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Right Triangle Trigonometry

Goals

I will be able to set up the 6 trigonometric ratios given a specified angle and legs.

I will be able to know the trigonometric values of special triangles.

I will be able to calculate the values of unknown sides of a triangle using trigonometric ratios of special angles.

I will be able to calculate the values of unknown sides of any right triangle using a calculator.

I will be able to calculate unknown angles of a right triangle using *inverse* trig functions.

Standards

Similarity, Right Triangles, and Trigonometry **G-SRT**
Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right.

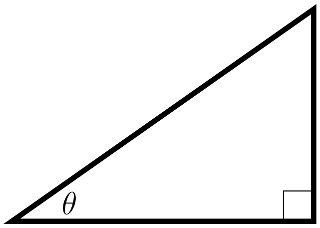
Connections

Before we learned the Pythagorean Theorem, the equation of a circle, the distance formula, and about the angles in a triangle all in preparation for right triangle trigonometry.

After we will apply what we know about right triangle trigonometry to trigonometry of any triangle, and from there to the unit circle.

Basic Trig Ratios

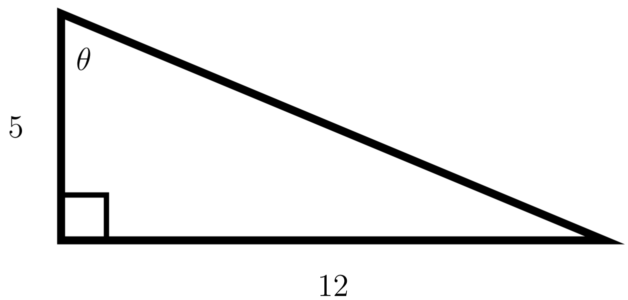
SOH - CAH - TOA



$\sin(\theta) = \frac{opp}{hyp}$ $\csc(\theta) = \frac{1}{\sin(\theta)} = \underline{\qquad}$

$\cos(\theta) = \frac{adj}{hyp}$ $\sec(\theta) = \frac{1}{\cos(\theta)} = \underline{\qquad}$

$\tan(\theta) = \frac{opp}{adj}$ $\cot(\theta) = \frac{1}{\tan(\theta)} = \underline{\qquad}$



Using the Pythagorean Theorem: Hyp=____________

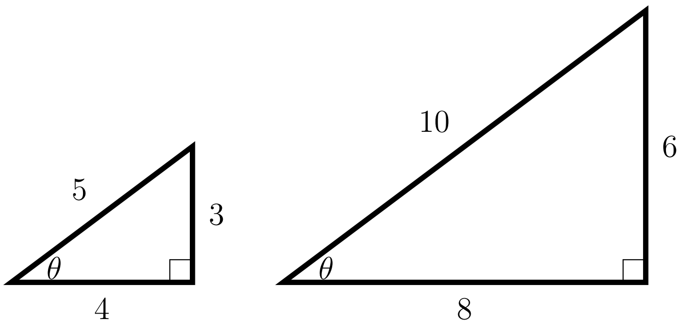
$\sin(\theta) = \underline{\qquad}$ $\csc(\theta) = \underline{\qquad}$

$\cos(\theta) = \underline{\qquad}$ $\sec(\theta) = \underline{\qquad}$

$\tan(\theta) = \underline{\qquad}$ $\cot(\theta) = \underline{\qquad}$

What is a Trig Ratio?

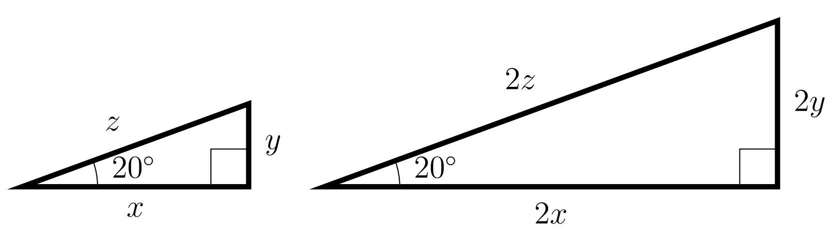
****The whole point of trig is the ratios of the sides of right triangles**** And as long as the triangles are proportional, the ratios will always be the same.



The two triangles above are similar. That means that the larger one is the same exact shape, but each side is twice as large.

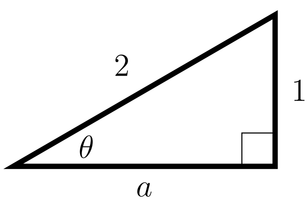
Small triangle	Big triangle
$\sin(\theta) = \frac{3}{5}$	$\sin(\theta) = \frac{6}{10} =$
$\cos(\theta) = \frac{4}{5}$	$\cos(\theta) = \frac{8}{10} =$
$\tan(\theta) = \frac{3}{4}$	$\tan(\theta) = \frac{6}{8} =$

Since the lengths of the sides are proportional (multiplied by 2), the trig ratios are the same.



Small triangle	Big triangle
$\sin(20°) = \frac{y}{z}$	$\sin(20°) = \frac{2y}{2z} =$
$\cos(20°) = \frac{x}{z}$	$\cos(20°) = \frac{2x}{2z} =$
$\tan(20°) = \frac{y}{x}$	$\tan(20°) = \frac{2y}{2x} =$

You Try Set up the six Trigonometric ratios for the triangle below.

