum(W, 1), [m, 1]) \* (W')) \* (W \* (sum(W, 1)')))) .\* 180) + (sum((sum((W.\* repmat(sum(W, 1), [n, 1])), 2).\* ((sum(W, 2).\*)repmat(sum(sum(W, 2), 1), [n, 1])) \* sum(repmat((sum(W, 1) .\* sum(W, 1), [n, 1], 2)), 1) .\* 360) +((sum((W.\*W), 1) \* ((W') \* sum(W, 2)).\* ((W') \* sum(W, 2))) .\* -360) + (sum((W.\*)W), 1) .\* ( (sum(W, 1) .\* sum(W, 1)) .\* (sum(W, 1) .\* sum(W, 1))), 2).\* 240) + ((sum(sum((W.\*W),(2), 1) \* sum(((sum(W, 1) \* sum(W, 1)) \* (sum(W, 1)) \* (1) \* sum(W, 1)), 2)) \* -30) + ((sum(( sum(W, 1) \*sum(W, 1), 2) .\* (sum((sum(W, 1) .\* sum(W, 1)), 2)\* sum(sum((W.\*W), 2), 1))).\* 45) + ((sum(sum(W, 45), 45)))2), 1) .\* (sum(sum(W, 2), 1) .\* sum(((sum(W, 1)).\*)sum(W, 1) \* sum((W.\*W), 1), 2)) \* -180) + (((sum(sum(W, 2), 1) \* sum(sum(W, 2), 1)) \* (sum((sum(W, 1) .\* sum(W, 1)), 2) .\* sum(sum(( W .\* W), (2), (1)) .\* 90) + (sum(sum(((repmat(sum((W.\*W), 2), [1, m]) .\* repmat(sum(( W .\* W), 1), [n, 1])) .\* ( W \* W), 2, 1 \* 360 + (sum((sum(( W \* W), 1).\* ( sum((W.\*W), 1) .\* repmat(sum((sum(W, 1).\* sum(W, 1), 2, [1, m])), 2).\* -90) + ((sum(sum((W.\*W), 2), 1).\*sum(((sum(W, 2).\*sum(W, 2))).\* (sum(W, 2) .\* sum(W, 2)), 1)) .\* -30) + ((sum((sum(W, 2) .\* sum(W, 2)), 1) .\* ( sum(( sum(W, 2) .\*sum(W, 2), 1) \* sum(sum((W.\*W), 2), 1))) \* 45)+ (sum((sum((W.\* repmat(sum(W, 2), [1, m])), 1).\* (sum(( W .\* repmat(sum(W, 2), [1, m])), 1) .\* repmat(sum(sum(( W .\* W), 2), 1), [1, m]))), 2) .\* 180)

+ (sum((sum((W.\*repmat(sum(W, 1), [n, 1])), 2)))\* (sum((W.\*repmat(sum(W, 1), [n, 1])), 2) .\* repmat(sum(sum(( W .\* W), 2), 1), [n, 1]))), 1) .\* 180) +  $(\operatorname{sum}(((\operatorname{sum}(W, 2))^* \operatorname{repmat}(\operatorname{sum}(\operatorname{sum}(W, 2), 1)),$ [n, 1]) .\* (sum((W.\*repmat(sum(W, 1), [n, 1])), 2).\* repmat(sum(sum(( W .\* W), 2), 1), [n, 1])), 1) .\* 360) + (sum((sum((W.\*repmat(sum(W, 1), [n, 1])),2) \* sum(( repmat(( sum(W, 1) \* sum(W, 1)), [n, 1]) .\* (W.\* repmat(sum(W, 1), [n, 1])), 2), 1).\* -240) + (sum(sum((W .\* repmat((repmat(sum(sum(W, 2),1), [1, m]) .\* (sum(W, 1) .\* sum(( ( repmat(sum(W, 1) ) ) ).\* 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* W), 1))), [n, 1])), 2), 1) .\* -240) + ((sum(( sum(W, 2) \* sum(W, 2), 1) \* sum(( ( sum(W, 1) .\* sum1)) \* (sum(W, 1) \* sum(W, 1)), 2)) \* -30) + (((sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1) .\* sum(((sum(W, 2) .\* sum(W, 2)) .\* sum((W.\*W), 2)),1)) \* -180) + ((( sum(sum(W, 2), 1) \* sum(sum(W, 2), 1)) .