Semidefinite

Monday, June 30, 2014 11:18 AM

Max cut. Find a (S,T) that maximizes $|E(S,T)|$.

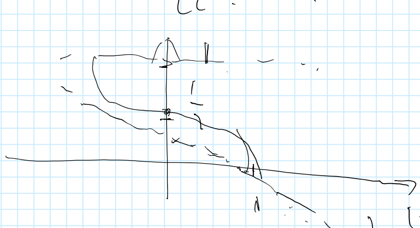
Need to show that $\Pr((v_i,v_j) \text{ is a cut}) \geq \alpha \cdot \frac{\|\alpha_i-\alpha_j\|^2}{4}$.

$\alpha = 0.878$.

$\Pr((v_i,v_j) \text{ is a cut}) = \arccos(\langle v_i,v_j\rangle)/\pi$. let $t = \langle u_i,u_j\rangle \in [-1,1]$.

$\alpha\cdot\frac{\|u_i-u_j\|^2}{4} = \alpha\cdot\frac{1-2\langle u_i,u_j\rangle+1}{4} = \alpha\cdot\frac{1-t}{2}$

wts $\frac{\arccos t}{\pi} \geq \alpha\cdot\frac{1-t}{2}$ find the max α.



$\alpha\cdot\frac{1-t}{2}$

~~wts~~ find α st. the line lies below the curve.

$\Rightarrow \alpha = 0.878...$

Prop $\varepsilon>0$. Let Z be the number of edges our algo cuts. wts

$\Pr(Z \geq (\alpha-\varepsilon)\cdot OPT) \geq \varepsilon/2$.

Pf: $E(Z) \geq \alpha\cdot OPT$. $E(Z) = E(Z \mid Z\geq(\alpha-\varepsilon)OPT)\cdot\Pr(Z\geq(\alpha-\varepsilon)OPT) + E(Z \mid Z<(\alpha-\varepsilon)OPT)\cdot\Pr(Z<(\alpha-\varepsilon)OPT)$.

$\alpha\cdot OPT$

$\Rightarrow E(Z) \leq OPT\cdot\Pr(Z\geq(\alpha-\varepsilon)OPT) + (\alpha-\varepsilon)OPT\cdot\Pr(Z<(\alpha-\varepsilon)OPT)$

$= OPT\cdot p + (\alpha-\varepsilon)(1-p)OPT$

$= (\alpha-\varepsilon)OPT + (1-\alpha+\varepsilon)OPT\cdot p$.

$\Rightarrow \varepsilon OPT \leq (1-\alpha+\varepsilon)OPT\cdot p \Rightarrow p \geq \frac{\varepsilon}{1-\alpha+\varepsilon}$

or $E(Z) \leq m\cdot p + (\alpha-\varepsilon)(1-p)OPT$.

$\Leftrightarrow \cancel{E(Z)}\ \varepsilon\cdot OPT \leq m\cdot\Pr(Z\geq(\alpha-\varepsilon)OPT)$

$\Rightarrow p \geq \frac{OPT}{m}\varepsilon$ since $\exists$ algo st. $OPT \geq \frac{m}{2}$

$\Rightarrow p \geq \frac{\varepsilon}{2}$.

$\max \sum_{(i,j)\in E} \frac{(x_i-x_j)^2}{4}$, where $x_i \in \{-1,1\}$

$\max \sum a_{ij}x_iy_j$. $x_i\in\{-1,1\}$, $y_j\in\{-1,1\}$. $A=(a_{ij})$ is an arbitrary $n\times n$ matrix.

$\max \sum a_{ij}\langle u_i,v_j\rangle$.

st. $\|u_i\|^2 = 1$. $\|v_j\|^2 = 1$.

this is relaxation of above problem since we can just take $u_i = x_i \cdot e$. $v_j = y_j \cdot e$. where e is a unit vector.

Round SDP solution. (Krivine's rounding 1977)

$u \longrightarrow X$ $\qquad \theta = \arccos \langle u, v \rangle$.

