Supplementary material: Efficient Computation Discovery for Polynomial Expressions

1. Computations

In the following sections we present computation for various expressions. This are results automatically generated by our software. It is really hard to format it into human readable form, so we have decided to post it as it is.

1.1. $\sum ABC$

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sum(A, 1) * (B * sum(C, 2))

1.2. $\sum AA^T$

sum(sum(A .* repmat(sum(A, 2), [1, m])))

1.3. $\sum AA^TA$

sum(sum(((repmat(sum(A, 2), [1, m]) .* repmat(sum(A, 1), [n, 1])) .* A), 2), 1)

1.4. $\mathbf{g}(\mathbf{x} \rightarrow \mathbf{x}^4, \mathbf{W})$

(((((sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .* (sum(sum(W, 2), 1) .* sum(sum(W, 2), 1))) .* 1) +(sum(((sum(W, 1) .* sum(W, 1)) .* (sum(W, 1)))).* sum(W, 1)), 2) .* -2) + ((sum((sum(W, 1)).*sum(W, 1), 2) .* sum((sum(W, 1) .* sum(W, 1)),2)) .* 3) + (sum(((sum(W, 1) .* sum(W, 1)) .*sum((W.*W), 1)), 2).*-12) + ((sum((sum(W, 1)* sum(W, 1), 2) * sum(sum((W.*W), 2), 1)) * 6)+ (sum((repmat(sum(sum(W, 2), 1), [n, 1]) .* sum(((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, m]))1])) .* W), 2)), 1) .* 24) + (((sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .* sum((sum(W, 1) .* sum(W, 1))1)), 2)) .* 6) + (sum(((sum(W, 2) .* sum(W, 2)) .* (sum(W, 2) .* sum(W, 2)), 1) .* -2) + ((sum((sum(W, 2) .* sum(W, 2), 1) .* sum((sum(W, 2) .*sum(W, 2), 1) .* 3) + ((sum(W, 2)) * ((W * (W') * sum(W, 2)) .* 12) + ((sum(W, 1) * (W')) * (W * (sum(W, 1)))) * 12) + (((sum(sum(W, 2), 1)))* sum(sum(W, 2), 1)) * sum((sum(W, 2) * sum(W, 2))(2), (3), (3), (4), sum((W.*W), 2)), 1).*-12) + ((sum((sum(W,1) * sum(W, 1)), 2) * sum((sum(W, 2) * sum(W, $\begin{array}{l} 2)),\,1))\,.^*\,6) + (\,\,\mathrm{sum}(\mathrm{sum}((\,\,(\,\,\mathrm{W}\,.^*\,\,\mathrm{W}\,)\,.^*\,(\,\,\mathrm{W}\,.^*\,\,\mathrm{W}\,)),\\ 2),\,1)\,.^*\,4) + (\,\,(\,\,(\,\,\mathrm{sum}(\mathrm{sum}(\mathrm{W},\,2),\,1)\,.^*\,\,\mathrm{sum}(\mathrm{sum}(\mathrm{W},\,2),\,1))\,.^*\,\,6) + (\,\,(\,\,\mathrm{sum}((\,\,\mathrm{sum}(\mathrm{W},\,2)\,.^*\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2),\,1))\,.^*\,\,6) + (\,\,(\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2),\,1)\,.^*\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2),\,1)\,.^*\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2),\,1)\,.^*\,\,3) + (\,\,\mathrm{sum}((\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,1)\,.^*\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,1)),\,2)\,.^*\,\,-6) + (\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2)\,.^*\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2)),\,1)\,.^*\,\,-6) + (\,\,\mathrm{sum}(\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2)\,.^*\,\,\mathrm{sum}((\,\,\mathrm{W}\,.^*\,\,\mathrm{W}),\,2)),\,1)\,.^*\,\,-6))) / \,\,256; \end{array}$

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1.5. $\mathbf{g}(\mathbf{x} \to \mathbf{x^5}, \mathbf{W})$

