Fast time-scale processes in the Perfect Plasticity Approximation (PPA) vegetation model

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1 Mass and energy balance equations

To represent the sub-grid scale heterogeneity of the land surface, each land grid cell can be split in a number of tiles, with each tile having distinct physical and biological properties, and its own exchange with the atmosphere. For example, one tile may represent natural vegetation, another – cropland, and yet another – secondary vegetation that was last disturbed certain amount of time ago. The energy and mass exchange is calculated separately for each tile, and the fluxes are aggregated to the grid cell level in the atmosphere.

In the presented manuscript, we only consider one tile (natural vegetation).

Within each tile the vegetation is represented by N cohorts arranged in L layers. Cohorts are numbered from the tallest to the shortest; layers are numbered from top of the canopy to the bottom. Each cohort k belongs to one layer (that is, cohorts do not straddle the boundaries between layers). Canopy of cohort k occupies fraction f_k of its layer area. Sum of all f_k is equal to 1 for each layer — that is, the cohort canopies fill the entire layer area:

$$\sum_{k \in L} f_k \equiv 1 \tag{1.1}$$

Each cohort k is composed of identical individuals, with density n_k individuals per unit area of tile. Each individual in the cohort has crown area a_k. We assume that the completely fill the layer space entirely. Since there is no guarantee that the sum of all crown areas (determined by allometric relationships) in the layer is going to be equal to the tile area, we need to stretch canopies of at least some layers. The fraction of the tile area covered by the canopy of k-th cohort is:

$$f_k = n_k a_k \times \Big(\sum_{i \in L} n_i a_i \Big)^{-1} \tag{1.2}$$ `cohort-frac`

where the summation is done over all cohorts that belong to layer L. The leaf area index (LAI) of each cohort needs to be calculated to preserve total area of cohort's leaves per unit tile area: $n_k A_k = \text{LAI}_k f_k$, where A_k is the total one-sided area of leaves per individual.

$$\mathrm{LAI}_k = \frac{n_k A_k}{f_k} = \frac{A_k}{a_k} \sum_{i \in L} n_i a_i \tag{1.3}$$

Each cohorts's canopy has its own temperature T_v, and amounts of intercepted water w_l and snow w_s. All cohorts exchange water, energy, and carbon dioxide with the common canopy air space of mass m_c, temperature T_c and specific humidity q_c.

With these assumptions, the energy balance of the canopy k can be expressed as:

$$\frac{d\mathbb{C}_k T_{v,k}}{dt} = R_{Sv,k} + R_{Lv,k} + \mathbb{H}_{v,k} - \mathbb{L}_{v,k} - L_f M_{i,k} \tag{1.4} \quad \boxed{\texttt{vegn-eb}}$$

where $\mathbb{C}_k$ is the total heat capacity of the canopy, $R_{Sv,k}$ and $R_{Lv,k}$ are the net short-wave and long-wave radiative balances of the k-th cohort canopy, $\mathbb{H}_{v,k}$ is the total sensible heat balance of the canopy, $\mathbb{L}_{v,k}$ is the total latent heat loss by the canopy, and $L_f M_{i,k}$ is the heat associated with the phase transitions of the intercepted water.

The total heat capacity of the cohort canopy $\mathbb{C}_k$ is sum of heat capacities of leaves $C_{v,k}$, intercepted water $w_{l,k}$, and intercepted snow $w_{s,k}$:

$$\mathbb{C}_k = C_{v,k} + w_{l,k} c_l + w_{s,k} c_s \tag{1.5}$$

c_l and c_s are the specific heat capacities of the liquid and frozen water, respectively,

Net radiative balances $R_{Sv,k}$ and $R_{Lv,k}$ are calculated using two-stream approximation, with the assumption of spherical leaf angular distribution **?**.

The sensible heat term $\mathbb{H}_{v,k}$ in equation (**??**) is:

$$\mathbb{H}_{v,k} = -H_{v,k} + H_{pl,i}\gamma_{il,k} + H_{ps,i}\gamma_{is,k} - c_l T_{v,k} D_{l,k} - c_s T_{v,k} D_{s,k} \tag{1.6}$$

where $H_{v,k}$ is the sensible heat flux due to turbulent exchange with the canopy air, $H_{pl,i}$ and $H_{ps,i}$ are the fluxes of heat carried by liquid and solid precipitation, intercepted by the k-th cohort canopy with coefficients $\gamma_{il,k}$, $\gamma_{is,k}$, and $D_{l,k}$, $D_{s,k}$ are the rates of water and snow drip from the canopy.

