

1406. Stone Game III (Recursion - Memo - Bottom Up - O(1)Space)

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1406. Stone Game III

Hint ⓘ

Hard 1.3K 31 ⚡

Companies

Alice and Bob continue their games with piles of stones. There are several stones arranged in a row, and each stone has an associated value which is an integer given in the array `stoneValue`.

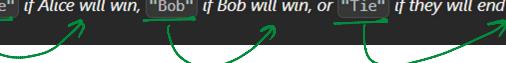
Alice and Bob take turns, with Alice starting first. On each player's turn, that player can take 1, 2, or 3 stones from the first remaining stones in the row.

The score of each player is the sum of the values of the stones taken. The score of each player is 0 initially.

The objective of the game is to end with the highest score, and the winner is the player with the highest score and there could be a tie. The game continues until all the stones have been taken.

Assume Alice and Bob play optimally.

Return "Alice" if Alice will win, "Bob" if Bob will win, or "Tie" if they will end the game with the same score.



Example 1:

Alice **Bob**

Input: values = [1,2,3,7]
Output: "Bob" *Bob wins*

Explanation: Alice will always lose. Her best move will be to take three piles and the score become 6. Now the score of Bob is 7 and Bob wins.

Example 2:

Alice **Bob**

Input: values = [1,2,3,-9]
Output: "Alice" *Alice wins*

Explanation: Alice must choose all the three piles at the first move to win and leave Bob with negative score.
If Alice chooses one pile her score will be 1 and the next move Bob's score becomes 5. In the next move, Alice will take the pile with value = -9 and lose.
If Alice chooses two piles her score will be 3 and the next move Bob's score becomes 3. In the next move, Alice will take the pile with value = -9 and also lose.
Remember that both play optimally so here Alice will choose the scenario that makes her win.

Example 3:

Input: values = [1, 2, 3, 6]