$v \longrightarrow Y$. $\qquad \|u\| = \|v\| = 1$.

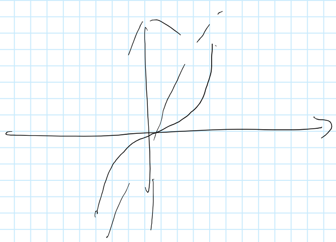
two vectors round to $\{+1, -1\}$. take arbitrary hyperplane belongs to upper +1 lower −1.

$$\begin{aligned}
\mathbb{E}(XY) &= Pr(XY=1) - Pr(XY=-1) \\
&= Pr(X=1, Y=1) + Pr(X=-1, Y=-1) - Pr(X=1, Y=-1) - Pr(X=-1, Y=1) \\
&= \frac{\pi - \theta - \theta}{\pi} = \frac{\pi - 2\theta}{\pi} \\
&= 1 - \frac{2}{\pi}\arccos\langle u, v\rangle = \frac{2}{\pi}\left(\frac{\pi}{2} - \arccos\langle u, v\rangle\right) \\
&= \frac{2}{\pi}\arcsin\langle u, v\rangle.
\end{aligned}$$

Attempt 1:

$$\mathbb{E}\left(\sum a_{ij} x_i x_j\right) = \sum a_{ij}\, \mathbb{E}(x_i y_j) = \sum a_{ij} \cdot \frac{2}{\pi} \cdot \arcsin\langle u_i, v_j\rangle$$

$$\overset{\text{wts}}{\geq} \alpha \cdot \underbrace{\sum a_{ij} \langle u_i, v_j\rangle}_{SDP} \geq \alpha \cdot OPT$$



no matter which α (line) we choose. $\langle u_i, v_j\rangle$ it lies both above / below arcsin. this is a problem.

Fail.

Lem. For $\forall$ set of unit vectors $u_1 \cdots u_n, v_1 \cdots v_n$ there exists unit vectors $\tilde{u}_1 \cdots \tilde{u}_n, \tilde{v}_1 \cdots \tilde{v}_n$ st.

$$\langle \tilde{u}_i, \tilde{v}_j \rangle = \sin(c \cdot \langle u_i, v_j \rangle). \text{ where } c = \ln(1+\sqrt{2}).$$

1) Solve the SDP.
2) transform vector.
3) apply GW procedure to vectors $\tilde{u}_1 \cdots \tilde{u}_n, \tilde{v}_1 \cdots \tilde{v}_n$.
4) obtain solution $x_1 \cdots x_n$, $y_1 \cdots y_n$.

$$\mathbb{E}\left(\sum a_{ij} x_i y_j\right) = \sum a_{ij}\, \mathbb{E}(x_i y_j) = \sum a_{ij} \cdot \frac{2}{\pi} \arcsin\langle \tilde{u}_i, \tilde{v}_j\rangle.$$

$$= \frac{2}{\pi} \sum a_{ij} \arcsin(\sin\langle u_i, v_j\rangle).$$

$$= \frac{2c}{\pi} \cdot \sum a_{ij} \langle u_i, v_j \rangle = \frac{2c}{\pi} \cdot SDP \geq \frac{2c}{\pi} \cdot OPT.$$

1) $u \in \mathbb{R}^n$. $v \in \mathbb{R}^n$.

$u \oplus v \in \mathbb{R}^{2n}$. $(\cdots) \oplus (\cdot —) = (\cdot —)(\cdot —)$.

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \oplus \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}$$

$\langle u_1 \oplus u_2, v_1 \oplus v_2 \rangle = \langle u_1, v_1 \rangle + \langle u_2, v_2 \rangle$.

$u \in \mathbb{R}^n$. $v \in \mathbb{R}^m$. $u \otimes v \in \mathbb{R}^{n \cdot m}$. $u = (a_1 \cdots a_n)$.

