Multilevel Logistic Regression Models

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Model Specification

Multilevel model for **binary dependent variable *Y***, measured on **person *i*** within **cluster *j***

$$\ln\left[\frac{P(y_{ij}=1)}{1-P(y_{ij}=1)}\right]=\operatorname{logit}[P(y_{ij}=1)]=\beta_0+\beta_1 x_{1ij}+u_{0j}+u_{1j}x_{1ij}$$

Fixed effects

Random effects

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Could use multilevel specification if desired!

Model Specification, cont'd

Same **distributional assumptions** about random cluster effects: normally distributed, mean vector 0, unique variances and covariances

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When we fit **generalized linear regression models** to non-normal outcomes and include random effects, **estimation is more difficult** mathematically ~ **<u>clear motivation is important!</u>**

Estimating the Model Parameters

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One approach = **adaptive Gaussian quadrature**
Simulation studies = works well in variety of scenarios
Deep dive: Reading by Kim et al. (2013)

Testing the Model Parameters

Compute confidence intervals or test hypotheses for model parameters

Test null hypotheses (e.g., fixed effect is zero, or variance component is zero – random effects don't vary!), can use **likelihood ratio testing**

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Reading this week: Provides specific details on how to perform these types of tests for parameters in multilevel models!

Revisiting NHANES Example

- Logistic regression to model probability of ever smoking 100 cigarettes as function of selected predictors.
- Assumed all NHANES observations independent of each other … Observations came from randomly sampled clusters *(geographic areas)*!

Not True!

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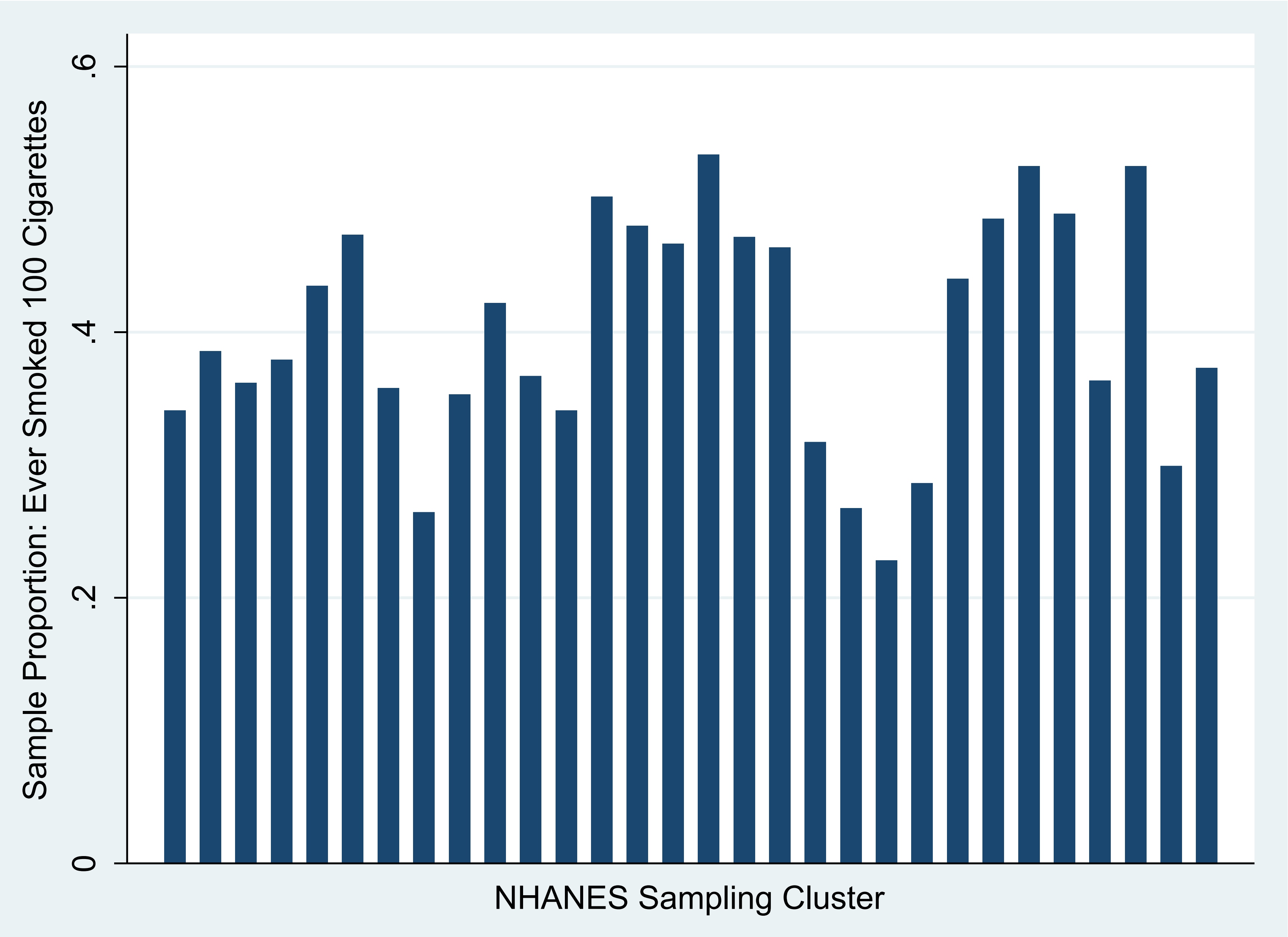
- If smoking observations correlated within areas, *standard errors* in "naïve" logistic regression analysis *likely understated*.
- Plus **explicit interest** in *estimating variance* between sampling clusters in terms of probability of smoking!

