Mr. Wolf
wolf-math.com

1 Lesson Plan – Sequences and Series

1.1 Goals & Objectives

1. **SWBAT** identify arithmetic sequences
2. **SWBAT** identify geometric sequences
3. **SWBAT** determine the n^{th} term of a sequences
4. **SWBAT** determine the rule of a sequence
5. **SWBAT** use proper sequence notation
6. **SWBAT** use Σ notation for series
7. **SWBAT** find the sum of a series

1.2 Standards

Interpreting Functions F-IF

Understand the concept of a function and use function notation

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n1$.

Building Functions **F-BF**

Build a function that models a relationship between two quantities

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Linear and Exponential Models **F-LE**

Construct and compare linear and exponential models and solve problems

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

1.3 Connections

Now we are learning about how patterns arise in mathematics.

Later we will learn about functions; how they are similar or different from sequences.

2 Sequences

A **Sequence** is a list of things.

2.1 Sequence Activity

1. Make a list of pictures, numbers, or letters.
2. Circle the second term.
3. What rules does your sequence follow?
4. Is your sequence an infinite sequence, or is it finite?
5. Could you change the -finity of your sequence?

So, as identified from the activity, a sequence is any list or progression of things. Usually sequences are numbers, but they can really be anything.

Examples:

$\{1, 2, 3, 4, 5, 6, ..., n\}$ – counting up by 1.

{c, d, e, f, g, ...} – The alphabet starting at "c."

$\{1, 0, 8, 08, 88...\}$ – the number of close spaces in each term.

{M, C, 2, S, T, E, M} – School name

$\{6, 5, 4, 3\}$ – Count backwards from 6 to 3.

$\{1, 2, 1, 2, 1, 2, 1, ...\}$ – Alternate 1, 2.

2.2 -Finity

Some sequences are infinite and other are finite. When listing a sequence infinity is determined by 3 dots at the end. $\{1, 2, 3, ...\}$. If it doesn't have 3 dots we assume it's finite. If we're using a mathematical formula (see **??**) we assume that it's infinite and it follows the mathematical rule forever.

2.3 Notation

Sequences have curly brackets around them, {}, and commas between the elements. Each element of the sequence is called a **term**. Sometimes they are called "elements", or "members."

Formulas are written with subscript. a_n. The title of the sequence is a, and n is the term number. a_n is the value of the term number.

Example: $a_n = 3n + 2$

$$
\begin{gathered}
a_1 = 3(1) + 2 = 5 \\
a_2 = 3(2) + 2 = 8 \\
a_3 = 3(3) + 2 = 11 \\
\vdots
\end{gathered}
$$

So, $a_n = 3n + 2 = \{5, 8, 11, ...\}$

2.4 Formulas

It's best to determine the formula for a sequence.

Take $\{3, 5, 7, 9, ...\}$, it's ok to say "start on 3 and add 2," but that makes it difficult to figure out the 823^{rd} term. So we should determine a mathematical formula. Different types of sequences have different ways of determining the formula. We will see them each in a little bit.

3 Arithmetic Sequence

An **Arithmetic Sequence** is a pattern of numbers through addition or subtraction.

ex 1) $\{1, 4, 7, 10, 13, 16, 19, 22, ...\}$
This sequence begins on 1 and increases by 3 each term. If you subtract any term by the term that succeeds it you should get 3. This number (3 in our example) is called the **common difference**.

We therefore could write a sequence as $\{a, a + d, a + 2d, a = 3d, ...\}$
, where a is the first term and d is the common difference.

You Try: Determine the sequences are arithmetic sequences. If so what's the common difference?

1. $\{2, 7, 12, 17, 22, ...\}$
   $d =$
2. $\{10, 8, 6, 4, 2, ...\}$
   $d =$
3. $\{1, 2, 4, 8, 16, 32, 64, ...\}$
   $d =$
4. $\{2, 6, 10, 14, 18, 22, ...\}$
   $d =$
5. $\{27, 9, 3, 1, ...\}$
   $d =$

3.1 Writing Formulas for Arithmetic Sequences

We use the general form

$$a_n = a + d(n - 1)$$

where a is the first term and d is the common difference. $(n - 1)$ is used because d is not in the first term.

