1. Introduction

Sensitivity is a measure on Boolean function such that something something. There is the sensitivity conjecture, which says block sensitivity is polynomially bounded by sensitivity. The largest known gap is quadratic and this is due to Rubinstein. Turan has shown that for a graph property on v vertices, $s(f) = \Omega(v)$, which implies the largest gap possible for graph properties is quadratic. In this paper, we prove that the bounds are tight. Specifically, we demonstrate a graph property f with $\Theta(v)$ sensitivity and $\Theta(v^2)$ block sensitivity. Moreover, we generalize this property to k-uniform hypergraphs and produce quadratic gaps for all k even.

For k odd, we don't have this nice quadratic gap. Instead, we make indirect progress by demonstrating k-uniform hypergraph properties with $O(v^{k/2})$ sensitivity. Conjecture by Kenyon and Kutin says that for a "nice" boolean function f, $s^0(f)s^1(f) = \Omega(n)$. However, Chakraborty showed that this was not true by giving a counterexample with sensitivity $\Theta(n^{1/3})$. Here, we conjecture that the inequality is holds for k-uniform hypergraph properties. Assuming this conjecture, we have a lower bound of $\Omega(v^{k/2})$ for the sensitivity of k-uniform hypergraphs and we give graph properties for k odd that show that this conjectured lower bound is tight. Let $f : \{0,1\}^n \to 0,1$ be a Boolean function. For any input $x \in \{0,1\}^n$, let $x^{(i)}$ be x with i^{th} bit flipped. Then the i^{th} bit is said to be *sensitive* for f if $f(x) \neq f(x^{(i)})$. The sensitivity of f on an input x, denoted by $s(f,x)$ is the number of sensitive bits for f on x.

Definition 1.1. The *sensitivity* of a Boolean function f, denoted by $s(f)$ is the maximum of $s(f,x)$ over all $x \in \{0,1\}^n$.

Similarly, given $x \in \{0,1\}^n$ and $B \subset [n]$, let x^B be x with i^{th} bit flipped for any $i \in B$. Then the "block" B is sensitive for f on x if $f(x) \neq f(x^B)$. And the *block sensitivity* of f on x, denoted by $bs(f,x)$ is the maximum number of *pairwise disjoint* sensitive blocks of f on x.

Definition 1.2. The *block sensitivity* of a Boolean function f, denoted by $bs(f)$ is the maximum of $bs(f,x)$ over all $x \in \{0,1\}^n$.

Obviously, for any Boolean function f, we have

$$bs(f) \geq s(f).$$

Definition 1.3. A Boolean function $f : \{0,1\}^{\binom{v}{2}} \to \{0,1\}$ is called a *graph property* if for every input $x = (x_{(1,2)}, ..., x_{(n-1,n)})$ and every permutation $\sigma \in S_v$, we have

$$f(x_{(1,2)}, ..., x_{(n-1,n)}) = f(x_{(\sigma(1),\sigma(2))}, ..., x_{(\sigma(n-1),\sigma(n))}).$$

Similarly, we can define k-uniform hypergraph property.

Definition 1.4. A Boolean function $f : \{0,1\}^{\binom{v}{k}} \to \{0,1\}$ is called a *k-uniform hypergraph property* if for every input $x = (x_{(1,2,...,k)}, ..., x_{(n-k+1,...,n-1,n)})$ and every permutation $\sigma \in S_v$, we have

$$f(x_{(1,2,...,k)}, ..., x_{(n-k+1...,n-1,n)}) = f(x_{(\sigma(1),\sigma(2),...,\sigma(k))}, ..., x_{(\sigma(n-k+1),...,\sigma(n-1),\sigma(n))}).$$