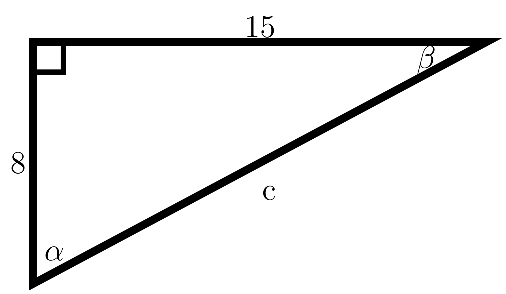


Using the Pythagorean Theorem: a=____________

$\sin(\theta) = \overline{}$ $\csc(\theta) = \overline{}$

$\cos(\theta) = \overline{}$ $\sec(\theta) = \overline{}$

$\tan(\theta) = \overline{}$ $\cot(\theta) = \overline{}$



c=____________

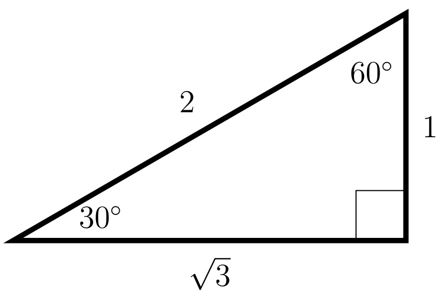
$\sin(\alpha) = \overline{}$ $\sin(\beta) = \overline{}$

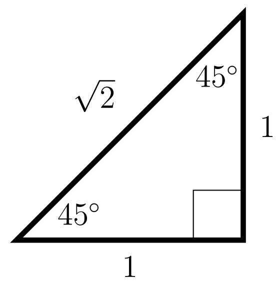
$\cos(\alpha) = \overline{}$ $\cos(\beta) = \overline{}$

$\tan(\alpha) = \overline{}$ $\tan(\beta) = \overline{}$

Special Triangles & Their Special Angles

There are two special triangles. These triangles are special because their angles are used to often. $30-60-90$, and $45-45-90$.





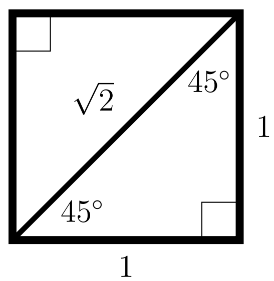
θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
30°			
45°			
60°			$\sqrt{3}$

θ	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
30°			
45°			
60°			$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

How did we get those numbers?

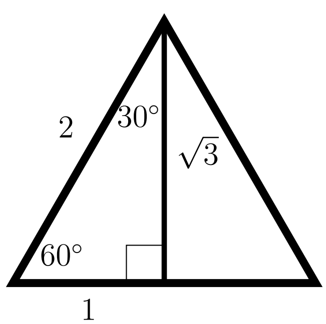
A $45-45-90$ is simply a square cut along its diagonal. If each side of the square was 1, then we'd use the Pythagorean Theorem to calculate the diagonal d.

$$
\begin{aligned}
1^2 + 1^2 &= d^2 \\
2 &= d^2 \\
d &= \sqrt{2}
\end{aligned}
$$



A $30-60-90$ triangle is an equilateral triangle, with side lengths of 2, cut by an altitude (segment from one angle, perpendicular to the opposite side). Again, two sides of the right triangle are known, so we apply the Pythagorean Theorem.

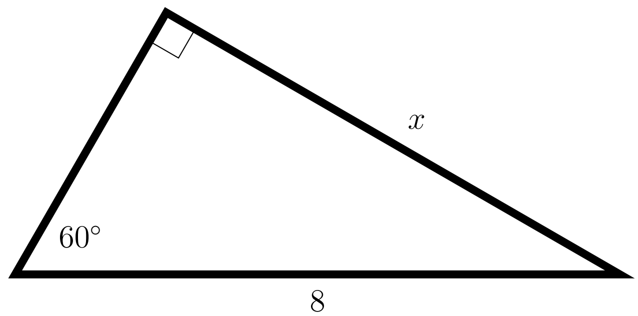
$$
\begin{aligned}
1^2+b^2 &= 2^2 \\
1+b^2 &= 4 \\
b^2 &= 3 \\
b &= \sqrt{3}
\end{aligned}
$$



Values of Unknown Sides

Since the trigonometric value of any two congruent angles are always the same (see above "What is a Trig Ratio"), that ratio can be used to help solve for unknown sides of the triangle.

Let's look at one of our special triangles first.



There are 3 questions to answer to solving for an unknown side

1. Which side are you solving for?
2. Which angle are you going to use?
3. Which trig function (proportion) to use?

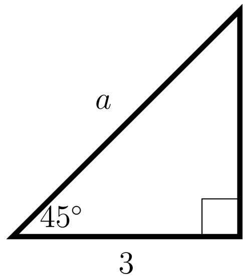
Use $\sin(60°)$

$$
\begin{array}{rrcll}
 & \sin(60°) & = & \frac{\sqrt{3}}{2} & \text{from the special triangle} \\[2em]
 & \sin(60°) & = & \frac{x}{8} & \text{from the triangle in front of us} \\[2em]
\Rightarrow & \frac{\sqrt{3}}{2} & = & \frac{x}{8} & \text{set equal to each other – solve proportion}
\end{array}
$$