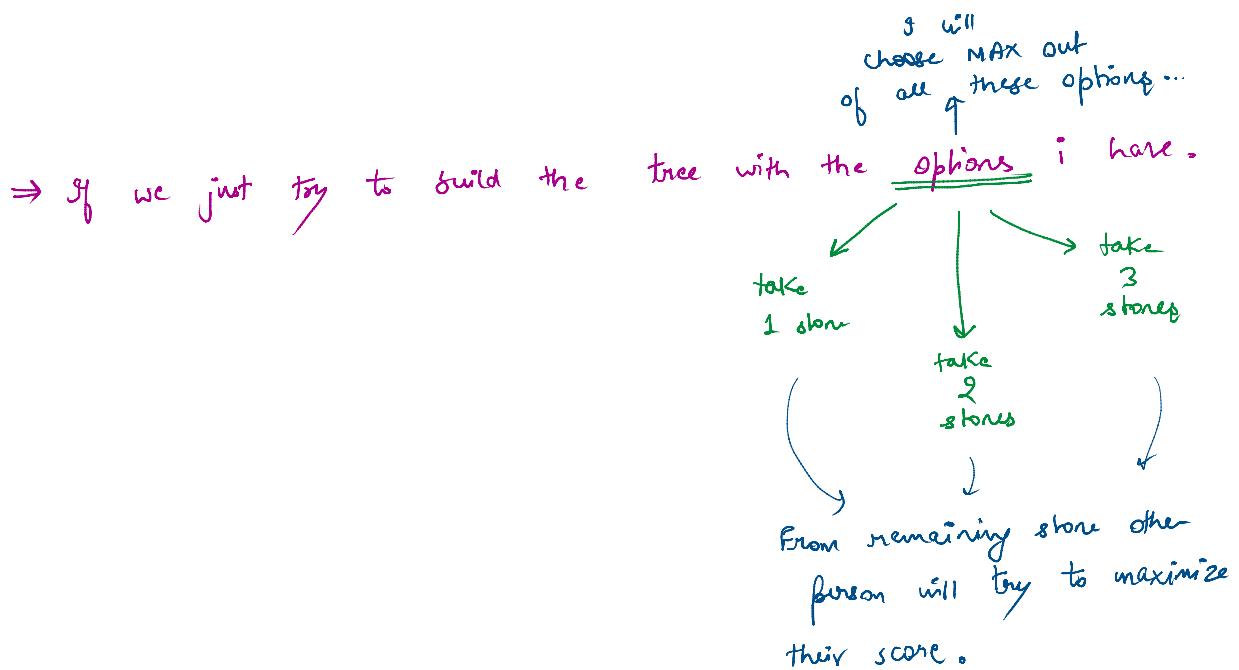
Output: "Tie" ↗ Tie

Explanation: Alice cannot win this game. She can end the game in a draw if she decided to choose all the first three piles, otherwise she will lose.

Constraints:

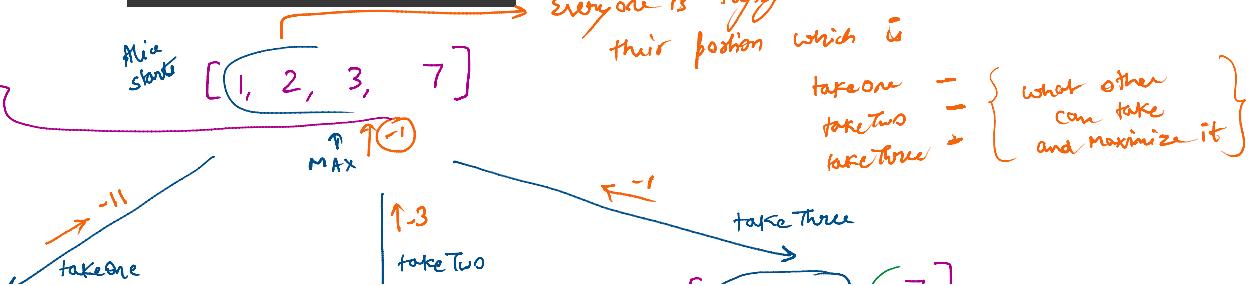
- $1 \leq \text{stoneValue.length} \leq 5 * 10^4$
- $-1000 \leq \text{stoneValue}[i] \leq 1000$

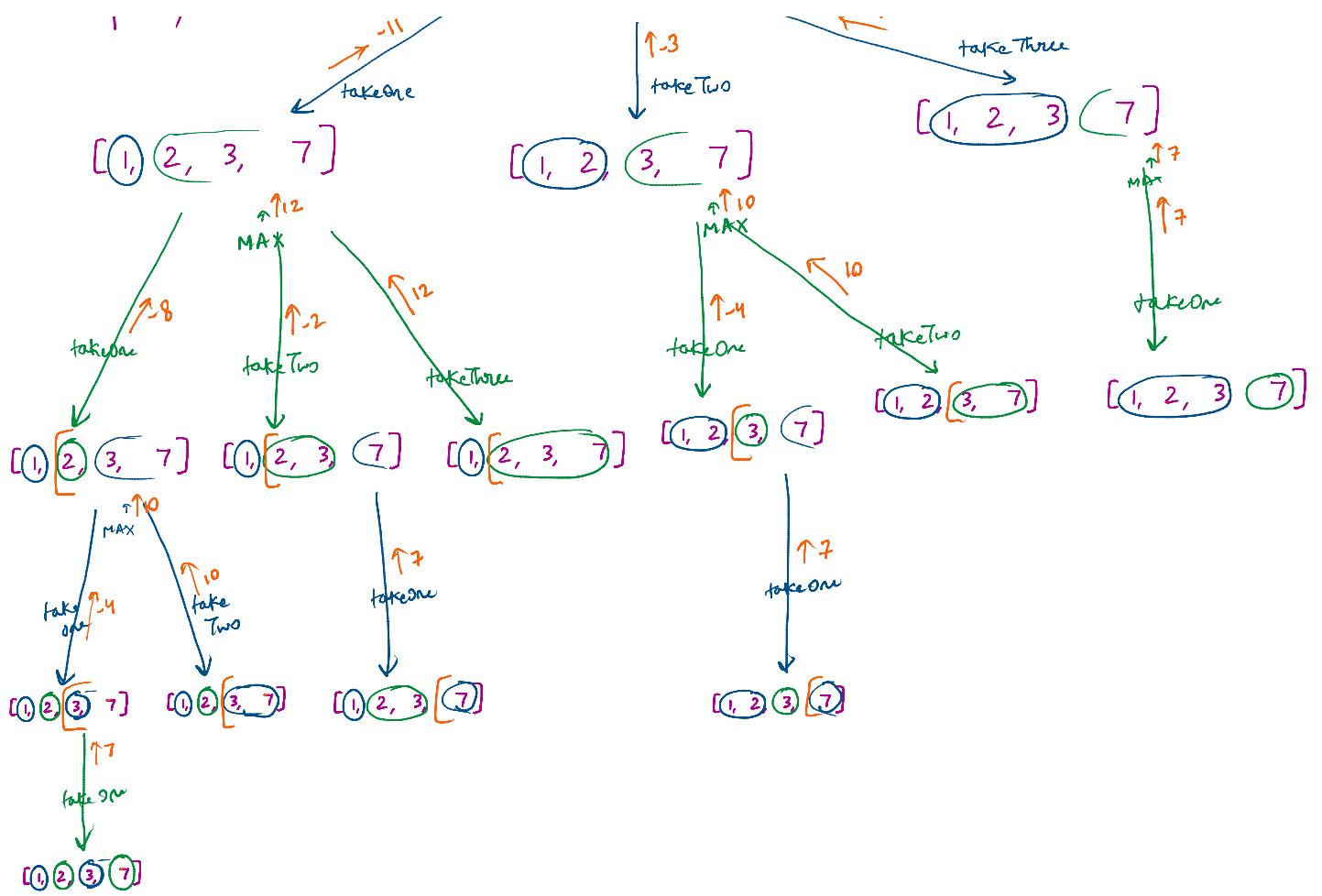
⇒ If we just try to build the tree with the options i have.



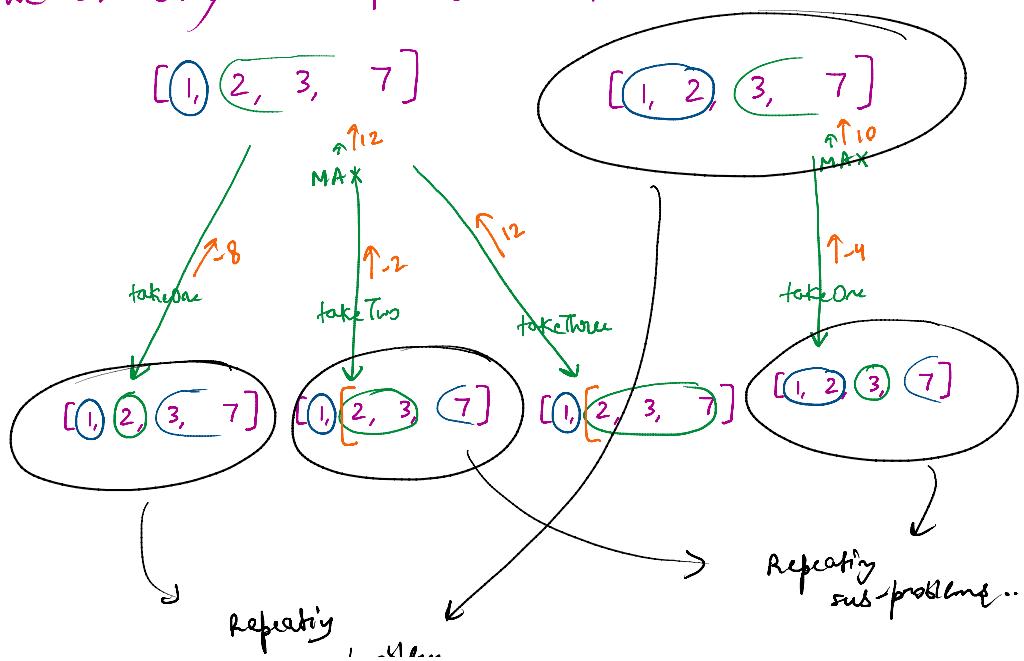
Input: values = [1, 2, 3, 7]

① is the max difference Alice can obtain if it plays optimally..





We can easily see repeating sub-problems :)



↳ *Repeating sub-problems..*

→ *Recuring sub-problems..*

Thus, we can think of Simple Recursion & then apply Memoization on top of it !!

Recursion

```
class Solution {
public:
    int stoneGame(const vector<int>& stoneValue, int i)
    {
        int n = stoneValue.size();
        if(i >= n) return 0;
        int takeOne = stoneValue[i] - stoneGame(stoneValue, i+1);
        int takeTwo = INT_MIN;
        if(i+1 < n) takeTwo = stoneValue[i] + stoneValue[i+1] - stoneGame(stoneValue, i+2);
        int takeThree = INT_MIN;
        if(i+2 < n) takeThree = stoneValue[i] + stoneValue[i+1] + stoneValue[i+2] - stoneGame(stoneValue, i+3);
        return max({takeOne, takeTwo, takeThree});
    }
    string stoneGameIII(vector<int>& stoneValue)
    {
        int value = stoneGame(stoneValue, 0);
        if(value > 0) return "Alice";
        else if(value < 0) return "Bob";
        else return "Tie";
    }
};
```

Base case when your index reaches beyond array bounds.

take one stone i.e. current stone also try to maximize their score !!.

Considering the fact that other person will also try to maximize their score !!.

take two stones i.e. (this) & (next).

take three stones i.e. (this), (next), (next to next).

↑ take max of what you can take.

Find the Max difference possible & As Alice is starting first then this difference of (Alice - Bob) :-

depending on difference

Memoization:

```

class Solution {
public:
    int stoneGame(vector<int>& dp, const vector<int>& stoneValue, int i)
    {
        int n = stoneValue.size();
        if(i>=n) return 0;

        if(dp[i] != INT_MAX) return dp[i];

        int takeOne = stoneValue[i] - stoneGame(dp, stoneValue, i+1);

        int takeTwo = INT_MIN;
        if(i+1 < n) takeTwo = stoneValue[i] + stoneValue[i+1] - stoneGame(dp, stoneValue, i+2);

        int takeThree = INT_MIN;
        if(i+2 < n) takeThree = stoneValue[i] + stoneValue[i+1] + stoneValue[i+2] - stoneGame(dp, stoneValue, i+3);

        return dp[i] = max({takeOne, takeTwo, takeThree});
    }

    string stoneGameIII(vector<int>& stoneValue)
    {
        vector<int> dp(stoneValue.size(), INT_MAX);
        int value = stoneGame(dp, stoneValue, 0);
        if(value > 0)
            return "Alice";
        else if(value < 0)
            return "Bob";
        else
            return "Tie";
    }
};

```

Just a 3 step Memoization

↓
 we initialize it with INT_MAX
 & not -1 because dp is the
 difference & it can be -ive
 also i.e. -1 also.

time : O(n)

space : O(n)

Code:

```

class Solution {
public:
    int stoneGame(vector<int>& dp, const vector<int>& stoneValue, int i)
    {
        int n = stoneValue.size();
        if(i>=n) return 0;
        if(dp[i] != INT_MAX) return dp[i];
        int takeOne = stoneValue[i] - stoneGame(dp, stoneValue, i+1);
        int takeTwo = INT_MIN;
        if(i+1 < n) takeTwo = stoneValue[i] + stoneValue[i+1] - stoneGame(dp, stoneValue, i+2);
        int takeThree = INT_MIN;
        if(i+2 < n) takeThree = stoneValue[i] + stoneValue[i+1] + stoneValue[i+2] - stoneGame(dp, stoneValue,
i+3);

        return dp[i] = max({takeOne, takeTwo, takeThree});
    }
    string stoneGameIII(vector<int>& stoneValue)
    {
        vector<int> dp(stoneValue.size(), INT_MAX);
        int value = stoneGame(dp, stoneValue, 0);
        if(value > 0)
            return "Alice";
        else if(value < 0)
            return "Bob";
        else
            return "Tie";
    }
};

```

⇒ Can we optimize it? → To optimize DP → we would need to convert to Tabulation (Bottom Up)

"Simple conversion of recursive code"

Only then we can think of optimizations :-

Bottom Up:

```
class Solution {
public:
    string stoneGameIII(vector<int>& stoneValue)
    {
        int n = stoneValue.size();
        vector<int> dp(stoneValue.size() + 1, 0);
        for (int i = n - 1; i >= 0; i--) {
            int takeOne = stoneValue[i] - dp[i + 1];
            int takeTwo = INT_MIN;
            if (i + 1 < n) takeTwo = stoneValue[i] + stoneValue[i + 1] - dp[i + 2];
            int takeThree = INT_MIN;
            if (i + 2 < n) takeThree = stoneValue[i] + stoneValue[i + 1] + stoneValue[i + 2] - dp[i + 3];
            dp[i] = max({takeOne, takeTwo, takeThree});
        }
        int value = dp[0];
        if (value > 0)
            return "Alice";
        else if (value < 0)
            return "Bob";
        else
            return "Tie";
    }
};
```

Exactly same as Recursion

Going from back as the values from back will be used

Exact same stuff.

time : O(n)
space : O(n)

CODE:

```
class Solution {
public:
    string stoneGameIII(vector<int>& stoneValue)
    {
        int n = stoneValue.size();
        vector<int> dp(stoneValue.size() + 1, 0);
        for (int i = n - 1; i >= 0; i--) {
            int takeOne = stoneValue[i] - dp[i + 1];
            int takeTwo = INT_MIN;
            if (i + 1 < n) takeTwo = stoneValue[i] + stoneValue[i + 1] - dp[i + 2];
            int takeThree = INT_MIN;
            if (i + 2 < n) takeThree = stoneValue[i] + stoneValue[i + 1] + stoneValue[i + 2] - dp[i + 3];
            dp[i] = max({takeOne, takeTwo, takeThree});
        }
        int value = dp[0];
        if (value > 0)
            return "Alice";
        else if (value < 0)
            return "Bob";
        else
            return "Tie";
    }
};
```

```

    }
};
```

→ we can easily see to
compute $dp(i)$ we only need

```

for (int i = n - 1; i >= 0; i--) {
    int takeOne = stoneValue[i] - dp[i + 1];
    ←
    int takeTwo = INT_MIN;
    if (i + 1 < n) takeTwo = stoneValue[i] + stoneValue[i + 1] - dp[i + 2];
    ←
    int takeThree = INT_MIN;
    if (i + 2 < n) takeThree = stoneValue[i] + stoneValue[i + 1] + stoneValue[i + 2] - dp[i + 3];
    ←
    dp[i] = max({takeOne, takeTwo, takeThree});
}
```

thus, we can only take space of ③
rather than taking entire space of ⑦

thus Reducing the
space from $O(n)$ to $O(1)$

*Better
optimized*

```

class Solution {
public:
    string stoneGameIII(vector<int>& stoneValue)
    {
        int n = stoneValue.size();
        vector<int> dp(3, 0);
        ←

        for (int i = n - 1; i >= 0; i--) {
            int takeOne = stoneValue[i] - dp[(i + 1) % 3];
            ←
            int takeTwo = INT_MIN;
            if (i + 1 < n) takeTwo = stoneValue[i] + stoneValue[i + 1] - dp[(i + 2) % 3];
            ←
            int takeThree = INT_MIN;
            if (i + 2 < n) takeThree = stoneValue[i] + stoneValue[i + 1] + stoneValue[i + 2] - dp[(i + 3) % 3];
            ←
            dp[i % 3] = max({takeOne, takeTwo, takeThree});
        }

        int value = dp[0];
        if (value > 0)
            return "Alice";
        else if (value < 0)
            return "Bob";
        else
            return "Tie";
    }
};
```

