Math 135 - Logic (part 2)

Announcements

- HW0 is due Friday

- Office hours tomorrow if you have questions, or use email.

- Worksheet today (in class)

$$\sum_{i=1}^{4} i = \overline{1 + 2 + 3 + 4}$$
$$= 10$$

A few useful tools:

Summations:

$$\sum_{i=1}^{n} 1 = \overbrace{(1 + 1 + \cdots + 1)}^{n \text{ times}} = n$$

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} i = (1 + 2 + 3 + \cdots + n) = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} c^i = \frac{c^{n+1} - c}{c - 1}$$

c is constant

Mod:

remainder

11 mod 12 = 11

13 mod 12 = 1

25 mod 12 = 1

rem

$$12 \overline{)11} \quad \begin{matrix} 0 \\ \end{matrix}$$

$$\begin{matrix} 0 \\ \widehat{11} \end{matrix}$$

$$\frac{0}{0}$$

Useful tools:

$$x^2 + x^2 = 2x^2$$ ~~(crossed out)~~

$$x^a \cdot x^b = x^{a+b}$$ ~~(crossed out)~~

$$(x^y)^a = x^{y \cdot a}$$

Logarithms:

What is $\log_2 16$? ∴ 4

$$\log_x (ab) = \log_x a + \log_x b$$

$$\log_x (y^a) = a(\log_x y)$$

$$\log_x (x^a) = a$$

$$\boxed{\frac{\log_x a}{\log_x b} = \log_b a}$$

Last time: Logic

- propositions + their negations

- truth tables

- logical equivalence

$p \to q$ - contrapositive, converse, inverse

$$\boxed{\neg q \to \neg p}$$

logically
equivalent
to $p \to q$

$q \to p$

$\neg p \to \neg q$

A more applied exercise:
truth tellers & liars

Suppose we meet two people, Alice and Bob.

Alice says: " Exactly one of us is lying."

Bob says: "At least one of us is telling
the truth."

How do you tell who is lying + who is honest?

Let p = "Alice is truthful"
q = "Bob is truthful"

p	q	Statement 1 Exactly 1 is lying	Statement 2 At least 1 is truthful
T	T	F	T
T	F	T	T
F	T	T	T
F	F	F	F

Which row is consistent? Both are liars.

Another one:
Alice says: "Exactly one of us is telling the truth."
Bob says: "We are all lying."
Cindy says: "The other two are lying."

Same principle, bigger table!

$p =$ Alice is truthful
$q =$ Bob
$r =$ Cindy

p	q	r	Exactly 1 truthful	All lying	Other 2 lying
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	F	F
F	F	F	F	T	T