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A fuzzy optimization model for planning integrated terrestrial carbon management networks

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Abstract
Biochar application and enhanced weathering are negative emission technologies (NETs) with the potential for large-scale deployment for the removal of CO_2 from the atmosphere. Biochar is a solid product of pyrolysis that can permanently store carbon when applied in soil due to its chemical recalcitrance. Enhanced weathering is based on the acceleration of the natural reaction of moisture, CO_2, and alkaline minerals. Both of these NETs rely on the application of pulverized material to different types of terrestrial sinks, which can include marginal and agricultural land. These two NETs can be used separately or concurrently, depending on local sink conditions. In some cases, simultaneous application of biochar and mineral powder to soil has the advantage of attaining additional beneficial effects of soil amendment. Although recent papers have reported the development of process integration models for optimizing carbon management networks based on either biochar application or enhanced weathering, none have reported models integrating these two NETs in the same system. To address this gap, a fuzzy mixed-integer linear programming model is developed that integrates biochar application and enhanced weathering for large-scale carbon sequestration. Fuzzy set theory provides a well-tested framework for integration of both subjectivity and uncertainty into mathematical programming. The model determines the optimal allocation of biochar and/or alkaline minerals from each source to each sink, while considering the application limits and CO_2 sequestration potential. An illustrative case study is solved that clearly demonstrates the application of the model. The case study shows interesting results that can guide how the full sustainable potential of these two technologies can be utilized in a carbon management network.

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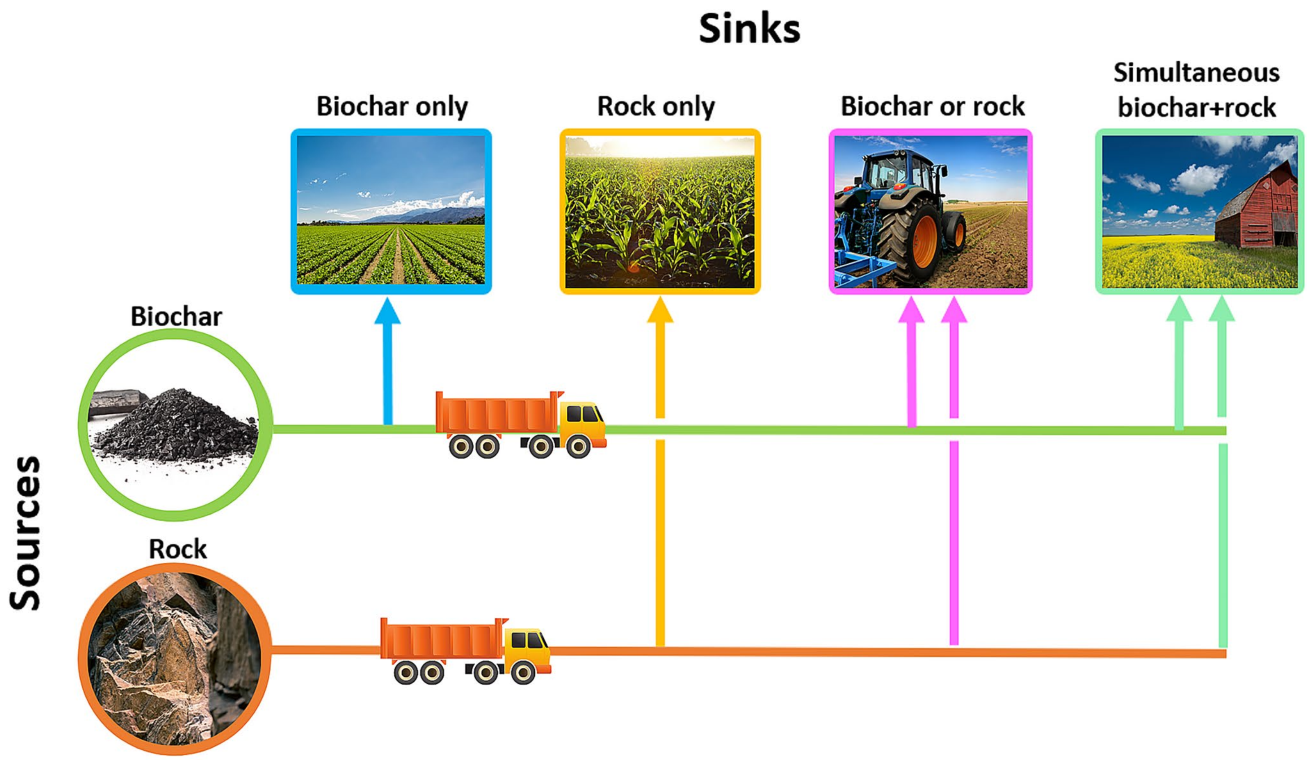
Graphic abstract



Keywords Fuzzy mixed-integer linear programming (FMILP) · Enhanced weathering (EW) · Biochar · Negative emission technologies (NET) · Optimization · Carbon drawdown

Abbreviations

Sets

I	Sources
J	Sinks
K	Contaminants
P	Time periods

Indexes

i	Source index ($i = 1,2,3 \ldots M$)
j	Sink index ($j = 1,2,3 \ldots N$)
k	Contamination index ($k = 1,2,3 \ldots Q$)
p	Time period index ($p = 1,2,3 \ldots T$)

Parameters

s_{ip}^{L}	Lower limit of production rate of powdered alkaline rocks or biochar at source i in period p (t/y)
s_{ip}^{U}	Upper limit of production rate of powdered alkaline rocks or biochar at source i in period p (t/y)
Q_{ikp}	Concentration of impurity k in biochar or crushed alkaline rocks produced from source i in period p (g/t)
Q_{jk}^{*}	Concentration of contaminant k in biochar or alkaline rocks applied in sink j (g/t)
Q_{jk}^{*U}	Upper limit of concentration of contaminant k in biochar or alkaline rocks applied in sink j (g/t)
Q_{jk}^{*L}	Lower limit of concentration of contaminant k in biochar or alkaline rocks applied in sink j (g/t)
D_{jp}	Annual application dosage of biochar, alkaline rock, and combined biochar and alkaline rock at sink j in period p (t/y)
L_j	Limiting storage capacity of sink j (t)
CS^U	Upper limit of cumulative carbon sequestration of the network (t)
CS^L	Lower limit of cumulative carbon sequestration of the network (t)
A_{ij}	Sequestration potential of biochar or alkaline rocks for each source–sink pair (t CO_2/t)
B_{ij}	CO_2 emissions factor for transporting, handling, and applying biochar and alkaline rocks from source i to sink j (t CO_2/t)

