Introductory Conformal Field Theory

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1   Conformal transformations

Conformal transformations are the coordinate transformations $x \to x'$ which lead to the metric $g_{\mu\nu}$ changes as,

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x'^\beta}{\partial x^\nu} g_{\alpha\beta}(x) = \Lambda(x) g_{\mu\nu}(x) \tag{1}$$

There are four kinds of transformations for dimenstion $d \geqslant 2$,

$$
\begin{aligned}
\text{(translation)} \quad & x'^\mu = x^\mu + a^\mu \\
\text{(dilation)} \quad & x'^\mu = \alpha x^\mu \\
\text{(rigid rotation)} \quad & x'^\mu = M^\mu{}_\nu x^\nu \\
\text{(special conformal transformation)} \quad & x'^\mu = \frac{x^\mu - b^\mu x^2}{1 - 2b \cdot x + b^2 x^2}
\end{aligned}
\tag{2}
$$