The latent heat term:

$$\mathbb{L}_{v,k} = E_{t,k} L_e(T_{u,k}) + E_{l,k} L_e(T_{v,k}) + E_{s,k} L_s(T_{v,k}) \tag{1.7}$$

where where $E_{t,k}$ is transpiration, $E_{l,k}$ – evaporation of liquid intercepted water, and $E_{s,k}$ – sublimation of intercepted snow. $L_e(T)$ and $L_s(T)$ are the temperature-dependent specific heats of evaporation and sublimation, respectively. $T_{u,k}$ is the temperature of the water the cohort uptakes from the soil.

Note that the terms of equation (**??**) are calculated per unit area of the stretched canopy. These units are convenient for the energy balance calculations, especially for radiative transport in the multy-cohohrt canopy.

Cohort liquid and solid water balance, respectively:

$$\frac{dw_{l,k}}{dt} = \gamma_{il,k} P_{l,i} - D_{l,k} - E_{l,k} + M_{i,k} \tag{1.8} \quad \boxed{\texttt{vegn-mb-l}}$$

$$\frac{dw_{s,k}}{dt} = \gamma_{is,k} P_{s,i} - D_{s,k} - E_{s,k} - M_{i,k} \tag{1.9} \quad \boxed{\texttt{vegn-mb-f}}$$

where $P_{l,i}$ and $P_{s,i}$ are liquid ans solid precipitation on top of layer i, $\gamma_{il,k}$ and $\gamma_{is,k}$ are the fractions of liquid and solid precipitation that cohort k intecepts, $D_{l,k}$ and $D_{s,k}$ are the rates of water and snow drip from the canopy, and $M_{i,k}$ is the rate of snow melt on the canopy of k-th cohort.

On top of the canopy P_l^1 is just the rainfall rate from the atmosphere, and P_s^1 is the snowfall rate. For layers $i > 1$ we can write:

$$P_{l,i} = \sum_{k \in i-1} f_k (1 - \gamma_{il,k}) P_{l,i-1} \tag{1.10}$$ `drip-l`

$$P_{s,i} = \sum_{k \in i-1} f_k (1 - \gamma_{is,k}) P_{s,i-1} \tag{1.11}$$ `drip-s`

where the summation is done over all cohorts in layer $i - 1$. Here we assume that the the drip from the canopy never gets intercepted by the layers below, and contributes directly to the water and energy balance of the underlying surface.

With these assumptions, canopy air water (specific humidity) balance equation is:

$$m_c \frac{dq_c}{dt} = \sum_{k=1}^{N} f_k E_{v,k} + E_g - E_a \tag{1.12}$$ `cana-mb`

where E_g is water vapor flux from the ground surface, E_a is the water vapor flux to the atmosphere. The total water vapor flux $E_{v,k}$ from k-th cohort canopy to the canopy air is a sum of three components:

$$E_{v,k} \equiv E_{t,k} + E_{l,k} + E_{s,k} \tag{1.13}$$

Canopy air energy balance:

$$m_c \frac{d}{dt} \left((1 - q_c) c_p T_c + q_c c_v T_c \right) = \sum_{k=1}^{N} f_k H_{v,k} + H_g - H_a$$
$$+ c_v \left(\sum_{k=1}^{N} f_k T_{v,k} E_{v,k} + T_g E_g - T_c E_a \right) \tag{1.14}$$ `cana-eb`

where c_p and c_v are specific heats of dry air and water vapor, respectively, $H_{v,k}$ is the flux of sensible heat from k-th cohort canopy to the canopy air, H_g is the sensible heat flux from the ground surface to the canopy air, H_a is the sensible heat flux from canopy air to the atmosphere.

Energy balance of the ground surface:

$$R_{Sg} + R_{Lg} - H_g - L_g E_g - G - L_f M_g = 0 \tag{1.15}$$ `grnd-eb`

2 Non-water-stressed photosynthesis

We assume that the whole canopy is isothermal with temperature T_v, and the air in the leaf interior is saturated with specific humidity equal to $q^*(T_v)$. However, the radiation absorbed by the leaves is not distributed uniformly through the entire

depth of the canopy, with upper parts of the canopy getting more light and the lower parts being shadowed by the upper ones. Therefore the net photosynthesis A_n and, accordingly, stomatal conductance for water vapor g_s are going to depend on vertical coordinate. Since the dependency of photosynthesis on light level is highly nonlinear, it is necessary to consider the vertical profile of A_n and g_s to get average canopy values, rather then base the calculations on average available radiation.

