Package ‘multifractal’

May 16, 2022

Type Package

Title Multifractal and Surrogate Analysis

Version 1.0.0

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Description The package provides functions for performing improved MF-DFA anaylsis, simulation of multifractal signals and surrogate algorithms (IAAFT, IAAWT) for testing the significance of multifractality.

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Encoding UTF-8

LazyData true

Imports waveslim, pracma, grDevices

R topics documented:

`ffGn` *Fast Fractional Gaussian Noise*

Description

A fast algorithm for simulating fractal Gaussian noise.

Usage

```
ffGn(n=1000, H=0.5, mu=0, sigma=1)
```

Arguments

`n`	length of the simulated series.
`H`	a numeric value within 0 and 1 denoting the Hurst exponent. This describes the persistence (i.e., level of linear auto-correlation). Above 0.5 is persistent, below 0.5 is anti-persistent.
`mu`	mean value of the series.
`sigma`	standard deviation of the series.

Value

Returns a series of simulated fractal Gaussian noise.

Note

Translated from Matlab into R. Original Matlab code by B. Scott Jackson.

Author(s)

Wolfgang Schadner

References

Mandelbrot, B. B., Fisher, A. J., & Calvet, L. E. (1997). A multifractal model of asset returns.

See Also

`mfsim`, `mfdfa`

Examples

```
x <- ffGn(H=0.8)

# fractal Brownian motion:
B <- cumsum(x)
plot(B)
```

`iaaft` *Iterated Amplitude Adjusted Fourier Transform*

Description

Performs the iterated amplitude adjusted Fourier transformation (IAAFT) upon a time-series `x`.

Usage

```
iaaft(x, xdist=x, N=1, ...)
```

Arguments

`x`	a numeric vector containing the values of the time-series.
`xdist`	a numeric vector containing the values of the distribution to match. Default is set to `xdist=x`.
`N`	number of surrogates to create. Default is 1.
`...`	additional parameters to control for accuracy.

Details

The algorithm can be used to remove a time-series' `x` non-linear correlation structure (multi-fractality) while keeping the degree of linear correlation (persistence). It can further transform \itemx to match any empirical distribution `xdist`.

Value

Returns a numeric vector of the surrogated time-series.

Note

Code is based on Henning Rust (2006).

Author(s)

Wolfgang Schadner

References

T. Schreiber and A. Schmitz (2000), Surrogate time series, *Physica D*, 142, pp.346-382.

See Also

`iaawt`

Examples

```
n <- 1000
# random normal variable with positive auto-correlation:
x <- sort(rnorm(n))
# random variable from t-distribution:
y <- rt(n, 5)
z <- iaaft(x, y)
```

`iaawt` *Iterated Amplitude Adjusted Wavelet Transform*

Description

Performs the iterated amplitude adjusted Wavelet transformation (IAAWT) upon a time-series `x`. The algorithm was translated from Chris Keylock's Matlab code into R.

Usage

```
iaawt(x, xdist=x, N=1, ...)
```

Arguments

`x`	a numeric vector containing the values of the time-series.
`xdist`	a numeric vector containing the values of the distribution to match. Default is set to `xdist=x`.
`N`	number of surrogates to create. Default is 1.
`...`	other parameters to control for the accuracy.

Details

The algorithm can be used to randomize a time-series `x` phase while keeping the non-linear correlation structure (point-wise Hölder regularity). It can further transform `x` to match any empirical distribution `xdist`.

Value

Returns a numeric vector or matrix of the surrogated time-series.

Note

Translated from Matlab into R. Original Matlab code by Chris Keylock.

Author(s)

Wolfgang Schadner

References

Keylock, C. J. (2017), Multifractal surrogate-data generation algorithm that preserves pointwise Holder regularity structure, with initial applications to turbulence, *Physical Review E*, 95(3), 032123.

See Also

`iaaft`

Examples

```
n <- 1000
# random normal variable with positive auto-correlation:
x <- sort(rnorm(n))
# random variable from t-distribution:
y <- rt(n, 5)
z <- iaawt(x, y)
```

`localH` *Local Hurst Exponent*

Description

Estimates the local Hurst exponent based on Ihlen and Vereijken (2014). This measures the current, temporal degree of persistence. A value larger 0.5 means positive auto-correlation and below 0.5 describes negative auto-correlation.

Usage

```
localH(x, mdl, scale=5:21, m=1, align="center")
```

Arguments

`x`	a numeric vector containing the values of the time-series.
`mdl`	an object of class `multifractal` estimated from the function `mfdfa`.
`scale`	an integer vector containing the time scales. These should be small to capture local correlation behavior, i.e. the current degree of persistence.
`m`	an integer representing the polynomial order of detrending, `m=1` defines linear detrending.
`align`	specifies the alignment within the estimation windows. Possible values are 'center', 'left' or 'right'.

