

# Minimum Cost to cut a board into squares

Given a board of dimensions  $n \times m$  that needs to be cut into  $n \times m$  squares. The cost of making a cut along a horizontal or vertical edge is provided in two arrays:

- **x[]**: Cutting costs along the vertical edges (length-wise).
- **y[]**: Cutting costs along the horizontal edges (width-wise).

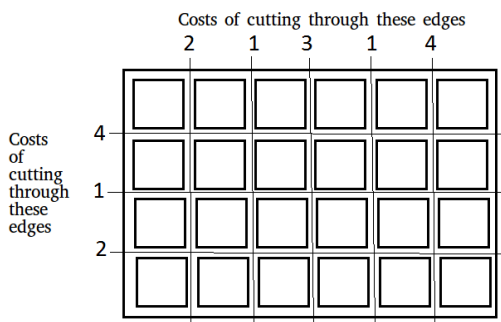
Find the **minimum total cost** required to cut the board into squares optimally.

**Examples:**

**Input:**  $n = 4, m = 6, x[] = [2, 1, 3, 1, 4], y[] = [4, 1, 2]$

**Output:** 42

**Explanation:**



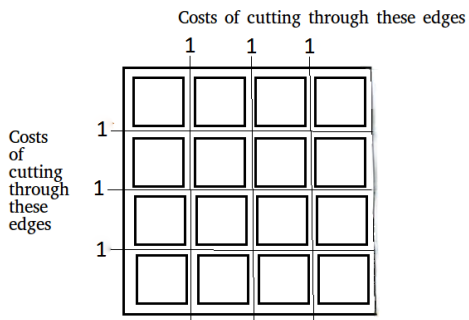
For above board optimal way to cut into square is:

1. Pick 4 (from x) -> vertical cut, Cost =  $4 \times$  horizontal segments = 4, Vertical segments = 2, Total = 4.
2. Pick 4 (from y) -> horizontal cut, Cost =  $4 \times$  vertical segments = 8, Horizontal segments = 2, Total = 12.
3. Pick 3 (from x) -> vertical cut, Cost =  $3 \times$  horizontal segments = 6, Vertical segments = 3, Total = 18.
4. Pick 2 (from x) -> vertical cut, Cost =  $2 \times$  horizontal segments = 4, Vertical segments = 4, Total = 22.
5. Pick 2 (from y) -> horizontal cut, Cost =  $2 \times$  vertical segments = 8, Horizontal segments = 3, Total = 30.
6. Pick 1 (from x) -> vertical cut, Cost =  $1 \times$  horizontal segments = 3, Vertical segments = 5, Total = 33.
7. Pick 1 (from x) -> vertical cut, Cost =  $1 \times$  horizontal segments = 3, Vertical segments = 6, Total = 36.
8. Pick 1 (from y) -> horizontal cut, Cost =  $1 \times$  vertical segments = 6, Horizontal segments = 4, Total = 42.

**Input:**  $n = 4$ ,  $m = 4$ ,  $x[] = [1, 1, 1]$ ,  $y[] = [1, 1, 1]$

**Output:** 15

**Explanation:**



For above board optimal way to cut into square is:

1. Pick 1 (from  $y$ )  $\rightarrow$  horizontal cut, Cost =  $1 \times$  vertical segments = 1,  
Horizontal segments = 2, Total = 1.
2. Pick 1 (from  $y$ )  $\rightarrow$  horizontal cut, Cost =  $1 \times$  vertical segments = 1,  
Horizontal segments = 3, Total = 2.
3. Pick 1 (from  $y$ )  $\rightarrow$  horizontal cut, Cost =  $1 \times$  vertical segments = 1,  
Horizontal segments = 4, Total = 3.
4. Pick 1 (from  $x$ )  $\rightarrow$  vertical cut, Cost =  $1 \times$  horizontal segments = 4,  
Vertical segments = 2, Total = 7.
5. Pick 1 (from  $x$ )  $\rightarrow$  vertical cut, Cost =  $1 \times$  horizontal segments = 4,  
Vertical segments = 3, Total = 11.
6. Pick 1 (from  $x$ )  $\rightarrow$  vertical cut, Cost =  $1 \times$  horizontal segments = 4,  
Vertical segments = 4, Total = 15.

**Constraints:**

$$2 \leq n, m \leq 10^3$$

$$1 \leq x[i], y[j] \leq 10^3$$