To begin with, let's consider a thin layer of the canopy that receives flux Q of the photosynthetically-active incident radiation per unit of leaf area. Obviously, Q will depend on the vertical coordinate, according to the radiation absorption and scattering relationships in the canopy.

The link between stomatal conductance g_s $\mathrm{mol_{H_2O}\,m^{-2}\,s^{-1}}$, the rate of net photosynthesis A_n, $\mathrm{mol_{CO_2}\,m^{-2}\,s^{-1}}$, intercellular concentration of CO_2 C_i, $\mathrm{mol_{CO_2}\,mol_{air}^{-1}}$ and difference in specific humidities in stomata and specific humidity in canopy air q_a, $\mathrm{kg_{H_2O}\,kg_{air}^{-1}}$ can be expressed as:

$$g_s = \frac{mA_n}{(C_i - \Gamma_*)(1 + (q^*(T_v) - q_a)/d_0)} \tag{2.1}$$ `E:Leuning`

where m is the slope of the stomatal conductance relationship, $\mathrm{mol_{H_2O}\,mol_{air}^{-1}}$, d_0 is a reference value of a water vapor deficit, $\mathrm{kg_{H_2O}\,kg_{air}^{-1}}$, Γ_* ($\mathrm{mol_{CO_2}\,mol_{air}^{-1}}$) is the CO_2 compensation point:

$$\Gamma_* = \alpha_c[O_2]\frac{K_C}{2K_O} \tag{2.2}$$

$\alpha_c = 0.21$ is the maximum ratio of oxygenation to carboxylation, $[O_2] = 0.209\,\mathrm{mol_{O_2}\,mol_{air}^{-1}}$ is the concentration of oxigen in canopy air, and K_C ($\mathrm{mol_{CO_2}\,mol_{air}^{-1}}$) and K_O ($\mathrm{mol_{O_2}\,mol_{air}^{-1}}$) are the Michaelis-Menten constants for CO_2 and O_2, respectively. K_C and K_O depend on temperature proportionally to Arrhenius function:

$$f_A(E_0, T) = \exp\left[E_{0,X}\left(\frac{1}{288.2\,\mathrm{K}} - \frac{1}{T}\right)\right] \tag{2.3}$$ `E:Arrhenius`

so that

$$K_C = D_C f_A(E_{0,C}, T_v) \tag{2.4}$$
$$K_O = D_O f_A(E_{0,O}, T_v) \tag{2.5}$$

with respective constants: $D_C = 1.5 \times 10^{-4}\,\mathrm{mol_{CO_2}\,mol_{air}^{-1}}$, $E_{0,C} = 6000\,\mathrm{K}$, $D_O = 0.25\,\mathrm{mol_{O_2}\,mol_{air}^{-1}}$, and $E_{0,O} = 1400\,\mathrm{K}$.

The equation (??) is a simplification of ? empirical relationship in the assumption of negligible cuticular conductance.

On the other hand, net photosynthesis A_n can also be expressed as a carbon dioxide diffusive flux between canopy air and the stomata space:

$$A_n = \frac{g_s}{1.6}(C_a - C_i) \tag{2.6}$$ `E:cotwodiff`

where C_a is the concentration of CO_2 in the canopy air; the factor 1.6 is the ratio of diffusivities for water vapor and CO_2. We assume that the diffusion of CO_2 is mostly limited by stomatal conductance and not by the leaf boundary layer conductance.