Take our arithmetic sequences from the "you try." Just plug in and simplify

$\{2, 7, 12, 17, 22, ...\}$ $\quad d = 5 \quad a_n = 2 + 5(n-1) = 5n - 3$

$\{10, 8, 6, 4, 2, ...\}$ $\quad d = -2 \quad b_n = 10 - 2(n-1) = -2n + 12$

$\{2, 6, 10, 14, 18, 22, ...\}$ $\quad d = 4 \quad c_n = 2 + 4(n-1) = 4n - 2$

Another way to do this is $a_n = dn + (a - d)$, where a is the first term and d is the common difference. Both ways are acceptable.

4 Summation of Arithmetic Sequences: Arithmetic Series

We know that $a_n = a_1, a_2, a_3, a_4...$ is a pattern that repeats at a constant rate. What if we wanted to add up all of these terms $(a_1 + a_2 + a_3...)$?

> **Example:** The following pattern of a pyramid is demonstrated by arithmetic sequence $b_n = 2n - 1$
>
> | | |
> |---|---|
> | Step 1 | X |
> | Step 2 | XXX |
> | Step 3 | XXXXX |
> | Step 4 | XXXXXXX |
> | $\vdots$ | $\vdots$ |
>
> To figure out how many X's are on the 8^{th} level, we would plug in to the formula, $b_8 = 2(8) - 1 = 15$. So there are 15 X's on the 8^{th} level. How many X's have been used total? All of the levels need to be added together to figure that out.
>
> We'll come back to this in a minute.

To indicate the **sum** of the terms of a sequence we use **sigma notation**.

$$\sum_{i=1}^{n} a_n = a_1 + a_2 + a_3 + a_4 + a_5 + ...$$

The Σ means that we are **adding** the parts of the pattern together.

i is the **starting term** from where we will begin adding things together.

n is the **ending term** from where we finish adding things together.

If we want to know how many X's are in the pyramid from 1^{st} term to the 8^{th} term we'd use Σ notation to indicate that sum.

$$\sum_{i=1}^{8} b_n$$

or since $b_n = 2n - 1$

$$\sum_{i=1}^{8} 2n - 1$$

In either case it would be $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64$

You Try:

1. $\sum_{n=1}^{5} 2n =$

2. $\sum_{b=1}^{4} 2b - 5 =$

3. $\sum_{g=3}^{10} 3g - 1 =$

$\sum_{n=1}^{5} 2n = 2(1) + 2(2) + 2(3) + 2(4) + 2(5) = 2 + 4 + 6 + 8 + 10 = 30$
$\sum_{b=1}^{4} 2b - 5 = [2(1) - 5] + [2(2) - 5] + [2(3) - 5] + [2(4) - 5] =$
$\sum_{g=3}^{10} 3g - 1 = [3(3) - 1] + [3(4) - 1] + [3(5) - 1] + [3(6) - 1] + [3(7) - 1] + [3(8) - 1] + [3(9) - 1] + [3(10) - 1] =$

The opposite can also be done. How would the following sum be written in Sigma notation?
$5 + 9 + 13 + 17 + 21 + 25$

> Since $h_p = 4p + 1$, and since we are adding from the 1^{st} term to the 7^{th} term we write:
>
> $$\sum_{p=1}^{7} 4p + 1$$
>
> or more simply (but less clearly ...
>
> $$\sum_{p=1}^{7} h_p$$

YOU TRY: Rewrite the following in sigma notation

$$1 + 3 + 5 + 7 + 9 + 11 + 13$$

BUT WAIT!!! There's a better way to do this. Let's look at the sum of all integers from 1 to 100. We could add $1 + 2 + 3 + 4 + 5 + ... + 98 + 99 + 100$, but that's awfully long. Let's add two sums together (vertically) in opposite order and see what happens.

$$
\begin{array}{ccccccccccccc}
1 & + & 2 & + & 3 & + & ... & + & 98 & + & 99 & + & 100 \\
+ & & & & & & & & & & & & \\
100 & + & 99 & + & 98 & + & ... & + & 3 & + & 2 & + & 1
\end{array}
$$

What do each of these columns add up to? Each one of them adds up to 101, and there are 100 columns. So if we add all of these together we get 10100. But that's double the amount of the sum, so just divide by 2 and get 5050!

