Lemma 0.1. *Let q be a prime power, and $d \leq q-1$. Then there exists a collection of sets $S_1, ..., S_m \subseteq [q^{l+1}]$ such that $|S_i| = q$ for all i and $|S_i \cap S_j| < d$ for $i \neq j$ and $m = q^{dl}$.*

Proof. Let $f : \mathbb{F}_q \to \mathbb{F}_q^l$ such that $f(x) = (f_1(x), ..., f_k(x))$ where each f_i is a degree $d-1$ polynomial over $\mathbb{F}_q$. Then each f corresponds to a set of $l+1$-tuples, $S_f = \{(x, f_1(x), ..., f_k(x)) \mid x \in \mathbb{F}_q\}$.

If $g \neq f$, then the sets S_f and S_g intersect at most $d-1$ points since the equation $f_1(x) = g_1(x)$ already has at most $d-1$ solutions in $x \in \mathbb{F}_q$. We can relabel S_f by associating $i \in [q^{l+1}]$ to each $f : \mathbb{F}_q \to \mathbb{F}_q^l$. There are q^d distinct polynomials over $\mathbb{F}_q$ of degree $d-1$, so there are $(q^d)^l$ distinct sets of $(l+1)$-tuples that satisfy the above property. Hence, we can construct a collection of sets $S_1, ..., S_{q^{dl}}$ such that $|S_i| = q$ for all i and $|S_i \cap S_j| < d$ for $i \neq j$. □

Corollary 0.2. *The upper bounds on 0-sensitivity of our graph properties are tight.*

Proof. For $s^0(f)$ of our first graph property (Theorem 3.2), let q be the prime power between $k+1$ and $2(k+1)$, $l = \lfloor \log_q(v) - 1 \rfloor \geq \log_q(v) - 2$ and $d = i$. Then we have $m = q^{dl} \geq \Omega(v^i q^{-2i}) = \Omega(v^i)$ many sets $S_1, ..., S_m$ such that $S_i \cap S_j < d$.

For second graph property (Theorem 4.2), let q be the prime power between $0.5v^t$ and v^t, $l = 1/t - 1$. Then we have $m = q^{dl} \geq \Omega(v^i q^{-2i}) = \Omega(v^{i(1-t)})$ many sets $S_1, ..., S_m$ such that $S_i \cap S_j < d$.

□