$u \otimes v = uv^T = (a_i \cdot b_j)_{ij}$. $v = (b_1, \cdots b_m)$

$n \times m$ matrix. tensor product.

$$\langle u_1 \otimes u_2, v_1 \otimes v_2 \rangle = \sum_{i,j} u_i^{(1)} v_i^{(1)} \cdot u_j^{(2)} \cdot v_j^{(2)}.$$

$$= \sum_{i,j} (u_i^{(1)} v_i^{(1)}) (u_j^{(2)} v_j^{(2)})$$

$$= \left(\sum_i (u_i^{(1)} v_i^{(1)}) \right) \left(\sum_j (u_j^{(2)} v_j^{(2)}) \right).$$

$$= \langle u_1, v_1 \rangle \cdot \langle u_2,$$

$$L(u) = u \oplus \frac{u^{\otimes 3}}{\sqrt{3!}} \oplus \frac{u^{\otimes 5}}{\sqrt{5!}} \oplus \cdots$$

$$R(v) = v \oplus - \frac{v^{\otimes 3}}{\sqrt{3!}} \oplus \frac{v^{\otimes 5}}{\sqrt{5!}} \oplus - \frac{v^{\otimes 7}}{\sqrt{7!}} \oplus \cdots$$

$$\langle L(u), R(v) \rangle = \sum_{k \text{ odd}} \frac{\langle u^{\otimes k}, v^{\otimes k} \rangle}{\sqrt{k!} \cdot \sqrt{k!}} \cdot (-1)^{\frac{k-1}{2}}$$

$$= \sum_k \frac{\langle u, v \rangle^k}{k!} \cdot (-1)^{\frac{k-1}{2}}$$

$$= \sin(\langle u, v \rangle).$$

$\|L(u)\| = 1$. want find

$u_i \longrightarrow L(u_i) \quad v_j \longrightarrow R(v_j)$.

6/30.

$\max \sum a_{ij} x_i y_j \quad x_i \in \{\pm 1\} \quad y_i \in \{\pm 1\}$.

$\max \sum a_{ij} x_i y_j$

$L(u) = u \oplus \frac{u^{\otimes 3}}{\quad} \oplus \frac{u^{\otimes 5}}{\quad} \oplus \cdots \quad R(v) = v \oplus - \frac{v^{\otimes 3}}{\quad} \oplus \frac{v^{\otimes 5}}{\quad} — \cdots$

$\max \sum a_{ij} x_i y_j$

$L(u) = u \oplus \frac{u^{\otimes 3}}{\sqrt{3!}} \oplus \frac{u^{\otimes 5}}{\sqrt{5!}} \oplus \cdots \quad R(v) = v \oplus \frac{v^{\otimes 3}}{\sqrt{3!}} \oplus \frac{v^{\otimes 5}}{\sqrt{5!}} \text{ --- } \cdots$

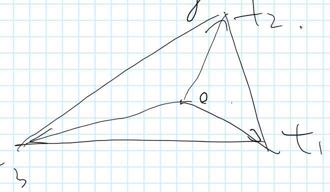
$\langle L(u), R(v) \rangle = \sin(\langle u, v \rangle)$

x, y, z generalization

$\sum a_{ij} x_i y_j + \sum b_{ik} x_i z_k + \sum c_{jk} y_j z_k$

can also generalized to n variables.

Ex.



$\langle t_i, t_i \rangle = 1$

$\langle t_i, t_j \rangle = -1/2$ for $i \neq j$

$\| t_1 + t_2 + t_3 \|^2 = 0$

$s_i = \frac{e}{\sqrt{2}}$ $\quad s_i$ are equal

Three transformations:

$A_x(u) = (u \otimes s_1) \oplus \frac{(u \otimes t_1)^{\otimes 3}}{\sqrt{3!}} \oplus \frac{(u \otimes s_1)^{\otimes 5}}{\sqrt{5!}} \oplus \cdots$

$A_y(u) = (u \otimes s_2) \oplus \frac{(u \otimes t_2)^{\otimes 3}}{\sqrt{3!}} \oplus \frac{(u \otimes s_2)^{\otimes 5}}{\sqrt{5!}} \oplus \cdots$

$u \longmapsto A_x(u)$

$\max \sum a_{ij} x_i x_j$: $\quad x_i \in \{\pm 1\}$. $\quad A = (a_{ij})$ psd.