Combining equations (**??**) and (**??**), we can get an expression for intercellular concentration of CO_2:

$$C_i = \frac{C_a + \Gamma_* \frac{1.6}{m} \left(1 + \frac{q^*(T_v) - q_a}{d_0}\right)}{1 + \frac{1.6}{m} \left(1 + \frac{q^*(T_v) - q_a}{d_0}\right)} \tag{2.7}$$

The mechanistic model of photosynthesis **?**, with extensions introduced in **??** expresses net photosynthesis as a difference between gross photosynthesis and leaf respiration. Furthermore, the gross photosynthesis is expressed as a minimum of several physiological process rates:

$$A_n = f_T(T_v) \begin{cases} \min(J_{E,C3}, J_C, J_j) - \gamma V_m(T_v) & \text{for C3 plants,} \\ \min(J_{E,C4}, J_C, J_{CO2}) - \gamma V_m(T_v) & \text{for C4 plants.} \end{cases} \tag{2.8}$$ `E:an`

where $f_T(T_v)$ is thermal inhibition factor, J_E is the light limited rate, J_C is the Rubisco limited rate, J_j is the export limited rate of carboxylation, J_C is a CO_2-limited rate, and $V_m(T_v)$ is the maximum velocity of carboxylase, mol_{CO_2} m^{-2} s^{-1}; the term $\gamma V_m(T_v)$ in both cases of equation (**??**) represents leaf respiration.

The thermal inhibition factor is expressed as

$$f_T(T_v) = \frac{1}{[1 + \exp(0.4(5\,°C - T_v))]\,[1 + \exp(0.4(T_v - 45\,°C))]} \tag{2.9}$$ `e:thermal-inhibition`

and affects carbon acquisition and respiration equally.

The maximum velocity of carboxylase V_m depends on the temperature of the leaf and on the leaf age, **?**:

$$V_m(T_v) = V_{\max} f_A(E_V, T_v) \exp\left(-\max\left[\frac{t - t_0}{\tau}, 0\right]\right) \tag{2.10}$$

where $V_{\max}$ is species-dependent constant, t is the age of leaf, t_0 is the time of leaf aging onset, and τ is aging rate. Note that in the configuration described in this manuscript the aging is only applied to temperate deciduous trees.

`E:c3limits` For C3 plants, **?**:

$$J_E = a\alpha Q \frac{C_i - \Gamma_*}{C_i + 2\Gamma_*} \tag{2.11a}$$

$$J_C = V_m(T_v) \frac{C_i - \Gamma_*}{C_i + K_C(T_v)\frac{p_{ref}}{p}\left(1 + \frac{p}{p_{ref}}\frac{[O_2]}{K_O(T_v)}\right)} \tag{2.11b}$$

$$J_j = \frac{V_m(T_v)}{2} \tag{2.11c}$$

where a is the leaf absorptance of photosynthetically-active radiation (PAR), Q is incident PAR, E m^{-2} s^{-1}, [1] α is intrinsic quantum efficiency of photosynthesis, mol_{CO_2} E^{-1},

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[1] 1 E(einstein)=6.022 × 10^{23} photons, regardless of the photon energy. In other words, one einstein is defined as one mole of photons.

p is atmospheric pressure and $p_{ref} = 1 \times 10^5$ Pa is the reference atmospheric pressure.

E:c4limits   For C4 plants, **?**:

$$J_E = a\alpha Q \tag{2.12a}$$

$$J_C = V_m(T_v) \tag{2.12b}$$

$$J_{CO2} = 18000 V_m(T_v) C_i \tag{2.12c}$$

While the solution of the equations (**??**)-(**??**) gives a photosynthesis rate for a thin canopy layer, given PAR flux Q incident to this layer, what we actually need is an average photosynthesis for the entire canopy. Assuming that PAR flux $Q = Q(L)$ monotonically decreases with canopy depth L (defined by equation (**??**) on page **??**), we can define a layer L_{eq} where the light-limited rate J_E is equal to the minimum of other limiting rates. The photosynthesis below the level L_{eq} will be a function of light availability, while above this level A_n will be function of other limiting rates. The net photosynthesis averaged over the entire canopy depth can be expressed as

$$\overline{A_n} = \frac{f_T(T_v)}{\text{LAI}}\left[J_{\min}L_{eq} + \int_{L_{eq}}^{\text{LAI}} J_E(L)\,dL\right] - f_T(T_v)\gamma V_m(T_v) \tag{2.13}$$ E:anbar

where

$$J_{\min} = \begin{cases} \min(J_C, J_j) & \text{for C3 plants,} \\ \min(J_C, J_{CO2}) & \text{for C4 plants.} \end{cases} \tag{2.14}$$