TL;DR: Add the first and last term, multiply by the number of terms, then divide by 2.

Or use this formula:

$$\sum_{i=1}^{k} a_n = \frac{k(a_1 + a_k)}{2}$$

where $n =$ the number of terms
$a_1 =$ the first term
$a_k =$ the last term.

For adding all the numbers from 1 to 100...

$$\frac{100(1 + 100)}{2}$$

You try:

1. $\sum_{i=1}^{10} 2n + 5$
2. $\sum_{i=1}^{7} 3n$
3. $\sum_{i=4}^{11} 6n - 6$

Quiz Review: Arithmetic Sequences and Series

Mr. Wolf NAME:__________________________________
2015-2016
mc^2 High School DATE:_______________________

Determine whether or not the following are arithmetic sequences.

1. $1, 2, 3, 4, ...$
2. $4, 3, 2, 1, ...$
3. $10, 1, 8, 2, 7, 3, ...$
4. $3, 9, 27, ...$

Determine the **common difference** of the sequence.

5. $9, 5, 1, -3, ...$
6. $2, 5, 8, 11, ...$
7. $100, 150, 200, 250, ...$
8. $25, -2, -29, -56, ...$

Create a formula for the sequence. $a_n = dn + (a_1 - d)$ or $a_n = a_1 + d(n-1)$

9. $1, 3, 5, 7, 9, ...$
10. $2, -4, -10, -16, ...$
11. $7, 10, 13, 16, ...$
12. $-10, -16, -22, -28, ...$

Calculate the value for n

13. $a_n = 7n + 13$, $n = 10$
14. $b_n = -9n + 100$, $n = 10$
15. $c_n = 5n - 12$, $n = 6$
16. $d_n = 4n - 4$, $n = 4$

Find the n^{th} term of the sequence.

17. $a_1 = 4,\ d = 3,\ n = 30$

18. $a_1 = -52,\ d = 10,\ n = 17$

19. $a_1 = -6,\ d = 13,\ n = 32$

20. $a_1 = -10,\ d = 3,\ n = 15$

Evaluate the arithmetic series. Use any method we learned to find the sum.

21. $\sum_{i=1}^{10} 2n - 1 =$

22. $\sum_{i=1}^{25} 5n =$

23. $\sum_{i=1}^{40} -4n + 44$

24. $\sum_{i=1}^{55} 3n - 27$

Evaluate the arithmetic series.

25. $a_1 = 10,\ a_n = 19,\ n = 3$

26. $b_1 = 19,\ b_n = 95,\ n = 20$

SCORE:____________

Quiz Review 2: Arithmetic Sequences and Series

Mr. Wolf NAME:__________________________________
2015-2016
mc^2 High School DATE:_______________________

Determine whether or not the following are arithmetic sequences.

1. $2, 4, 6, 8, ...$
2. $8, 5, 2, -1, ...$
3. $1, 11, 111, 1111, 11111, ...$
4. $4, 4, 4, ...$

Determine the **common difference** of the sequence.

5. $13, 7, 1, -5, ...$
6. $2, 5, 8, 11, ...$
7. $10, 13, 16, 19, ...$
8. $5, -2, -9, -16, ...$

Create a formula for the sequence. $a_n = dn + (a_1 - d)$ or $a_n = a_1 + d(n - 1)$

9. $-11, -13, -15, -17, -19, ...$
10. $5, -4, -13, -22, ...$
11. $0, 3, 6, 9, ...$
12. $10, 7, 4, 1, -2, ...$

Calculate the value for n

13. $a_n = -38n + -94$, $n = 75$
14. $b_n = 69n + -91$, $n = 15$
15. $c_n = 66n + 100$, $n = -71$
16. $d_n = -61n + -61$, $n = -67$

Find the n^{th} term of the sequence.

17. $a_1 = 2,\ d = 3,\ n = 15$

18. $a_1 = -42,\ d = 18,\ n = 16$

19. $a_1 = -8,\ d = 12,\ n = 17$

20. $a_1 = -13,\ d = 5,\ n = 13$

Evaluate the arithmetic series. Use any method we learned to find the sum.