(((sum(sum((repmat((sum(W, 1) .* sum(W, 1)), [n,1]) .* ((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1]) * W)), 2), 1) * -40) + (sum(((sum(W, 1))))1) .* sum(W, 1)) .* sum((repmat(sum(W, 1), [n, 1]) .* (repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])), 1)), 2) .* -10) + ((sum(sum(W, 2), 1) .* sum(((sum(W, 2) .* sum(W, 2)) .* sum((W .* W), 2)), 1)) .* -60) + (sum((repmat(sum(sum(W, sum(W, sum(W,2), 1), [n, 1]) .* (repmat(sum(sum(W, 2), 1), [n, 1]) * sum(((repmat(sum(W, 2), [1, m]) * repmat(sum(W, 1), [n, 1])) .* W), 2))), 1) .* 60) + (sum((sum((W.*repmat(sum(W, 1), [n, 1])), 2).*sum((repmat(sum(W, 1), [n, 1]) .* (repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])), 2)), 1) .* 60) + (sum(sum(((repmat(sum(W, 2), [1, m]).* repmat(sum(W, 1), [n, 1])) .* (W .* (W .* W))), 2), 1) .* 80) + (sum((sum(W, 2) .* (sum(W, 2) .* sum(((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1),[n, 1]) .* W), 2))), 1) .* -40) + (sum((repmat(sum)(sum(W, 2) .* sum(W, 2)), 1), [n, 1]) .* sum(((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])) (*W), (2), (1), (*W'), (1), (*W'), (*W'),W * (W')) * sum(W, 2))) .* 120) + (sum(((sum(W, 2))) .* 120) + (sum(((sum(W, 2))) .* 120)) + (sum(((sum(W, 2))) .* 120))))))2) .* sum(W, 2)) .* sum((repmat(sum(W, 2), [1, m]) .* (repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1),[n, 1])), 2), 1) .* -10) + ((sum(sum(W, 2), 1) .*sum(((sum(W, 1) .* sum(W, 1)) .* sum((W .* W),1)), 2)) .* -60) + (sum((sum(repmat((sum(W, 2) .*sum(W, 2), [1, m], 1).* sum((repmat(sum(W, 2), [1, m]), 1)). m]) .* (repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])), 1)), 2) .* 15) + ((sum(sum(W, 2), 1)* (sum(W, 2)')) * ((W * (W')) * sum(W, 2))) .* 60) + ((sum(sum(W, 2), 1) * (sum(W, 1) * (W'))) * (W * (sum(W, 1)'))) .* 60) + ((sum(sum(W, 2), 1) .* (sum((sum(W, 1) * sum(W, 1)), 2) * sum((sum(W, 1)), 3) * sum((sum1) .* sum(W, 1), 2))) .* 15) + ((sum(sum(W, 2), 1).* ((sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .* (sum(sum(W, 2), 1) .* sum(sum(W, 2), 1))) .* 1) +((sum((sum(W, 1).*sum(W, 1)), 2).*(sum(sum(W, 1)), 2).*)(2), (3) * (3) 1)))) .* 10) + (sum((sum((repmat(sum(W, 2), [1,

m]) .* repmat(sum(W, 1), [n, 1])) .* W), 2) .* repmat(sum(sum((W.*W), 2), 1), [n, 1])), 1).*60) +((sum((sum(W, 2).*sum(W, 2)), 1).*(sum(sum(W, 2)), 1).*)(2), 1) .* (sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)))).* 10) + ((sum(sum((W .* W), 2), 1) .* (sum(sum(W, (2), 1) * (sum(sum(W, 2), 1) * sum(sum(W, 2), 1))).* 10) + $(\operatorname{sum}((\operatorname{sum}(\operatorname{repmat}((\operatorname{sum}(W, 1)).* \operatorname{sum}(W,$ 1)), [n, 1]), 2) .* sum((repmat(sum(W, 2), [1, m]) .* (118 repmat(sum(W, 2), [1, m]) * repmat(sum(W, 1), [n, m])119 1))), 2)), 1) .* 30) + (sum(sum((((repmat(sum(W,120 2), [1, m]) .* repmat(sum(W, 1), [n, 1])) .* W) .* repmat(sum((W .* W), 1), [n, 1])), 2), 1) .* -120) + ((sum(sum(W, 2), 1) .* (sum((sum(W, 1) .*sum(W, 1), 2) .* sum(sum((W.*W), 2), 1))) .*124 30) + (sum(sum(((repmat(sum(W, 2), [1, m]).* rep-125 mat(sum((W .* W), 1), [n, 1])) .* repmat(sum((W .* W), 1), [n, 1]), 2), 1).*-30) + ((sum(sum(W, 2), 1).* (sum((sum(W, 2).*sum(W, 2)), 1).*sum(sum((W 127 128 * W, 2, 1) * * 30 + (sum((repmat(sum(sum(W, 2), 1), [n, 1]) .* (sum((W .* W), 2) .* sum((W .* 129 (W), (2)), (1) * -30) + (sum((repmat(sum(sum(W, 2), 1))))1), [n, 1]) .* sum(((W .* W) .* (W .* W)), 2)), 1) .* 20) + ((sum(sum(W, 2), 1) .* (sum(sum((W.*)W), 2), 1) * sum(sum((W * W), 2), 1))) * 15) + $(\operatorname{sum}((\operatorname{sum}((\operatorname{W}.^*\operatorname{W}), 2).^*\operatorname{sum}((\operatorname{repmat}(\operatorname{sum}(\operatorname{W},$ 2), [1, m]) * repmat(sum(W, 1), [n, 1])) * W), 2)), 1) 136 .* -120) + (sum(((W * (W')) .* (W * (W'))) *repmat(sum(sum(W, 2), 1), [n, 1])), 1).* 30))) / 1024; 138

