

CS2100

AVL Trees



This week + next:

- HW due today - given website error, may submit tomorrow.
- Next HW:
 - remove in a BST
 - next Tues remove in an AVL
 - Wed/Thurs next week
- tomorrow will be a work (no lab) day
- Today: office hours at 2pm (noon-2:30)
- Likely not be a lab next Tuesday - lecture instead
- Midterm 2: April 16

Last time:

Binary Search trees.

What's left - runtimes?

Fn L:

$O(n)$

①

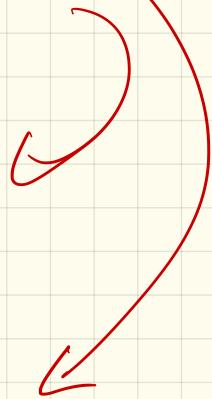
②

③

④

⑤

insert:

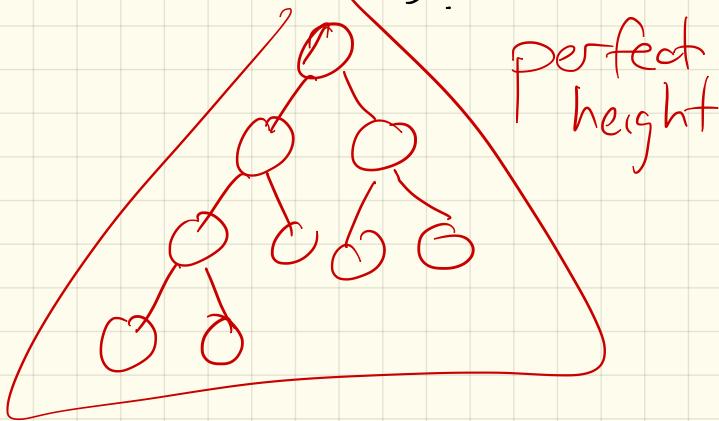


remove:

Alternative: $O(\text{height}(T))$

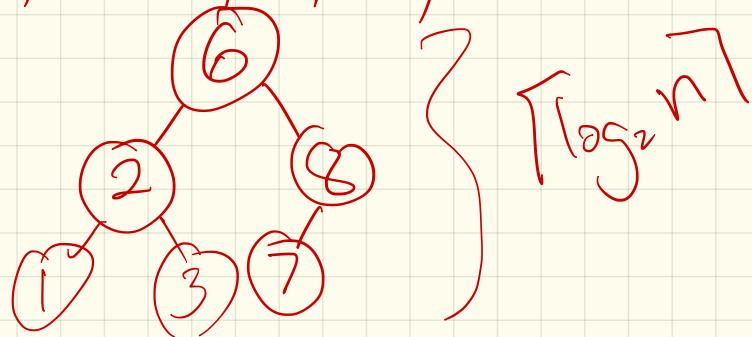
Today: How to make better?

Well, why were heaps $O(\log n)$ instead of $O(n)$?



Goal: How can we balance?

Ex: $1, 3, 8, 7, 6, 2$



Balanced BSTs

Many kinds:

- Red-black trees : $1.4 \lceil \log_2 n \rceil$

- Splay trees :

- AVL trees : $2 \lceil \log_2 n \rceil$

⋮

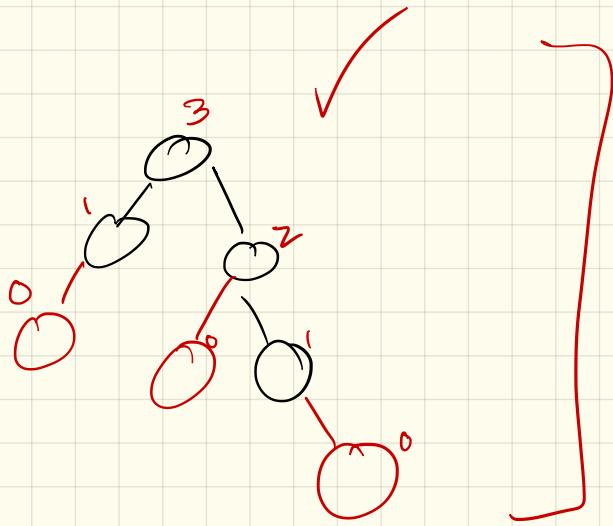
Goal of them all:

$O(\log_2 n)$

AVL trees :

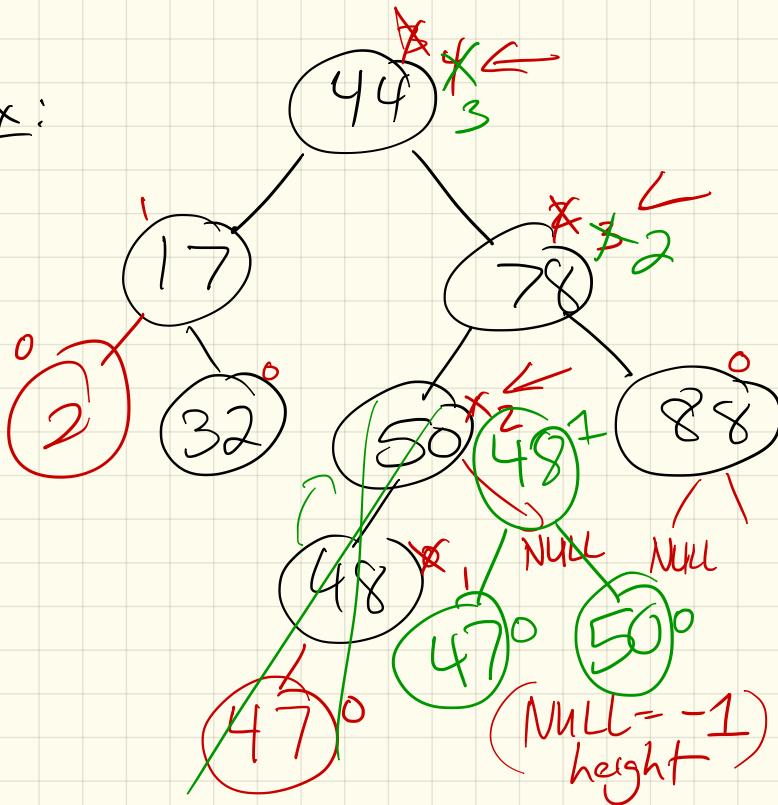
Height balance property :

For every node x in T ,
the heights of x 's
children differ
at most 1.



$$\Rightarrow \text{max height} \leq 2 \lceil \log_2 n \rceil$$

Ex:



Now: how can we mess this up?

insert(2) ← OK

Ex: insert(47) ← NO!

Fix it! How?

rotate (or pivot)
48 up

Consider the lowest node

which fails the Height-Balance
property.

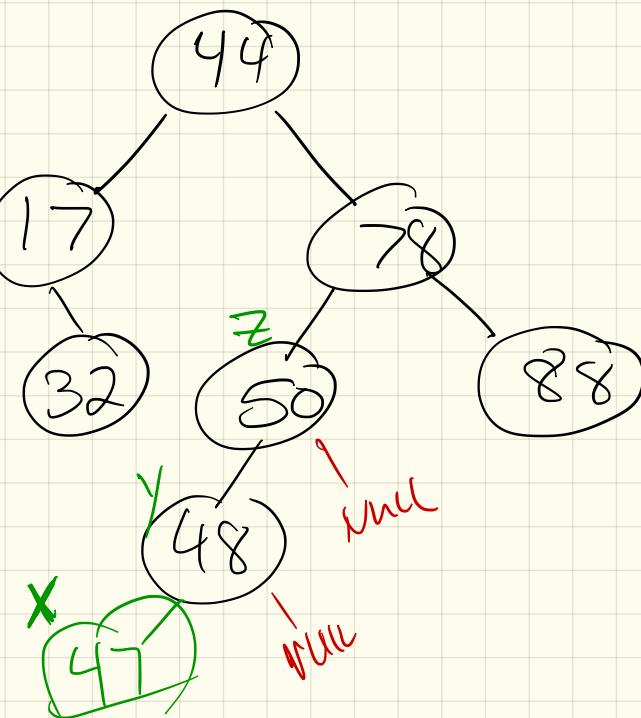
↙ Call this Z

Let y be child w/ bigger height.