$\max \sum a_{ij} x_i y_j$ $\quad$ if $x_i = y_i \ \forall i$. then solved problem above.

output the better of the two solution x_i and y_i.

Prove $x^T A x \geq x^T A y$ or $y^T A y \geq x^T A y$

$0 \leq (x-y)^T A (x-y) = x^T A x + y^T A y - 2 x^T A y$

Max k-CSP problem with domain size $\leq q$

$x_1, \cdots x_n \in [q] = \{0, \cdots q-1\}$.

Given a set of predicator (Boolean functions)

$f_1, \ldots, f_m$: each depends on at most k variables.

Our Goal is to find an assignment that maximize the

# of satisfied predicates

E.g. 3SAT problem

$x_1, \dots x_n$. $(x_i \vee \bar{x}_j \vee x_k)$

Max k-CSP(f) problem, special version.

$f_1 \dots f_n$ predicates and each predicate looks like

$f(x_{i_1}+e_1, x_{i_2}+e_2, \dots x_{i_k}+e_k)$

$f(x_1, \dots x_k)$ $\qquad \frac{|\mathrm{Supp}(f)|}{q^k}$

$q=2$ domain size 2

$C \cdot \frac{k}{2^k}$, $\quad 2 \cdot \frac{k}{2^k}$ approx.

$q>2$ if $q > \log k$ we can get $\frac{C \cdot kq}{q^k}$

if $k \geq q$. $\frac{C \cdot kq}{q^k}$ $\qquad k < q$. $\frac{C \cdot q^2}{q^k}$

$\boxed{\text{Always. } C \cdot \frac{q}{q^k}}$

Monday

$\exists ? x$. $\quad Ax \geq b$. $x \geq 0$

$a_1 x \geq b \dots a_m x \geq b$

$x^{(t)}$. $a_1 x^{(t)} - b \dots a_m x^{(t)} - b$

$p_1^{(t)}, \dots p_m^{(t)} \geq 0$. ask $\exists ? x$ st. $\sum p_i^{(t)} (a_i x - b_i) \geq 0$.

$|a_i x - b_i| \leq \rho \ \forall i$

$T = O\left(\frac{\rho^2 \log m}{\varepsilon}\right)$ $\qquad x \to x^{(t+1)}$

Set Cover

$U = \{1, \dots n\}$. $S_1, \dots S_m \subset U$. pick minimal # of sets to cover U.

$\min \sum_S x_S$. $\quad (x_S \in \{0,1\})$ $\quad 0 \leq x_S \leq 1$. $\forall S$.

$\forall i.\ \sum_{i \in S} x_S \geq 1.$

Oracle: given $p_1 \dots p_m$. one for every set.

guess optimal g. try to find x achieves g.

$x_S \geq 0$ st. $\sum_{i \in [m]} p_i \cdot (\sum_{i \in S} x_S - 1) \geq 0$. $\sum x_S = g$.

$$\Leftrightarrow \sum_S x_S \cdot (\underbrace{\sum_{i \in S} p_i}) \geq 1$$

S st. $\sum_{i \in S} p_i$ maximum $x_S = \frac{1}{\sum_{i \in S} p_i}$ or just g.

weight $(w(S))$.

Oracle: For S which maximizes $\sum_{i \in S} p_i$ check if $g(\sum p_i) \geq 1$

if yes. $x_S = g$ for this set S. and 0 for other S.

if no. output Fail.

we want ρ. st. $\forall i$. $|a_i \cdot x - b| \leq \rho$.

$|\sum x_S - 1| \leq g - 1 \leq m$.