1.6. $g(x \to x^6, W)$

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(((sum(sum(((W .* W) .* ((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])) .* (repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1]))), 2), 1) * 360) + (sum(sum(((W .* repmat(sum(W, 2), [1, m])) .* ((W .* repmat(sum(W, 2), [1, m])) .* repmat(sum((W.*W), 1), [n, 1]))), 2), 1) .* 360) +(sum(((sum(W, 1) .* sum(W, 1)) .* ((sum(W, 1) $* \operatorname{sum}(W, 1)$ * ($\operatorname{sum}(W, 1)$ * * $\operatorname{sum}(W, 1)$)), 2) * 16) + ((sum((sum(W, 1) .* sum(W, 1)), 2) .* sum(((sum(W, 1) .* sum(W, 1)) .* (sum(W, 1) .* sum(W, 1))1))), 2)) .* -30) + ((sum(W, 1) * ((W') * sum(W, 1) * (W') * (W'2))) * (sum(W, 1) * ((W') * sum(W, 2)))) .* 360) + (((sum(W, 1) * (W')) * ((W.* (W.* W)) * (sum(W, 1) * (W'))) * (sum(W, 1) * (W')) * (W')) * (W') * (W') * (W') * (W') * (W') * (W') * (W')) * (W') *1)'))) .*480) + (sum(((sum(W, 2) .* sum(W, 2)) .* ((sum(W, 2) .* sum(W, 2)) .* (sum(W, 2) .* sum(W, 2))(2))), 1) .* 16) + (sum(sum(((W.*repmat(sum(W,1), [n, 1])) .* ((W .* repmat(sum(W, 1), [n, 1])) .* repmat(sum((W .* W), 2), [1, m]))), 2), 1) .* 360) + ((sum((sum(W, 1) .* sum(W, 1)), 2) .* sum(((sum(W, 1) .* sum(W, 1)), 3) .* sum(((sum(W, 1) .* sum(W, 1)) .* sum((W.*W), 1)), 2))*-180) + (sum((repmat(sum(sum(W, 2), 1), [n, 1]).* sum(((repmat(sum(W, 2), [1, m]).*repmat(sum(W, 1), [1, m]))))1), [n, 1])) .* (W .* (W .* W))), 2)), 1) .* 480) +