Between-Cluster Variance in Smoking



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Logistic model including random effects of randomly sampled clusters
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- **Same inferences** regarding which predictors significant
- **Slight changes** in estimated fixed effects
- Standard errors of estimates are now **larger!**

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Even after adjusting for predictors, randomly sampled clusters still vary in terms of smoking prevalence!

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Q: Look at distribution of **predicted** values of random interviewer effects, or EBLUPs … **outliers**?

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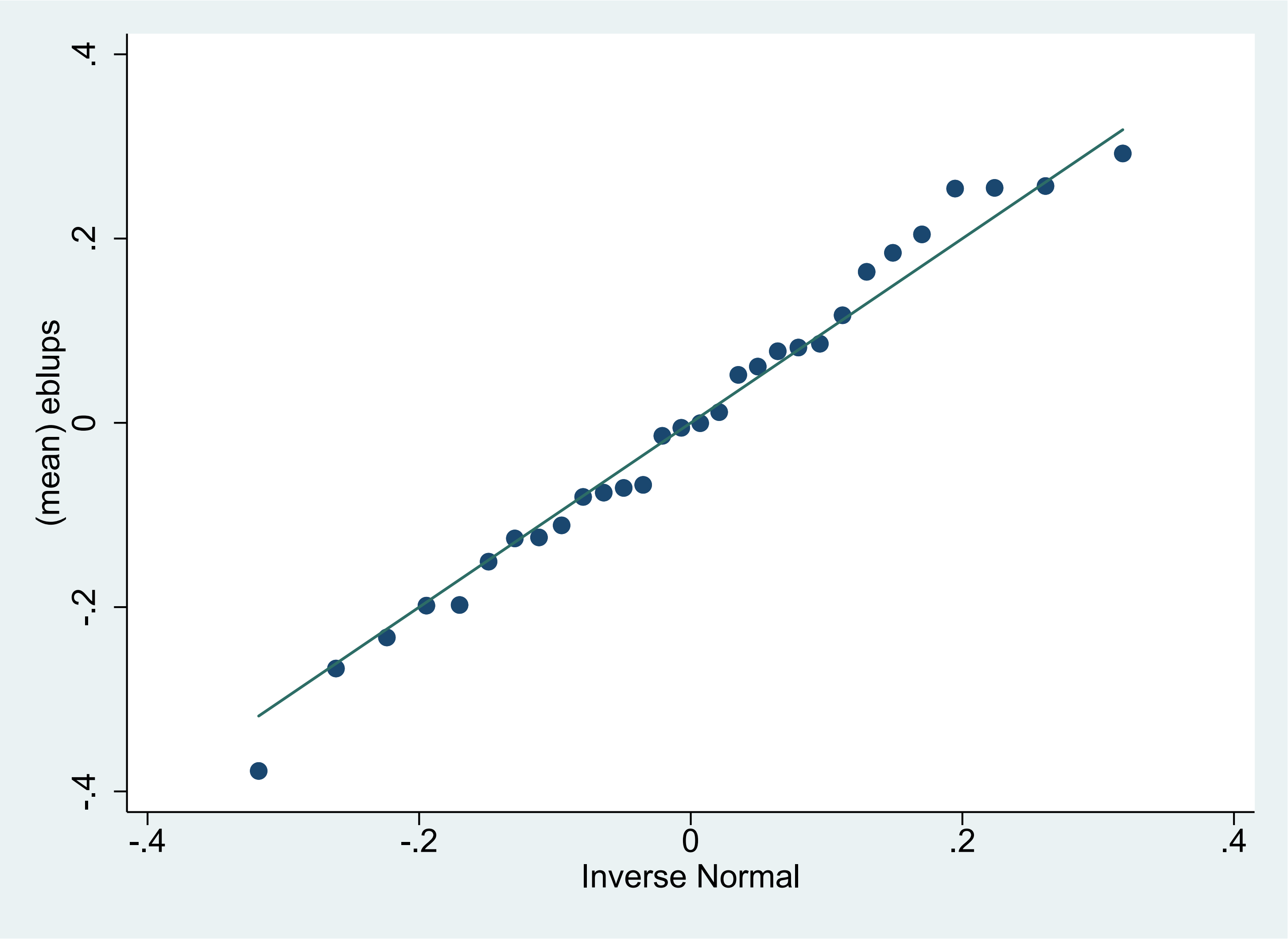
Another Consideration:

Center continuous predictor variables so intercept is interpretable!

EBLUPs for Random Intercepts



QQ plot suggests
random effects on intercept normally distributed + no outliers!

Conclusions from the Example

- Same predictors of smoking still important!
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- Given significant unexplained variance in random cluster effects → explain variance by including fixed effects of cluster-level predictors (e.g., SES)
- HOWEVER: When comparing variance components between multilevel models with different cluster-level fixed effects, **both models must include same respondent-level fixed effects**

Deeper Dive: Multilevel Analysis: Techniques and Applications, Hox et al, 3rd Edition, Section 6.5

What's Next?

- **Full example:** fitting multilevel models to longitudinal data with Python + making inference
- **Marginal models** for dependent data + alternatives for modeling **clustered** and **longitudinal** data sets