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min $c \cdot x$. Primal

$a_i x \geq b_i, \dots x \geq 0$. $y_i = a_i x - b_i \Leftrightarrow y_i \geq 0 \dots x \geq 0$.

$\sum_{i \in [m]} y_i (a_i \cdot x) \geq b \cdot y$.

$$A \begin{cases} a_1 x \geq b_1 & \} y_1 \geq 0 \\ \vdots & \vdots \\ a_m x \geq b_m & \} y_m \geq 0 \end{cases} \qquad x \geq 0.$$

$\sum y_i (a_i \cdot x) \geq b \cdot y \Rightarrow y^T A x \geq b \cdot y$

if $y^T A \leq c$ then. $c \cdot x \geq y^T A x \geq b \cdot y$.

max by. $y^T A \leq c$ $y \geq 0$. best lower bound.

Dual

$a_1 x \geq b_1 \} y_1$

$\vdots \qquad \vdots$

$a_m x \geq b_m$ 3 y_m

Oracle': $\sum p_i (a_i x - b_i) \geq -\epsilon'$

$p_1 \cdots p_m$

$p_1^{(t)} \cdots p_m^{(t)}$ oracle' gives x st $\sum p_i^{(t)} (a_i x - b_i) \geq -\epsilon'$

$\hookrightarrow x^{(t+1)}$

take $\bar{x} = \frac{1}{T} \sum_t x^{(t+1)}$ for $T = O\left(\frac{\rho^2 \log m}{\epsilon^2}\right) \Rightarrow \forall i$ $a_i \bar{x} - b_i \geq -\epsilon'$

$\forall i$ we know $\frac{1}{T} \sum p^{(t)} L^{(t)} \leq \frac{1}{T} \sum l_i^{(t)} + \varepsilon$ $T \geq \frac{4\rho^2 \log m}{\varepsilon^2}$

$\frac{1}{T} \sum_t \sum_i p_i^{(t)} (a_i x^{(t+1)} - b_i) \leq \frac{1}{T} \sum_i (a_i x^{(t+1)} - b_i) + \varepsilon$

min $C \cdot X$

$A_1 \cdot X \geq b_1$

$\vdots$

$A_m \cdot X \geq b_m$

$X \succeq 0$ psd $\longleftarrow$ all constraints linear except this

linear program

min $c \cdot x$

$a_i \cdot x \geq b_i$ ---- $x \geq 0$

$\exists$? $A_1 \cdot x \geq b_1$

$\vdots$

$A_m \cdot x \geq b_m$

$x \succeq 0$ oracle handles the p.s.d restraint

$w_i^{(t)} = w_i^{(t-1)} \exp(-\varepsilon l_i^{(t)}) = \exp(-\varepsilon \sum_{\tau=1}^{t} l_i^{(\tau)})$

$p_i^{(t)} = \frac{w_i^{(t)}}{\sum_{j=1}^{m} w_j^{(t)}}$ $l_i^{(t)} = A_i \cdot x - b_i$

Oracle find x st $\sum p_i^{(t)} (A_i x - b_i) \geq 0$ ~~$x \succeq 0$~~ x psd

and $\forall i$ $|A_i x - b_i| \leq \rho$

$A' \cdot X \geq b'$ A | —— 0 —— | (| |) | b_{ij}

X ~~...~~ 0

$$\begin{cases} A \cdot X \geq b \\ X \succeq 0 \\ tr(X) = 1 \end{cases} \qquad A \begin{pmatrix} \text{———}\circ\text{———} \\ \\ \end{pmatrix} \begin{pmatrix} \\ \\ \end{pmatrix} \begin{pmatrix} b_{ij} \\ \\ \end{pmatrix}$$

$$\sum a_{ik} \cdot X_{kj} = b_{ij}$$

$a_{il} = \max a_{ik}$. $\cancel{\sum a_{il}}$. $X_{lj} = \dfrac{b_{ij}}{a_{il}}$. $\sum X_{ii} \overset{?}{\leq} 1$

$$A' \cdot X = \begin{pmatrix} A_1 \cdot X \\ \vdots \\ A_m \cdot X \end{pmatrix}.$$

$$A' \cdot X = \sum_{ij} A'_{ij} X_{ij}.$$

Hint: eigenvalues

$$X = \lambda_1 u_1 u_1^T + \lambda_2 u_2 u_2^T + \cdots + \lambda_k v_k v_k^T.$$

