Math 135: Discrete Mathematics, Fall 2010
Worksheet 10

1. (a) How many bit strings of length 7 either begin with two 0's or end with three 1's?

   (b) A palindrome is a string whose reversal is identical to the string. How many bigstrings of length n are there?

   (c) How many positive integers less than 100 are divisible by 7? How many are divisible by 7 but not 11? How many are divisible by both 7 and 11?

   (d) A group contains n men and n women. How many ways are there to arrange the people in a row if the men and women must alternate?

   (e) How many ways are there for 8 men and 5 women to stand in a line so that no two women stand next to each other? (Hint: First position the men and then consider positions for the women.)

2. A computer network consists of 6 computers. Each computer is directly connected to 0 or more of the other computers. Show that there are at least 2 computers in the network connected to the same number of other computers.

3. One hundred tickets, numbered 1 to 100, are sold to 100 different people in a drawing. Four different prizes are awarded, including a grand prize trip to Hawaii. How many ways are there to award prizes if:

   (a) there are no restrictions?

   (b) the person holding ticket 47 wins the grand prize?

   (c) the person holding 47 does not win any prize?

   (d) the people holding tickets 19 and 47 both win a prize?

   (e) the people holding 19, 47, and 73 all win prizes?

   (f) none of the people holding 19, 47, and 73 win prizes?

   (g) the grand prize winner is a person holding 18, 47, 73, or 97?

4. Show that $k\binom{n}{k} = n\binom{n-1}{k-1}$ using a combinatorial proof. (Hint: Consider choosing a committee with a chairperson.)