Assuming that the dependence of light on canopy depth can be described by Beer-Lambert-Bouguer law:

$$Q(L) = Q_0 \exp(-\kappa L) \tag{2.15}$$ E:bouger

we can obtain the following expressions for the integral in the right-hand side of (**??**):

$$\int_{L_{eq}}^{\text{LAI}} J_E(L)\,dL = a\alpha' Q_0 \frac{\exp(-\kappa L_{eq}) - \exp(-\kappa \,\text{LAI})}{\kappa} \tag{2.16}$$

and for L_{eq}:

$$L_{eq} = \frac{1}{\kappa} \log\left(\frac{a\alpha' Q_0}{J_{\min}}\right) \tag{2.17}$$

where

$$\alpha' = \begin{cases} \alpha\frac{C_i - \Gamma_*}{C_i + 2\Gamma_*} & \text{for C3 plants,} \\ \alpha & \text{for C4 plants.} \end{cases} \tag{2.18}$$ E:aprime

Average stomatal conductance is calculated from (**??**) similar to equation (**??**)

$$\overline{g_s} = \begin{cases} \frac{m\overline{A_n}}{(C_i - \Gamma_*)(1 + (q^*(T_v) - q_a)/d_0)} & \overline{A_n} > 0 \\ g_{s,\min} & \overline{A_n} \le 0 \end{cases} \tag{2.19}$$

where $g_{s,min} = 0.01\,\mathrm{mol_{H_2O}\,m^{-2}\,s^{-1}}$ is the minimum stomatal conductance allowed in the model.

The model applies some further corrections to the calculated net photosynthesis and stomatal conductance. If there is water or snow on the canopy, the photosynthesis is reduced proportionally to the covered fraction of leaves:

$$\overline{A_n} = \overline{A_n}\left(1 - (f_s + f_w)\alpha_{\text{wet}}\right) \tag{2.20}$$

$$\overline{g_s} = \overline{g_s}\left(1 - (f_s + f_w)\alpha_{\text{wet}}\right) \tag{2.21}$$

where f_l and f_s are the fractions of canopy covered by liquid water and snow, respectively; α_{wet} is the down-regulation coefficient assumed to be equal 0.3. That means that the photosynthesis of the leaf fully covered by water or snow will be reduced by 30% compared to the dry leaf.

The model then imposes an upper limit on the value of stomatal conductance: if the calculated $\overline{g_s}$ is higher than the limit $g_s^{\text{max}} = 0.25\,\mathrm{mol_{H_2O}\,m^{-2}\,s^{-1}}$, then the stomatal conductance and net photosynthesis are adjusted:

$$\overline{A_n} = \overline{A_n} \times \begin{cases} g_s^{\text{max}}/\overline{g_s}, & \overline{A_n} > 0 \\ 1, & \overline{A_n} \leq 0 \end{cases} \tag{2.22}$$

$$\overline{g_s} = g_s^{\text{max}} \tag{2.23}$$

3   Effect of water on photosynthesis

Last, but not least, the stomatal conductance and photosynthesis need to be adjusted for available water limitations. Given stomatal conductance, the water demand per unit area of land, $\mathrm{mol_{H_2O}\,m^{-2}\,s^{-1}}$ can be calculated as

$$U_d = \overline{g_s}\,\mathrm{LAI}\left(q^*(T_v) - q_a\right)\frac{M_{\text{air}}}{M_{\text{H}_2\text{O}}} \tag{3.1}$$

where the factor $M_{\text{air}}/M_{\text{H}_2\text{O}}$ is used to convert water vapor deficit to units of $\mathrm{mol\,mol^{-1}}$, to be compatible with the units of $\overline{g_s}$.

Given the maximum water supply rate U_{max}, the net photosynthesis and stomatal conductance are adjusted as:

$$\overline{A_n} = \overline{A_n} \times \min(U_{\text{max}}/U_d, 1) \tag{3.2}$$

$$\overline{g_s} = \overline{g_s} \times \min(U_{\text{max}}/U_d, 1) \tag{3.3}$$

The maximum soil-controlled uptake by the root is defined as that with root water potential at the plant permanent wilting point. To calculate this maximum rate of water uptake we use the standard 2D radial flow model formulation **?**, in the quasi-steady flow approximation.

Let u be the water uptake rate per unit length of fine root, $\mathrm{kg\,m^{-1}\,s^{-1}}$, R – characteristic radial half-distance to the next root, m, r_r – root radius, m, and r – "microscopic" distance from root axis, m.

