HW Assignment #1 – Game Theory Basic Definitions

General Instructions

- Due to: 17/11/2013 16:00
- Individual submission via Moodle (upload a single file). Please work on your own.
- For any questions or advice, feel free to contact me: lirifink@gmail.com
- Recitation will be on 17/11 (after HW submission). At the recitation, we will solve the HW questions together, on the board.

Question 1

Two players are claiming simultaneously their share of 100$. Each claim is an integer in the range {1,2,…,100}. The payment to the players is set according to the following rules:

1. If the sum of claims is not higher than 100, each one gets his claim.
2. If the sum of claims is higher than 100, and the claims are equal, each player gets 50$.
3. If the sum of claims is higher than 100, and the claims are not equal, then the player who asked for less (smaller claim) gets his claim, and the other one gets what remains of the 100$.

Find a Nash Equilibrium by iterated deletion of dominated strategies. Specify the deletion process.

Question 2

A seller wants to sell a product in an auction. Each bidder has a private valuation for the product (denote bidder i's valuation by v_i). The bidder has to decide how to conduct the auction, and she wants that for each bidder truthfulness will be a weakly dominant strategy (weakly dominating ALL his other strategies).

Prove or dispute: "In any auction that the seller plans, she has to give the product to the bidder who gave the highest bid".

Question 3

Find a Bayesian-Nash equilibrium in the first-price auction, when players' values are independently drawn from the uniform distribution on $[a,b]$, for any $b > a > 0$.

Hints for one possible solution:
a. Assume that the equilibrium bid function is $b(z)=\alpha+\beta\cdot z$, and that $b(a)=a$.
b. Start by writing the utility function $u(x,z)$ that denotes a player's utility when her value is x and she bids $b(z)$, for arbitrary x,z.
c. Since $b(z)$ is an equilibrium, it follows that a player's utility is maximized when she bids $b(x)$. This should give you a first-order condition on the bid function that will lead you to get an exact expression for α and β, which gives you the bid function.

Question 4

In the setting of the previous question, suppose there are 3 players with values independently drawn from the uniform distribution on [5,10]. Suppose player 1 has value v_1=8. What is her equilibrium bid? Show that she will not improve her expected utility by declaring b_1=6 or by declaring b_1=8.

Good Luck!
Liri