Let x be grandchild w/ larger height.

Fix!

move
up
balance



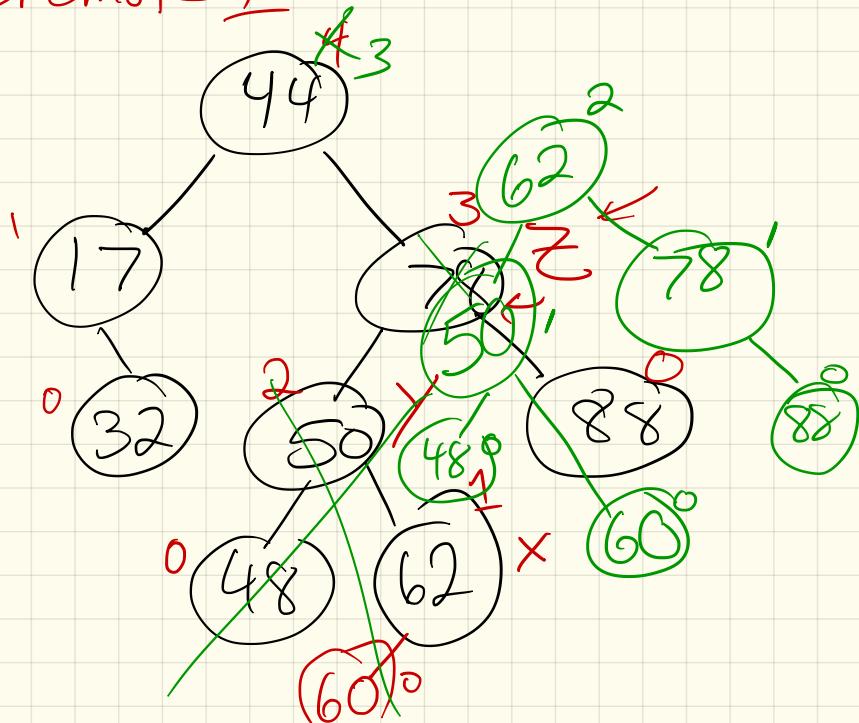
Another: insert(60) below
Consider the lowest node

which fails the Height-Balance
property.
→ call this Ξ

Let y be child w/ bigger height.