 $(\operatorname{sum}((\operatorname{repmat}(\operatorname{sum}(\operatorname{Sum}(\operatorname{W}, 2), 1), [n, 1])))$ * $(\operatorname{sum}(\operatorname{W}, 2), (\operatorname{Sum}(\operatorname{W}, 2), 1))$ 2) .* (sum(W, 2) .* sum(((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])) .* W), 2))), 1) .* -240) + (sum((sum((W.* repmat(sum(W, 1), [n, 1])), 2).* (sum(W, 2) .* sum(repmat(sum((repmat(sum(W, 2), [1, m]) .* (repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])), 1), [n, 1], 2)), 1) .* 360) +((sum(sum(W, 2), 1) * (sum(W, 1) * (W'))) * ((W * (W')) * sum(W, 2))) .* 720) + (((sum(sum(W, 2))) .* 720) + ((sum(W, 2)) .* 720) + ((sum(W, 2)) .* 720) + ((sum(W, 2)) .* 720) + ((sum(W, 2))) .* 720) + ((sum(W, 2)) .* 72(2), (1) * (1) 1) * sum(sum(W, 2), 1)) * (sum(sum(W, 2), 1) * sum(sum(W, 2), 1)))) * 1) + ((sum((sum(W, 1) .* sum(W, 1)), 2) .* ((sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .* (sum(sum(W, 2), 1) .* sum(sum(W, 2), 1))) .* 15) + (sum((sum((W.*)repmat(sum(W, 1), [n, 1])), 2) .* (sum(W, 2) .* repmat((sum(sum(W, 2), 1) .* (sum(sum(W, 2), 1) .*sum(sum(W, 2), 1)), [n, 1])), 1) * 120) + ((sum((sum(W, 2) .* sum(W, 2), 1) .* ((sum(sum(W, 2),1) * sum(sum(W, 2), 1)) * (sum(sum(W, 2), 1) *sum(sum(W, 2), 1))).* 15) + ((sum(sum((W.*W),(2), (1) * ((sum(sum(W, 2), 1) * sum(sum(W, 2), 1))1)) .* (sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)))* 15) + (sum(sum((repmat((sum(W, 2) .* sum(W, 2)), [1, m]) .* ((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])) * (repmat(sum(W, 2), [1, [m]) .* repmat(sum(W, 1), [n, 1]))), 1), 2) .* -30) + (sum((sum((repmat(sum(W, 2), [1, m])).* (repmat(sum(W, 2), [1, m]) * repmat(sum(W, 1), [n, 1]))), 1) .* sum((repmat(sum(W, 2), [1, m]) .* (repmat(sum(W, 2), [1, m]).* repmat(sum(W, 1), [n, 1]))), 1)), 2) .* 45) + (sum((sum((W .* repmat(sum(W, 2), [1, m])), 1) .* (sum((W .* repmat(sum(W, 2), [1, m]), 1) .* repmat(sum((sum(W, 1) .* sum(W, 1)), 2), [1, m]))), 2) .* 180) + (sum((sum((W.*repmat(sum(W, 2), [1, m]), 1) .* (sum(W, 1) .* sum(((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1),[n, 1]) .* W), 1))), 2) .* -360) + ((sum((sum(W, 1) .* sum(W, 1), 2) .* (sum((sum(W, 1)) .* sum(W, 1)1)), 2) * sum((sum(W, 1) * sum(W, 1)), 2))) *15) + (sum((sum((repmat(sum(W, 1), [n, 1])) * (repmat(sum(W, 2), [1, m]) * repmat(sum(W, 1), [n, (1)), 1) * sum((repmat(sum(W, 1), [n, 1]) .* (repmat(sum(W, 2), [1, m]).* repmat(sum(W, 1), [n, 1]))), 1)), 2) $\cdot * -30$) + ((sum(sum(W, 2), 1) $\cdot *$ (sum(sum(W, (2), 1).*(sum((sum(W, 1).*sum(W, 1)), 2).*sum((sum(W, 1) * sum(W, 1), 2))) * 45) + (((sum(W, 1), 2))) * 45) + (((sum(W, 1), 2), 2))) * 45) + ((sum(W, 1), 2))) * ((sum(W, 1), 2)) * ((sum(W, 1), 2))) * ((sum(W, 1), 2)) * ((sum(W, 1), 2))1) .* sum(W, 1)) * ((repmat(sum(W, 1), [m, 1]) * (W')) * (W * (sum(W, 1)')))) .* 180) + (sum((sum((W.* repmat(sum(W, 1), [n, 1]), 2).* ((sum(W, 2).*)repmat(sum(sum(W, 2), 1), [n, 1])) .* sum(repmat((sum(W, 1) .* sum(W, 1)), [n, 1]), (2))), (1) .* (360) +((sum((W.*W), 1) * ((W') * sum(W, 2)).* ((W') * sum(W, 2)))) .* -360) + (sum((sum((W .*

