

# Parsing

Grammars

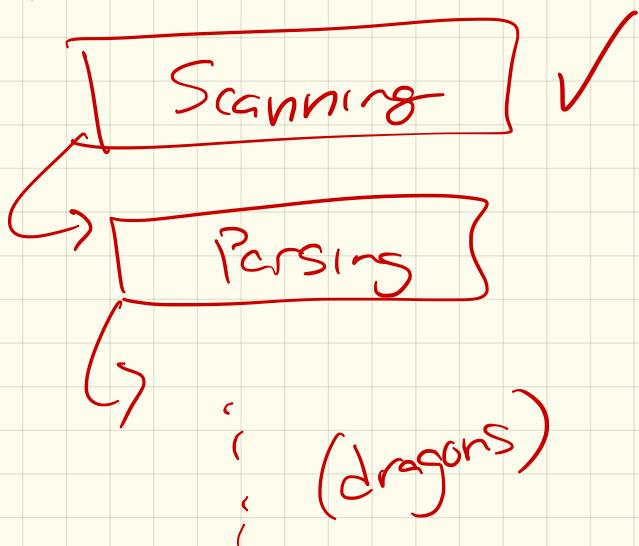
Parse trees

left vs right

## Recap:

- HW due today
- Next HW - flex Scanner  
(due next Thursday)
- Git repo guide

## Compilation



The limits of regular expressions:

Certain languages can never be recognized by regular expressions.

Simple example:  $0^n 1^n$

$$L = \{\epsilon, 01, 0011, 000111, \dots\}$$

Only option: \*

$0^* 1^*$  → this accepts  
011

More real world: math equations

$$(((x+7)^* 2) + 3) - 1$$

Why? At heart,  
 $(\dots)^n$  - .

So, in our next level down, we need something stronger

### Parsing:

- Given string of input tokens, a parser must determine if the tokens generate a valid program

The basis of these are context free grammars (CFGs):

- terminals: tokens (via scanner)  
*ids, +, (, ), -*
- nonterminals (one a start symbol)  
*(usually capital letter)*
- production rules

$$\begin{array}{c} E \rightarrow ( E ) \\ \rightarrow \underline{id} + \underline{id} \end{array}$$

Example :  $0^n 1^n$

terminals; 0 + 1 (+ε)

$$L: S \rightarrow 0S1 \quad (\leftarrow)$$

$$S \rightarrow \epsilon \quad (\leftarrow)$$

( nonterminals : S  
also start )

Show  $000111 \in L$

$$\begin{aligned} S &\xrightarrow{(1^{\text{st}})} 0S1 \\ &\rightarrow 00S11 \quad (2^{\text{nd}}) \\ &\rightarrow 000S111 \quad (1^{\text{st}}) \\ &\rightarrow 000111 \quad (2^{\text{nd}}) \end{aligned}$$

Ex: (bigger) non terms:  $E, A$

term: id, -, +, /, ↑

$E \rightarrow E$	$E$	$A$	$E$	1
$E \rightarrow (E)$	$(E)$	$A$		2
$E \rightarrow -E$	$-E$			3
$E \rightarrow id$	<u>id</u>			4

$A \rightarrow +$			5
$A \rightarrow -$			6
$A \rightarrow *$			7
$A \rightarrow /$			8
$A \rightarrow \uparrow$			9

Derivation: The process by which a grammar parses & defines a language.