For steady flow toward the root,

$$u = 2\pi r K \frac{d\psi}{dr} \tag{3.4}$$

where $K = K(\psi)$ is unsaturated hydraulic conductivity $kg\,m^{-2}\,s^{-1}$,

$$K(\psi) = \begin{cases} K_s \left(\frac{\psi}{\psi_*}\right)^{-(2+3/b)} & \psi \le \psi_* \\ K_s & \psi > \psi_* \end{cases} \tag{3.5} \quad \boxed{\text{K:soil}}$$

where ψ is the soil water matric head, m, and ψ_* is the air entry water potential. Note that since the flow is assumed to be steady-state, u doesn't depend on r.

Integrating from root-soil interface to "bulk" soil (with matric head ψ_s at the distance R from the root axis, and ψ_r at the root surface):

$$\int_{r_r}^{R} \frac{u\,dr}{2\pi r} = \int_{\psi_r}^{\psi_s} K(\psi)d\psi \tag{3.6} \quad \boxed{\text{u0}}$$

or, equivalently:

$$u = \frac{2\pi}{\ln(R/r_r)} \int_{\psi_r}^{\psi_s} K(\psi)d\psi \tag{3.7} \quad \boxed{\text{u1}}$$

This relationship is assumed to hold at a macroscopic point, i.e., a model layer in our case. The integral in the righ-hand side of equation (**??**) is sometimes called *matric flux potential*, **?**.

On the other hand, the water flux through the root skin per unit length of root is

$$u = 2\pi r_r K_r (\psi_r - \psi_x) \tag{3.8} \quad \boxed{\text{u:root}}$$

where K_r is permeability of root membrane per unit membrane area, $kg\,m^{-3}\,s^{-1}$, and ψ_x is the water potential inside the root (xylem water potential) m.

The characteristic half-distance between roots R can be expressed in terms of total root length per unit volume of soil. Suppoze that cohort k has the specific root length λ_k, $m\,kg_C^{-1}$ (SRL, length of fine roots per unit mass of carbon) and the volumetric density of root biomass $b_{r,k}$, $kg_C\,m^{-3}$. The total length of roots of all cohorts per unit volume is $\sum n_k \lambda_k b_{r,k}$; therefore the area of soil cross-section surrounding the root is $A = \pi R^2 = 1/\sum n_k \lambda_k b_{r,k}$, giving

$$R = \left(\pi \sum n_k \lambda_k b_{r,k}\right)^{-1/2} \tag{3.9}$$

Combining (**??**), (**??**), and (**??**), after tedious algebraic transformations we get:

$$r_r K_r (\psi_r - \psi_x) = \frac{2\pi K_s}{\ln(R/r_r)} \left\{ \frac{\psi_*}{n} \left[\left(\frac{\min(\psi_s, \psi_*)}{\psi_*}\right)^n - \left(\frac{\min(\psi_r, \psi_*)}{\psi_*}\right)^n \right] \right.$$

$$\left. + \max(0, \psi_s - \psi_*) - \max(0, \psi_r - \psi_*) \right\} \tag{3.10} \quad \boxed{\text{u:final}}$$

where we introduced notation $n = -(1 + 3/b)$.

Given xylem water potential ψ_x and soil water potential ψ_s, we can solve the equation (**??**) to get the water potential at the root-soil interface ψ_r, and, consequently, the water flux per unit root length $u = u(\psi_r, \psi_s)$.

To calculate the total water uptake, we should note that the xylem potential increases with depth so that $\psi_x = \psi_{x0} + z$, where ψ_{x0} is the xylem potential at the surface. The total uptake will be then the sum of layer values, properly weighted:

$$U(\psi_{x0}) = \sum_1^N u(\psi_{x0} + z_i, \psi_i) L_i S_i \qquad (3.11) \quad \boxed{\texttt{u:total}}$$

where z_i is the depth of the layer, ψ_i is the soil water potential in the layer, and L_i is the total length of roots in the layer. The additional factor S_i is used to turn off uptake when certain conditions are met: when there is ice in the layer; when the uptake is negative (optional, when one-way-uptake is requested); and when the soil is saturated (optional, when uptake-from-sat is not requested).

The maximum soil water supply to the vegetation $U_{\max}$ is calculated as the value of the uptake for the xylem water potential at the surface equal to the permanent wilting point ψ_{wilt}: $U_{\max} = U(\psi_{wilt})$.