Or $\cos(30°)$

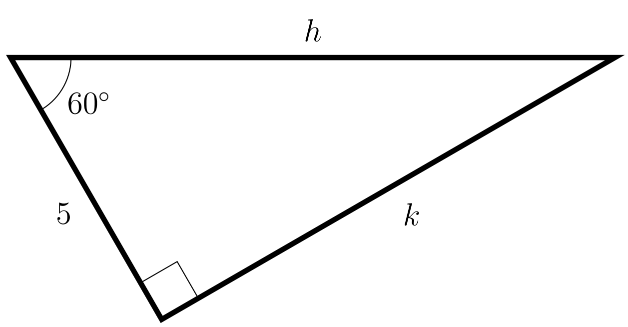
$$
\begin{array}{rrcll}
 & \cos(30°) & = & \frac{\sqrt{3}}{2} & \text{from the special triangle} \\[2em]
 & \cos(30°) & = & \frac{x}{8} & \text{from the triangle in front of us} \\[2em]
\Rightarrow & \frac{\sqrt{3}}{2} & = & \frac{x}{8} & \text{set equal to each other – solve proportion}
\end{array}
$$

Example 2: Solve for a using a trig ratio.



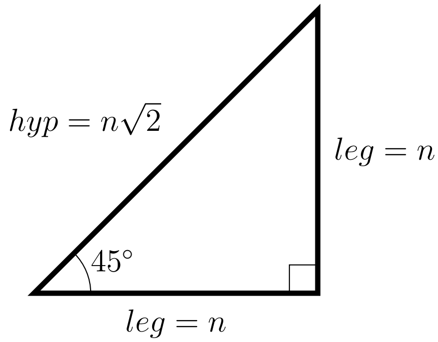
1. Which side are you solving for?
2. Which angle are you going to use?
3. Which trig function are you going to use?

You Try: Solve for k using a trig ratio.



Tips and Tricks

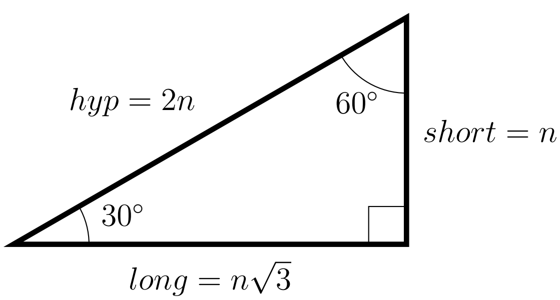
45-45-90



angle	45°	45°	90°
Opposite side	n	n	$n\sqrt{2}$

angle	90°	45°	45°
Opposite side	c	$\frac{c\sqrt{2}}{2}$	$\frac{c\sqrt{2}}{2}$

30-60-90



angle	30°	60°	90°
Opposite side	n	$n\sqrt{3}$	$2n$

angle	90°	30°	60°
Opposite side	c	$\frac{c}{2}$	$\frac{c\sqrt{3}}{2}$

angle	60°	30°	90°
Opposite side	b	$\frac{b\sqrt{3}}{3}$	$\frac{2b\sqrt{3}}{3}$

Right Triangle Trigonometry Quiz Review

NAME:__________________________________

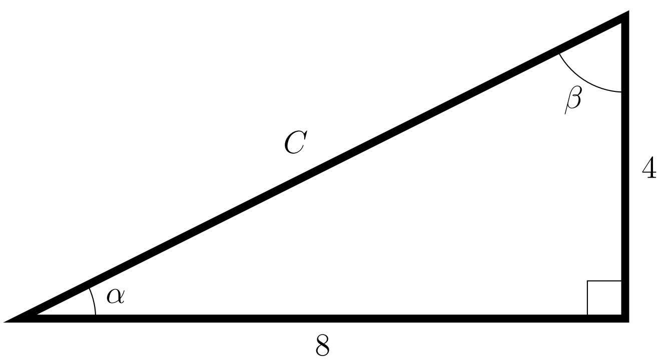
DATE:_______________________

SOH-CAH-TOA

Directions: Find the trigonometric ratios of the triangle below. Keep track of which angle is which. Reduce your fractions to lowest terms.

$$a^2 + b^2 = c^2$$

$$\sin(\theta) = \frac{opp}{hyp} \qquad \cos(\theta) = \frac{adj}{hyp} \qquad \tan(\theta) = \frac{opp}{adj}$$



1. $C =$ _____

2. $\sin(\alpha) =$ _____

3. $\cos(\alpha) =$ _____

4. $\tan(\alpha) =$ _____

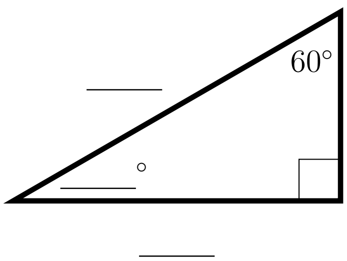
5. $\sin(\beta) =$ _____

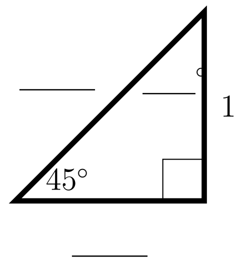
6. $\cos(\beta) =$ _____

7. $\tan(\beta) =$ _____

Special Triangles

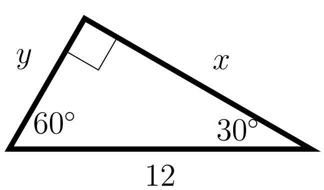
8. Fill in the missing parts of the special triangles and the chart.(1 pt each)