Details

The intertemporal persistence is estimated from the interpolating routine of Ihlen and Vereijken (2014). The multifractal model should be estimated in a first step using `mfdfa`.

Value

Returns a two-column matrix of local Hurst exponent and related interquartile range.

Author(s)

Wolfgang Schadner

References

Ihlen E. A. F., & Vereijken B. (2014). Detection of co-regulation of local structure and magnitude of stride time variability using a new local detrended fluctuation analysis. *Gait Posture*, 39(1), p.466-471.

See Also

mfdfa

Examples

```
n <- 1024

# random t-distributed variable:
x <- rt(n, 3)

mdl <- mfdfa(x, overlap=T)

Ht <- localH(mdl)

plot(Ht)
```

---

mfdfa *Multifractal Detrended Fluctuation Analysis*

Description

Performs the multifractal detrended fluctuation analysis upon a time-series x. Allows the improvement of overlapping windows.

Usage

```
mfdfa(x, scale=NA, q=-5:5, m=1, overlap=FALSE)
```

Arguments

x	a numeric vector containing the values of the time-series.
scale	an integer vector containing the time scales. It is recommended to use a log-equally spaced sequence for scale, for example via the function logseq(). When set to NA then it is calculated as scale = round(logseq(from=32,to=length(x)/10,n=20)).
q	a numeric vector containing values of moment orders.
m	an integer representing the polynomial order of detrending, m=1 defines linear detrending.
overlap	a boolean indicating if using overlapping windows or not. If set to TRUE then the estimation is more robust and reliable, if set to FALSE then the computational time is faster.

Details

The algorithm allows a basic but also improved (overlapping windows) estimation of a time-series' multi-scaling characteristica. It comes with the methods plot() and print() to easily visualize/access the multifractal spectra and statistics. The function significance() allows to easily test the significance of multifractality.

Value

Returns an object of class multifractal containg multifractal statistics and more.

Author(s)

Wolfgang Schadner

References

Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A: Statistical Mechanics and its Applications*, 316(1-4), 87-114.

See Also

`significance`

Examples

```
n <- 1024

# random t-distributed variable:
x <- rt(n, 3)

mdl <- mfdfa(x, overlap=T)
plot(mdl)
significance(mdl)
```

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mfsim — *Simulation of Multifractal Brownian Motion*

Description

Simulates a multifaractal Brownian motion based on Mandelbrot's "Multifractal Model of Asset Returns" (MMAR) using a lognormal cascade.

Note: the series has a length of b^k.

Usage

```
mfsim(b=2, k=10, H=0.5, mu=0, sigma=1)
```

Arguments

`b`	an integer representing the number of subdivision (i.e., 2 for the binomial model).
`k`	an integer representing the number of iterations. Note: the series has a length of b^k.
`H`	a numeric value within 0 and 1 denoting the Hurst exponent. This describes the persistence (i.e., level of linear auto-correlation). Above 0.5 is persistent, below 0.5 is anti-persistent.
`mu`	the mean value of the normal cascade.
`sigma`	the standard deviation of the normal cascade.

Value

Returns a simulated multifractal series.

Note

Translated from Matlab into R. Original Matlab code by Christian Wengert.

Author(s)

Wolfgang Schadner

References

Mandelbrot, B. B., Fisher, A. J., & Calvet, L. E. (1997). A multifractal model of asset returns.

See Also

mfdfa, ffGn

Examples

```
# multifractal B.M.:
B <- mfsim()
plot(B)

# multifractal noise:
r <- diff(B)  # e.g., stock returns
```

---

significance — *Significance of MF-DFA*

Description

Tests the significance of multifractality for an object of class `multifractal` based on `iaaft` surrogates.

Usage

```
significance(model, size, pval, ...)
```

Arguments

`mdl`	an object of class `multifractal`. See mfdfa for further details.
`size`	a numeric value indicating the number of surrogates.
`pval`	a numeric value indicating the confidence level (p-value). Default is 0.05, i.e. 95% confidence.

Details

The significance of multifractality is computed by creating IAAFT surrogates and computing the corresponding multifractal spreads. The surrogates have same distribution, length and linear-correlation as the original series but are not exposed to multifractality. Therfore, the spread as computed on the surrogates represent the spourious degree of multifractality.

Value

Returns a matrix of the original and spourious spreads of Hurst and Holder exponents.

Author(s)

Wolfgang Schadner

See Also

mfdfa

Examples

```
n <- 1024

# random t-distributed variable:
x <- rt(n, 3)

mdl <- mfdfa(x, overlap=T)
plot(mdl)
significance(mdl)
```

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