Variables

λ Overall degree of satisfaction

x_{ijp} Allocation of powdered alkaline rocks or biochar for sink j from source i in period p (t/y)

s_{ip} Production rate of powdered alkaline rocks or biochar at source i in period p (t/y)

b_{ip} Binary variable used to signify the presence or absence of source i in period p

CS Cumulative carbon sequestration of the network

b_{ijp} Binary variable for biochar allocated from source i to sink j in period p

Introduction

Climate change due to greenhouse gas (GHG) emissions is undoubtedly one of the most critical problems the world is facing. The atmospheric concentration of CO_2 exceeded 410 ppm in 2019 and continues to increase (Plenio 2020). In order to mitigate climate change and meet the targets of the 2015 Paris Agreement, there is a need for a significant reduction in atmospheric CO_2 concentration. Haszeldine et al. (2018) suggest the deployment of carbon capture and storage (CCS) and negative emission technologies (NETs) to achieve net-zero CO_2 emissions. Global GHG emissions will need to be cut to zero by 2050 to keep climate change at a manageable level. To reach this goal, NETs will be necessary in order to offset the GHG emissions of sectors that are inherently difficult to decarbonize (IPCC 2018). Different NETs have been studied for their potential to offset emissions (McLaren 2012). Two recent review papers give surveys of the NET research landscape (Minx et al. 2018) as well as their risks and potentials (Fuss et al. 2018). The resource-constrained global potential of different NETs has also been assessed by Smith et al. (2016). Among these NETs, biochar and enhanced weathering are two of the promising options that can potentially remove 130 Gt CO_2 (Woolf et al. 2010) and 300 Gt CO_2 (Strefler et al. 2018) by the end of the twenty-first century. Both of these NETs have the advantage of relying on mature component technologies, although neither has been proven at the system-level for large-scale carbon sequestration. However, biochar application and enhanced weathering also require the use of land (Smith et al. 2019). Process integration (PI) models can be used to rationalize the allocation of land resources to different NET options (Tan et al. 2020) via the concept of carbon management networks (CMNs) (Tan and Foo 2018).

Biochar is a carbon-rich solid product of pyrolysis. It can store carbon for hundreds of years due to its chemical recalcitrance and can be used in agriculture for soil amendment. It has gained considerable attention in the past decade as shown by ample research studies that have been published. One of the recent advances in biochar research is the production and modification of biochar properties to make it suitable for the intended application. Doped biochar contains improved nutrient (phosphorus and potassium) content for soil enhancement (Buss et al. 2020). Tomczyk et al. (2020) reported improvement in soil quality due to the addition of biochar produced at various temperatures. Bong et al. (2020) found that biochar produced from lignocellulosic biomass has the advantage of greater carbon sequestration, while those derived from manure and food waste are more efficient in improving soil nutrients. Techno-economic and environmental assessment of biochar is another main aspect of present research on biochar. An increase in crop yield and profitability is projected from the use of biochar in agricultural lands, while the total global emission is expected to decrease due to avoided emissions from land use change (Dumortier et al. 2020). Struhs et al. (2020) proposed a stochastic optimization model for determining the total cost and carbon emissions for an onsite biochar-based refinery. The role of process systems engineering (PSE) in the large-scale deployment of biochar systems is also emerging (Belmonte et al. 2017). Tan (2016) developed the first mixed-integer linear programming model (MILP) for the large-scale deployment of biochar-based carbon management network (BCMN). The model determines the optimal allocation of biochar to various application sites that maximizes the system-wide carbon sequestration. Belmonte et al. (2018) developed an improved version of the mathematical model formulation by performing a bi-objective optimization of BCMN that maximizes profit and carbon sequestration. The complex interactions between biochar and the soil are accounted in the model to properly assess the full potential of the network for climate change mitigation. Belmonte et al. (2019) published the first paper that considers the design of BCMN that generates different grades of biochar to fit local soil conditions. Ong et al. (2020) proposed a MILP model as a decision support tool in designing BCMN integrated in agro-industrial supply chains. The optimal network was determined based on operational cost, CO_2 emission reduction, water and nutrient retention, land constraints, and transport routes. Aviso et al. (2020) developed a fuzzy MILP model based on direct and indirect biomass co-firing with biochar production as a byproduct in existing coal power plants. The optimal carbon management strategy reduces GHG emissions via coal displacement coupled with carbon sequestration in biochar. A decision framework was developed by Li et al. (2019) for the optimal design of negative emission hybrid renewable energy systems containing biochar production for soil amendment. The system incorporates carbon sequestration, waste treatment, and renewable energy production. This multi-objective stochastic optimization model considered maximizing energy production while minimizing GHG emissions by the optimal sizing of the process units for a stand-alone rural island case study. A decision support tool for applying biochar to soil was evaluated by Phillips et al. (2020). The method includes inputting soil characteristics, determining the crop's nutrient requirements, identifying the appropriate biochar, and solving the amount of biochar needed to achieve desired nutrient and pH levels. The review paper by Matustík et al. (2020) focused on the life cycle assessment (LCA) of applying biochar to soil as a carbon management strategy. Aside from these benefits, the co-products of biochar production can be utilized for energy production. These studies prove the great potential of biochar as a carbon management and soil amendment strategy. However, more research is necessary on mass and energy balance aspects of large-scale biochar application (Yang et al. 2020).

