1 Gauge theory

I will not use the language of fibre bundle to construct it strictly, but just simplely write down main ideas.

1.1 Non-Abelian Gauge Theory

We have a Lie-algebra-valued one-form $A^a_\mu t^a \mathrm{d}x^\mu \in \mathfrak{g} \times \Lambda^1$, which is called a *connection*. Thus $A_\mu \equiv A^a_\mu t^a \in \mathfrak{g}$. We have a Lie-algebra-valued two-form

$$F_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu = \partial_\mu A_\nu \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu - \partial_\nu A_\mu \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu - i[A_\mu, A_\nu]\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \tag{1}$$

which is called a *curvature*. The Lie-algebra-valued component is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$.

In a representation R of the Lie algebra $\mathfrak{g}$, the *covariant derivative* for representation-space-vector-valued scalar (0-form) ψ

$$\mathcal{D}_\mu \psi = \partial_\mu \psi - iA_\mu \psi \tag{2}$$

The components,

$$\mathcal{D}_\mu \psi^i = \partial_\mu \psi^i - iA^a_\mu R(t^a)^i{}_j \psi^j, \qquad i, j = 1, 2, \ldots, \dim R \tag{3}$$

For a Lie-algebra-valued object $\phi = \phi^a t^a$, the covariant derivative,

$$\mathcal{D}\phi = \partial\phi - i[A_\mu, \phi]. \tag{4}$$

ϕ^a can be a scalar, one-form or any other. We can get,

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] = -iF_{\mu\nu} \tag{5}$$

I gather some elements of gauge theory above, now we begin the physical part.

The action of Yang-Mills,

$$S_{YM} = \frac{1}{-2g}\int d^4x \, \mathrm{tr}(F^{\mu\nu}F_{\mu\nu}). \tag{6}$$

The classical e.o.m for this action is,

$$\mathcal{D}_\mu F^{\mu\nu} = 0 \tag{7}$$

The action has a symmetry, for

$$\Omega(x) \in G \tag{8}$$

The action is invariant under,

$$A_\mu \to \Omega(x)A_\mu\Omega^{-1}(x) + i\Omega(x)\partial_\mu\Omega^{-1}(x) \tag{9}$$

leading to,

$$F^{\mu\nu} \to \Omega(x)F^{\mu\nu}\Omega^{-1}(x) \tag{10}$$

Wilson lines and **Wilson loops**

1.2 Quantization of Gauge Theory

1.3 Renormalization in Gauge Theory