\* sum(sum(( ( W .\* W) .\* ( W .\* W)), 2), 1)) \* 60) + (sum((sum((repmat(sum(W, 2), [1, m]) .\* ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1]))), 2) .\* sum((repmat(sum(W, 2), [1, m]) .\* ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])), 2)), 1) .\* -30) + ((sum(sum(W, 2), 1) .\*  $\operatorname{sum}((\operatorname{sum}(\operatorname{repmat}((\operatorname{sum}(W, 2) .* \operatorname{sum}(W, 2)), [1,$ [m], 1) .\* [m] sum((repmat(sum(W, 2), [1, m]) .\* (repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 1), [n, 1]))), 1)), 2)) .\*45) + (sum((sum(( W .\* repmat(sum(W, 2), [1, m]), 1).\* (sum((W.\*repmat(sum(W, 2), [1, m])), 1) .\* repmat(( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)1)), [1, m])), 2) .\* 180) + ((( sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)) \* (sum((sum(W, 1) \* sum(W, 1))) \* (sum(W, 1) \* sum(W, 1)) \* (sum(W, 1) \* su1)), 2) .\* sum((sum(W, 2) .\* sum(W, 2)), 1))) .\* 90) + (sum((sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) .\* (sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) .\* repmat((sum(sum(W, 2), 1) .\* sum(sum(W, 2), 1)), [n,1])), 1) .\* 180) + ( ( sum(W, 1) \* ((W') \* W)) \* (((W') \* W) \* (sum(W, 1)'))) .\* 360) + (sum((sum((W.\* repmat(sum(W, 1), [n, 1])), 2) .\* (sum(( W .\* repmat(sum(W, 1), [n, 1])), 2) \* repmat(sum(( sum(W, 2) \* sum(W, 2)), 1), [n, 1])), 1) \* 180) + ((sum(( sum(W, 1) .\* sum(W, 1), 2) .\* (sum((sum(W, 1) .\*sum(W, 1), 2) .\* sum((sum(W, 2) .\* sum(W, 2)), 1))).\*45) + (((sum(W, 2)') \* W) \* ((W') \* W) \* ((W') \* sum(W, 2))) .\* 360) + (sum((sum((W.\* repmat(sum(W, 2), [1, m])), 1).\* sum((repmat((sum(W, 2) .\*  $\operatorname{sum}(W,2)),$  [1, m]) .\* ( W .\*  $\operatorname{repmat}(\operatorname{sum}(W,2),$ [1, m])), 1), 2) .\* -240) + ((( (sum(W, 2)') \* W) \* ((W') \* (repmat(sum(W, 2), [1, n]) \* (sum(W, 2).\*)sum(W, 2)))) .\* 180) + ( ( (sum(W, 2)') \* W) \* (((W.\*(W.\*W))')\*sum(W, 2)).\*480) + ((sum((sum(W, 2) \* sum(W, 2), 1) \* sum(( ( sum(W, 2))) \* sum(( ( sum(W, 2))) \* sum(( ( sum(W, 2)))) \* sum(( ( sum(W, 2)))) \* sum(( sum(W, 2))) \* sum((.\* sum(W, 2)) .\* (sum(W, 2) .\* sum(W, 2)), 1)) .\* -30) + ((sum((sum(W, 2)).\* sum(W, 2)), 1).\* (sum((sum(W, 2) .\* sum(W, 2)), 1) .\* sum((sum(W, 2), 2)), 1) 2) \* sum(W, 2)), 1))) \* 15) + ((sum(( sum(W, 2) \*sum(W, 2), 1) \* sum(( (sum(W, 2) \* sum(W, 2)) \*sum((W.\*W), 2), 1).\*-180) + (sum((Sum((W.\*V), 2)), 1)). W), 2) .\* sum(( W .\* W), 2)) .