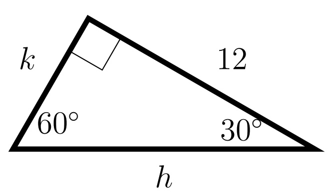


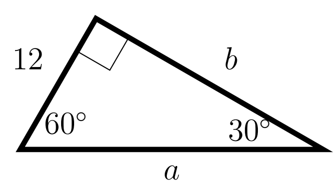


θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
30°			
45°			
60°			

Solve for the missing sides







9. $x =$ ____________

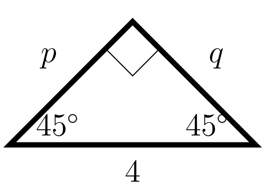
10. $y =$ ____________

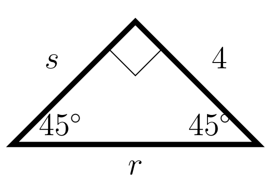
11. $h =$ ____________

12. $k =$ ____________

13. $a =$ ____________

14. $b =$ ____________





15. $p =$ ____________

16. $q =$ ____________

17. $r =$ ____________

18. $s =$ ____________

Right Triangle Trigonometry Quiz

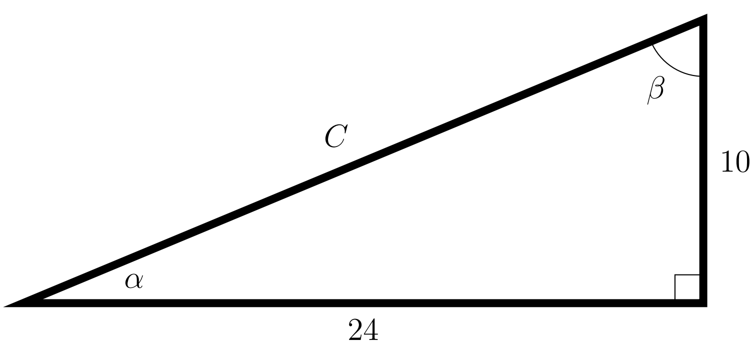
NAME:__________________________________

SOH-CAH-TOA

Directions: Find the trigonometric ratios of the triangle below. Keep track of which angle is which. Reduce your fractions to lowest terms.

$$a^2 + b^2 = c^2$$

$$\sin(\theta) = \frac{opp}{hyp} \qquad \cos(\theta) = \frac{adj}{hyp} \qquad \tan(\theta) = \frac{opp}{adj}$$



1. $C =$ _____

2. $\sin(\alpha) =$ _____

3. $\cos(\alpha) =$ _____

4. $\tan(\alpha) =$ _____

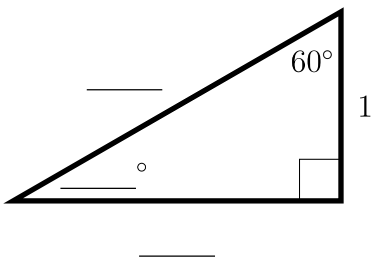
5. $\sin(\beta) =$ _____

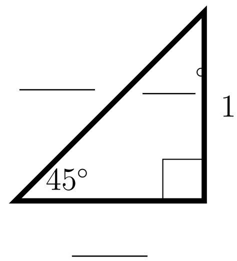
6. $\cos(\beta) =$ _____

7. $\tan(\beta) =$ _____

Special Triangles

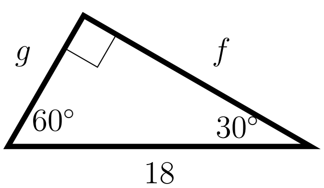
8. Fill in the missing parts of the special triangles and the chart.(1 pt each)

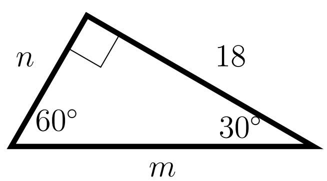


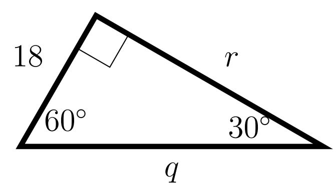


θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
30°			
45°			
60°			

Solve for the missing sides







9. $f =$ ____________

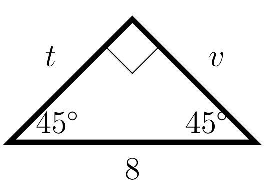
10. $g =$ ____________

11. $m =$ ____________

12. $n =$ ____________

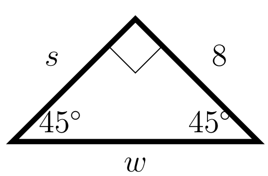
13. $q =$ ____________

14. $r =$ ____________



15. $t =$ ____________

16. $v =$ ____________



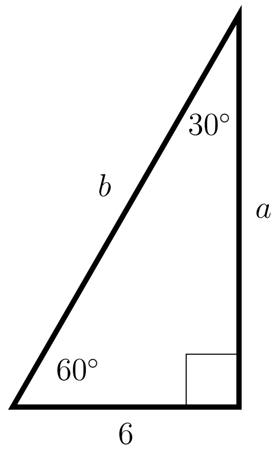
17. $w =$ ____________

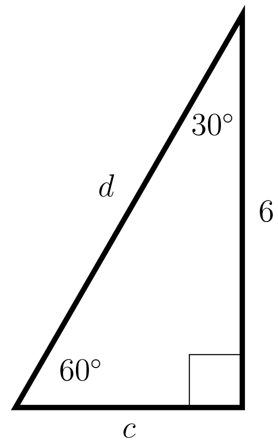
18. $s =$ ____________

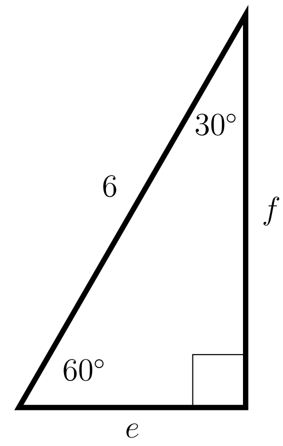
Post Quiz Extra Credit

NAME:______________________________

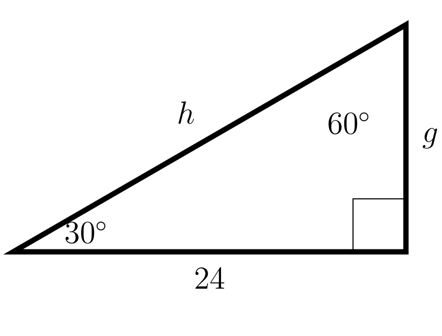
Directions: Solve the unknown sides of each of the triangles. All questions must be correct for credit.

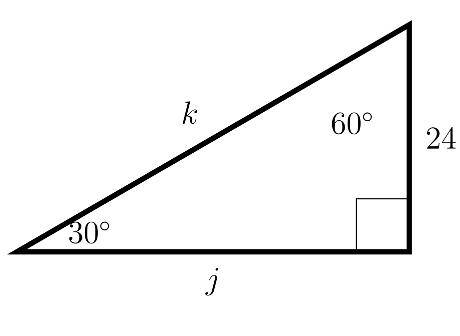




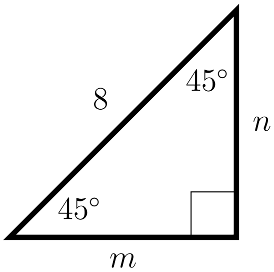


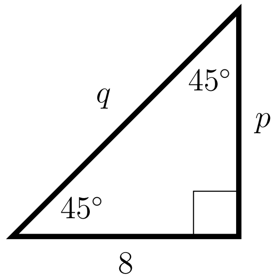
1. $a =$____________
2. $b =$____________
3. $c =$____________
4. $d =$____________
5. $e =$____________
6. $f =$____________





7. $g =$____________
8. $h =$____________
9. $j =$____________
10. $k =$____________





11. $m =$____________

12. $n =$____________

13. $p =$____________

14. $q =$____________

Right Triangle Trigonometry Re-Quiz

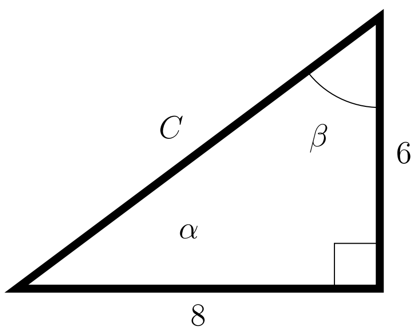
NAME:____________________________________

SOH-CAH-TOA

Directions: Find the trigonometric ratios of the triangle below. Keep track of which angle is which. Reduce your fractions to lowest terms.

$$a^2 + b^2 = c^2$$

$$\sin(\theta) = \frac{opp}{hyp} \qquad \cos(\theta) = \frac{adj}{hyp} \qquad \tan(\theta) = \frac{opp}{adj}$$



1. $C =$ _____

2. $\sin(\alpha) =$ _____

3. $\cos(\alpha) =$ _____

4. $\tan(\alpha) =$ _____

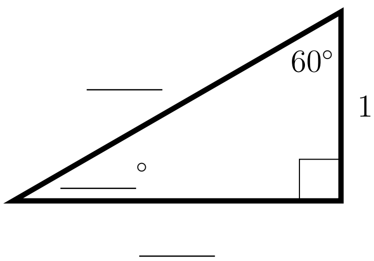
5. $\sin(\beta) =$ _____

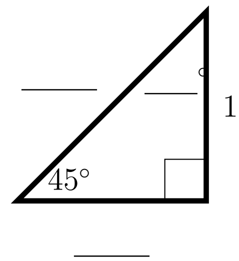
6. $\cos(\beta) =$ _____

7. $\tan(\beta) =$ _____

Special Triangles

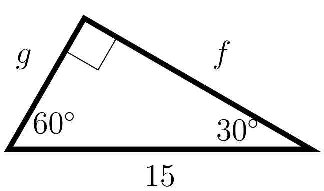
8. Fill in the missing parts of the special triangles and the chart.(1 pt each)

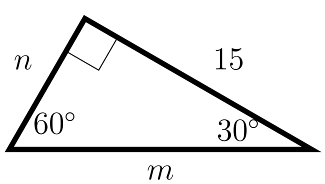


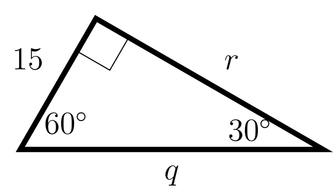


θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
30°			
45°			
60°			

Solve for the missing sides







9. $f =$ ____________

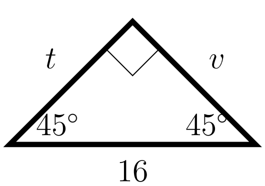
10. $g =$ ____________

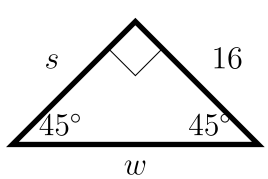
11. $m =$ ____________

12. $n =$ ____________

13. $q =$ ____________

14. $r =$ ____________





15. $t =$ ____________

16. $v =$ ____________

17. $w =$ ____________

18. $s =$ ____________

Solving for Any Right Triangle

Decimals

To solve for any right triangle you will need either a trig table or a calculator

A trig ratio is a fraction that compares the sides of a triangle. Any fraction can be written (or approximated) as a decimal.

