

# Geometry for Physics Students

Qing-Jie Yuan

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# Chapter 1

## Smooth manifolds

For  $f \in C^\infty(M, N)$ , we define the differential of  $f$  at point  $m \in M$  as a map  $f_{m*} : T_m M \rightarrow T_{f(m)} N$ , satisfying

$$f_{m*}(v)h = v(h \circ f) \tag{1.0.1}$$

for all  $v \in T_m M$  and all  $h \in C^\infty(N, \mathbb{R})$ . For

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## Chapter 2

# Riemannian geometry

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# Chapter 3

## Fiber bundles

### 3.1 Principal bundles

Let  $\{(U_\alpha, \phi_\alpha)\}$  be an atlas of the manifold  $M$  and  $g_{\beta\alpha}$  a cocycle  $g_{\beta\alpha} : U_\alpha \cap U_\beta \rightarrow G$  for a given Lie group  $G$ . The cocycle condition is

$$g_{\gamma\alpha}(m) = g_{\gamma\beta}(m)g_{\beta\alpha}(m)$$

for all  $m \in U_\alpha \cap U_\beta \cap U_\gamma$ . We can also define smooth functions:

$$\tilde{g}_{\beta\alpha} : \phi_\alpha(U_\alpha \cap U_\beta) \times G \rightarrow \phi_\beta(U_\alpha \cap U_\beta) \times G$$

by

$$\tilde{g}_{\beta\alpha}(\phi_\alpha(m), g) := (\phi_\beta(m), g_{\beta\alpha}g) \tag{3.1.1}$$

for all  $m \in U_\alpha \cap U_\beta$  and all  $g \in G$ .

The space

$$\bigsqcup_\alpha \phi_\alpha(U_\alpha) \times G$$

with equivalence relation

$$(\phi_\alpha(p), g) \sim (\phi_\beta(m), g_{\beta\alpha}(m)g)$$

can be made into a smooth manifold  $P$ , called the principal  $G$ -bundle. It is easy to see there is a map  $\pi : P \rightarrow M$ , such that  $\tilde{\phi}_\alpha : \pi^{-1}(U_\alpha) \rightarrow \phi_\alpha(U_\alpha) \times G$ .