Let x be grandchild w/ larger height.
→ promote x

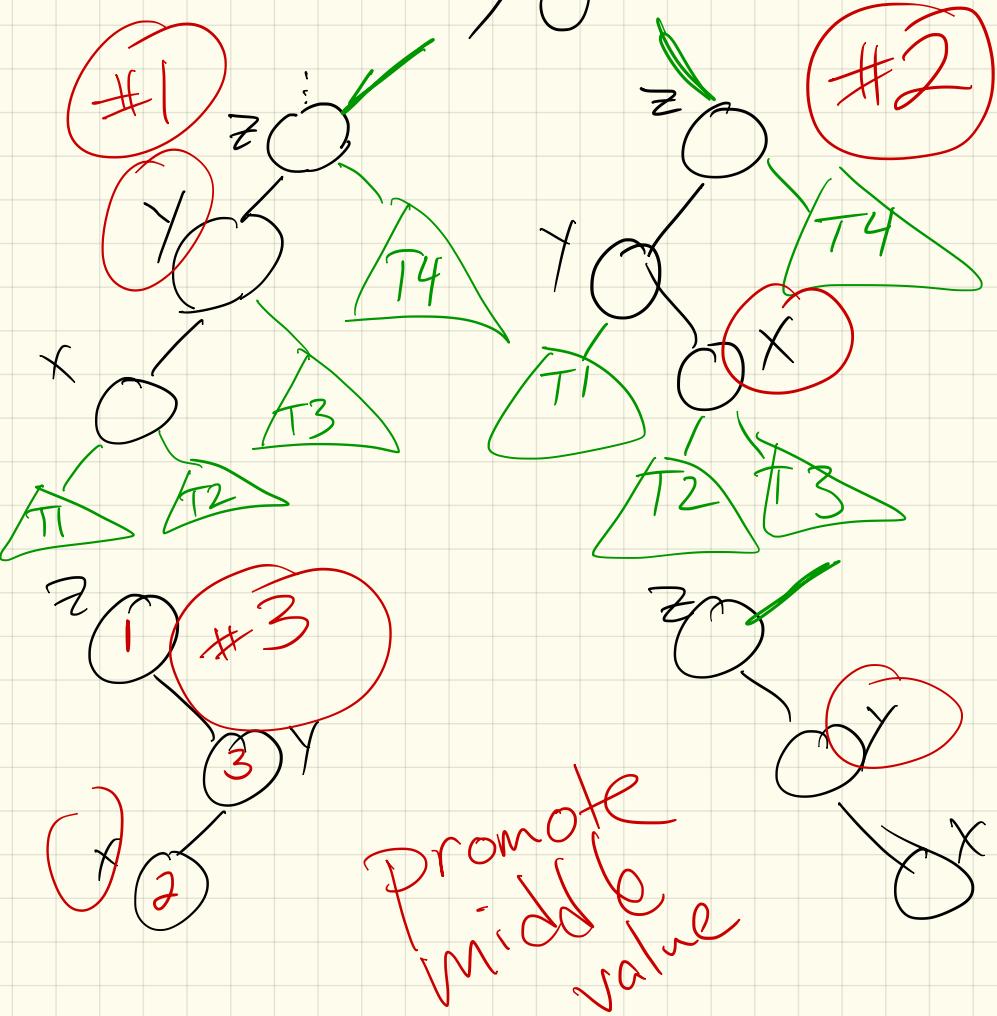
Fix!