Silicate minerals have long been utilized to increase soil pH and amend unproductive acidic lands (Dietzen et al. 2018). Seifritz (1990) was the first to propose the idea of utilizing pulverized silicate minerals to remove CO_2 from the atmosphere. These fine particles when applied to soil react with CO_2 in the presence of rainwater. The resulting bicarbonates may be leached out from the soil and carried to the ocean by runoff water, which in turn offsets ocean acidification (Dietzen et al. 2018). The use of enhanced weathering on agricultural land can yield multiple Gt/y of CO_2 removal (Beerling et al. 2020) along with agricultural co-benefits (Edwards et al. 2017). The use of silicate minerals as a substitute for agricultural lime to increase soil pH can also eliminate GHG emissions from lime production (Dietzen et al. 2018). The potential of enhanced weathering to capture CO_2 is dependent on several factors such as the application site being considered, average rainfall, type of rock used, and rate of application (Lefebvre et al. 2019). Environmental conditions may limit the CO_2 sequestration potential since these factors dictate the weathering rate. It is also important to choose the silicate-bearing rocks that can weather easily. The maximum sequestration potential can be achieved from alkaline rocks with 1.1 t CO_2/t rock (Strefler et al. 2018). Proper determination of the soil saturation limit is also necessary since additional application could decrease the dissolution rate (Dietzen et al. 2018) and lead to deposition of mineral residues (Pullin et al. 2019). The potential of enhanced weathering is generally limited by available land area rather than mineral availability (Renforth 2012). In addition to alkaline minerals (e.g., olivine) and rocks (e.g., basalt), different types of alkaline industrial waste (e.g., slag) are also potentially suitable for enhanced weathering (Renforth 2019). The need for a systematic framework to properly match potential mineral sources and application sites was first addressed by Tan and Aviso (2019) using a linear program (LP) for the optimal matching of sources and sinks; the model is similarly structured to those previously developed for biochar. The formulation was then extended to incorporate fuzzy goals and constraints for the consideration of uncertainties in the application rates and sequestration potential characteristics (Aviso and Tan 2020).

Carbon management literature deals with biochar application and enhanced weathering as separate technologies. The simultaneous application of biochar in combination with alkaline minerals may be highly beneficial. Synergy relationship is possible, since rock powder is a source of mineral nutrients while biochar catalyzes microbial activity and nutrient exchange (Hunt 2015). Biochar prevents the leaching of these minerals while allowing beneficial microbes to thrive (Cascade Minerals 2016). Soil benefits of applying both biochar and alkaline minerals can have positive implications for the sustainable development goals (Smith et al. 2019). Simultaneous application of biochar with powdered rock supports soil biota and speeds up transfer of nutrients to plant roots (Cutler 2020). The water retention properties of biochar increase the contact time of the rainwater with the rock powder to facilitate enhanced weathering. However, biochar and powdered rock can both contain trace contaminants that may cause adverse effects on soil quality. This risk imposes an upper limit on application, and thus carbon sequestration, rates. The state-of-the-art literature clearly indicates the possible synergistic use of these two NETs. However, the publications focus on effects at the application sites and do not consider a system-level perspective. Simultaneous use of these technologies can relieve some of the land footprint constraints on terrestrial carbon drawdown. Despite the potential for the simultaneous use of biochar application and enhanced weathering for achieving negative emissions, the absence of any published work on optimization models to aid in planning such CMNs indicates a clear research gap.

To address the research gap, this study develops a novel optimization model for integrated biochar and enhanced weathering systems. Simultaneous use of these two approaches can help ease land footprint limitations, while also optimizing soil co-benefits to yield incremental gains in carbon sequestration. Previously published models do not address these aspects. The model is formulated as a fuzzy mixed-integer linear program (FMILP) and considers land sinks where the two NET options can or cannot be used simultaneously. For any given site, the biochar or mineral application limit is much lower than the theoretical physical limit due to the complex interplay of technological, environmental, and socioeconomic factors (Renforth 2012). The limit is hard to define precisely due to the inherently subjective nature of risk tolerance for adverse effects such as dust emissions or excessive changes in soil chemistry (Tan 2016). Fuzzy set theory provides a well-tested framework for integration of both subjectivity and uncertainty into mathematical programming (Luhandjula 2014). This work is also the first to explore the potential benefits of utilizing both biochar and powdered alkaline rocks in a CMN while addressing the challenges associated with its implementation with the aid of mathematical programming. The paper is organized as follows. In Sect. 2, the formal problem statement is given. The formulation of the fuzzy optimization model is described in Sect. 3. Section 4 presents the illustrative case study that demonstrates the applicability of the fuzzy MILP model. The discussion of results is then presented in Sect. 5. Finally, conclusions and future perspectives are provided in Sect. 6.

Statement of the problem

The problem statement can be formally expressed as follows:

- Given m sources ($i \in \mathrm{I}$) of biochar and alkaline minerals (biomass processing plants or crushing/milling facilities for rocks, $i = 1, 2, 3 \ldots m$) whose annual supply limits and contaminant levels $k \in K$ ($k = 1, 2, 3 \ldots q$) are known throughout the planning period;
- Given n sinks ($j \in J$) such as croplands or agricultural lands where biochar or powdered rocks are to be applied ($j = 1, 2, 3 \ldots n$) and whose total storage capacity limits, annual biochar or rock acceptance limits, and fuzzy contaminant (k) limits are known;
- For each potential source–sink match, the amount of carbon emitted due to handling, transportation, and application of biochar or crushed alkaline rocks is known;
- The planning period consists of time intervals $p \epsilon P$ ($p = 1, 2, 3 \ldots t$);
- Given that the biochar and powdered alkaline rocks are characterized by fuzzy carbon sequestration potential.

The goal is to find the optimum allocation of biochar or alkaline powdered rocks from each source i to each sink j for each period p that attains the highest cumulative CO_2 sequestration while simultaneously considering the uncertainties in contaminant limits and CO_2 sequestration potential. The schematic representation of the problem is shown in Fig. 1.