λ_i eigenvalues. v_i eigenvectors

when $X = uu^T$. $\max A' uu^T = \sum A'_{ij} u_i u_j$.

$$= u^T A' u = \lambda_{max}(A')$$

$\lambda_{max}(A')$ we get

SDP for max cut. Klein Lamp. 6.

$$\max_{u_1 \cdots u_n \in \mathbb{R}^n} \sum_{(i,j) \in E} \frac{\|u_i - u_j\|^2}{4}$$

$$\text{st.} \quad \|u_i\|^2 = 1. \quad \forall i \in [n].$$

$\max. \frac{1}{4} (E_{ii} + E_{jj} - 2E_{ij}) \cdot X.$ Laplacian of the graph.

$$E_{ii} \cdot X \cancel{\geq} 1.$$

$$tr(X) = n.$$

Ex Solve above SDP. find width.

$\mathcal{L} = D - A$

D: diagonal. $D_{ii} = \deg(i)$.

Ex. prove $\mathcal{L} \cdot X = \sum_{(i,j) \in E} (E_{ii} + E_{jj} - 2E_{ij}) \cdot X$.

$\max \frac{1}{4} \mathcal{L} \cdot X$.

st. $E_{ii} \cdot X \geq 1$. $tr(X) = n$. X p.s.d.

$\max \frac{n}{4} \mathcal{L} \cdot Z$. st. $nE_{ii} \cdot Z \geq 1$. $\forall i \in [n]$. $tr(Z) = 1$. Z p.s.d.

$\exists? Z$. $\frac{n}{4g} \mathcal{L} \cdot Z \geq$ } $p_0^{(t)}$.

$nE_{ii} \cdot Z \geq 1$ } $p_i^{(t)}$ $\forall i$

$tr(Z) = 1$.

find Z st. $tr(Z) = 1$. $Z \succeq 0$.

$(p_0^{(t)} \frac{n}{4g} \mathcal{L} + \sum_{i=1}^{n} p_i^{(t)} n E_{ii}) \cdot Z \geq 1$. $A^{(t)} \cdot Z \geq 1$.

Oracle.

- Find. $\lambda_{max}(A^{(t)})$.
- $\lambda_{max} \geq 1$. } $O\left(\frac{m \ln n}{\epsilon}\right)$. with approximately

$Z = uu^T$, where u is the first eigenvector

$|\underbrace{n \cdot E_{ii} \cdot Z}_{Z_{ii} \leq 1} - 1| \leq n - 1$

$\mathcal{L} \cdot u^T u = u^T \mathcal{L} u$.

$\left|\frac{n}{4g} \mathcal{L} \cdot Z - 1\right| = O(n)$.

$T = O\left(\frac{\rho^2 \ln n}{\epsilon^2}\right)$. actually can get $O\left(\frac{\rho \ln n}{\epsilon^2}\right)$

$= O\left(\frac{\rho \ln n}{\epsilon^2} + \text{oracle}\right) = O\left(\frac{mn \ln n}{\epsilon^2}\right)$.

$= O\left(\frac{\rho \ln n}{\epsilon^2} + \text{oracle}\right) = O\left(\frac{mn \ln n}{\epsilon^2}\right)$.

find $\cancel{X}$. $\frac{1}{4g} L \cdot X \geq 1 - \epsilon$.

$E_{ii} \cdot X \geq 1 - \epsilon$.

$tr(X) = n$.

$1 - \epsilon \leq X_{ii} \leq 1 + n\epsilon$.