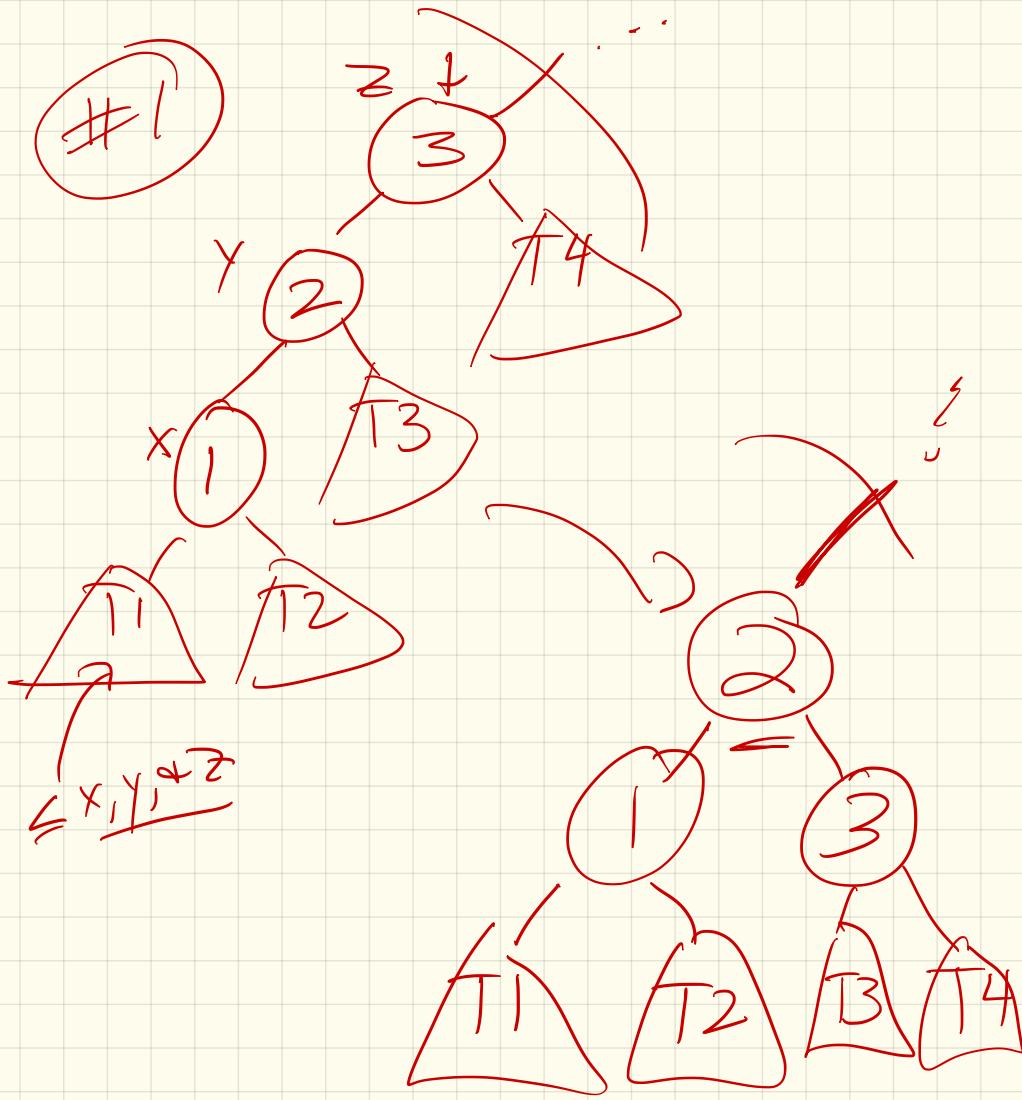


So: algorithm to insert:

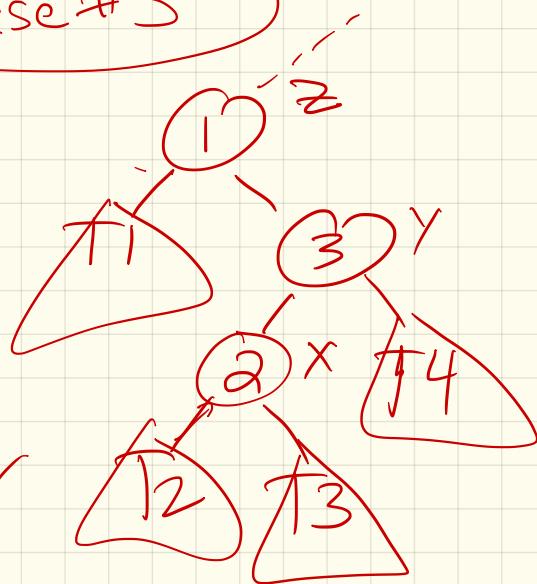
- First run BST insert

- Then find lowest unbalanced node z , + deeper child, grand child.