21. $\sum_{i=1}^{10} 5n - 25 =$

22. $\sum_{i=1}^{25} 6n - 12 =$

23. $\sum_{i=1}^{100} -4n + 44$

24. $\sum_{i=1}^{250} n - 250$

Evaluate the arithmetic series.

25. $a_1 = 60,\ a_n = 810,\ n = 11$

26. $b_1 = -9,\ b_n = 81,\ n = 11$

Accelerated Objectives: Evaluate the series.

27. $-7 - 2 + 3 + 8 ... 88$

28. The sum of an arithmetic series is 374. If $a_1 = -2$, and $d = 3$, how many terms are being added together?

SCORE:____________

Quiz – Arithmetic Sequences & Series

Mr. Wolf
2015-2016
mc^2 High School

NAME:____

DATE:__________________________

Determine whether or not the following are arithmetic sequences.

1. $1, 3, 5, 7, ...$
2. $10, 7, 4, 1, ...$
3. $1, 20, 300, 4000, 50000, ...$
4. $4, 5, 4, 5, ...$

Determine the **common difference** of the sequence.

5. $14, 8, 2, -4, ...$
6. $1, 4, 7, 10, ...$
7. $12, 15, 18, 21, ...$
8. $9, 2, -5, -12, ...$

Create a formula for the sequence. $a_n = dn + (a_1 - d)$ or $a_n = a_1 + d(n-1)$

9. $-2, -4, -6, -8, -10, ...$
10. $4, -4, -12, -20...$
11. $1, 2, 3, 4, ...$
12. $0, -2, -4, -6, ...$

Calculate the n^{th} term.

13. $a_n = 11n + 2$, $n = 7$
14. $b_n = -6n + 10$, $n = 20$
15. $c_n = 9n - 90$, $n = 5$
16. $d_n = 4n - 3$, $n = 15$

Find the n^{th} term of the sequence.

17. $a_1 = 12$, $d = 13$, $n = 5$

18. $a_1 = -4$, $d = 1$, $n = 6$

19. $a_1 = -28$, $d = 2$, $n = 13$

20. $a_1 = -12$, $d = 6$, $n = 12$

Evaluate the arithmetic series. Use any method we learned to find the sum.

21. $\sum_{i=1}^{11} n - 11 =$

22. $\sum_{i=1}^{16} 8n - 32 =$

23. $\sum_{i=1}^{100} -10n + 100$

24. $\sum_{i=1}^{3} 2n - 4$

Evaluate the arithmetic series given the first and last terms.

25. $a_1 = 18$, $a_n = 54$, $n = 20$

26. $b_1 = -17$, $b_n = 68$, $n = 5$

Accelerated Objectives: Evaluate the series.

27. $-8 - 4 + 0 + 4 + ... + 108$

28. If $a_1 = -1$, and $a_{10} = 107$, what is the formula for a_n? What is the sum of the first 10 terms?

SCORE:____________

Quiz:: – Arithmetic Sequences & Series

Mr. Wolf NAME:______________________________
2015-2016
mc^2 High School DATE:____________________

Determine whether or not the following are arithmetic sequences.

1. $-1, 1, 3, 5, ...$
2. $2, 4, 6, 8, ...$
3. $11, 9, 7, 5, ...$
4. $3, 2, 3, 2, ...$

Determine the **common difference** of the sequence.

5. $14, 7, 0, -7, ...$
6. $12, 16, 20, 24, ...$
7. $1, 5, 9, 13, ...$
8. $7, 2, -3, -8, ...$

Create a formula for the sequence. $a_n = dn + (a_1 - d)$ or $a_n = a_1 + d(n-1)$

9. $-1, -3, -5, -7, ...$
10. $5, -5, -15, -25, ...$
11. $1, 2, 3, 4, ...$
12. $0, 6, 12, 18, ...$

Calculate the n^{th} term.

13. $a_n = 11n + 2$, $n = 9$
14. $b_n = -6n + 10$, $n = 28$
15. $c_n = 9n - 90$, $n = 6$
16. $d_n = 4n - 3$, $n = 16$

Find the n^{th} term of the sequence.

17. $a_1 = 12,\ d = 13,\ n = 6$

18. $a_1 = -4,\ d = 1,\ n = 16$

19. $a_1 = -28,\ d = 2,\ n = 14$

20. $a_1 = -12,\ d = 6,\ n = 2$

Evaluate the arithmetic series. Use any method we learned to find the sum.