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W), 1) .* ((sum(W, 1) .* sum(W, 1)) .* (sum(W, 1) .* sum(W, 1))), 2) .* 240) + ((sum(sum((W.*W),222 (2), 1) * sum(((sum(W, 1) * sum(W, 1)) * (sum(W, 1)) * (223 1) * sum(W, 1))), 2)) * -30) + ((sum((sum(W, 1) *224 sum(W, 1), 2).* (sum((sum(W, 1).*sum(W, 1)), 2)* sum(sum((W.*W), 2), 1))).* 45) + ((sum(sum(W, 45), 45)))226 (2), 1).* (sum(sum(W, 2), 1).* sum(((sum(W, 1).*)))227 sum(W, 1)) * sum((W.*W), 1), 2))) * -180) + (((228 sum(sum(W, 2), 1) * sum(sum(W, 2), 1)) * (sum((229 sum(W, 1) .* sum(W, 1)), 2) .* sum(sum((W .* W),(2), 1)) * (90) + (sum(sum(((repmat(sum((W.*W),2), [1, m]) .* repmat(sum((W .* W), 1), [n, 1])) .* (231 232 W.*W), 2), 1).*360) + (sum((sum((W.*W), 1)233 .* (sum((W .* W), 1) .* repmat(sum((sum(W, 1) 234 $* \operatorname{sum}(W, 1), 2, [1, m]), 2 .* -90 + ((\operatorname{sum}(\operatorname{sum}(($ 235 W .* W), 2), 1) .* sum(((sum(W, 2) .* sum(W, 2)))236 .* (sum(W, 2) .* sum(W, 2)), 1)) .* -30) + ((sum((237 sum(W, 2) .* sum(W, 2), 1) .* (sum((sum(W, 2) .*238 sum(W, 2), 1) * sum(sum((W.*W), 2), 1)) * 45)239 + (sum((sum((W.*repmat(sum(W, 2), [1, m])), 1).* (sum((W.*repmat(sum(W, 2), [1, m])), 1) .* rep-241 mat(sum(sum((W .* W), 2), 1), [1, m]))), 2) .* 180) + (sum((sum((W.*repmat(sum(W, 1), [n, 1])), 2))243 .* (sum((W .* repmat(sum(W, 1), [n, 1])), 2) .* rep-244 mat(sum(sum((W .* W), 2), 1), [n, 1])), 1) .* 180) 245 + $(\operatorname{sum}(((\operatorname{sum}(W, 2) .* \operatorname{repmat}(\operatorname{sum}(\operatorname{sum}(W, 2), 1),$ [n, 1]) .* (sum((W.*repmat(sum(W, 1), [n, 1])), 2)247 .* repmat(sum(sum((W .* W), 2), 1), [n, 1]))), 1) .* 248 360) + (sum((sum((W.*repmat(sum(W, 1), [n, 1])),249 2) * sum((repmat((sum(W, 1) * sum(W, 1)), [n, 1]) 250 .* (W.* repmat(sum(W, 1), [n, 1])), 2)), 1).* -240) 251 + (sum(sum((W .* repmat((repmat(sum(sum(W, 2), 252 1), [1, m]) .* (sum(W, 1) .* sum(((repmat(sum(W, 253 2), [1, m]) .* repmat(sum(W, 1), [n, 1])) .* W), 1))), [n, 1])), 2), 1) .* -240) + ((sum((sum(W, 2) 254 * sum(W, 2), 1) * sum(((sum(W, 1) * sum(W, 1)) * sum(1)) .* (sum(W, 1) .* sum(W, 1)), 2)) .* -30) + (((256 257 sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .* sum(((sum(W, 2) .* sum(W, 2)) .* sum((W .* W), 2)), 258 259 1)) .* -180) + (((sum(sum(W, 2), 1) .* sum(sum(W, 260 2), 1)) .* sum(sum(((W .* W) .* (W .* W)), 2), 1)) .* 60) + (sum((sum((repmat(sum(W, 2), [1, m]) 261 * (repmat(sum(W, 2), [1, m]) * repmat(sum(W, 263 1), [n, 1]))), 2) .* sum((repmat(sum(W, 2), [1, m]) 264 .* (repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [1, m])265 1), [n, 1])), 2)), 1) .* -30) + ((sum(sum(W, 2), 1) 266 .* $\operatorname{sum}((\operatorname{sum}(\operatorname{repmat}((\operatorname{sum}(W, 2) .* \operatorname{sum}(W, 2)), [1,$ 267 m]), 1) .* sum((repmat(sum(W, 2), [1, m]) .* (rep-268 mat(sum(W, 2), [1, m]).* repmat(sum(W, 1), [n, 1]))), 269 1)), 2)) .*45) + (sum((sum((W.*repmat(sum(W, 2), 270 [1, m]), 1) .* (sum((W .* repmat(sum(W, 2), [1, m])), 271 1) .* repmat((sum(sum(W, 2), 1) .* sum(sum(W, 2), 272 1)), [1, m])), 2) .* 180) + (((sum(sum(W, 2), 1) .* 273 sum(sum(W, 2), 1)).* (sum((sum(W, 1)).* sum(W, 1). 274

