Notes of Statistical Physics

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Contents

1 Ensembles

A system may exist in a certain state with a specific probability. When we macroscopically observe a system in thermal equilibrium, the value of a macroscopic quantity should be the weighted average of the values corresponding to these states. However, this implicitly assumes that the theory will traverse all these states. Intuitively, the system changes extremely rapidly, and macroscopic observations occur over a finite period of time, during which the system has already undergone a vast number of states. Therefore, it can be considered to have already achieved traversal. Thus, the actual observed value, interpreted as a time average, can be regarded as equivalent to the theoretical average, which is later referred to as the ensemble average. This issue can be discussed in greater depth, but that will not be addressed here.

So what is this theoretical probability? Here, another assumption is still needed: each state occurs with equal probability. Many different states can have the same observable value. So we prefer to consider the probability of a class of states, such as states with a certain energy or, for a continuous spectrum, states within a certain energy range. Of course, energy is just one example; other quantities can also be used.

Since the probability of each individual state is uniform, this becomes a counting problem. How exactly this is done depends on the constraints we impose on the system, whether it is adiabatic or isothermal, closed or open. Below, we will analyze this using ensemble theory.

1.1 Microcanonical ensemble

The microcanonical ensemble is the set of the states with the same energy E. The probability for one state, labled by n, is just,

$$p(n) = \frac{1}{\Omega(E)}, \tag{1}$$

where $\Omega(E)$ is the number of states with energy E. This result is directly from our assumption.

1.2 Canonical ensemble

The canonical ensemble is the set of the states with the same temperature T. Here I just write down the result. The probability for states with the energy E_n is,

$$p(E_n) = \frac{e^{\beta E_n}}{\sum_m e^{-\beta E_m}}, \tag{2}$$

where $\beta \equiv 1/k_B T$. This is Boltzmann distribution. The denominator is defined as the partition function,

$$Z = \sum_n e^{-\beta E_n}. \tag{3}$$

2 Quantum gases

A general method of handling problems in statistical physics is to find a variable for which the density of states is easy to compute, such as momentum. Then, using the relation between this variable and the energy, one can determine the density of energy states, and from there obtain the partition function. Once the partition function is known, many other quantities become straightforward to calculate.

3 Examples

3.1 Conductivity

For metals, the Drude model can give good predictions.