Algorithms in Finite Groups

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10/07/14

Exercise 1.1. $S_k^{(2)} \leq S_{\binom{k}{2}}$ is primitive for $k \geq 5$.

Exercise 1.2. Peterson's graph is isomorphic to the line graph of K_5.

Exercise 1.3. Automorphism group of Dodecahedron is not isomorphic to S_5.

Exercise 1.4. $\mathrm{Aut}^+(Dodecahedron) \cong A_5$ and $\mathrm{Aut}(Dodecaheron) \cong A_5 \times C_2$.

Exercise 1.5. If A is an orientation preserving congruence of $\mathbb{R}^3$, then $A \in \mathrm{SO}(3)$, which is the set of orthogonal matrices of determinant 1. Hence A is a rotation.

Exercise 1.6. Given $A \in \mathrm{O}(3) \setminus \mathrm{SO}(3)$. Then A can be represented by rotation and reflection in a plane perpendicular to the rotation axis.

Exercise 1.7. Realize 4-cycles in $\mathrm{Aut}(Tetrahedron)$ as rotational reflection.

Exercise 1.8. $\mathrm{Aut}(Q_n)$ includes $\mathbb{Z}_2^n$ and $\mathbb{Z}_2^n \triangleleft \mathrm{Aut}(Q_n)$.

Exercise 1.9. $\mathrm{Aut}(Q_n)/\mathbb{Z}_2^n \cong S_n$.

Exercise 1.10. $\mathrm{Aut}(Q_n) \cong \mathbb{Z}_2^n \wr S_n$.

Exercise 1.11. Is $\mathrm{Aut}(Q_n)$ primitive?

Exercise 1.12. If A is primitive and not of prime order and B is primitive. Then $A \wr B$ in product action is primitive.

Exercise 1.13. $G_{x \to y} = \{g : x^g = y\} = G_x \cdot g_0 = g_0 \cdot G_y$, where $g_0 \in G$ such that $x^{g_0} = y$.

Exercise 1.14. $[R_G, L_G] = 1$. R_G, L_G centralize each other.

Exercise 1.15. $C_{\mathrm{Sym}(G)}(R_G) = L_G$.

Exercise 1.16. L_G and R_G are permutationally isomorphic. In particular, every regular permutation group any regular permutation group $H \leq \mathrm{Sym}(G)$ is isomorphic to R_G.

Exercise 1.17. If G is semiregular, then $C_{\mathrm{Sym}(\Omega)}(G)$ is transitive.

Exercise 1.18. If G is transitive, then $C_{\mathrm{Sym}(\Omega)}(G)$ is semiregular.

Exercise 1.19. G transitive and $N \triangleleft G$, then orbits of N form a G-invariant partition of Ω.

Exercise 1.20. G primitive and has a normal subgroup N which is not trivial, then N is transitive.

Exercise 1.21. Transitive abelian group is regular.

Exercise 1.22. Give $H \leq G$, there exists a unique permutation action of G such that G_x corresponds to H.

Exercise 1.23. The kernel of this permutation action is $\mathrm{Core}(H) = \cap_{g \in G} H^g$.