1)), 2) * sum((sum(W, 2) * sum(W, 2)), 1))) * 90) + (sum((sum((W.* repmat(sum(W, 1), [n, 1])), 2).* (sum((W .* repmat(sum(W, 1), [n, 1])), 2) .* repmat((sum(sum(W, 2), 1).*sum(sum(W, 2), 1)), [n,1]))), 1) .* 180) + (((sum(W, 1) * ((W') * W)) * (((W') * W) * (sum(W, 1)')) .* 360) + (sum((sum((W, 1)')))) .* 360) + (sum((W, 1)'))) .* 360) + (sum((W, 1)')) .* 360) + (.* repmat(sum(W, 1), [n, 1])), 2) .* (sum((W .* repmat(sum(W, 1), [n, 1])), 2) .* repmat(sum((sum(W, 2) * sum(W, 2)), 1), [n, 1])), 1) * 180) + ((sum((sum(W, 1) .* sum(W, 1)), 2) .* (sum((sum(W, 1) .*sum(W, 1), 2) * sum((sum(W, 2) * sum(W, 2)), 1))).*45) + (((sum(W, 2)) * W) * ((W) * W) * ((W') * sum(W, 2))) .* 360) + (sum((sum((W.* repmat(sum(W, 2), [1, m])), 1).* sum((repmat((sum(W, 2), [1, m]))), 1). 2) .* sum(W, 2)), [1, m]) .* (W .* repmat(sum(W, 2), [1, m])), 1), 2) .* -240) + ((((sum(W, 2)') * W) * ((W') * (repmat(sum(W, 2), [1, n]) * (sum(W, 2).* sum(W, 2))))) .* 180) + (((sum(W, 2)') * W) * (((W.*(W.*W))')*sum(W, 2))).*480) + ((sum((sum(W, 2) .* sum(W, 2)), 1) .* sum(((sum(W, 2)))).* sum(W, 2)) .* (sum(W, 2) .* sum(W, 2))), 1)) .* -30) + ((sum((sum(W, 2) .* sum(W, 2)), 1) .* (sum((sum(W, 2).*sum(W, 2)), 1).*sum((sum(W, 2), 1).*sum((sum(W, 2), 1).*sum((sum(W, 2), 1), 1).*sum(2) * sum(W, 2)), 1))) * 15) + ((sum((sum(W, 2) *sum(W, 2), 1) * sum(((sum(W, 2) * sum(W, 2)) * * sum(W, 2)) * * sum(W, 2)) * * sum(W, 2) * sum(W, 2sum((W.*W), 2)), 1)).*-180) + (sum(((sum((W.*W), 2) .* sum((W .* W), 2)) .* repmat(sum((sum(W, 2) * sum(W, 2)), 1), [n, 1])), 1) * -90) + ((sum((sum(W, 1) .* sum(W, 1)), 2) .* (sum((sum(W, 2) .*sum(W, 2), 1) .* sum(sum((W.*W), 2), 1))) .* 90)+ (sum(((sum(W, 1).*sum(W, 1)).*sum(((W.**sum(W, 1))).*sum(((W.**sum(W, 1)).*sum(((W.**sum(W, 1))).*sum(((W.**sum(W, 1)))).*sum(((W.**sum(W, 1)))).*sum(((W.**suW) .* (W .* W)), 1)), 2) .* -480) + ((sum((sum(W, 1) .* sum(W, 1)), 2) .* sum(sum(((W .* W) .* (W .* W)), 2), 1)) .* 60) + (sum((w .* w), 1).* sum(((W .* W) .* (W .* W)), 1)), 2) .* -480) + $(\operatorname{sum}((\operatorname{sum}(\operatorname{repmat}((\operatorname{sum}(W, 2)), \operatorname{sum}(W, 2)), [1,$ m]), 1) .* sum(((W .* W) .* (W .* W)), 1)), 2) .* 60) + ((sum(W, 1) .* sum((W.* W), 1)) * (((W') * W) * (sum(W, 1)'))) .* -720) + ((sum(sum(W, 1)'))) .* -720) + ((sum(W, 1)')) .* -720) + ((sum(W, 1)') .* -720) + ((sum(W, 1)')) .* -720) + ((sum(W, 1)')) .* -720) +2), 1) .* sum(sum((((repmat(sum(W, 2), [1, m]) .* repmat(sum(W, 1), [n, 1])) .* W) .* repmat(sum((W .* W), 1), [n, 1]), 2), 1)) .* -720) + ((sum((sum(W, 2) * sum(W, 2)), 1) * sum(((sum(W, 1) * sum(W, 1)) * sum((W * W), 1)), 2)) * -180) + ((sum(sum((W.*W), 2), 1).*sum(((sum(W, 1).*sum(W, 1)).*sum((W.*W), 1)), 2)).*-180) + (sum((sum(W, ...))))1) .* sum(W, 1)) .* (sum((W .* W), 1) .* sum((W .* W), 1))), 2) .* 720) + (sum((Sum((W .* W), 1)).* (sum((W .* W), 1) .* sum((W .* W), 1))), 2) .* 240) + (sum(sum((repmat(sum((W .* W), 1), [n, 1]) \cdot * (repmat((sum(W, 2) \cdot * repmat(sum(sum(W, 2), 1), [n, 1])), [1, m]) .* repmat(sum((W .* W), 1), [n, 1))), 2), 1) .* -90) + (sum((sum(repmat((sum(W, 2).* sum(W, 2)), [1, m]), 1) .* (sum((W .* W), 1) .*