21. $\sum_{i=1}^{12} n - 12 =$

22. $\sum_{i=1}^{16} 4n - 16 =$

23. $\sum_{i=1}^{200} -20n + 200$

24. $\sum_{i=1}^{3} 5n - 15$

Evaluate the arithmetic series given the first and last terms.

25. $a_1 = -1,\ a_n = 66,\ n = 20$

26. $b_1 = -8,\ b_n = 8,\ n = 5$

Accelerated Objectives: Evaluate the series.

27. $-2 + 14 + 30 + 46 + ... + 110$

28. If $a_1 = 8$, and $a_{10} = 62$, what is the formula for a_n? What is the sum of the first 10 terms?

SCORE:____________

5 Geometric Sequences

A **Geometric Sequence** is a sequence that increases of decreases by multiplication[1] of a constant.

Example:

$$\{1, 2, 4, 8, 16, 32, 64, 128, ...\}$$

The general form of a geometric sequence is

$$\{a, a \cdot r, a \cdot r^2, a \cdot r^3, a \cdot r^4, ...\}$$

where a is the first term and r is the common ratio (the number they're getting multiplied by.

For the above example $a = 1$ because the first term is 1, and $r = 2$ because each term is being multiplied by 2. So plugging into the formula we get

$$\{a, a \cdot r, a \cdot r^2, a \cdot r^3, a \cdot r^4, ...\}$$
$$\{1, 1 \cdot 2, 1 \cdot 2^2, 1 \cdot 2^3, 1 \cdot 2^4, ...\}$$
$$\{1, 2, 4, 8, 16, ...\}$$

5.1 Writing Formulas for Geometric Sequences

The general form for a geometric sequence is $x_n = ar^{(n)}$. a_1 is the first term.
Example:

$$10, 30, 90, 270, 810, 2430, ...$$

$a_1 = 10$ and $r = 3$

So,

$$a_n = 10 \cdot 3^{(n)}$$

$\therefore$

$$a_{10} = 10 \cdot 3^{(10-1)} = 10 \cdot 3^9 = 10 \cdot 19683 = 196830$$

Geometric sequences can also get smaller by multiplying by a fraction or a decimal. For example,

$$b_n = 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$$

The common ratio $r = \frac{1}{2}$

[1] remember that dividing is the same as multiplying by a fraction

the first term $a_1 = 4$

So we can make our formula $b_n = 4 \cdot \left(\frac{1}{2}\right)^n$

Be careful that $r \neq 0$. If $r = 0$ then it's not a geometric sequence.

6 Geometric Series

Since the geometric sequence

$$a_n = \{a_1, a_1 \cdot r, a_1 \cdot r^2, a_1 \cdot r^3, ...\}$$

then

$$\sum_{k=0}^{n-1}(a_1) = a_1 + a_1 \cdot r + a_1 \cdot r^2 + a_1 \cdot r^3 + ... + a_1 \cdot r^{(n-1)}$$

Just like with arithmetic series, the geometric series can be summed up with its own equation equation,

$$\sum_{k=0}^{n-1} a_1 \cdot r^{n-1} = a\left(\frac{1-r^n}{1-r}\right)$$

Exponential Functions

Goals

SWBAT identify exponential functions, and differentiate between exponential and power functions.

SWBAT graph exponential functions by using a t-chart.

SWBAT graph exponential functions without using a t-chart.

SWBAT determine the end behavior of an exponential function.

SWBAT identify translations of the exponential graph by identifying key parts of the equation.

SWBAT calculate compound interest given the interest rate, number of years, and number of times per year, and the principal.

SWBAT use the number e as an exponential base for continuously compounded interest.

Standards

Creating Equations A -CED Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Interpreting Functions F-IF Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Introduction to Exponential Functions

An exponential function is whenever there is a variable in the exponent. This is different from a power function where there is a variable with a rational number exponent.