Then can produce SDP solution with value $(1 - O(r))g$.

if $1 - r \leq X_i \leq 1 + r$.

Ex. $\|(1+\alpha)u_i - (1+\beta)u_j\|^2 \geq (1 - O(r))^2 \|u_i - u_j\|^2$ for

$\|u_i\|^2 = \|u_j\|^2 = 1$ and $|\alpha|, |\beta| \leq r$.

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max. $C \cdot X$.

st. y_i { $A_i \cdot X \leq b_i$ $i = 1, \ldots m$.

X p.s.d.

Dual for this S.DP.

$(\sum y_i A_i) \cdot X \leq b \cdot y$.

Ex. for A, B p.s.d. $A \cdot B$ p.s.d

Dual:

min $b \cdot y$.

st. $\sum y_i A_i - C$ p.s.d. $y \geq 0$. results not exactly equal for S.DP.

time t. have $X^{(t)}$, $tr(X^{(t)}) = 1$.

guess g is OPT.

Oracle: find $y \geq 0$ st.

$\sum y_i A_i \cdot X^{(t)} - C \cdot X^{(t)} \geq 0$.

linear program for y.

$b \cdot y \leq g.$

$\|\sum y_i A_i - C\| \leq \rho.$ weight

where $\|M\| = \max\{|\lambda_{max}(M)|, |\lambda_{min}(M)|\}.$

$W^{(1)} = I.$ $X^{(1)} = \dfrac{W^{(1)}}{tr(W^{(1)})}$

$M^{(t)} = \dfrac{\sum y_i^{(t)} A_i - C + \rho I}{2\rho}.$ $W^{(t)} = \exp(-\varepsilon \sum_{\tau=1}^{t-1} M^{(\tau)}).$

$X^{(t)} = \dfrac{W^{(t)}}{tr(W^{(t)})}.$

$\exp(A) = I + A + \dfrac{A^2}{2!} + \cdots$

Ex. If A symmetric so is $\exp(A)$.

Ex For any symmetric A. $\exp(A)$ p.s.d.

$tr(e^{A+B}) \neq tr(e^A \cdot e^B)$. matrices need not to commute.

<u>Golden-Thompson</u>: $tr(\exp(A+B)) \leq tr(e^A \cdot e^B).$

Ex Analyze matrix MW algorithm.

Ex. $\lambda_n(\sum \bar{y}_i A_i - C) \geq -O(\varepsilon).$ for T large enough

[AK 08]

Max-Cut.

$x_i, x_j \in \{\pm 1\}.$ $(i,j) \in E.$ $x_i x_j = -1$. is a cut.

$\max \sum_{(i,j) \in E} \left(\dfrac{1 - x_i x_j}{2}\right).$

Max 3-XOR.

$x_i \in \{\pm 1\}.$ $x_i x_j x_k = b$ (1 or -1)

$\hookrightarrow$ $1 + b x_i x_j x_k$

$$X_{(i,j)} \overset{2}{=} x_i x_j \Rightarrow x_i x_j x_k = X_{(i,j)} x_k.$$

$$\frac{1+b x_i x_j x_k}{2} = \frac{1+b X_{(i,j)} x_k}{2} \longrightarrow \frac{1+b \langle u_{(i,j)}, u_k \rangle}{2}$$

$$\langle u_\phi, u_{(i,j)} \rangle = \langle u_i, u_j \rangle.$$

$$\{u_S\}_{S \subset [n], |S| \le t} \quad \|u_S\| = 1.$$

$$\langle u_S, u_T \rangle = \langle u_{S'}, u_{T'} \rangle \text{ when } S \triangle T = S' \triangle T'.$$

$$\max \sum_{i,j,k} \frac{1+b \langle u_{\{i,j\}}, u_{\{k\}} \rangle}{2} \quad \text{relaxation of original problem.}$$

Ex. $t = n$. exact.

Ex. Write relaxation for 0/1 variables.