Lecture 1

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What is the Logical pt of view?

Math Logic: Model Theory, Set Theory, Computibility, Proof Theory.

Some celebrated theorem.

- Cantor 1874. $\aleph_0 < 2^{\aleph_0}$.

  Corolleary: transcendental numbers exist. i.e. there are real numbers which are not roots of any non-zero poly w/ rational coeff.
- Hilbert first question asked IS $2^{\aleph_0} = \aleph_1$?
- Ax 1968 (Borel?)

  Any injective polynomial map $\mathbb{C}^n \to \mathbb{C}^n$ is surjective.

A set is infinite if it's possible to put a bijection with a proper sbset

- Godel 1939 Whether or not $2^{\aleph_0} = \aleph_1$ is independent of the axioms of ZFC.
- G 1931 Any "sufficiently complex" set of axioms, which can be effectively linked is either incomplete or inconsis destroy's Hilbert's program to find complete, consistent axioms in math.
- Morley 1965. Any complete countable theory which has one model up to isom, of some uncountable size has one model up to isom, in every uncountable size.
- Shelah. If $2^{\aleph_n} < \aleph_\omega$ for all n, then $2^{\aleph_\omega} < \aleph_{\omega_4}$. early 90s

Counting

Def Call a real number $x \in \mathbb{R}$ algc if it's the root of a non zero polynomial with rational coefficients

Call a set countable if it may be put in 1-1 corres with a sbset of rational numbers

Rk: This equip a set with a enumeration.

Def A set is infinite if it may be put a 1-1 corresp with a proper sbset of itself.

Obs 1 $\mathbb{Z}$ is countable

More generally, a union of two countable sets is countable

Obs 2 The lattic $\mathbb{N} \times \mathbb{N}$ is countable.

Obs 3 The countable union of countable sets is countable.

Obs 4. The set of k elements sequences of elements of a countable set is countable.
Obs 5. The set of finite sequences of elements of countable set is countable
Obs 6 The set of polynomials in 1 var w/ rational coefficients is countable.

Thm. The set of algc numbers is countable.
Thm. (Cantor) The set of real numbers is not countable.
Thm. Let X be any set, Then X cannot be put in bijection with $P(X)$.
Pf: sufficient to prove there's no surjection $f: X \to P(X)$
Suppose by contradiction $\exists$ such ~~f~~. let $Y = \{y \mid y \notin f(y)\}$.
$Y \subset X \Rightarrow Y \in P(X) \Rightarrow \exists z \in X$ st. $f(z) = Y$.
if $z \in Y$ then $z \notin f(z) = Y \Rightarrow$ contradiction.
if $z \notin Y$ then $z \in f(z) = Y \Rightarrow$ contradiction.