\* repmat(sum(( sum(W, 2) \* sum(W, 2)), 1), [n, 1])), 1) \* -90) + ((sum(( sum(W, 1) .\* sum(W, 1)), 2) .\* (sum((sum(W, 2) .\*sum(W, 2), 1) \* sum(sum((W.\*W), 2), 1)) \* 90)+ (sum(((sum(W, 1) .\* sum(W, 1)) .\* sum(((W .\*W) .\* (W .\* W)), 1)), 2) .\* -480) + ((sum((sum(W, 1) .\* sum(W, 1)), 2) .\* sum(sum(( ( W .\* W) .\* ( W (\* W), 2), 1)) (\* 60) + (sum((W \* W), 1).\* sum(( ( W .\* W) .\* ( W .\* W)), 1)), 2) .\* -480) +  $(\operatorname{sum}((\operatorname{sum}(\operatorname{repmat}((\operatorname{sum}(W, 2) .* \operatorname{sum}(W, 2)), [1,$ m]), 1) .\* sum(( ( W .\* W) .\* ( W .\* W)), 1)), 2) .\* 60) + ((sum(W, 1) .\* sum((W.\*W), 1)) \* (((W') \* W) \* (sum(W, 1)')) .\* -720) + ((sum(sum(W, 1)'))) .\* -720) + ((sum(W, 1)')) - ((sum(W, 1)')2), 1) .\* sum(sum((( ( repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1])) .\* W) .\* repmat(sum(( W .\* W), 1), [n, 1]), 2), 1)) .\* -720) + ((sum(( sum(W, 2) \* sum(W, 2)), 1) \* sum(( (sum(W, 1) \* sum(W, 1)) \* sum(( W \* W), 1)), 2)) \* -180) + ((sum(sum(( W.\*W), 2), 1).\*sum(((sum(W, 1).\*sum(W, 1)).\*sum((W.\*W), 1)), 2)).\*-180) + (sum((sum(W, ...))))1) .\* sum(W, 1)) .\* ( sum(( W .\* W), 1) .\* sum(( W (\* W), (1)), (2) (\* 720) + (sum((sum((W.\* W), 1)))).\* ( sum(( W .\* W), 1) .\* sum(( W .\* W), 1))), 2) \* 240) + (sum(sum((repmat(sum(( W .\* W), 1), [n, 1)  $\cdot$ \* (repmat((sum(W, 2)  $\cdot$ \* repmat(sum(sum(W, 2), 1), [n, 1])), [1, m]) .\* repmat(sum(( W .\* W), 1), [n, 1))), 2), 1)  $\cdot * -90$ ) + (sum((sum(repmat((sum(W, 2) .\* sum(W, 2)), [1, m]), 1) .\* ( sum(( W .\* W), 1) .\* sum((W.\*W), 1)), 2).\*-90) + (sum((sum((W.\*W), 2).\* (sum((W.\*W), 2).\* repmat((sum(Sum(Sum), 2).\* 2), 1) \* sum(sum(W, 2), 1)), [n, 1])), 1) \* -90) + W.\*W), 2), 1).\*sum(sum((W.\*W), 2), 1))).\*45)+ (((sum(sum(W, 2), 1) \* sum(sum(W, 2), 1)) \* (sum(sum((W.\*W), 2), 1).\*sum(sum((W.\*W), 2),1))) .\*45) + ((( sum(sum(W, 2), 1) .\* sum(sum(W, (2), 1) .\* (sum((sum(W, 2) .\* sum(W, 2)), 1) .\*sum(sum((W.\*W), 2), 1))).\*90) + (sum((Sum((W.\*W), 2), 1))).\*90).\* W), 2) .\* ( (sum(W, 2) .\* sum(W, 2)) .\* (sum(W, 2))2) \* sum(W, 2))), 1) \* 240) + (sum(( (sum(W, 2) .\* sum(W, 2)) .\* sum(((W.\*W).\*(W.\*W)), 2)),1) .\* -480) + (sum(( (sum(W, 2) .\* sum(W, 2)) .\* ( sum(( W .\* W), 2) .\* sum(( W .\* W), 2)), 1) .\* 720) + ((sum(sum(W, 2), 1) .\* sum((sum((W.\*W), 2) .\*sum(((repmat(sum(W, 2), [1, m]).\* repmat(sum(W, 2), [1, m])))1), [n, 1]) .\* W), 2)), 1)) .\* -720) + ( ( (sum(W, 2))) \* ( W \* (W'))) \* ( sum(W, 2) .\* sum(( W .\* W), 2))) .\* -720) + (sum((sum((W.\*W), 2).\*sum((W.\*W), 2))).\* W) .\* (W .\* W)), 2)), 1) .\* -480) + (sum((sum(( W.\* W), 2).\* (sum((W.\* W), 2).\* sum((W.\* (W), (Sum)), (Sum)) 1)), 2) \* sum(( (sum(W, 2) \* sum(W, 2)) \* sum((