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} \qquad \text{we know from our chart}$$
$$\sin(60^\circ) \approx 0.8660254... \qquad \text{we obtain from the calculator}$$
$$\frac{\sqrt{3}}{2} \approx 0.8660254... \qquad \text{therefore this must be true}$$

These decimals are approximations. For the non-special angles the approximation is just fine, but for the special angles keep to the exact radical/fraction form unless otherwise noted.

Any angle can be plugged into the trig functions.

Example 1: 25°

```
sin(25) = 0.422618626...
cos(25) = 0.906307787...
tan(25) = 0.466307658...
```

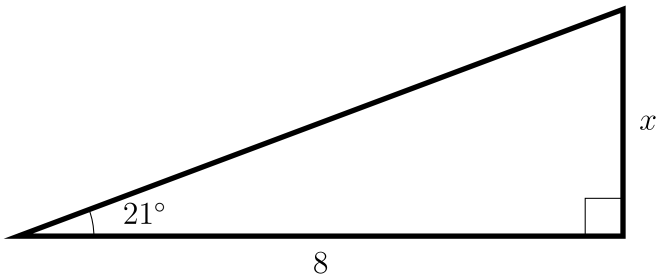
You Try: Evaluate the trig functions of the following angles on your calculator. Round to 4 decimal places.

1. (a) $\sin(50^\circ) =$
   (b) $\cos(50^\circ) =$
   (c) $\tan(50^\circ) =$
2. (a) $\sin(37^\circ) =$
   (b) $\cos(37^\circ) =$
   (c) $\tan(37^\circ) =$
3. (a) $\sin(90^\circ) =$
   (b) $\cos(90^\circ) =$
   (c) $\tan(90^\circ) =$
4. (a) $\sin(0^\circ) =$
   (b) $\cos(0^\circ) =$
   (c) $\tan(0^\circ) =$

Did you notice anything weird with 3 and 4?

Using Decimals to Solve Triangles

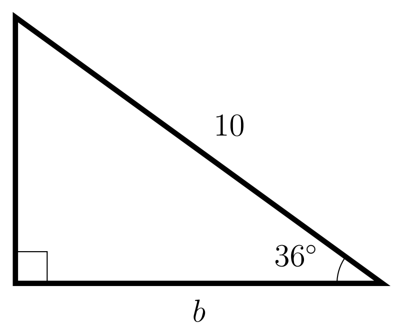
The calculator (or trig table) can determine a trig value for any angle. Therefore we can substitute the trig function with a number and determine how long an unknown side is.



This triangle has a known side *adjacent* to 21°, and since we want to know the value of x, *opposite* of 20° we will use **Tangent**.

$$\begin{aligned} \tan(21°) &= 0.383864035... && \text{from calculator} \\ \tan(21°) &= \frac{x}{8} && \text{from triangle} \\ 8\tan(21°) &= x && \text{solved for } x \\ 8 \cdot 0.383864035... &= x && \text{subtitute out} \tan(21°) \end{aligned}$$

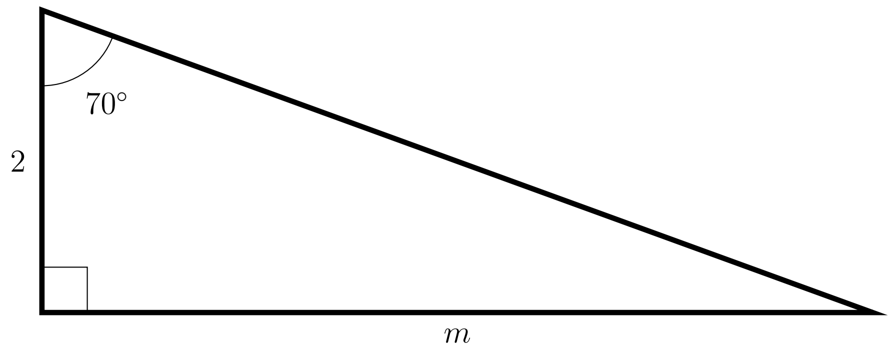
$$x \approx 3.0709$$



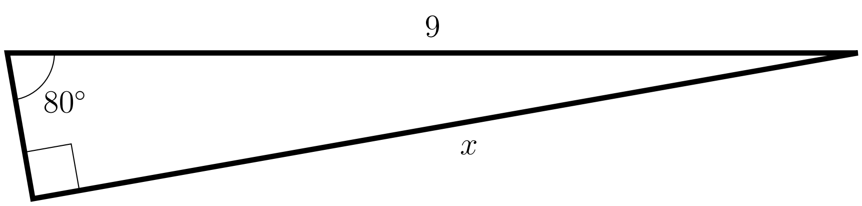
$$\begin{aligned} \cos(36°) &= 0.809016994... && \text{from calculator} \\ \cos(36°) &= \frac{b}{10} && \text{from triangle} \\ 10\cos(36°) &= b && \text{solved for } b \\ 10 \cdot 0.809016994... &= b && \text{subtitute out} \cos(36°) \end{aligned}$$

$$b \approx 8.0902$$

You Try: Solve for the missing side using your calculator or a trig table.