Compare Linear vs. Power vs. Exponential functions

Linear

x	$y = 2x$
0	0
1	2
2	4
3	6
4	8
5	10
$\vdots$	$\vdots$

Power

x	$y = x^2$
0	0
1	1
2	4
3	9
4	16
5	25
$\vdots$	$\vdots$

Exponential

x	$y = 2^x$
0	1
1	2
2	4
3	8
4	16
5	32
$\vdots$	$\vdots$

Notice that *linear functions* always increase by the same amount by **addition**, *Power functions* always increase by the *square* of a number, and *Exponential Functions* always increase by **multiplying** by the same amount.

Example 2: Determine which each chart represents: linear, power, or exponential.

x	$y = 2x^3$
-2	-16
-1	-2
0	0
1	2
2	16
3	54
$\vdots$	$\vdots$

x	$y = 2x - 3$
-2	-7
-1	-5
0	-3
1	-1
2	1
3	3
$\vdots$	$\vdots$

x	$y = 2 \cdot 3^x$
-2	$\frac{2}{9}$
-1	$\frac{2}{3}$
0	2
1	6
2	18
3	54
$\vdots$	$\vdots$

Determine an Exponential Function from a Chart

x	-1	0	1	2	3	...
y	$\frac{1}{10}$	1	10	100	1000	...

The rule for this chart is easy to determine. Since each x increases the y value by multiplying 10, the equation is therefore

$$y = 10^x$$

But what about this...

x	-1	0	1	2	3	...
y	$\frac{11}{10}$	2	11	101	1001	...

The pattern goes away! There is no number to multiply by to get the next term. For this one we need to look at the first chart from $y = 10^x$. Notice that the y term for the second chart is always 1 more than the y value of the first chart. Besides that everything is the same.

Graphing Exponential Functions

$$a \cdot b^{x-h} + k$$

The size of the base affects the vertical stretch of the function.

Compare $f(x) = 2^x$ and $g(x) = 3^x$

x	$f(x) = 2^x$		$g(x) = 3^x$
-2	$\frac{1}{4}$		$\frac{1}{9}$
-1	$\frac{1}{2}$		$\frac{1}{3}$
0	1		1
1	2		3
2	4		9
3	8		27
4	16		81
5	32		243
6	64		729
$\vdots$	$\vdots$		$\vdots$

Notice that the two functions overlap at $(0, 1)$. Negative of here the larger base is smaller and the smaller base is bigger. Positive of here the larger base is bigger and the smaller base is smaller.

The two graphs will not go below the line $y = 0$. This is called an **ASYMPTOTE**. It is a way of describing the *end behavior* of a function. As the functions move towards negative infinity they will get smaller and smaller, but they will never actually reach 0.

Translations

A translation is when a function is moved around the graph. To move the graph up or down, add or subtract from the function.

If there is no $+h$ on the back of the function, then the asymptote is at $y = 0$. In these cases the h's are $+2$ and -1.

If there is a $+h$ or $-h$ then the asymptote is at $\pm h$

To move the function side to side add or subtract from the exponent. Be careful because it goes in the opposite direction of the sign. Notice the asymptote doesn't move

Method for Graphing

Graph exponential equations. Easy method.

1. Identify the asymptote. It's always the $\pm$ at the *end* of the function
2. Set the exponent equal to 0
3. Find the y value when the exponent is equal to zero. Graph that point.
4. Follow the asymtote until it passes through the point, and graph the exponentiality.

Example 1: graph $y = 2^{x-2} + 3$

Example 2: graph $y = \left(\frac{1}{2}\right)^x - 7$

Example 3: graph $y = -2^{x-2} + 3$

Example 4: graph $y = 2^{-x} - 7$

You Try: Graph the exponential Functions

$y = 3^x - 5$

$y = (-2)3^{x-1} + 6$

$y = \left(\frac{1}{3}\right)^{x+4} + 1$

$y = 3^{-x}$

Compound Interest

Compounding interest is an exponential expression because the exponent changes as time goes on.

$$A = P(1+r)^t \qquad A = P\left(1+\frac{r}{n}\right)^{nt}$$

$A =$ Actual amount – what you'll end up with
$P =$ Principle – what you started with
$r =$ rate – as a decimal, not as a fraction
$n =$ number of times per year
$t =$ time – years

The *independent* variable is t the amount of time that goes by. The *dependent* variable is the actual amount. This is because the actual amount depends on how much time goes by. If we set this up as a relationship between t and A, we would get an exponential function.