W \* W), 2), 1) \* -180) + ((sum(sum(( W \* W),2), 1) \* sum(( ( sum(W, 2) \* sum(W, 2)) \* sum(( W.\*W), 2)), 1)).\*-180) + (sum((sum((W.\*repmat(sum(W, 1), [n, 1])), 2) .\* (sum(W, 2) .\* sum(( (repmat(sum(W, 2), [1, m]) .\* repmat(sum(W, 1), [n, 1)) .\* W), 2))), 1) .\* -360) + ( ( ( sum(W, 1) \* (W')) .\* (sum(W, 1) \* (W'))) \* sum((W.\*W), 2)).\* -360) + (sum((sum(repmat((sum(W, 1) .\* sum(W, 1)), [n, 1]), 2) .\* ( sum(( W .\* W), 2) .\* sum(( W .\* 2)), 1) .\* ( sum(sum(( W .\* W), 2), 1) .\* sum(sum(( W .\* W), 2), 1))) .\* 45) + (sum(sum(( ( W .\* W).\* ( ( W .\* W) .\* ( W .\* W))), 2), 1) .\* 256) + ( ( sum(sum(( W .\* W), 2), 1) .\* sum(sum(( ( W .\* W) .\* (W.\*W), 2), 1)) .\* 60) + ((sum(sum((W.\*W),2), 1) .\* ( sum(sum(( W .\* W), 2), 1) .\* sum(sum(( W \* W), 2), 1))) \* 15) + (sum(sum((repmat(sum((W.\* W), 1), [n, 1]).\* (repmat(sum((W.\* W), 2), [1, m]) \*\* repmat(sum(( W .\* W), 1), [n, 1])), 1), 2) .\* -90) + ((repmat(sum(sum(W, 2), 1), [1, m]) \* (((W') \* W) .\* ((W') \* W)) \* repmat(sum(sum(W, W))) \* repmat(sum(W, W))) \* repmat(sum(Sum(W, W))) \* repmat(sum(Sum(W, W))) \* repmat(sum(Sum(W, W))) \* repmat(sum(W, W))) \* repmat(sum(Sum(W, W))) \* repmat(sum(W, W))) \* repmat(sum(Sum(W, W))) \* repmat(sum(Sum(W, W))) \* repmat(sum(W, W)) \* repmat(sum(W, W))(2), 1), [m, 1])) .\* (90) + (sum(sum((repmat(sum((W.\* W), 2), [1, m]) .\* ( repmat(sum(( W .\* W), 2), [1, m]) .\* repmat(sum(( W .\* W), 1), [n, 1]))), 2), 1) .\* -90) + (sum(( ( ( W \* (W')) .\* ( W \* (W'))) \* ( sum(W, 2) .\* sum(W, 2)), 1) .\* -360) + (sum(( ( (W \* (W')) .\* ( W \* (W'))) \* sum(( W .\* W), 2)), 1) \* -360) + (sum(( ( ( W \* (W')) .\* ( W \* (W'))) \*sum(repmat((sum(W, 1) .\* sum(W, 1)), [n, 1]), 2)),1) .\* 90) + (sum(( ( ( W \* (W')) .\* ( W \* (W'))) \* repmat(sum((sum(W, 2) .\* sum(W, 2)), 1), [n, 1])),1) .\* 90) + ( sum(( ( sum(W, 1) .\* sum(W, 1)) \* (((W') \* W) .\* ((W') \* W)), 2) .\* -360) + (sum((sum((W.\*W), 1)\*((W')\*W).\*((W')\*W)),2) .\* -360) + (( repmat(sum(sum(( W .\* W), 2), 1), [1, m] \* sum(( ( (W') \* W) .\* ( (W') \* W)), 2)) .\* 90) + (sum(sum((W.\*((W\*(W')))\*(W.\*(W  $(W^*(W))$ , 2), 1)  $(W^*(W))$ \* W) .\* ( ( W \* (W')) \* W)), 2), 1) .\* 120))) / 4096;