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sum((W.*W), 1)), 2).*-90) + (sum((sum((W.*
               W), 2).* (sum((W.*W), 2).* repmat((sum(Sum(Sum), 2).*
               2), 1) * sum(sum(W, 2), 1)), [n, 1])), 1) * -90) +
               ((sum((sum(W, 1) .* sum(W, 1)), 2) .* (sum(sum((
               + (((sum(sum(W, 2), 1) .* sum(sum(W, 2), 1)) .* (
               sum(sum((W.*W), 2), 1).*sum(sum((W.*W), 2),
               1))) .*45) + ((( sum(sum(W, 2), 1) .* sum(sum(W,
               2), 1)) .* ( sum(( sum(W, 2) .* sum(W, 2)), 1) .*
339
               sum(sum((W.*W), 2), 1))).*90) + (sum((Sum((W.*W), 2), 1)))
340
               .* W), 2) .* ( (sum(W, 2) .* sum(W, 2)) .* (sum(W, 2))
               2) * sum(W, 2))), 1) * 240) + (sum(( (sum(W, 2)
               .* sum(W, 2)) .* sum(((W.*W).*(W.*W)), 2)),
               1) .* -480) + (sum(( (sum(W, 2) .* sum(W, 2)) .* (
               sum(( W .* W), 2) .* sum(( W .* W), 2))), 1) .* 720)
               + ((sum(sum(W, 2), 1) .* sum((sum(( W .* W), 2) .*
               sum(((repmat(sum(W, 2), [1, m]).* repmat(sum(W, 1), [1, m]))))
               1), [n, 1]) .* W), 2)), 1)) .* -720) + ( ( (sum(W, 2)')
               * (W * (W')) * (sum(W, 2) .* sum((W .* W), 2)))
                .* -720) + ( sum(( sum(( W .* W), 2) .* sum(( ( W
349
350
               .* W) .* (W .* W)), 2)), 1) .* -480) + (\operatorname{sum}((\operatorname{sum}((
               W .* W), 2) .* ( sum(( W .* W), 2) .* sum(( W .*
               (W), (2)), (1) * (240) + ((sum((sum(W, 1) .* sum(W, 1) 
               1)), 2) .* sum(( ( sum(W, 2) .* sum(W, 2)) .* sum((
               2), 1) * sum(( ( sum(W, 2) * sum(W, 2)) * sum((
               W.*W), 2)), 1)).*-180) + (sum((sum((W.*rep-
               mat(sum(W, 1), [n, 1])), 2) .* (sum(W, 2) .* sum(( (
358
               repmat(sum(W, 2), [1, m]) * repmat(sum(W, 1), [n, m])
359
               1])) * W), 2))), 1) * -360) + ( ( ( sum(W, 1) *
360
               (W')) .* (sum(W, 1) * (W'))) * sum((W.*W), 2))
                * -360) + (sum((sum(repmat((sum(W, 1) * sum(W,
               1)), [n, 1]), 2) .* ( sum(( W .* W), 2) .* sum(( W .*
363
               2)), 1) .* ( sum(sum(( W .* W), 2), 1) .* sum(sum((
364
               W .* W), 2), 1))) .* 45) + (sum(sum(( ( W .* W)
               .* ( ( W .* W) .* ( W .* W))), 2), 1) .* 256) + ( (