Formulation of the optimization model

The formulation developed here is based on the symmetric fuzzy model of Zimmermann (1978), where (a) objectives are replaced with goals with predefined thresholds, (b) constraints are given margins of allowable partial violation, and (c) the fuzzy constraints and goals are mathematically indistinguishable from each other. This versatile formulation lends itself to many practical applications in green engineering (Arriola et al. 2020) and is applied here to the case of planning terrestrial CMNs. The advantage of utilizing fuzzy optimization in this work is twofold. First, it enables the representation of uncertainties in the data parameters. Second, unlike other approaches for multi-objective optimization, fuzzy optimization is non-compensatory and works using the principle of max–min aggregation. This results in the simultaneous consideration of all objectives which maximizes the satisfaction of the least satisfied objective.

Equation (1) depicts the objective function which is to maximize λ (overall degree of satisfaction as shown in Eq. (5)). The fuzzy membership function of the constraints and goals will vary depending on whether a higher value (or lower value) is more desirable. Figure 2a illustrates a fuzzy membership function for goals which must be maximized;

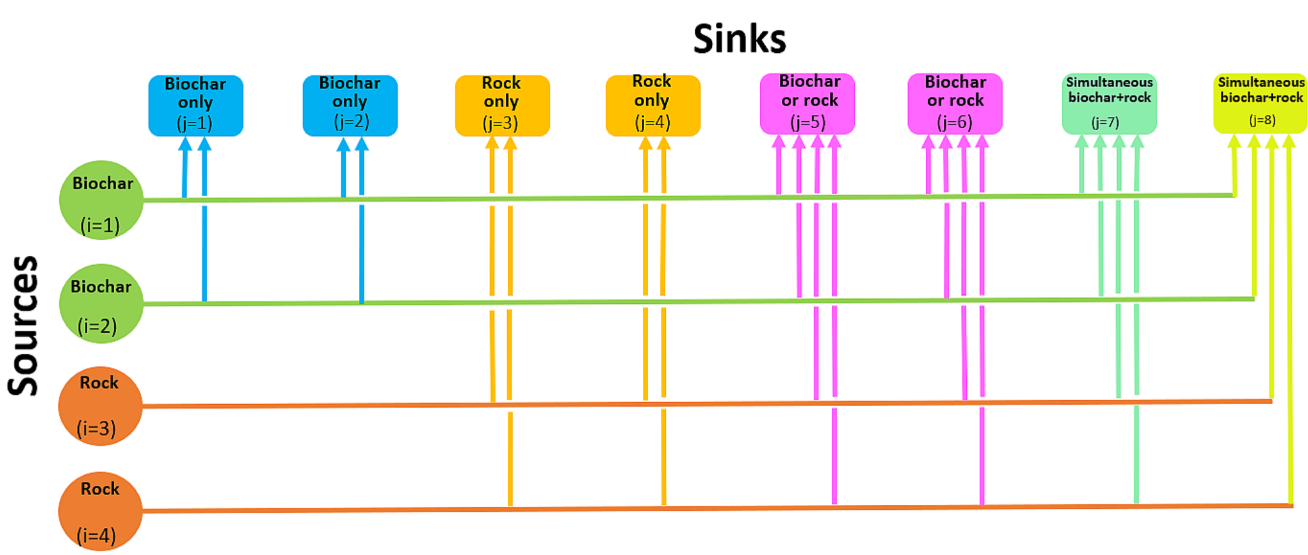


Fig. 1 Superstructure of carbon management network utilizing biochar and powdered alkaline rocks