Example 1: How much will you have after investing \$500,000.00 compound yearly at 3% for 12 years?

Example 2:
How much will you have if you invest \$100 in a bond at %5 compound monthly for 15 years.

Example 3: How much will you have after inveesting \$15.00 compound quarterly at 4% since 1945 (the end of WW2)?

Exponential (radioactive) Decay

Exponential decay is the same as exponential growth except for a minus sign.

$$A = P(1 - r)^t$$

The Number e

e is an irrational number discovered by Leonhard Euler (pronounced oiler). It is commonly called Euler's Number, but more commonly as the natural base. $e \approx 2.7182818...$ It is used as the base for many exponential functions, and has many applications in calculus.

e is calculated by the following expression as n gets larger and larger

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.7182818$$

I hope this expression looks a little familiar.

---

e can be used a a regular exponential base.

So we can graph $y = e^x$

What it comes down to is that e is a number just like any other, and we can use it as such.

So what's so special about this number e?

Continuously Compounded Interest

What is so special about e is that we can compound interest continuously. The nice thing about that is that I get more money when interest is compounded more frequently.

The graph above shows the same interest rate, but when the interest is compounded more often there are more significant gains made.

To calculate continuously compound interest we use the formula

$$A = Pe^{rt}$$

Where
A = Actualized amount
P = Principal
r = rate
t = time

So basically all the letters are the same (remember e is a number, not a variable), and it's a much tighter, neater package.

Example: You deposit \$1500.00 in an account compound continuously at 3%. How much will you have after 3 years?

How much will you have after 10 years?

How does this compare to compounded semi-annually (twice a year) for 10 years?

Example 2: How much will you have after investing \$2.00 compounded continuously since the year 1776?

Quiz Review

Mr. Wolf
Pre-Calculus
MC^2 STEM HS
2015-2016

NAME:______________________________

DATE:____________________

Graphing Exponentials & Exponential Growth

Due Monday 2014-10-13

Graph the exponential function

5. graph $y = 2^{x-2} + 3$

6. graph $y = \left(\frac{1}{2}\right)^x - 7$

7. graph $y = -2^{x-2} + 3$

8. graph $y = 2^{-x} - 7$

Compound the Interest

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

9. How much money would you have after investing \$17 compounded daily at 10% for 25 years?

10. (Population Growth) A rabbit population increases at 60% compounded quarterly. If there were 20 rabbits to start with, how many rabbits will there be after 10 years?

11. (a) How much would you have after investing \$400 compounded monthly at 5% for 10 years?

    (b) How much would you have after investing \$400 compounded monthly at 5% for 20 years?

    (c) Even though twice as much time went by between (a) and (b), was twice as much money made?

Continuously Compound Interest

$$A = Pe^{rt}$$

12. How much money will you have after investing \$600 compounded continuously at 4% for 8 years?

13. (a) How much money will you have if you invest \$500 compounded continuously at 12% for 10 years?

    (b) What's the difference between (a) and if you compounded twice per year for 10 years.

Quiz

Mr. Wolf NAME:__________________________________
Pre-Calculus
MC^2 STEM HS DATE:______________________
2015-2016

Graphing Exponentials & Exponential Growth
OPEN NOTE

Graph the exponential function

14. graph $y = 2^{x+2} - 3$
15. graph $y = \left(\frac{1}{2}\right)^x - 8$
16. graph $y = -2^{x+5} - 3$
17. graph $y = 2^{-x} + 5$

Compound the Interest

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

18. How much money would you have after investing \$17 compounded daily at 10% for 25 years?

19. (Population Growth) A rabbit population increases at 60% compounded quarterly. If there were 20 rabbits to start with, how many rabbits will there be after 10 years?

20. (a) How much would you have after investing \$1000 compounded monthly at 6% for 10 years?

    (b) How much would you have after investing \$1000 compounded monthly at 6% for 30 years?

    (c) Even though triple the time went by between (a) and (b), was triple the money made?

Continuously Compound Interest

$$A = Pe^{rt}$$

21. How much money will you have after investing \$800 compounded continuously at 4% for 6 years?

22. (a) How much money will you have if you invest \$1200 compounded continuously at 10% for 14 years?

    (b) What's the difference between (a) and compounded twice per year for 14 years.

**Now you're done :-)