Ex: Show by the above grammar:  $\underline{-}(x+5)$  is accepted

$$E \Rightarrow -E \Rightarrow -(E)$$

(3)  $\Rightarrow$

$$\Rightarrow -(E A E) \Rightarrow -(E+E)$$

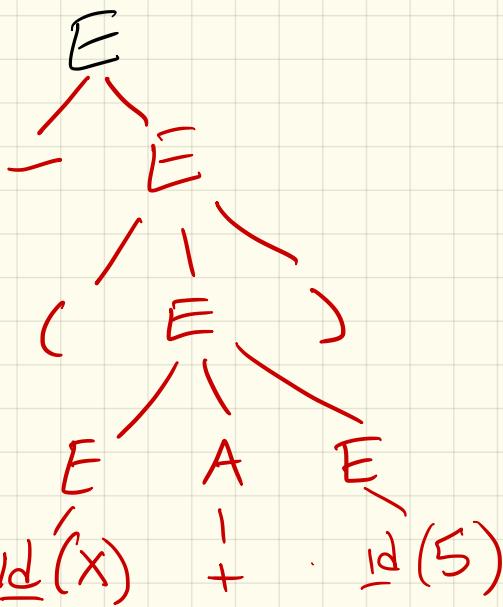
(1)  $\Rightarrow$

$$\Rightarrow -(id+E) \Rightarrow -(id+id)$$

(5)  $\Rightarrow$

$$5$$

Parse tree: A graphical representation of this derivation:



Each parent/child shows one step of the derivation

- leaves are terminals
- root is start non-terminal

## Leftmost vs rightmost

Leftmost derivation: one where  
the leftmost nonterminal is  
replaced in each step

Ex:  $- (\underline{id} + \underline{id})$

$$E \Rightarrow - E \Rightarrow - (E)$$

$$\Rightarrow - (\underline{EA} E) \Rightarrow - (\underline{id} \underline{AE})$$

$$\Rightarrow - (\underline{id} + \underline{E}) \Rightarrow - (\underline{id} + \underline{id})$$

- Rightmost: always the one  
on the right

(Often we'll fix one way, since  
each free has a unique  
left & right most derivation.)

Note: Not necessarily unique!

Ex:  $5 - 2 * 6$

①

$E \Rightarrow E A E$

$\Rightarrow E^{\perp} A E A E$

$\Rightarrow id(5) A E A E$

$\Rightarrow id(5) + E A E$

$\Rightarrow id(5) + id(2) * id(6)$

②

$E \Rightarrow E A E$

## Ambiguity:

- Any grammar that produces  
More than one parse tree  
for some sentence is  
ambiguous.

This can be very undesirable!

We'll spend time trying to rule  
this possibility in our  
grammars.

Note: Any regular expression can also be written as a grammar (CFG) nonterm → char

Ex:  $(a \mid b)^* abb$

Grammar:

$$S \rightarrow Aabb$$

$$A \rightarrow \epsilon$$

$$\begin{aligned} &\rightarrow aA \\ &\rightarrow bA \end{aligned}$$

(However, the reverse is not true!)

Still do regular expressions;  
Simpler + faster

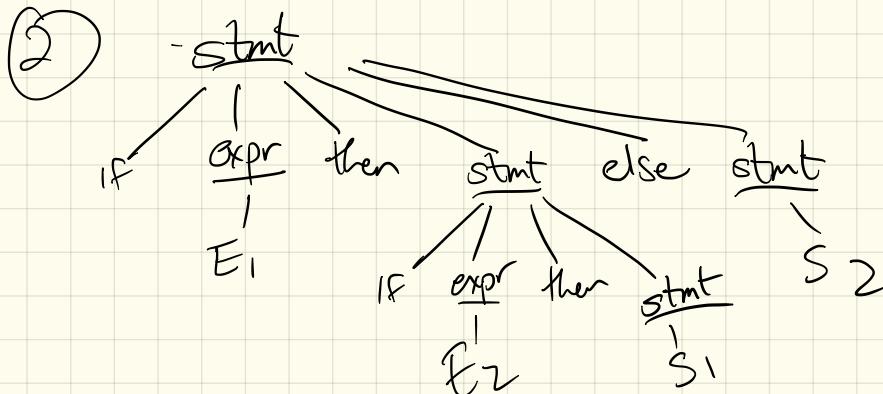
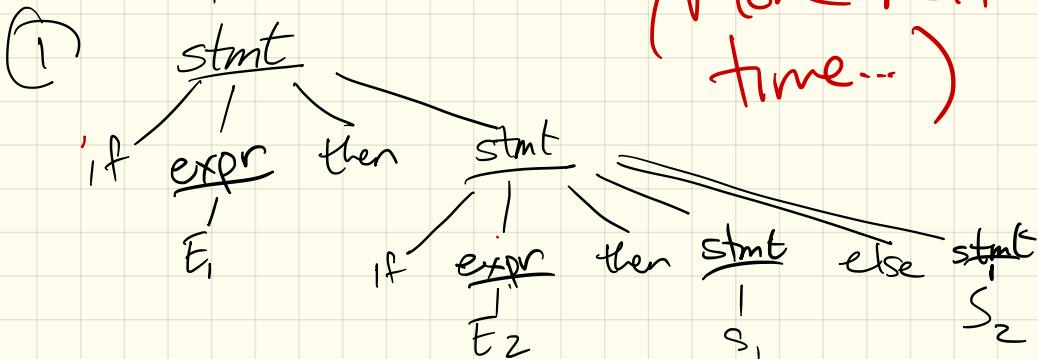
A more complex example: If Statements



