Anomalies and Spontaneous Symmetry Breakings

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1 ABJ anomaly

For a theory of massless Dirac fermions coupled to an electromagnetic gauge field in $d = 3 + 1$ dimensions, the action for the fermions is

$$S = \int d^4x \; i\bar{\psi}\not{D}\psi. \tag{1}$$

Here the gauge field is considered as a fixed backgroud field. The symmetries for this action are,

$$\psi \to e^{i\alpha}\psi \tag{2}$$

with the corresponding current,

$$j^\mu = \bar{\psi}\gamma^\mu\psi, \tag{3}$$

and

$$\psi \to e^{i\alpha\gamma^5}\psi \tag{4}$$

with the corresponding current,

$$j^\mu_A = \bar{\psi}\gamma^\mu\gamma^5\psi. \tag{5}$$

(**Aside.**) The action (2) can be written as

$$S = \int d^4x \; i\bar{\psi}\not{\partial}\psi + j^\mu A_\mu. \tag{6}$$

Thus if the action is invariant under the gauge transformation $A_\mu \to A_\mu + \partial_\mu\alpha$, we have $\partial_\mu j^\mu = 0$.

In classial theory, we should have $\partial_\mu j_A^\mu = 0$. It is true, while in quantum theory,

$$\partial_\mu j_A^\mu = \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}. \tag{7}$$

This is ABJ or chiral anomaly.

1.1 Deriving the chiral anomaly

2 Gauge anomalies

Anomalies in gauge symmetries kill all physics completely.

3 't Hooft anomalies

> A global symmetry with a 't Hooft anomaly remains a symmetry in the quantum theory. You only run into trouble if you couple the symmetry to a background gauge field, in which case the charge is no longer conserved. You run into real trouble if you try to couple the symmetry to a dynamical gauge field because then the 't Hooft anomaly becomes a gauge anomaly and the theory ceases to make sense. In other words, the 't Hooft anomaly is an obstruction to gauging a global symmetry.