Only code changed to use
only space of ③ → constant
rather than the space of ⑦

time: $O(n)$

space : O(1)

CODE:

```
class Solution {
public:
    string stoneGameIII(vector<int>& stoneValue)
    {
        int n = stoneValue.size();
        vector<int> dp(3, 0);
        for (int i = n - 1; i >= 0; i--) {
            int takeOne = stoneValue[i] - dp[(i + 1) % 3];
            int takeTwo = INT_MIN;
            if (i + 1 < n) takeTwo = stoneValue[i] + stoneValue[i + 1] - dp[(i + 2) % 3];
            int takeThree = INT_MIN;
            if (i + 2 < n) takeThree = stoneValue[i] + stoneValue[i + 1] + stoneValue[i + 2] - dp[(i + 3) % 3];
            dp[i % 3] = max({takeOne, takeTwo, takeThree});
        }
        int value = dp[0];
        if(value > 0)
            return "Alice";
        else if(value < 0)
            return "Bob";
        else
            return "Tie";
    }
};
```

C++

```
class Solution {
public:
    string stoneGameIII(vector<int>& stoneValue)
    {
        int n = stoneValue.size();
        vector<int> dp(3, 0);
        for (int i = n - 1; i >= 0; i--) {
            int takeOne = stoneValue[i] - dp[(i + 1) % 3];
            int takeTwo = INT_MIN;
            if (i + 1 < n) takeTwo = stoneValue[i] + stoneValue[i + 1] - dp[(i + 2) % 3];
            int takeThree = INT_MIN;
            if (i + 2 < n) takeThree = stoneValue[i] + stoneValue[i + 1] + stoneValue[i + 2] - dp[(i + 3) % 3];
            dp[i % 3] = max({takeOne, takeTwo, takeThree});
        }
        int value = dp[0];
        if(value > 0)
            return "Alice";
        else if(value < 0)
            return "Bob";
        else
            return "Tie";
    }
};
```

JAVA

```

class Solution {
    public String stoneGameIII(int[] stoneValue) {
        int n = stoneValue.length;
        int[] dp = new int[3];
        for (int i = n - 1; i >= 0; i--) {
            int takeOne = stoneValue[i] - dp[(i + 1) % 3];
            int takeTwo = Integer.MIN_VALUE;
            if (i + 1 < n)
                takeTwo = stoneValue[i] + stoneValue[i + 1] - dp[(i + 2) % 3];
            int takeThree = Integer.MIN_VALUE;
            if (i + 2 < n)
                takeThree = stoneValue[i] + stoneValue[i + 1] + stoneValue[i + 2] - dp[(i + 3) % 3];
            dp[i % 3] = Math.max(Math.max(takeOne, takeTwo), takeThree);
        }
        int value = dp[0];
        if (value > 0)
            return "Alice";
        else if (value < 0)
            return "Bob";
        else
            return "Tie";
    }
}

```

PYTHON:

```

class Solution:
    def stoneGameIII(self, stoneValue):
        n = len(stoneValue)
        dp = [0] * 3
        for i in range(n - 1, -1, -1):
            takeOne = stoneValue[i] - dp[(i + 1) % 3]
            takeTwo = float('-inf')
            if i + 1 < n:
                takeTwo = stoneValue[i] + stoneValue[i + 1] - dp[(i + 2) % 3]
            takeThree = float('-inf')
            if i + 2 < n:
                takeThree = stoneValue[i] + stoneValue[i + 1] + stoneValue[i + 2] - dp[(i + 3) % 3]
            dp[i % 3] = max(takeOne, takeTwo, takeThree)
        value = dp[0]
        if value > 0:
            return "Alice"
        elif value < 0:
            return "Bob"
        else:
            return "Tie"

```