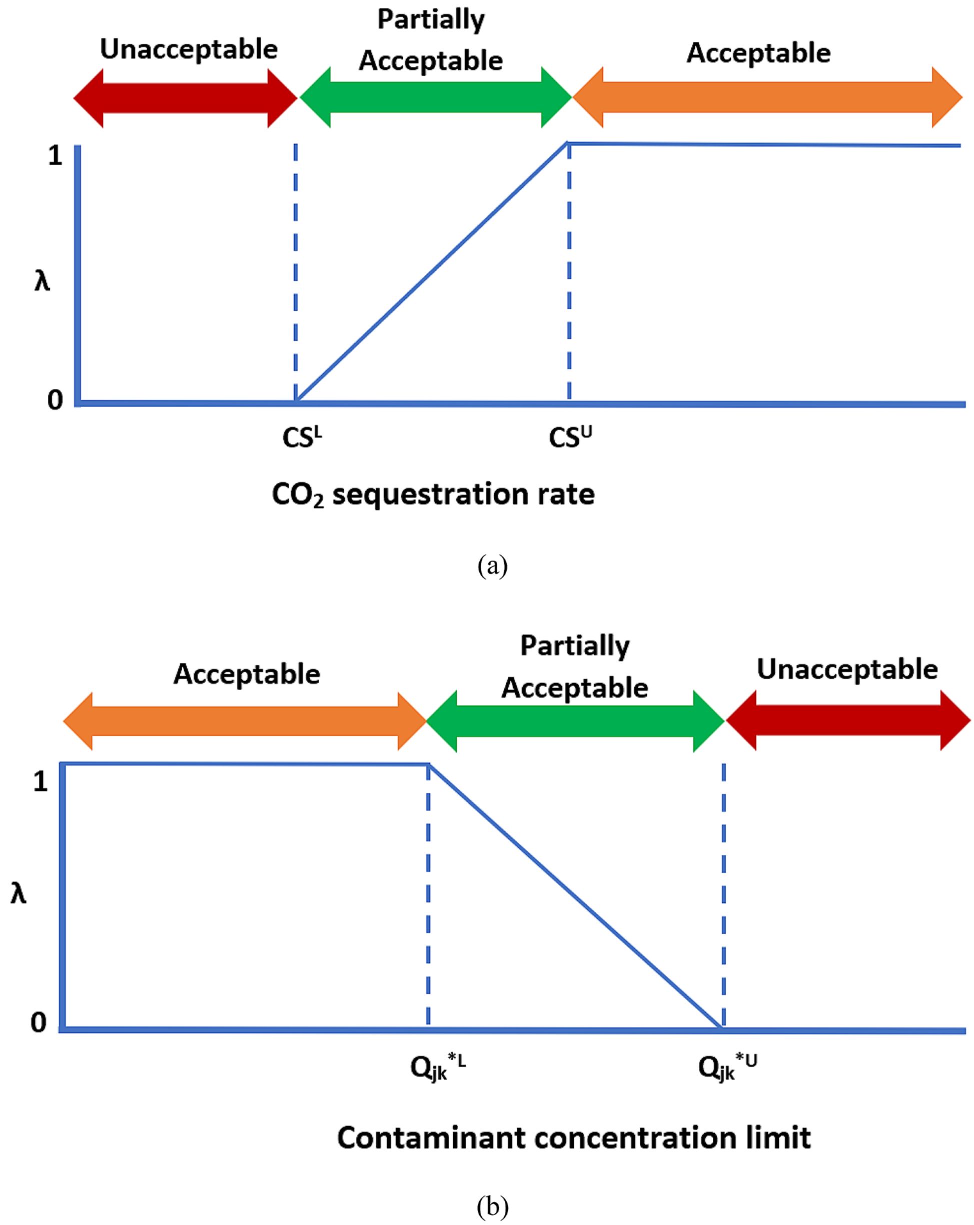


Fig. 2 Fuzzy membership function for (**a**) carbon sequestration rate (**b**) contaminant concentration limit

an example is the desire to maximize the sequestration rate. The degree of satisfaction, λ, increases linearly from 0 to 1 as the parameter value increases from the lower fuzzy limit to the upper fuzzy limit of the parameter. Figure 2b on the other hand illustrates a fuzzy membership function for goals which must be minimized; an example is the contaminant application limit. For this scenario, the degree of satisfaction, λ, decreases linearly from 1 to 0 as the parameter value increases from the lower fuzzy limit to the upper fuzzy limit.

$$max\lambda \tag{1}$$

Equation (2) gives the source balance where x_{ijp}(t/y) denotes the quantity of powdered alkaline rocks or biochar supplied by source i to site j during the period p. The total amount of powdered alkaline rocks or biochar produced from source i during p is represented by s_{ip}(t/y) which is subject to lower (s_{ip}^{L}) and upper (s_{ip}^{U}) bounds shown in Eq. (3). These bounds address specific operational limitations such as minimum economical rate of production or maximum output supply capacity per year which may be based on the availability of biomass and silicate-bearing rocks.

$$\Sigma_j x_{ijp} = s_{ip} \quad \forall_{i,p} \tag{2}$$

$$b_{ip}s_{ip}^{L} \leq s_{ip} \leq b_{ip}s_{ip}^{U} \quad \forall_{i,p} \tag{3}$$

The binary variable b_{ip} in Eq. (4) is used to signify the presence ($b_{ip} = 1$) or absence ($b_{ip} = 0$) of source i at time p.

$$b_{ip} \in \{0, 1\} \quad \forall_{i,p} \tag{4}$$

$$\lambda = \frac{\Sigma_i \Sigma_p s_{ip}}{\Sigma_i \Sigma_p s_{ip}^U} \tag{5}$$

Equation (6) accounts for the limiting amount (t/y) of biochar, alkaline rock, and combined biochar and alkaline rock $\left(D_{jp}\right)$ that sink j can accept during the period p. It is necessary to properly assess the type of sinks in which these pulverized materials are to be applied in order to determine the suitable value of D_{jp}.

$$\Sigma_i x_{ijp} \leq D_{jp} \quad \forall_{j,p} \tag{6}$$

Equation (7) indicates that every sink j is subject to a maximum limit to cumulative application L_j (t) throughout the planning horizon. To determine L_j, there are factors that need to be considered such as topsoil depth, land area, and synergistic biochar-alkaline powdered rocks-soil blending rate that will not cause adverse effects at the application site.

$$\Sigma_i \Sigma_p x_{ijp} \leq L_j \quad \forall_j \tag{7}$$

The parameter Q_{ikp} in Eq. (8) is the amount (g/t) of impurity in biochar or crushed alkaline rocks from source i at time p. The parameter Q_{jk}^* is the maximum level (g/t) of contaminant k present in biochar or alkaline rocks that can be applied to the soil in sink j which is subject to fuzzy contaminant limits as can be seen in Eq. (8). Biochar and powdered rocks can both contain trace elements or contaminants that may cause adverse effects in soil when applied in amounts that the soil cannot tolerate. The concurrent application of biochar and mineral powder may thus impact the plants and other organisms living in soil. The tolerable level of contaminants for each soil type may differ (Belmonte 2021). In practice, the determination of tolerable safe levels of such trace elements in the soil is challenging (Tan 2016) due to several environmental factors; limited studies on single species and the laboratory conditions differ from the actual field conditions (Ashraf et al. 2014). These risks suggest that these unintended adverse effects should be minimized by minimizing the contaminant concentration that the soil can receive. Equation (8) implies that it is desired to minimize the contaminant limits.

$$\Sigma_i x_{ijp} Q_{ikp} \leq D_{jp} \left(Q_{jk}^{*U} - \lambda \left(Q_{jk}^{*U} - Q_{jk}^{*L} \right) \right) \quad \forall_{j,k,p} \tag{8}$$

The cumulative carbon sequestration (CS) of the network is accounted by Eq. (9) where A_{ij} gives the sequestration potential (t CO_2/t) of biochar or alkaline rocks for each source–sink pair. The parameter B_{ij} gives the amount of CO_2 emissions (t CO_2/t) associated with transporting, handling, and applying biochar and alkaline rocks. Since it is intended to achieve the highest possible value of CS, Eq. (10) shows that as λ approaches 1, CS reaches the upper limit CS^U.

$$CS = \Sigma_i \Sigma_j \Sigma_p A_{ij} x_{ijp} - \Sigma_i \Sigma_j \Sigma_p B_{ij} x_{ijp} \tag{9}$$

$$CS \leq CS^L + \lambda (CS^U - CS^L) \tag{10}$$

Limits are then set on all streams for all possible source–sink matches as indicated in Eq. (11). If a link between a source and a sink is activated, b_{ijp} takes the value of 1 as indicated in Eq. (12). Binary variables permit certain topological constraints to be incorporated in the mathematical model when there is a need to limit the number of sinks that can be linked to a particular source (i), such as when the connection between a source (i) and a sink (j) is forbidden, or when the connection between a source (i) and a sink (j) must exist. The succeeding illustrative case study demonstrates the application of this optimization model.

$$0 \leq x_{ijp} \leq b_{ijp} s_{ip}^U \quad \forall_{i,j,p} \tag{11}$$

$$b_{ijp} \in \{0, 1\} \quad \forall_{i,j,p} \tag{12}$$

Illustrative case study

A case study is given in this section to illustrate a carbon management network (CMN) that utilizes biochar and alkaline rocks for carbon sequestration. The CMN depicts eight application sites (sinks), four sources (two biochar sources and two alkaline rock sources), and a planning period of ten years. Three impurities that may be present in biochar or alkaline rocks are considered in this study which are sodium (Na), magnesium (Mg), and calcium (Ca). Salts of these elements may hinder crop productivity when present in excessive amounts. It is assumed that sources 1, 2, and 3 are operational within the ten-year period while source 4 starts operating in the second year. The limiting data for all the sources are given in Table 1. Sources 1 and 2 are supplying different grades of biochar, while sources 3 and 4 are producing powdered alkaline rocks. The minimum and maximum supplies are also specified in the table. The impurity contents for biochar sources are based from the previous study of Belmonte et al. (2018), while the paper of Styles et al. (2014) is used to estimate the impurity contents in sources 3 and 4. Table 2 provides the CO_2 sequestration factors for all possible matches between a source (i) and a sink (j) and are based from Belmonte et al. (2018) for biochar and Strefler et al. (2018) for crushed alkaline rocks. Table 3 supplies the data for the sinks. As can be seen in the table, the fuzzy contaminant limits are provided in the last three columns. These limits represent the prescribed amounts that can be applied without causing adverse effects to the soil.

Table 1 Data for the sources (impurity limits are adapted from Belmonte et al. (2018) and Styles et al. (2014))

Sources	Minimum rate of production, s_{ip}^{L} (t/y)	Maximum rate of production, s_{ip}^{U} (t/y)	Sodium (Na) content, Q_{i1p} (g/t)	Magnesium (Mg) content, Q_{i2p} (g/t)	Calcium (Ca) content, Q_{i3p} (g/t)
1 (biochar)	1000	3500	80	150	4600
2 (biochar)	1200	3500	2900	10,400	5000
3 (alkaline rocks)	1200	3500	0	25,000	10,000
4 (alkaline rocks)	2500	10,000	0	20,000	7500

Table 2 CO_2 sequestration potential used in the study (data are adapted from Belmonte et al. (2018) and Strefler et al. (2018))

Sources	CO_2 sequestration factor, A_{ij} (t CO_2/t)							
	Sinks							
	1	2	3	4	5	6	7	8
1 (biochar)	3.86	4.29	4.72	5.15	3.86	4.29	4.72	5.15
2 (biochar)	2.25	2.50	2.75	3.00	2.25	2.50	2.75	3.00
3 (alkaline rocks)	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
4 (alkaline rocks)	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30

Table 3 Data for the sinks

Sinks	Area (ha)	Prescribed application rate, (t/ha)	Storage capacity, L_j (t)	Limit for flowrate, D_{jp} (t/y)	Fuzzy limits for Na content, $Q_{j1}^{*U} - Q_{j1}^{*L}$ (g/t)	Fuzzy limits for Mg content, $Q_{j2}^{*U} - Q_{j2}^{*L}$ (g/t)	Fuzzy limits for Ca content, $Q_{j3}^{*U} - Q_{j3}^{*L}$ (g/t)
1 (suitable for biochar only)	1000	35	35,000	3500	750–500	2600–1600	1250–1000
2 (suitable for biochar only)	3500	10	35,000	3500	1000–400	3000–2000	1500–500
3 (suitable for rocks only)	1000	50	50,000	5000	7250–5250	26,000–16,000	12,500–10,000
4 (suitable for rocks only)	2000	25	50,000	5000	5500–4500	30,000 – 20,000	7000–5000
5 (suitable for either biochar or rocks alone)	1750	20	35,000	3500	1500–1200	5200–4400	500–2000
6 (suitable for either biochar or rocks alone)	875	40	35,000	3500	2000–1500	4000–3200	1500–1000
7 (suitable for simultaneous biochar + rocks)	625	80	50,000	5000	2900–2000	10,400–8200	5000–4500
8(suitable for simultaneous biochar + rocks)	1000	50	50,000	5000	3000–2500	7500–5000	4000–3000

These values are assumed since the extent to which the soil can tolerate salinity depends on physiological factors that cannot be easily determined. In this hypothetical case study, the prescribed application rates are considered conservative estimates since they are based from the published literature on biochar (5–50 t/ha) and generally much lower than the assumption made by Strefler et al. (2018) for alkaline rocks. The case study takes into account four kinds of sink according to their suitability to biochar and rock application: suitable for biochar only, suitable for alkaline rocks only, suitable for either biochar or rocks alone, and suitable for simultaneous application of biochar and powdered rocks. It is important to note that the storage capacity of a particular sink depends on the kind of soil as can be seen in Table 3. The ratio of application may also significantly affect the soil chemistry when biochar is applied in combination with crushed alkaline rocks. For demonstration purposes, it is assumed that the appropriate ratio of biochar to powdered rock that will maximize the beneficial effects is 1:3 (500 t/y biochar and 1500 t/y rock) at sink 7 and 1:1 (1000 t/y biochar and 1000 t/y rock) at sink 8. The CO_2 emission factors for source–sink pairs for transporting, handling, and applying biochar or alkaline rocks are indicated in Table 4. These were calculated based from the CO_2 footprint of 0.1 kg CO_2/t/km (Tan 2016) wherein the emissions from handling and application are assumed to be subsumed within this value. The upper limit for the system-wide CO_2 sequestration potential of the CMN is obtained by maximizing *CS* using the highest possible contaminant concentration limit set for the sinks. The lower limit is equivalent to zero which represents the baseline scenario where the CMN does not exist or operate.

Topological constraints shown in the following equations are imposed in the optimization model. Equations (13–14) signify that a connection between alkaline rock sources and sink 1 is not allowed since it is only suitable for biochar application.

$$b_{31p} = 0 \quad \forall_p \tag{13}$$

$$b_{41p} = 0 \quad \forall_p \tag{14}$$

Likewise, a connection between alkaline rock sources and sink 2 is not allowed as shown in Eq. (15–16).

$$b_{32p} = 0 \quad \forall_p \tag{15}$$

$$b_{42p} = 0 \quad \forall_p \tag{16}$$

Equations (17–18) indicate that a connection between biochar sources and sink 3 is not permitted since it is fit for powdered rock application only.

$$b_{13p} = 0 \quad \forall_p \tag{17}$$

$$b_{23p} = 0 \quad \forall_p \tag{18}$$

Similarly, a connection between biochar sources and sink 4 is not permitted as indicated by Eq. (19–20).

$$b_{14p} = 0 \quad \forall_p \tag{19}$$

$$b_{24p} = 0 \quad \forall_p \tag{20}$$

Equations (21–24) show that biochar and alkaline rocks are not allowed to be mixed in sink 5. The same condition applies for sink 6 as depicted in Eq. (25–28).

$$b_{15p} + b_{35p} \le 1 \quad \forall_p \tag{21}$$

$$b_{15p} + b_{45p} \le 1 \quad \forall_p \tag{22}$$

$$b_{25p} + b_{35p} \le 1 \quad \forall_p \tag{23}$$

$$b_{25p} + b_{45p} \le 1 \quad \forall_p \tag{24}$$

$$b_{16p} + b_{36p} \le 1 \quad \forall_p \tag{25}$$

$$b_{16p} + b_{46p} \le 1 \quad \forall_p \tag{26}$$

$$b_{26p} + b_{36p} \le 1 \quad \forall_p \tag{27}$$

$$b_{26p} + b_{46p} \le 1 \quad \forall_p \tag{28}$$

Equations (29–33) are also added in the mathematical model since it is assumed that the appropriate ratio of biochar to powdered rock is 1:3 (500 t/y biochar and 1500 t/y rock) at sink 7 and 1:1 (1000 t/y biochar and 1000 t/y rock) at sink 8.

$$x_{17p} + x_{27p} = 500 \quad \forall_p \tag{29}$$

$$x_{37p} + x_{47p} = 1500 \quad \forall_p \tag{30}$$

$$x_{18p} + x_{28p} = 1000 \quad \forall_p \tag{31}$$

$$x_{38p} + x_{48p} = 1000 \quad \forall_p \tag{32}$$

Table 4 CO_2 emissions (t CO_2/t) due to transporting, handling, and applying biochar and alkaline rocks

Sources	CO_2 emission factor, B_{ij} (t CO_2/t)							
	Sinks							
	1	2	3	4	5	6	7	8
1 (biochar)	0.0257	0.0303	0.0238	0.0232	0.0257	0.0303	0.0238	0.0232
2 (biochar)	0.00935	0.0173	0.0099	0.0265	0.00935	0.0173	0.0099	0.0265
3 (alkaline rocks)	0.00652	0.0123	0.0122	0.021	0.00652	0.0123	0.0122	0.021
4 (alkaline rocks)	0.00652	0.0123	0.0122	0.021	0.00652	0.0123	0.0122	0.021

Results and discussion

Results generated an overall degree of satisfaction equivalent to 77.7% ($\lambda = 0.777$) which corresponds to a cumulative carbon sequestration (*CS*) of 199,949 t throughout the 10-year period. This optimal quantity is much closer to the fuzzy upper limit ($CS^U = 257{,}334t$). On the other hand, the optimal concentration of contaminants used in the sinks (see Table 5) is much closer to the fuzzy lower limits. The degree of satisfaction is the solution's degree of membership in the fuzzy set defined by the intersection of all the fuzzy objectives and fuzzy constraints. The optimal solution provides a satisficing solution in consideration of all identified fuzzy goals (Zimmermann 1978). The optimal allocation network is given in Table 6. Each cell in the table contains two values; the first value provides the allocation during the first year of operation, and the second value provides the allocation from the second to the tenth years. The results show that the maximum available amount of biochar from sources 1 and 2 is 66.91% and 79.63% utilized, respectively, during the first year. On the other hand, the maximum available amount of powdered alkaline rocks from source 3 is fully consumed in year 1. From the second to the tenth year, the maximum available amount of biochar from sources 1 and 2 is 76.14% and 72.26% utilized. When source 4 becomes operational from the second year onward, the percentage of powdered rock utilized from source 3 is reduced to 42.86%; and the powdered alkaline rock from source 4 is 91.88% utilized. Figure 3 shows that the allocation for the sinks in year 1 is 22.94%, 14.46%, 0% 20%, 44.06%, 22.22%, 40%, and 40% of the annual application rate limit at sinks 1,2,3,4,5,6,7, and 8, respectively. On the other hand, the operation of source 4 in year 2 onward results in a modified allocation particularly at sinks 3, 4, and 5, wherein the allocation has changed to 91.14%, 72.62%, and 45.91% of the annual application rate limits, respectively. In year 1, the model suggests blending of biochar (from sources 1 and 2) at sink 5, blending of biochar (from source 2) and alkaline rock (from source 3) at sink 7, and blending of biochar (from sources 1 and 2) and alkaline rock (from source 3) at sink 8. In year 2 onward, the model recommends blending of biochar (from source 2) and alkaline rock (from source 3) at sink 7, and blending of biochar (from sources 1 and 2) and alkaline rock (from source 4) at sink 8. Considering sink 8 in year 1 as an example, blending 743 t biochar from source 1 (that contains 80 g/t Na, 150 g/t Mg and 4600 g/t Ca), 257 t biochar from source 2 (that contains 2900 g/t Na, 10,400 g/t Mg and 5000 g/t Ca), and 1000 t alkaline rock powder from source 3 (that contains 0 g/t Na, 25,000 g/t Mg and 10,000 g/t Ca) results to 2000 t of combined biochar and alkaline rock powder that contains 402 g/t Na, 13,892 g/t Mg, and 7351 g/t Ca. The resulting mixture now contains lower levels of the contaminants than the specified limits set on sink 8. The net carbon sequestration in year 1 is 17,924 t/y. This value is obtained by subtracting the amount of CO_2 emissions due to transporting, handling, and applying biochar and alkaline rocks from the CO_2 sequestered by biochar and alkaline rocks (18,084 t/y–160 t/y). During the last nine years of operation, the net carbon sequestration rate is 20,225 t/y (20,511 t/y–286 t/y). Therefore, the total net carbon sequestration throughout the ten-year period is 199,949 t (17,924 t/y * 1y + 20,225 t/y * 9y).

