

CS3100

More reductions



Announcements

- HW due Wednesday

For HW: Show A is NP-Complete

- ① in NP - "yes" solution can be checked in poly time
- ② NP-Hard :.

reduce known NP-Hard problem to A

In general: may use any listed problems in lecture notes

Last time :

Graph reduction:

- Ind. Set
- Clique
- Vertex Cover

In lecture notes:

- Hamiltonian cycle
- Traveling Salesman

↳ weighted graph, design
a tour that visits
every vertex of
minimum length

Subset Sum :

Given a set of numbers

$X = \{x_1, x_2, x_3, \dots, x_n\}$
and some subset of X , does it sum to t ?

Ex: (actually did this one!

See lecture on Sept 8)

Runtime: exponential
 2^n

Subset Sum is NP-Hard.

Reduction: Vertex Cover

Input: Graph G & size k .

Construct a set of numbers: X

Label each edge $0, \dots, m-1$

Add a number to X for each e_i^0 : $4^i = b_i^0$

Add a number to X for each vertex v :

$$a_v : 4^m + \sum_{\substack{e_i^0 \text{ adjacent} \\ \text{to } v}} 4^i$$

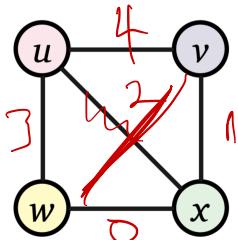
View these as base 4 #s:

edges e_i^0 : $\underbrace{\text{0} \dots 0}_{m-1} \overbrace{|}^{i} \text{0} \dots 0$

v : $\underbrace{1}_{m} \underbrace{\dots}_{\text{is adj for edges}}$

reduction cont:

Ex:



$$\begin{aligned}a_u &:= 111000_4 = 1344 \\a_v &:= 110110_4 = 1300 \\a_w &:= 101101_4 = 1105 \\a_x &:= 100011_4 = 1029\end{aligned}$$

$$\begin{aligned}b_{uv} &:= 010000_4 = 256 \\b_{uw} &:= 001000_4 = 64 \\b_{vw} &:= 000100_4 = 16 \\b_{vx} &:= \underline{000010}_4 = 4 \\b_{wx} &:= \underline{000001}_4 = 1\end{aligned}$$

Nice feature: for $i < m$,
only 3 1's anywhere in X

\Rightarrow no carrying for any
subset of these!

$$\text{Set } t = k \cdot 4^m + \sum_{i=0}^{m-1} 2 \cdot 4^i$$

Poly time conversion:
 $n+m$ numbers

\Rightarrow : reduct. cont.

if cover of size k
 \Rightarrow subset of value t

Take a vertex cover in G .

For each v , choose a_v
is subset of X .

$$\text{have } \geq k \cdot 4^m + \sum_{i=0}^{m-1} 1 \cdot 4^i$$

For every edge w only 1 endpoint in cover, also
add b_i to subset.

$$\Rightarrow \text{Sum is exactly } k \cdot 4^m + \sum_{i=0}^{m-1} 2 \cdot 4^i$$

\Leftarrow : Supp. Subset $X' \subseteq X$
sums to $t = k \cdot 4^m + \sum_{i=0}^{m-1} 2 \cdot 4^i$

Know close exactly k of a_v 's,
since lower terms can't carry.

These are a cover, since including
 b_i isn't enough to hit $2 \cdot 4^i$ \square

Another: Partition

Given a set of n numbers,

Can you partition into 2 sets $X + Y$ so that

$$\sum_{x \in X} x = \sum_{y \in Y} y ?$$

Easy reduction:

On worksheet...

Set Cover:

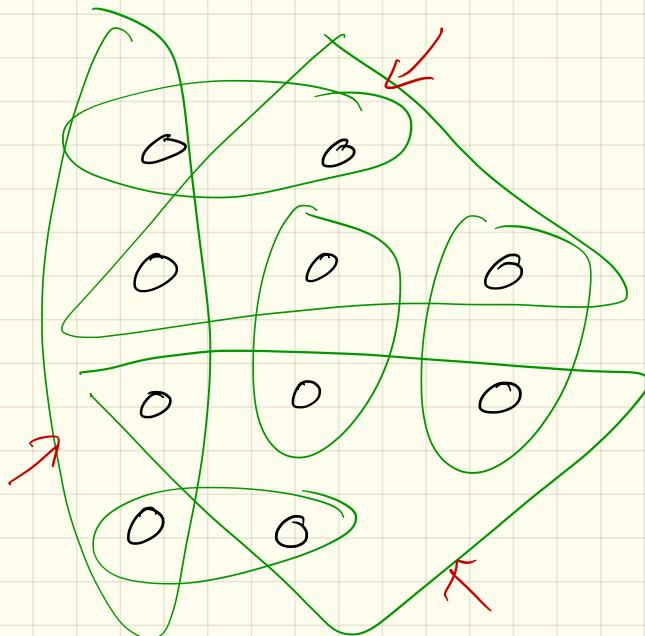
Given a set U of n elements,
a collection S_1, S_2, \dots, S_m of
subsets of U , & a number k ,
is there a collection of k
of the S_i 's whose union is
all of U ?

Ex:

elements
in U :

Subsets
 S_1, \dots, S_7

& $k=3$.



Answer? Yes!