$m =$ ____________



$x =$ ____________

Inverse Trig Functions

Use inverse trig functions to find the angle from the side lengths. They cancel out the trig function so that it will go away and we can get whatever's inside of the parenthesis.

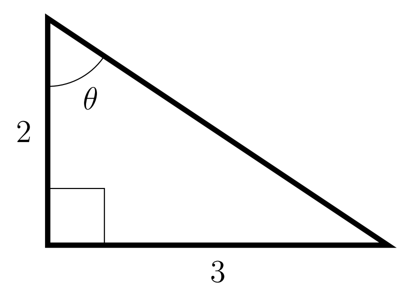
Recall from function notation that

$$f^{-1}(f(x)) = x$$

The same holds true with trig functions. They are used to find out the value of θ.

$$\sin^{-1}(\sin(\theta)) = \theta \qquad \cos^{-1}(\cos(\theta)) = \theta \qquad \tan^{-1}(\tan(\theta)) = \theta$$

Example:



To remove the θ from the *tan* we use *inverse tan* on both sides.

$$\begin{aligned}
\tan(\theta) &= \frac{3}{2} \\
\tan^{-1}(\tan(\theta)) &= \tan^{-1}\left(\frac{3}{2}\right) \\
\theta &= \tan^{-1}\left(\frac{3}{2}\right) \\
\theta &\approx 56.31^\circ
\end{aligned}$$

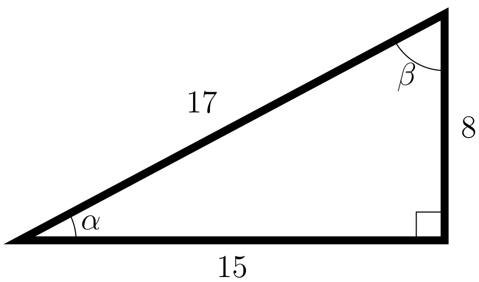
Another word for *inverse sine* is **arcsine**. Also *inverse cosine* is **arccos**, and *inverse tangent* is **arctan**. This is likely how they are labelled on your cellphone calculators.

Right Triangle Trig Test Review

Mr. Wolf NAME:__________________________________

Trig Ratios

$$\sin(\theta) = \frac{opp}{hyp} \qquad \cos(\theta) = \frac{adj}{hyp} \qquad \tan(\theta) = \frac{opp}{adj}$$



1. $\sin(\alpha) =$ _____

2. $\cos(\alpha) =$ _____

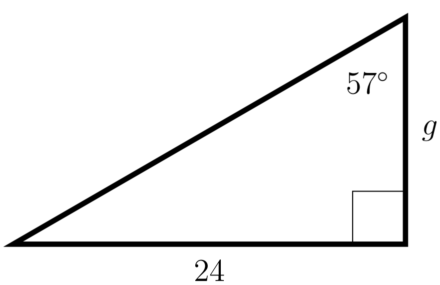
3. $\tan(\alpha) =$ _____

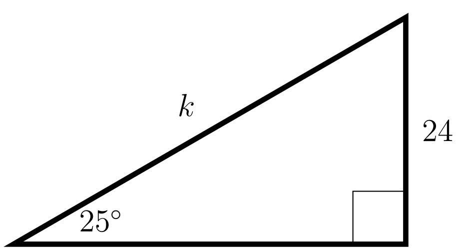
4. $\sin(\beta) =$ _____

5. $\cos(\beta) =$ _____

6. $\tan(\beta) =$ _____

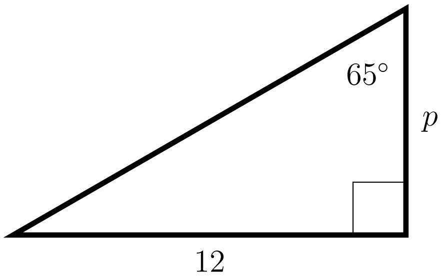
Solve for Missing Sides

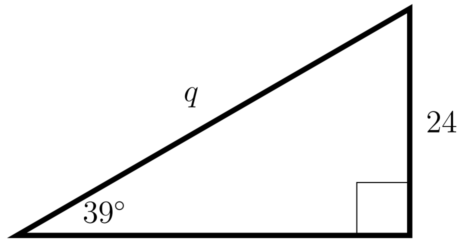




7. $g =$____________

8. $k =$____________

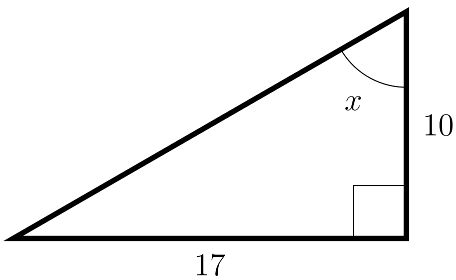


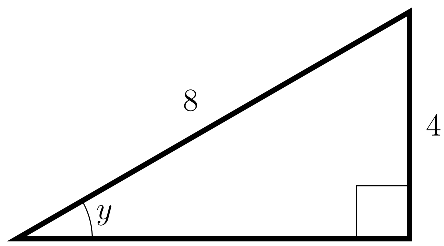


9. $p =$____________

10. $q =$____________

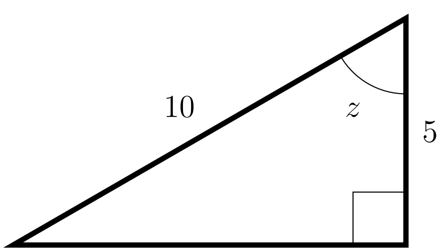
Solving for an Angle

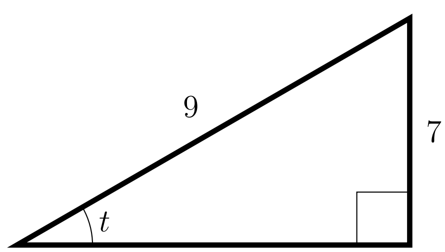




11. $x =$____________

12. $y =$____________





13. $z =$____________

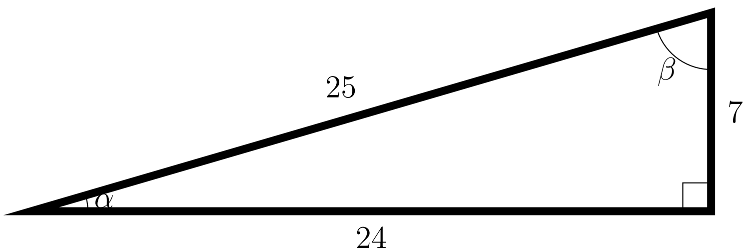
14. $t =$____________

Right Triangle Trig Test

Mr. Wolf NAME:______________________________

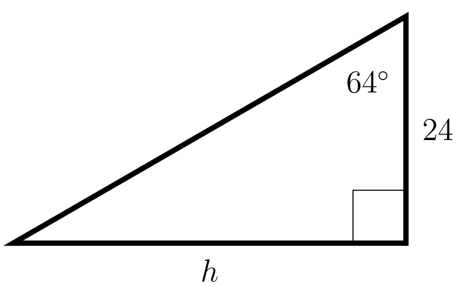
Trig Ratios

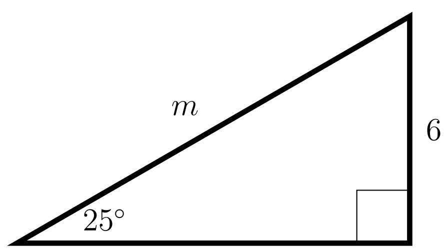
$$\sin(\theta) = \frac{opp}{hyp} \qquad \cos(\theta) = \frac{adj}{hyp} \qquad \tan(\theta) = \frac{opp}{adj}$$



1. $\sin(\alpha) =$ _____
2. $\cos(\alpha) =$ _____
3. $\tan(\alpha) =$ _____
4. $\sin(\beta) =$ _____
5. $\cos(\beta) =$ _____
6. $\tan(\beta) =$ _____

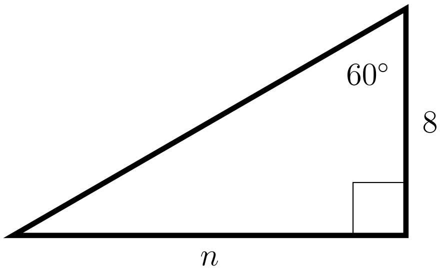
Solve for Missing Sides



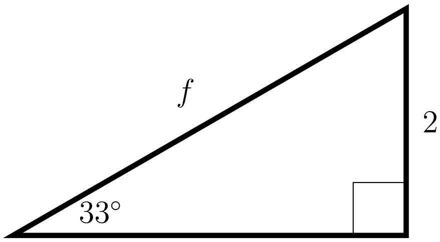


7. $h =$____________

8. $m =$____________

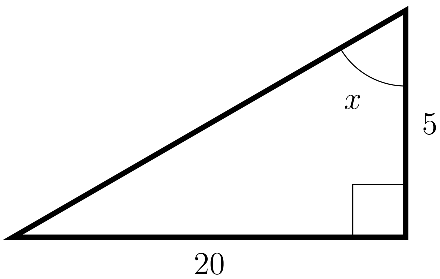


9. $n =$____________

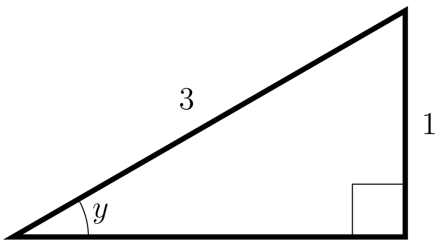


10. $f =$____________

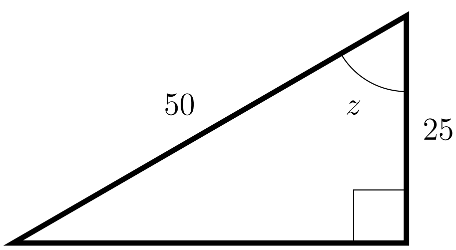
Solving for an Angle



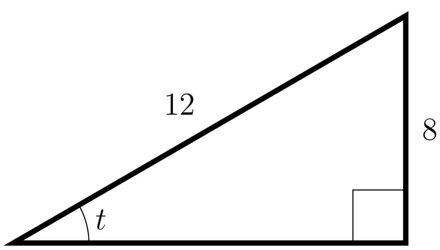
11. $x =$____________



12. $y =$____________



13. $z =$____________



14. $t =$____________