Classical Mechanics

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Contents

1 Newton mechanics

$$\sum_i \mathbf{F}_i = m\ddot{\mathbf{x}}. \tag{1.1}$$

2 Lagrangian mechanics

Find the generalized coordinates q_i and write the Lagrangian

$$L(q_i, \dot{q}_i; t) = T(q_i, \dot{q}_i; t) - V(q_i, \dot{q}_i; t). \tag{2.1}$$

Euler-Lagrangian equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \tag{2.2}$$

The action functional is defined as

$$S\{q_i(t)\} = \int_{t_i}^{t_f} L(q_i(t), \dot{q}_i(t); t)\mathrm{d}t. \tag{2.3}$$

The Hamilton principle:

$$\delta S = 0 \tag{2.4}$$

3 Hamilton mechanics

The canonical momenta

$$p_i = \frac{\partial L}{\partial \dot{q}_i}. \tag{3.1}$$

The Hamilton functional

$$H(q_i, p_i; t) \equiv \sum_{i=1}^{S} p_i \dot{q}_i - L(q_i, \dot{q}_i; t). \tag{3.2}$$

The Hamilton's equations of motion (or called canonical equations)

$$\begin{aligned}\dot{q}_i &= \frac{\partial H}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial H}{\partial q_i}\end{aligned} \tag{3.3}$$

The Poisson bracket

$$\{f, g\}_{\mathbf{q},\mathbf{p}} \equiv \sum_{j=1}^{S} \left(\frac{\partial f}{\partial q_j} \frac{\partial g}{\partial p_j} - \frac{\partial f}{\partial p_j} \frac{\partial g}{\partial q_j} \right). \tag{3.4}$$

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \{f, H\}_{\mathbf{q},\mathbf{p}} + \frac{\partial f}{\partial t} \tag{3.5}$$

$$\{p_i, p_j\}_{\mathbf{q},\mathbf{p}} = \{q_i, q_j\}_{\mathbf{q},\mathbf{p}} = 0, \quad \{p_i, q_j\}_{\mathbf{q},\mathbf{p}} = \delta_{ij} \tag{3.6}$$

4 Hamilton-Jacobi theory