Math 135: More Logic	8/31/2012
Announcements	
-HWI will be up after class  (due next Friday)	

Ex: Truth tellers a liars Alice: "Exactly one of us is telling the truth." Bob: "We are all lyne." Cindy: "The other Awo are lying." Same idea - bigger table! P: Alice is truthful 9: Bob is truttand v: Cindy is truthful

	a	1/~	Exactly 1 trutha	1/ All lying	A4 6 2 1
		<b>V</b>	Tracing I want	11 / 11   19   11	Other 2 lying
XT			<b>—</b>		
XT		F	P	F	F
XT	F	1	F	F	F
十二	F	F		F	F
XF	1	1	F	F	F
XF	1	F			F
XF		T		E	
XF	F	F	7	,	
	X	,			
	Alic	e 15	only hone	st person	

Worksheet example:

Kevin Pan 1 Dan 15 lying "Both teling truth"

Paradox: It is possible to have no consistent rows - this is known as a paradox.

It is also possible to have multiple possible rows - in this case, can't decide who is that ful.

0< x < 10

Predicates:

Propositions which depend on a variable: Ex: P(x): x ≥ 0 Pend on a variable:

Negating predicates:

$$\frac{1}{2}\left(\frac{x>0}{p}\right) \wedge \left(\frac{x<10}{q}\right) = \frac{1}{2}\left(\frac{p}{p}\right) \wedge \frac{q}{q}$$

$$\frac{1}{2}\left(\frac{x>0}{p}\right) \wedge \frac{q}{q}$$

Quantifiers

R=real

R=real

R=real

N=ratifal #S

(in universe)

Thiversal

P(x) is true  $E_{x}$ : Let P(x) = "x+1>x" Q(x) = "x+2"Give truth values for: Vx E [R, P(x): Far all real #5x, () Yx E 12, O(x): Forall x, XL2. (F)

Fx P(x): There exists an x (in univose) such that P(x) is true. Ex: Let P(x) = "x+1=x" Q(x) = "x < 2"Gue truth values for: Fx ER, P(x): F)

Shere is an x with x+1=x." Fix & IR, O(x): (T) Ex: 1
There is an x with x<2

These can get more complicated.

(3) (P(x) 1 Q(x)) V (AxR(x)) Which quantifier holds where? Jx (P(x) ^ Q(x)) V Hy R(y) Vegations: How Should we negate quantifiers? Consider: P(x) = "x has taken collège algebra."HxP(x): Everyone has taken Collège algebra. What is  $7(\forall x P(x))$ ? Someone has not taken college  $7(\forall x P(x)) = \exists x 7P(x)$  angelone. What about  $\exists x P(x)^*$ .

Some one has taken college algebra.

- ( ]x P(x)): No one has taken collège algebra.

 $\neg (\exists x P(x)) = \forall x \neg P(x)$  $\neg (\forall \times P(x)) = \exists x \neg P(x)$ And they "stack":  $\neg(\forall x \forall y P(x,y)) = \exists x \exists y P(x,y)$  $\neg \left( \forall x \left( P(x) \vee Q(x) \right) \right) = \exists x \, \neg \left( P(x) \vee Q(x) \right)$  $=\exists x (\neg P(x) \land \neg Q(x))$ 

Nested quantifiers: YXER, JYER, (X+Y=0): translate: For any x, there is a y, = y \x (x+y=0): (= There is ay s.t. for all x, X+Y=O

Suppose are universe is IR. (T)

Translate:  $\forall x \ \forall y ((x>0) \land (y<0)) \rightarrow (xy<0)$ For all x and all y, if x is positive and y is negative, er XY 15 negative

Wegating implications
What is 7 (p -> 9) eguvalent So:
What  $5 - (4 \times (P(x) \rightarrow Q(x)))$ ?  $\exists x \neg (P(x) \rightarrow Q(x))$   $\exists x (P(x) \land \neg Q(x))$ Ex: Write the negation of  $4 \times 20$ , if  $3 \times 2 = 1$ . There is an x > 0
with x2 = 1 and x3 + 1.

A theorem (or lemma proposition, etc.) is a statement that can be rigourously shown to be true. Generally some thing like:

"If n is even, then it is divisible by?"

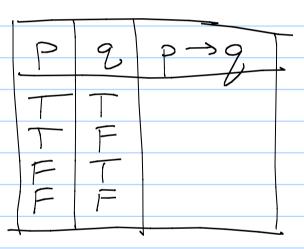
#2>0 350 such that if |f(x)-f(y)| < E, 1

then |x-y| < 8." he sequence of statements giving that arguement is called a proof.

Direct proofs

Think about p > 9

When is it false



Ex: If n is an odd integer, then n2 is even. true or false?

Ex: If n is an even integer, then (Assume p is true, & show g can't be failse.)