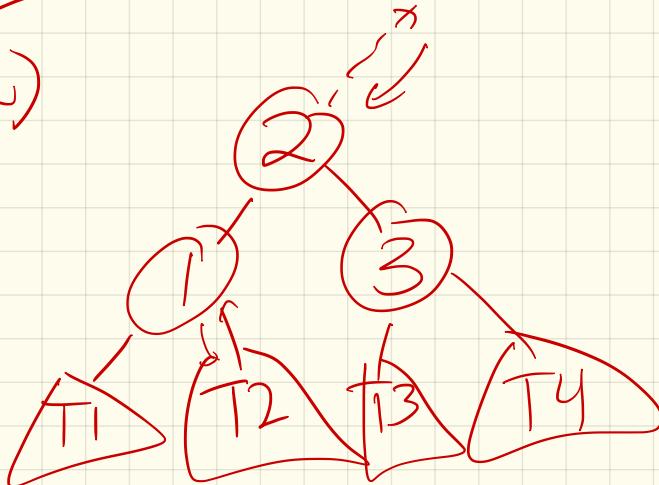


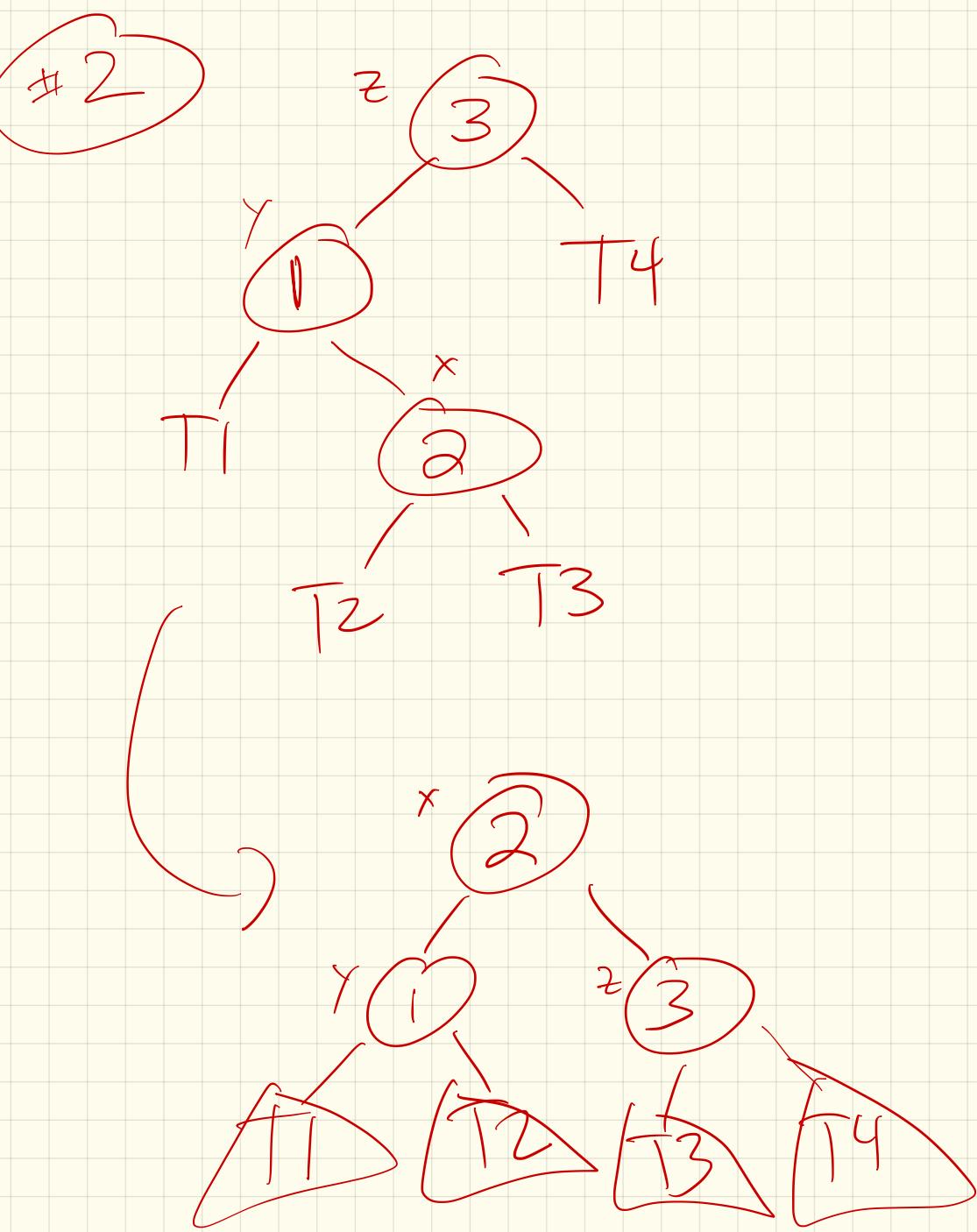


Case #3

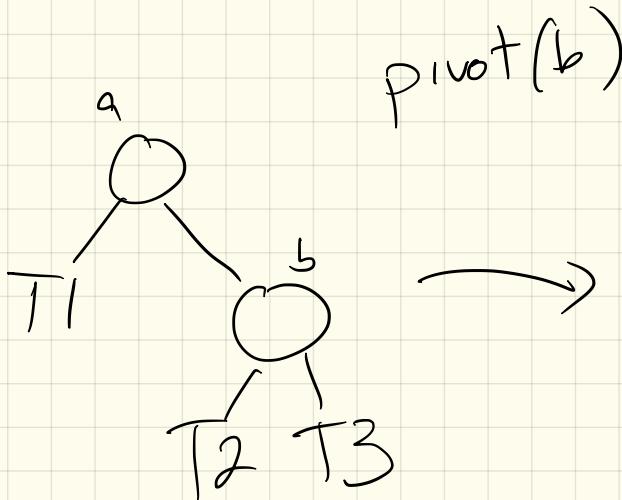
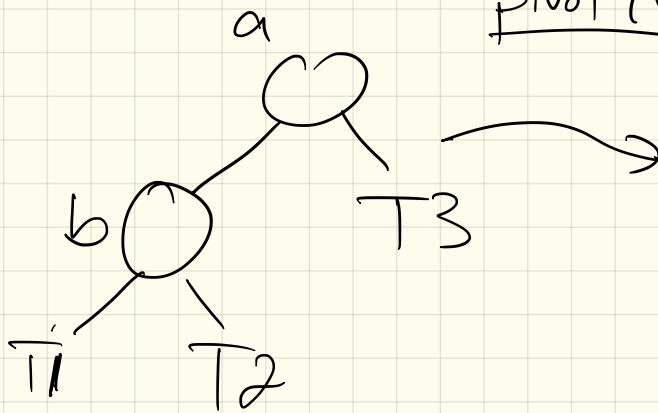


fix: make \times the root

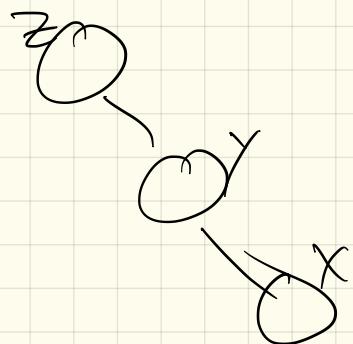
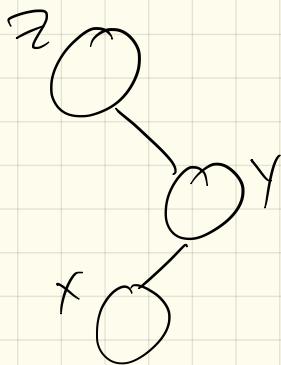
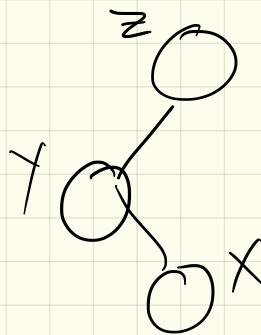
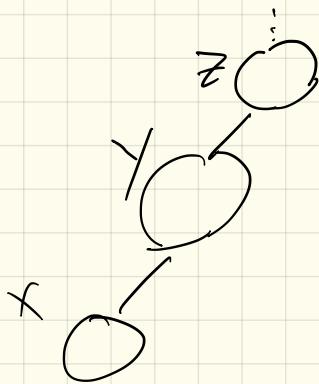




Key operation : pivot



Then: implement w/ pivot!



Bigger example:

Insert : 1, 2, 3, 4, 5, 6, 7