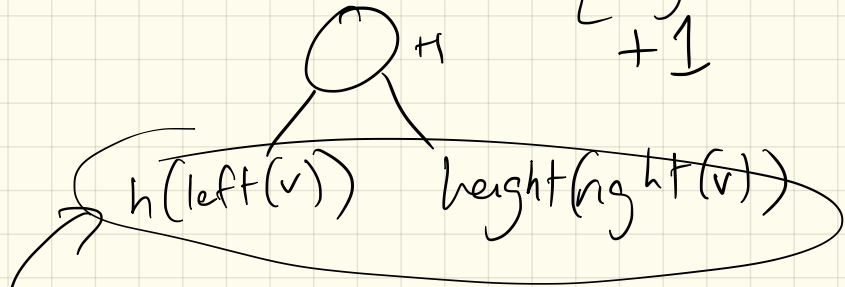# CS2100

Last day of BST

Recap

- HW due last night
- AVL tree HW (# 9) due next Thursday
- Review next Friday
- MT2 following Monday

Last time

Remove in an AVL tree.

AVL key: height-balance property

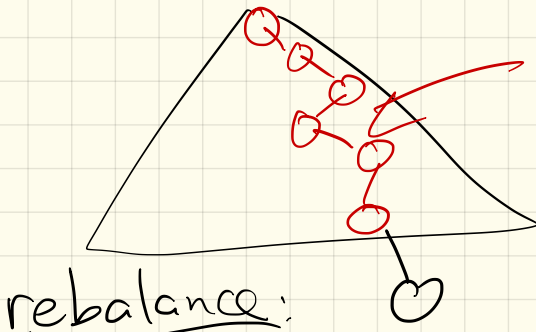$$h(v) = \max\{\text{height of child}\} + 1$$

$h(\text{left}(v))$ $\text{height}(\text{right}(v))$

these must be ≤ 1 apart

(Nulls are -1 height)

Insert :
→ Do BST insert
  ↳ this gives a new leaf, v

height might <u>change</u>
(& it may not
satisfy   h-b prop)

<u>rebalance</u>:
  v = v → parent v
  while (not above root) {
    reset v's height
    if v is unbalanced
        z = v
        y = v's higher child
        x = y's higher child
        pivot (y) <u>or</u> pivot(x) twice
    else
        v = v. up   (could have
                        done a break)
  3

Remove

→ Do BST remove

Need it = parent of actual node removed
(lower node)

reset it's height

loop to travel up
+ rebalance (don't do a break)

Note one difference:

Each insert will trigger
at most 1 set of pivots

In remove, may have to
pivot at every level

Runtimes:

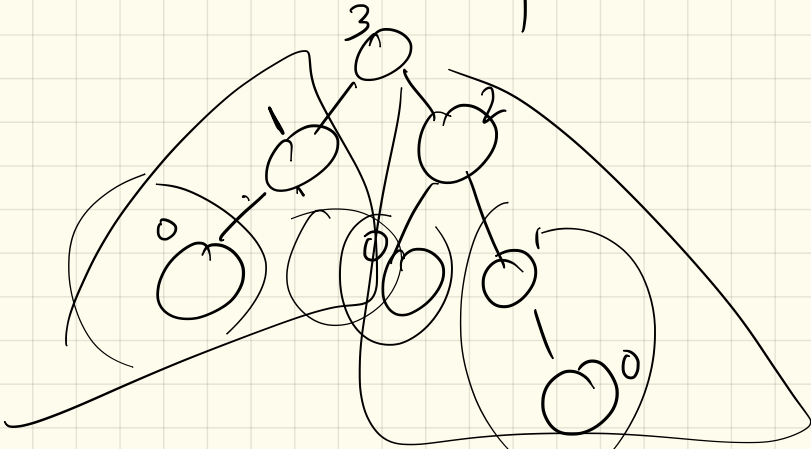In each of insert/remove:
$$O(height(t)) \rightarrow \text{for BST}$$

Then, starting at a leaf,
travel up to root $\leftarrow O(1)$
each time, reset height
+ do ≤ 2 pivots

loop that
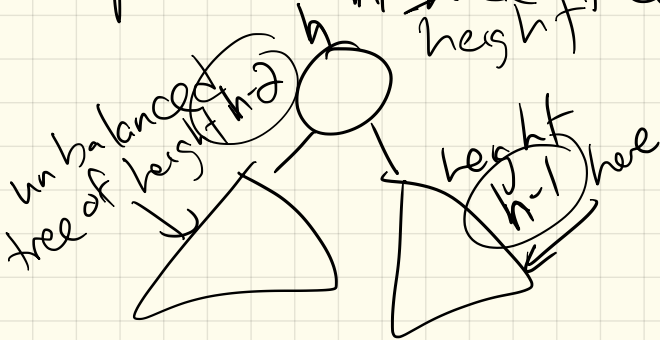repeats O(height)
times

$\searrow O(1)$ time

Key — height?

Worst case:

every node has children 1 apart

Note: know h(tree w/n nodes)
$\geq$ h(tree w/n-1 nodes)

root   worst

possible h ~~node~~ tree:

height tree:

unbalanced tree of height h-2

height h-1 tree

\#nodes in entire tree =

1 + \#nodes in height h-1 tree

+ \#nodes in height h-2 tree

$$\geq 1 + 2^0 \left(\begin{array}{c} \text{\#nodes in h-2} \\ \text{height tree} \end{array} \right)$$

$$T(h) \geq 1 + 2T(h-2)$$

$$\geq 2\underline{T(h-2)}$$

know $T(h-2) \geq 2T(h-4)$

So: $T(h) \geq 2\left(2T(h-4)\right)$

$$\underset{n}{\geq} \cdots 2^{h/2}$$

$$\Rightarrow n > 2^{h/2}$$

$$\log_2 n \geq \frac{h}{2} \Rightarrow \boxed{h < 2 \lg n}$$

Red - black trees

insert new node, color
it red

If red node has a
red child, rebalance

Color both nodes black

$\rightarrow h \leq 1.4 \log_2 n$

<u>Note</u>: search for applets
or demos