

Parallel Courses III

You are given an integer n , which indicates that there are n courses labeled from 1 to n . You are also given a 2D integer array `relations` where `relations[j] = [prevCoursej, nextCoursej]` denotes that course `prevCoursej` has to be completed **before** course `nextCoursej` (prerequisite relationship). Furthermore, you are given a **0-indexed** integer array `time` where `time[i]` denotes how many **months** it takes to complete the $(i+1)^{\text{th}}$ course.

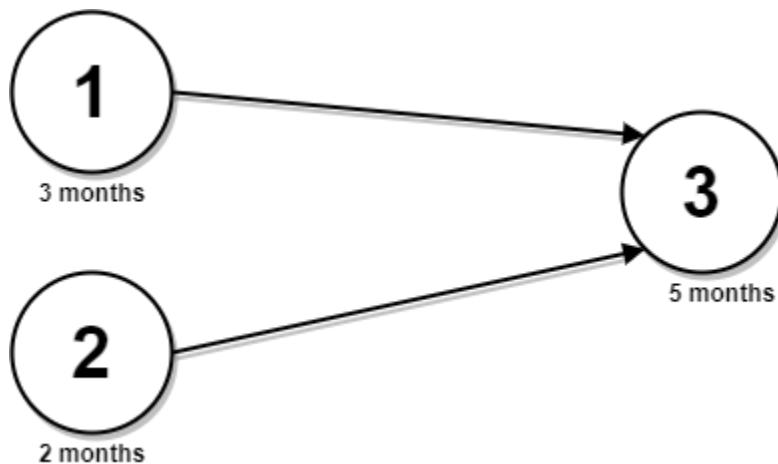
You must find the **minimum** number of months needed to complete all the courses following these rules:

- You may start taking a course at **any time** if the prerequisites are met.
- **Any number of courses** can be taken at the **same time**.

Return the **minimum** number of months needed to complete all the courses.

Note: The test cases are generated such that it is possible to complete every course (i.e., the graph is a directed acyclic graph).

Example 1:



Input: $n = 3$, `relations = [[1,3],[2,3]]`, `time = [3,2,5]`

Output: 8

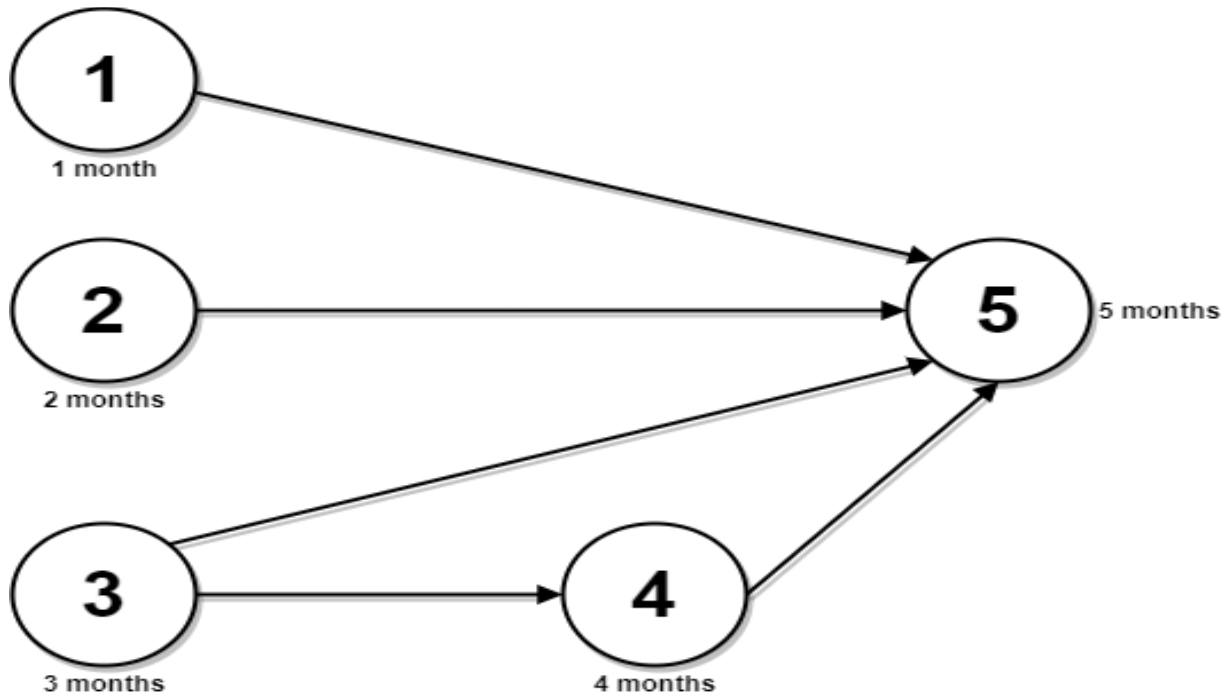
Explanation: The figure above represents the given graph and the time required to complete each course.

We start course 1 and course 2 simultaneously at month 0.

Course 1 takes 3 months and course 2 takes 2 months to complete respectively.

Thus, the earliest time we can start course 3 is at month 3, and the total time required is $3 + 5 = 8$ months.

Example 2:



Input: $n = 5$, relations = $[[1,5],[2,5],[3,5],[3,4],[4,5]]$, time = $[1,2,3,4,5]$

Output: 12

Explanation: The figure above represents the given graph and the time required to complete each course.

You can start courses 1, 2, and 3 at month 0.

You can complete them after 1, 2, and 3 months respectively.

Course 4 can be taken only after course 3 is completed, i.e., after 3 months. It is completed after $3 + 4 = 7$ months.

Course 5 can be taken only after courses 1, 2, 3, and 4 have been completed, i.e., after $\max(1,2,3,7) = 7$ months.

Thus, the minimum time needed to complete all the courses is $7 + 5 = 12$ months.

Constraints:

- $1 \leq n \leq 5 \cdot 10^4$
- $0 \leq \text{relations.length} \leq \min(n \cdot (n - 1) / 2, 5 \cdot 10^4)$
- $\text{relations}[j].\text{length} == 2$
- $1 \leq \text{prevCourse}_j, \text{nextCourse}_j \leq n$
- $\text{prevCourse}_j \neq \text{nextCourse}_j$
- All the pairs $[\text{prevCourse}_j, \text{nextCourse}_j]$ are **unique**.
- $\text{time.length} == n$
- $1 \leq \text{time}[i] \leq 10^4$
- The given graph is a directed acyclic graph.