367
               sum(sum(( W .* W), 2), 1) .* sum(sum(( ( W .* W)
368
               *(W.*W), 2, 1) *60 +((sum(sum((W.*W),
369
              2), 1) .* ( sum(sum(( W .* W), 2), 1) .* sum(sum((
               W * W), 2), 1))) * 15) + (sum(sum((repmat(sum((
               W.* W), 1), [n, 1]) .* ( repmat(sum(( W.* W), 2),
               [1, m]) ** repmat(sum(( W .* W), 1), [n, 1])), 1), 2)
               .* -90) + ((repmat(sum(sum(W, 2), 1), [1, m]) * (
               ((W') * W) .* ((W') * W)) * repmat(sum(sum(W, W))) * repmat(sum(W, W))) * repmat(sum(Sum(W, W))) * repmat(sum(W, W))) * repmat(sum(Sum(W, W))) * repmat(sum(Sum(W, W))) * repmat(sum(W, W))) * repmat(sum(W, W)) * repmat(sum(W, W)) * repmat(sum(W, W)) * repmat(sum(W, W))) * repmat(sum(W, W)) * repmat(sum(W
375
               2), 1), [m, 1]))) .* 90) + (sum(sum((repmat(sum(( W
               .* W), 2), [1, m]) .* ( repmat(sum(( W .* W), 2), [1,
               m]) .* repmat(sum(( W .* W), 1), [n, 1]))), 2), 1) .*
378
              -90) + (sum(( ( ( W * (W')) .* ( W * (W'))) * (
379
               sum(W, 2) * sum(W, 2)), 1) * -360) + (sum(( ( (
380
               W * (W')) .* (W * (W'))) * sum((W .* W), 2)), 1)
381
               * -360) + (sum(((W * (W')) .* (W * (W'))) *
382
               sum(repmat((sum(W, 1) .* sum(W, 1)), [n, 1]), 2)),
383
               1) .* 90) + (sum(( ( ( W * (W')) .* ( W * (W'))) *
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 $\begin{array}{l} \operatorname{repmat}(\operatorname{sum}((\ \operatorname{sum}(W,\ 2)\ .\ *\ \operatorname{sum}(W,\ 2)),\ 1),\ [n,\ 1])), \\ 1)\ .\ *\ 90)\ +\ (\ \operatorname{sum}((\ (\ \operatorname{sum}(W,\ 1)\ .\ *\ \operatorname{sum}(W,\ 1))\ *\ (\ (\ W')\ *\ W)),\ 2)\ .\ *\ -360)\ +\ (\ \operatorname{sum}((\ W\ .\ *\ W),\ 1)\ *\ (\ (W')\ *\ W)),\ 2)\ .\ *\ -360)\ +\ ((\ \operatorname{repmat}(\operatorname{sum}(\operatorname{sum}((\ W\ .\ *\ W),\ 2),\ 1),\ [1,\ m])\ *\ \operatorname{sum}((\ (\ W')\ *\ W)\ .\ *\ (\ W')\ *\ W),\ 2))\ .\ *\ 90)\ +\ (\ \operatorname{sum}(\operatorname{sum}((\ W\ .\ *\ (W')\)\ *\ (W\ .\ *\ (W\ .\ *\ W)))),\ 2),\ 1)\ .\ *\ 480)\ +\ (\ \operatorname{sum}(\operatorname{sum}((\ (\ W\ *\ (W')\)\ *\ W)),\ 4096;\ W)\ .\ *\ (\ W\ *\ (W\ .\ W)))\ /\ 4096; \end{array}$

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