Then: if  $E_1$ , then if  $E_2$  then  $S_1$ , else  $S_2$

2 parse trees:

(More next time...)



General rule:

Match each else w/ closest unmatched then

How?

- Rewrite so any statement between an "else" + a "then" must be matched (so no if-then w/o else)

Grammar:

stmt → matched\_stmt  
| unmatched\_stmt

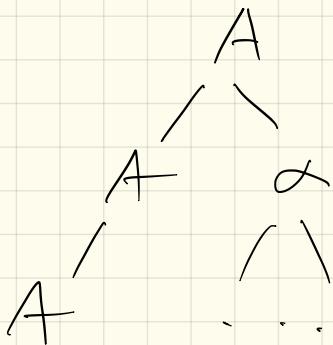
matched\_stmt → if expr then matched\_stmt else  
| matched\_stmt  
| other

unmatched\_stmt → if expr then stmt  
| if expr then matched\_stmt  
| else unmatched\_stmt

Dfn: A grammar is left-recursive  
if it has a non-terminal A  
with some rule

$$A \rightarrow A \alpha$$

These are bad for parsers:



When scanning tokens &  
trying to build a tree,  
not sure when to stop!

Ex:  $E \rightarrow E + T \mid T$   
 $T \rightarrow T * F \mid F$   
 $F \rightarrow (E) \mid \text{id}$

Parse:  $x + y * 10$

However, we do have left recursion!

To eliminate:

$$A \rightarrow A\alpha \mid \beta$$

$$\hookleftarrow A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' \mid \epsilon$$

On

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid \text{id}$$



## Back to the practical:

- Any CFG can be parsed

↳ Chomsky Normal Form  
CYK algorithm

Run time:

This is too slow!

Most modern parsers look  
for certain restricted  
families of CFGs.

Result:

## Top down parsing

Called predictive parsing.

Works well on LL(1) grammars.

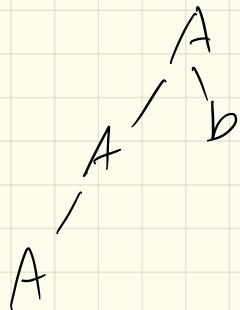
Ex:  $S \rightarrow cAd$   
 $A \rightarrow ab/a$

Parse cad:

Rule: Starting w/ S,  
apply rules until  
one matches the  
next input  
(back track if there  
is a mistake)

Note: Left recursion is  
very bad on these!

$$A \rightarrow A b$$



∴  never matches an input or hits a conflict

So never forced to back track.

How predictive parsing works:

- the input string  $w$  is in an input buffer.
- Construct a predictive parsing table for  $G$ .
- if you can match a terminal, do it  
(+ move to next character)
- otherwise, look in table for rule to get transition that will eventually match

Hard part (next time):  
the table