Solving the same case study on the assumption that biochar and rocks cannot be applied simultaneously in any sink gives the network shown in Table 7. The overall

Table 5 Optimal concentration of contaminants (g/t) throughout the ten-year period

Sink	Optimal concentration of contaminants (g/t)		
	Na	Mg	Ca
Sink 1	556	1823	1058
Sink 2	534	2223	723
Sink 3	5696	18,230	10,558
Sink 4	4723	22,230	5446
Sink 5	1267	4578	2112
Sink 6	1612	3378	1112
Sink 7	2201	8691	4612
Sink 8	2612	5558	3223

Table 6 Optimal CMN (flowrates in t/y)

Sink	Source				Total
	1 (biochar)	2 (biochar)	3 (alkaline rocks)	4 (alkaline rocks)	
1 (biochar only)	803				803
	803				803
2 (biochar only)		506			506
		506			506
3 (rock only)				0	0
				4557	4557
4 (rock only)			1000	0	1000
			0	3631	3631
5 (biochar or rock)	796	746			1542
	1607	0			1607
6 (biochar or rock)		778			778
		778			778
7 (simultaneous)		500	1500		2000
		500	1500		2000
8 (simultaneous)	743	257	1000	0	2000
	255	745	0	1000	2000
Total	2342	2787	3500	0	
	2665	2529	1500	9188	

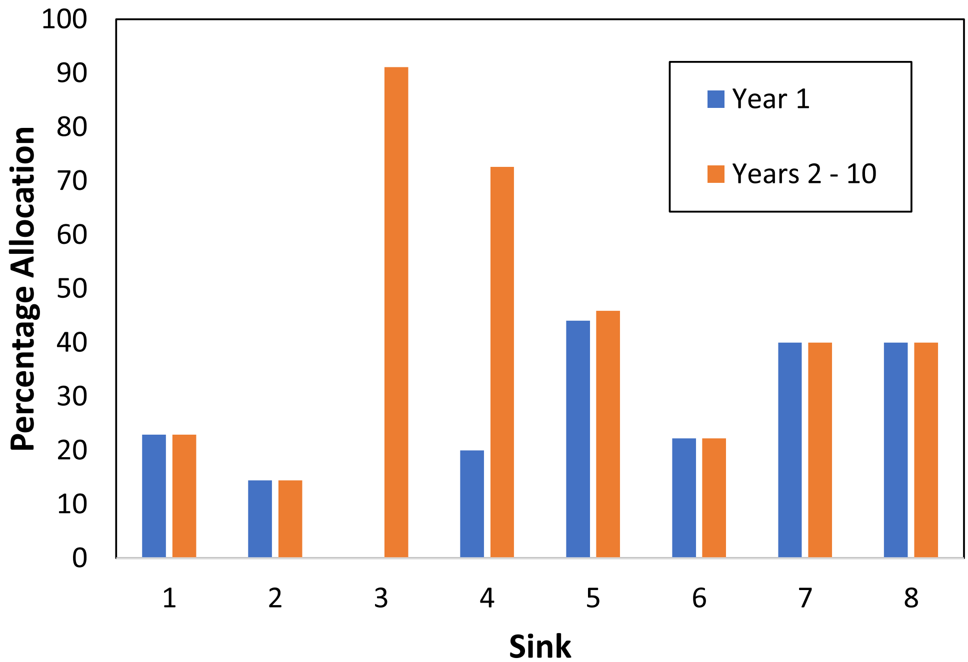


Fig. 3 Percentage allocation for the sinks throughout the ten-year period

degree of satisfaction achieved is 74.3% ($\lambda = 0.743$) which corresponds to a cumulative carbon sequestration (CS) of 158,376 t throughout the planning horizon. This quantity is 20.8% lower than what is achieved in the first solution, for which biochar and alkaline rocks can be applied simultaneously to a particular sink within the network. The result of the case study presented here is interesting because it shows how the full sustainable potential of these two technologies can be utilized in a carbon management network.

Table 7 Optimal CMN for the case when biochar and rocks cannot be applied simultaneously (flowrates in t/y)

Sink	Source				Total
	1 (biochar)	2 (biochar)	3 (alkaline rocks)	4 (alkaline rocks)	
1 (biochar only)	810				810
	810				810
2 (biochar only)		530			530
		530			530
3 (rock only)			0	0	0
			1536	2722	4258
4 (rock only)			2757	0	2757
			0	3676	3676
5 (biochar only)	1619				1619
	1619				1619
6 (biochar only)		790			790
		790			790
7 (rock only)			743	0	743
			0	2191	2191
8 (rock only)				0	0
				1411	1411
Total	2429	1320	3500	0	
	2429	1320	1536	10,000	

Conclusion

A novel FMILP model was developed in this work for the optimal planning of CMNs based on concurrent use of biochar application and enhanced weathering technologies. The model accounts for any incompatibilities as topological constraints, while the integration of these NETs in a CMN provides additional beneficial effects due to the synergistic relationship of biochar and rock powder when simultaneously applied to suitable agricultural lands. The illustrative case study presents a representative scenario that can happen during the implementation of a terrestrial CMN using these two NETs. The findings provide insights to aid decision-making for the implementation of the final carbon management network that utilizes biochar and powdered alkaline rocks for carbon sequestration. The model creates the capability to plan terrestrial CMNs based on two technologies that will play important roles in carbon drawdown, particularly in agro-industrial contexts. In future work, the model can further be extended to incorporate economic aspects and other features not accounted in the mathematical model wherein new variables, constraints, and parameters can be added in the original formulation. The presence of multiple agents or decision makers can also be integrated into future extensions.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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