Set Cover is NP-Hard:

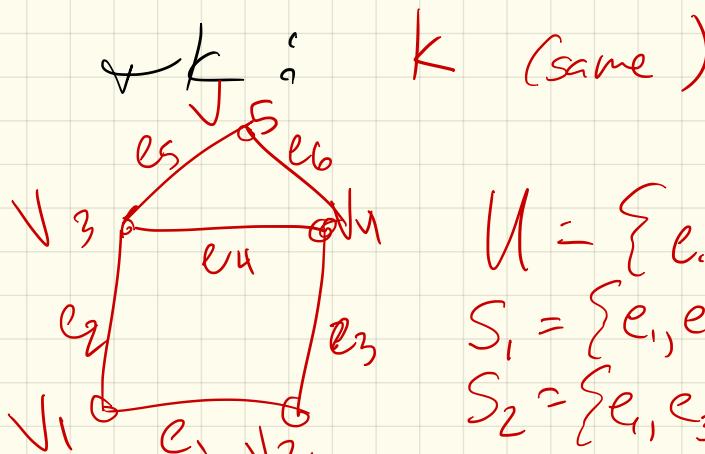
Reduction from vertex cover,

so input is $G \& k$.

Construct:

$$\mathcal{U} = \text{edges} = \{e_1, \dots, e_m\}$$

$$S_i^o \text{ 's : } S_i = \{\text{edges adjacent to vertex } v_i\}$$



$$\mathcal{U} = \{e_1, \dots, e_{10}\}$$

$$S_1 = \{e_1, e_2\}$$

$$S_2 = \{e_4, e_5\}$$

;

Vertex cover of size k
 \iff Set cover
of size k

Some fun examples

arXiv.org > cs > arXiv:1203.1895 Search or Article ID inside arXiv All papers Broaden your search usin
(Help | Advanced search)

Computer Science > Computational Complexity

Classic Nintendo Games are (Computationally) Hard

Greg Aloupis, Erik D. Demaine, Alan Guo, Giovanni Viglietta

(Submitted on 8 Mar 2012 ([v1](#)), last revised 8 Feb 2015 (this version, v3))

We prove NP-hardness results for five of Nintendo's largest video game franchises: Mario, Donkey Kong, Legend of Zelda, Metroid, and Pokemon. Our results apply to generalized versions of Super Mario Bros. 1-3, The Lost Levels, and Super Mario World; Donkey Kong Country 1-3; all Legend of Zelda games; all Metroid games; and all Pokemon role-playing games. In addition, we prove PSPACE-completeness of the Donkey Kong Country games and several Legend of Zelda games.

Comments: 36 pages, 36 figures. Fixed some typos. Added NP-hardness results (with proofs and figures) for American SMB2 and Zelda 2

Subjects: Computational Complexity (cs.CC); Computer Science and Game Theory (cs.GT)

Cite as: [arXiv:1203.1895 \[cs.CC\]](#) (or [arXiv:1203.1895v3 \[cs.CC\]](#) for this version)

Submission history

From: Alan Guo [[view email](#)]
[v1] Thu, 8 Mar 2012 19:37:20 GMT (627kb,D)
[v2] Thu, 6 Feb 2014 18:24:15 GMT (3330kb,D)
[v3] Sun, 8 Feb 2015 19:45:26 GMT (3425kb,D)

Which authors of this paper are endorsers? I Disable MathJax (What is MathJax?)

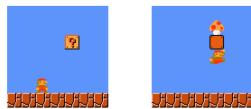
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Left: Start gadget for Super Mario Bros. Right: The item block contains a



Figure 10: Variable gadget for Super Mario Bros.

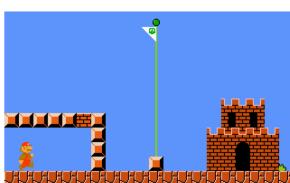


Figure 9: Finish gadget for Super Mario Bros.

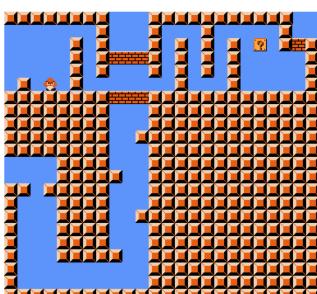


Figure 12: Crossover gadget for Super Mario Bros.

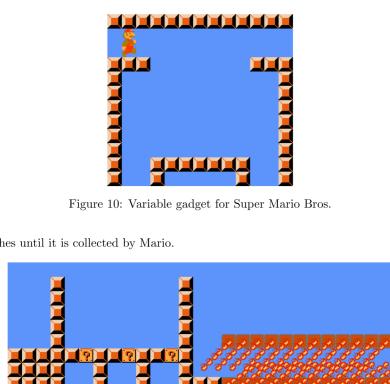
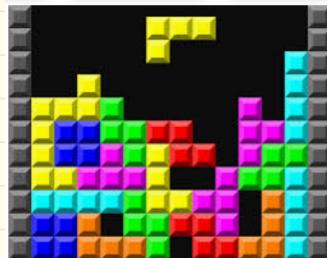


Figure 11: Clause gadget for Super Mario Bros.

shes until it is collected by Mario.

Another: Tetris



NP-Hard:
Reduce 3-partition

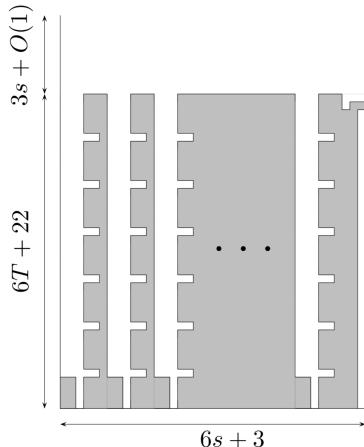


Fig. 2. The initial gameboard for a Tetris game mapped from an instance of 3-PARTITION.

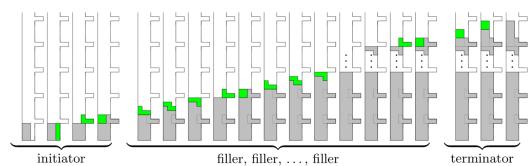


Fig. 3. A valid sequence of moves within a bucket.

Next time: