What Do We Mean by "Fitting Models to Data"?

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Fitting Models to Data

Goal: How to fit **statistical models** to **data**
to help answer research questions

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Fit Data → Models

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Fitting Models to Data

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Fit Data → Models

Specify based on theory or subject knowledge

Fit Models → Data

Variables follow distributions and have certain relationships

Fitting Models to Data

Why do we fit models to data?

- **Estimate** distributional properties of variables, potentially conditional on other variables

means
variances
quantiles

- Concisely **summarize relationships** between variables, and make inferential statements about those relationships
- **Predict** values of variables of interest conditional on values of other predictor variables, and characterize prediction uncertainty

Fitting Models to Data

- Focus on **parametric models** → estimating **parameters** that describe the distributions of variables

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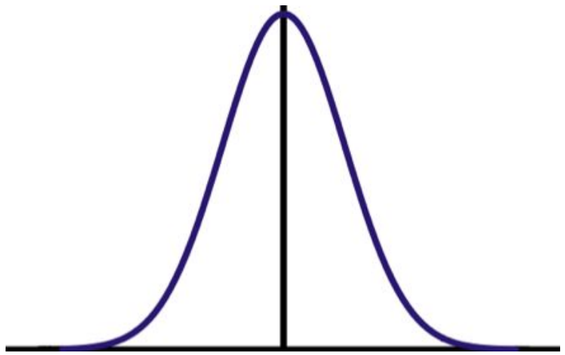

e.g. normal distribution

Fitting Models to Data

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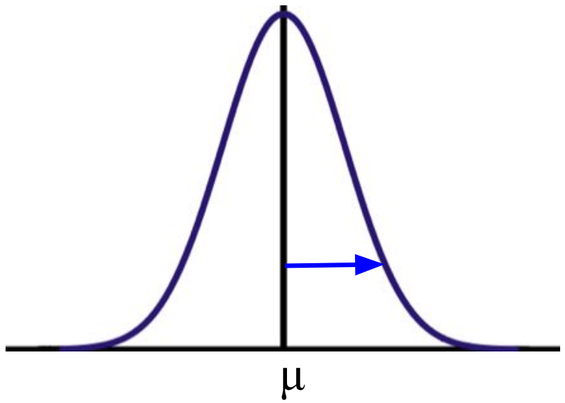

e.g. normal distribution

- Given data, suggest **variable of interest** follows certain **probability model**

  → estimate parameters that define that model

e.g. mean and variance

Fitting Models to Data

Estimate model parameters + sampling variance

= **make inference about parameters**

by testing hypotheses or generating confidence intervals

Course 2!

Fitting Models to Data

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Course 2!

Up next …

Example of specifying probability model (given a research question)

and estimating the parameters of that model

Idea of **assessing model fit**: *Does model seem to fit observed data well?*

Example: Test Performance and Age

Variable of Interest
Test Performance
(0 – 8 points)

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Possible Predictor
Age
(Standardized)

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Believe age has **curvilinear** relationship with performance

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moderate values of age → performance best
smaller or larger values of age → performance tends to be worse

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Goals: 1) estimate **marginal mean** of performance across all ages

2) estimate mean performance **conditional** on age

Example: Test Performance and Age

"mean-only" model for performance

Performance follows **normal distribution** overall
defined by mean and variance (two parameters)

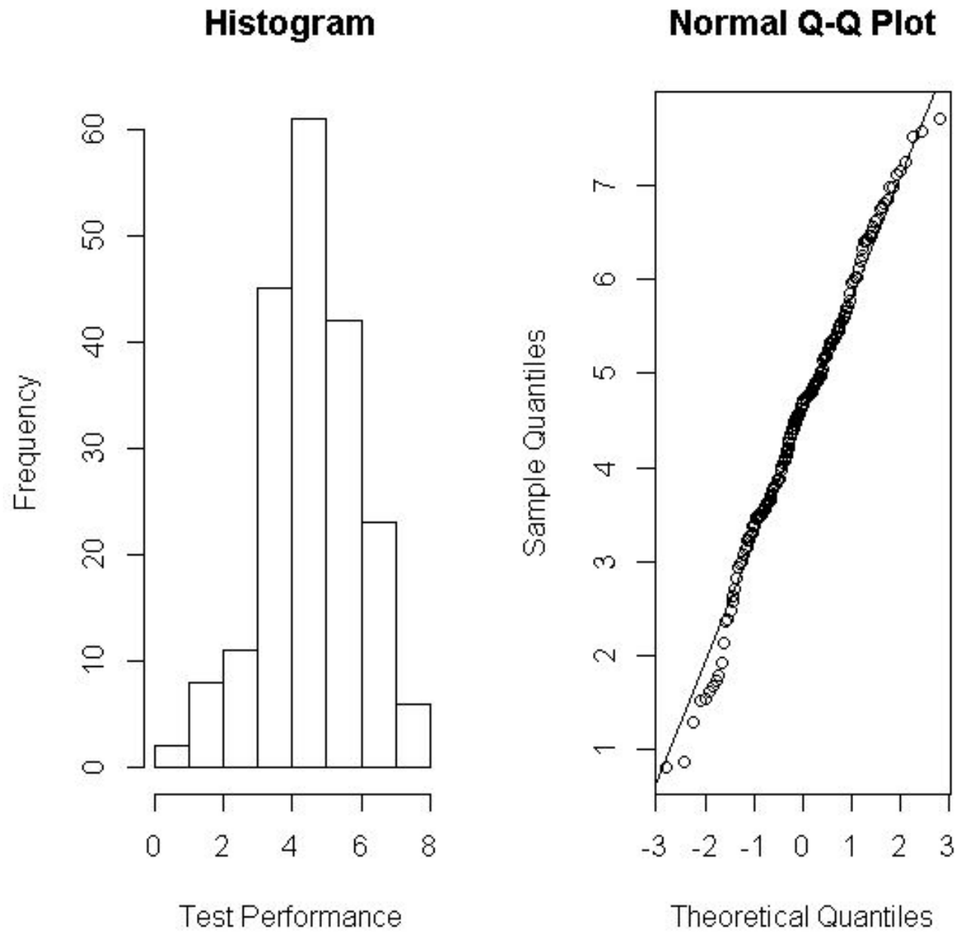
Conditional on age, performance follows **normal distribution**
mean defined by a **quadratic function** of age
$a + b*age + c*age^2$ (three parameters: a, b, and c)
and variance σ^2 (one parameter)

Conditional model for performance

The Data: Performance

Examine marginal distribution of performance (*n* = 200) via Histogram and Normal Q-Q plot

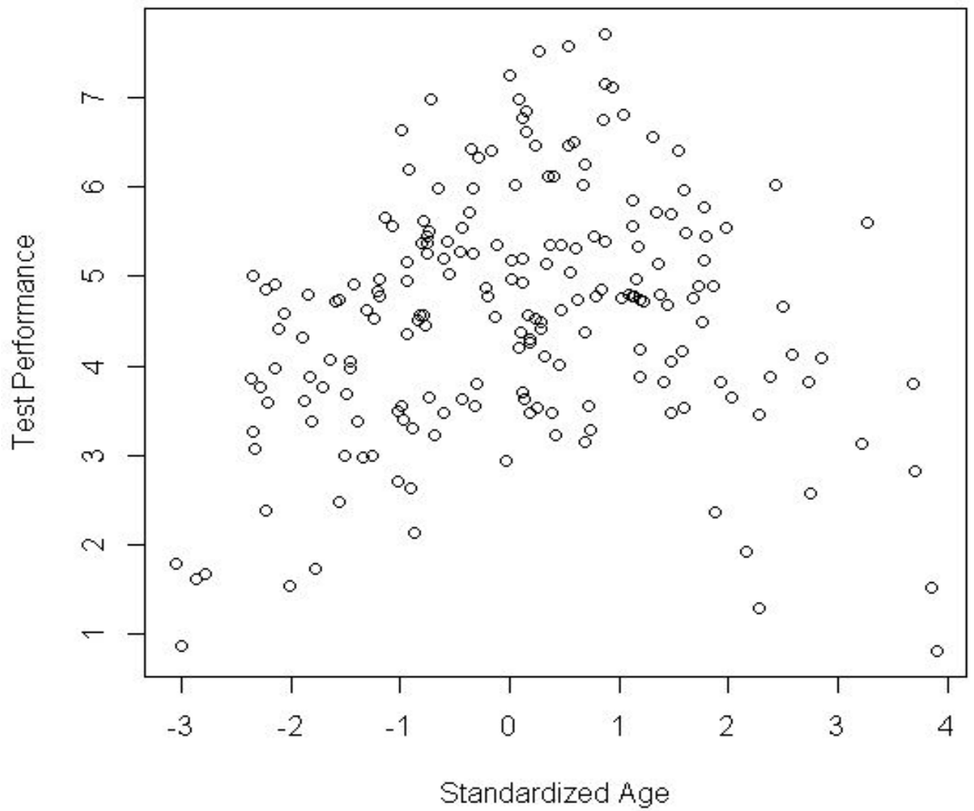
Does the **normal distribution** seem like a **reasonable** model for performance?



The Data: Performance and Age

Visualize relationship
between age and test performance
via scatter plot

Support for our theory regarding **curvilinear** relationship?



Fit the "Mean-Only" Model

Fit **regression model** to performance data

perf = m + e

1st parameter
(*unknown constant*)

m = marginal mean

Fit the "Mean-Only" Model

Fit **regression model** to performance data

perf = m + e

1st parameter
(*unknown constant*)

m = marginal mean

e = random error defining each observation's deviation from the overall mean m,

Errors are **normally distributed** with mean 0 and variance σ^2

where e ~ N(0, σ^2)

2nd parameter

Fit the "Mean-Only" Model

Fit **regression model** to performance data

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Parameter Estimates

Estimate of overall (marginal) mean m = 4.57 points

with SE = 0.10 points

→ support overall mean non-zero!

Estimate of σ^2 = 1.82 (points^2)

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Need to check this!

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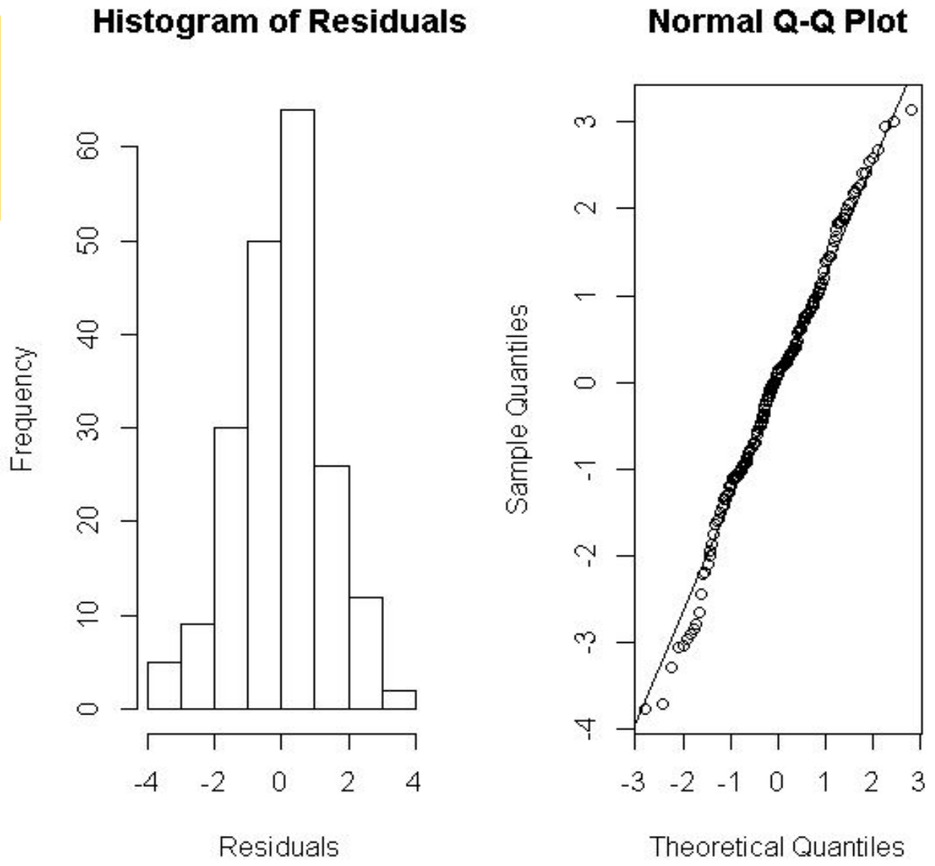
Assess Fit of "Mean-Only" Model

Residuals = realized values of random errors
= observed performance – estimated mean m

- **Examine realized residuals** via **histogran and normal Q-Q plot**
  to see if normal model is good fit for data
- **If normal model was not good fit**, would see large deviations from normality in realized residuals.



Fit the Conditional Model

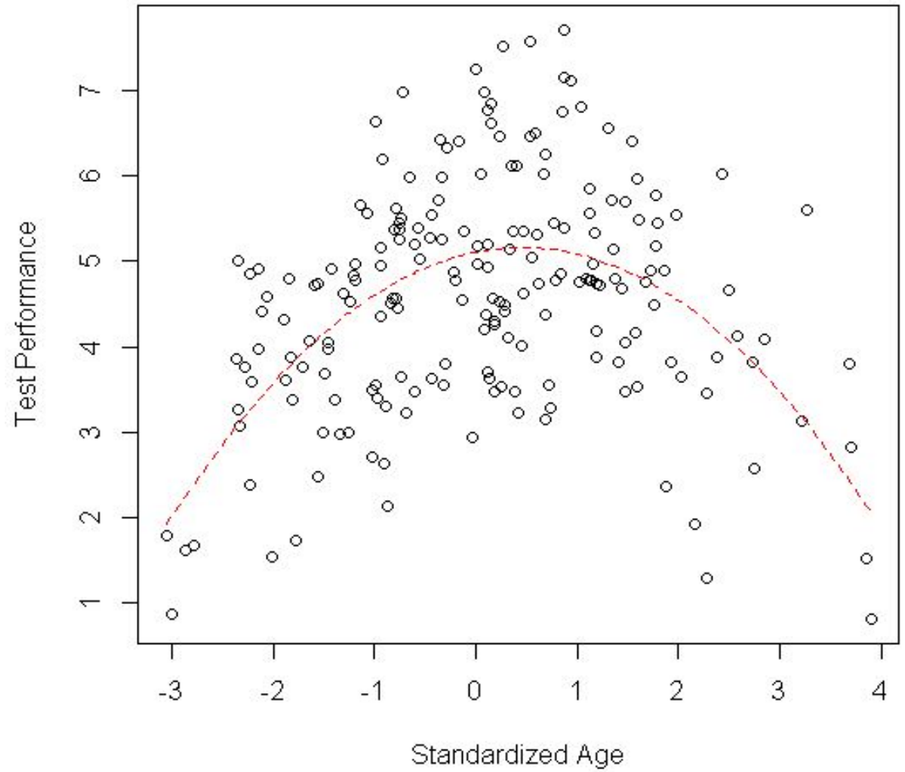
Fit **regression model:** regress performance on age and age^2

perf = a + b*age + c*age^2 + e, where e ~ N(0, σ^2)

a, b, and c = three parameters

e = random error

Errors are **normally distributed**



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Estimate of a = 5.11 (SE = 0.10)

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Estimate of c = -0.26 (SE = 0.03)

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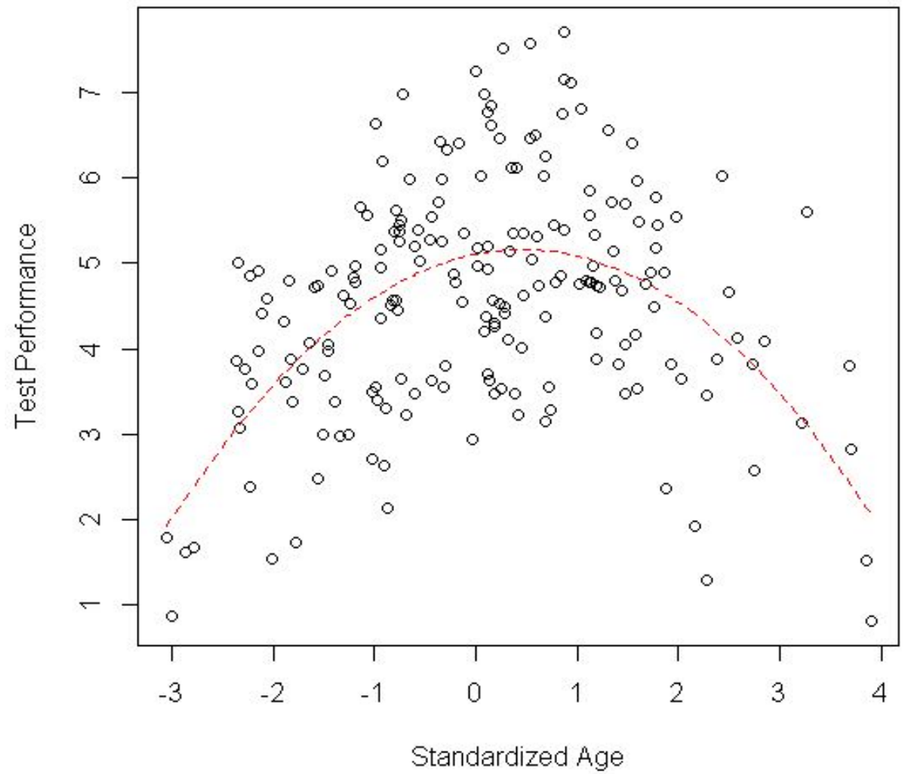
Estimate of σ^2 = 1.29 (points^2)



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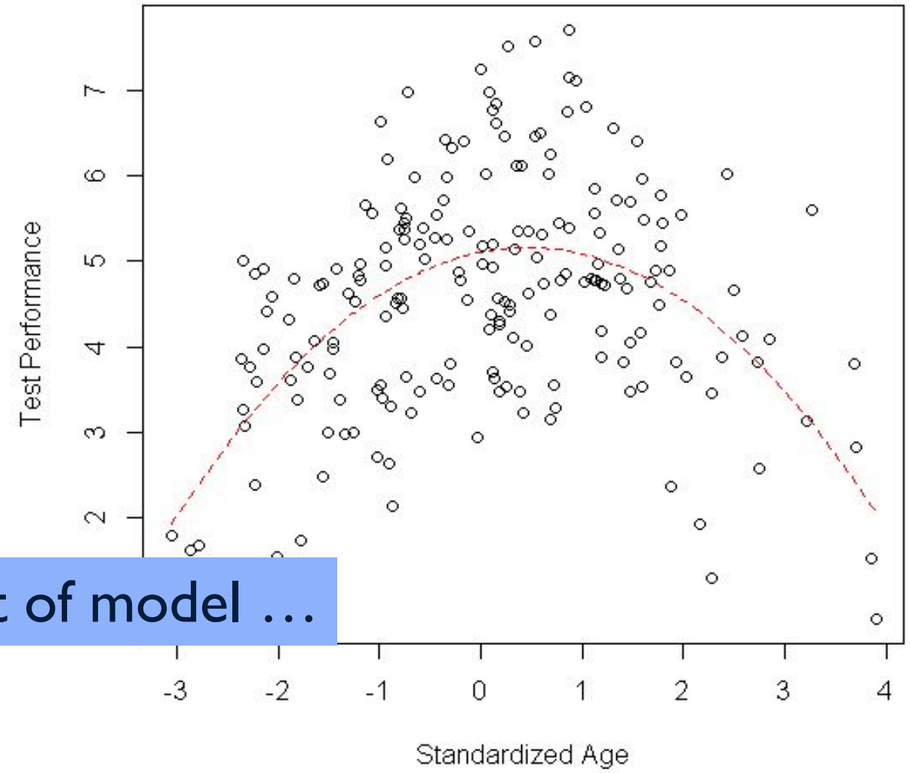
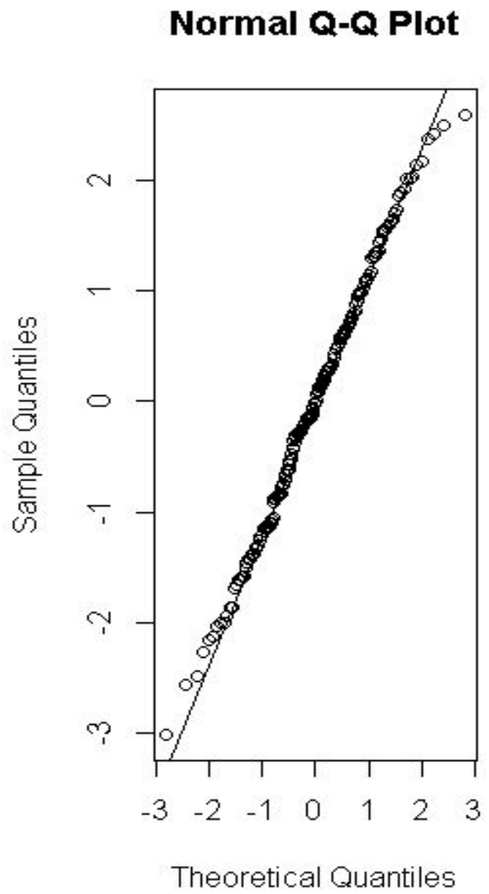


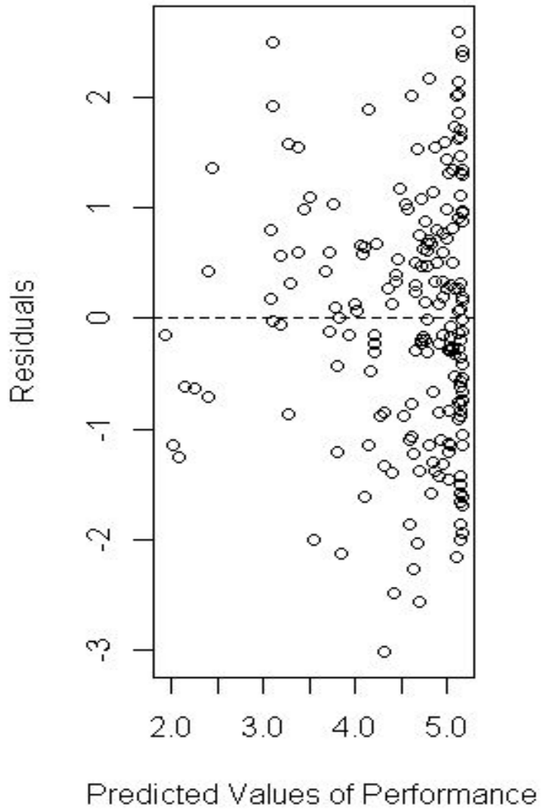
Assess fit of model …

Assess Fit of Conditional Model

See if residuals (realized values of e):

- appear to be normally distributed





Assess Fit of Conditional Model

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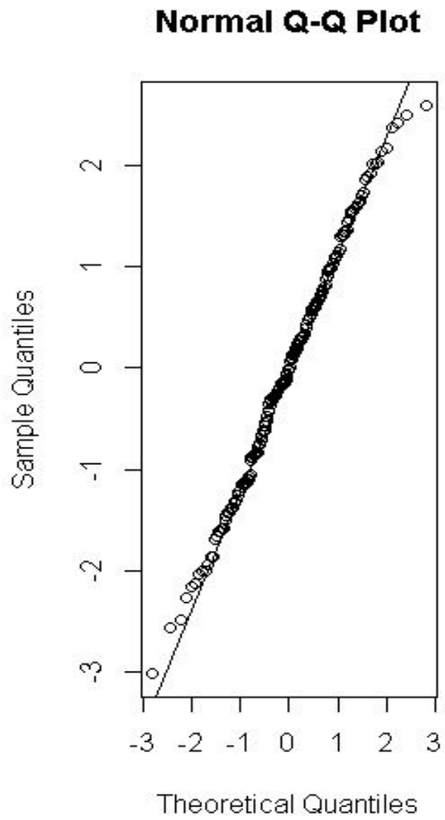
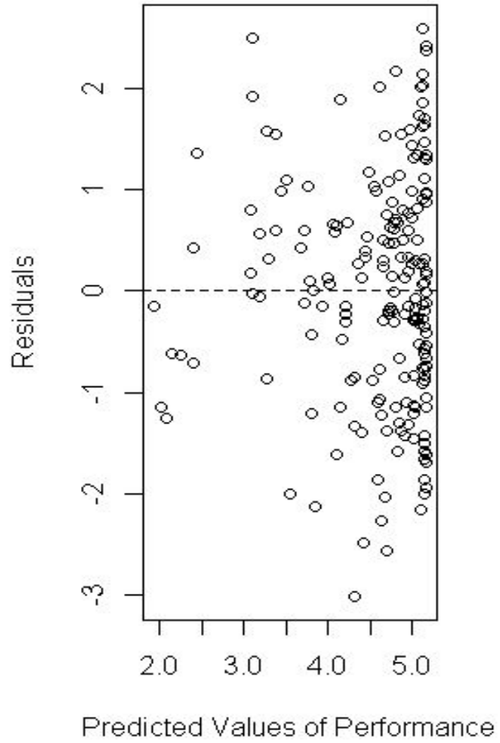
- are symmetrically distributed around zero with **constant variance**
  (as function of **predicted values** of performance, given estimates of parameters a, b, and c)





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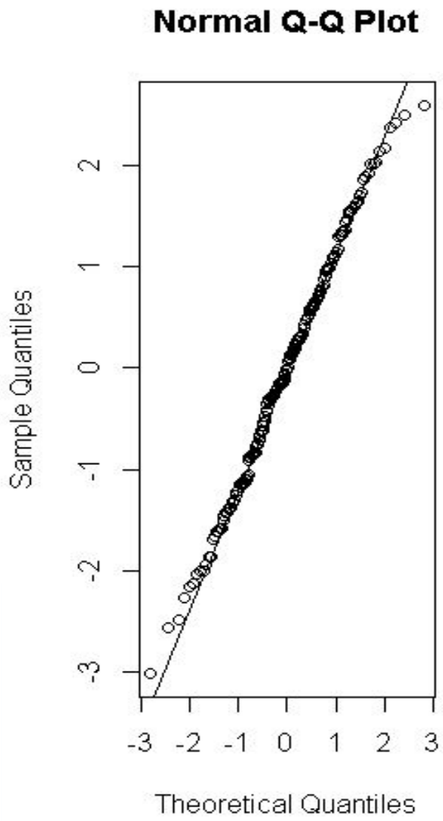
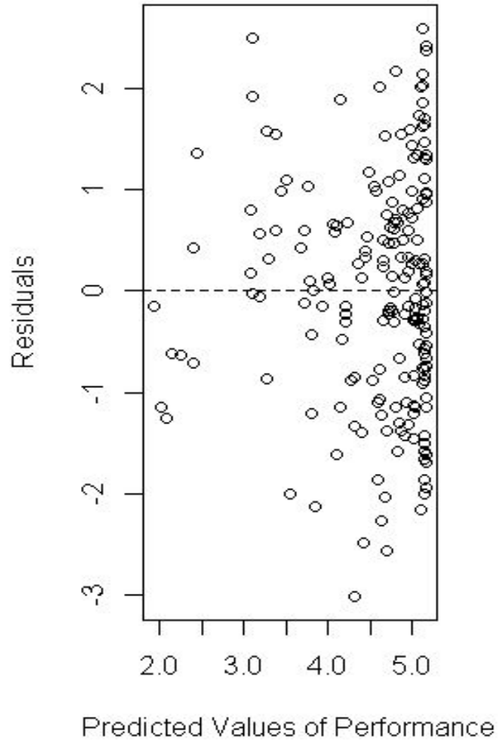
Model fit looks good!
Could predict performance well,
given standardized age ... can we do better?

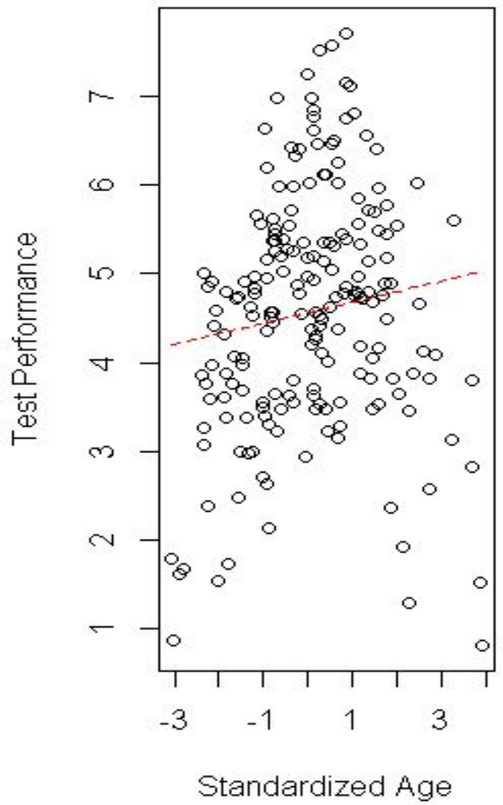


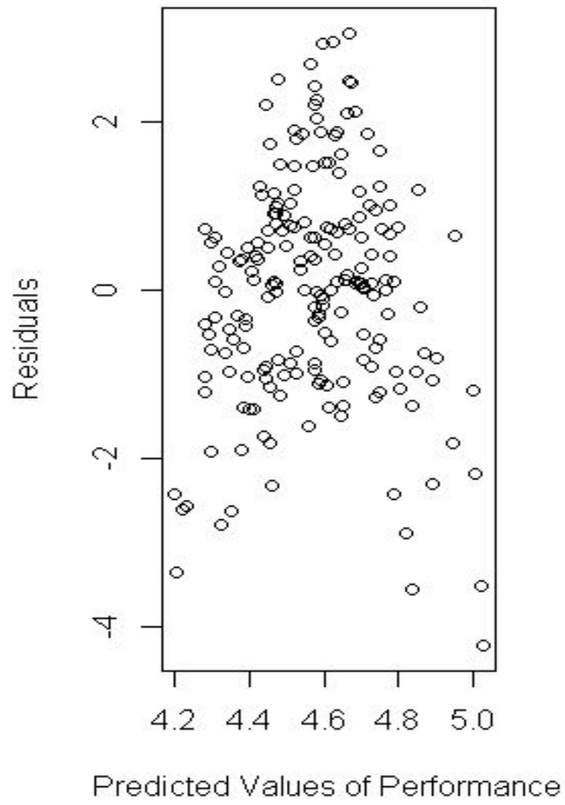


A model that does not fit well…

What if …?
Fit **misspecified model** to data, assuming a **linear** relationship of performance and age?

Model fit looks poor …
residuals are **not** symmetrically scattered around zero;
poor predictions at low and high values of age, higher σ^2





What did we see and what is next?

- **We have** … introduced idea of fitting parametric models to data and assessing model fit
- **We will** … talk about different types of variables interested in modeling, different types of data sets depending on study design (and implications for modeling), and different approaches to estimation and inference when fitting models
- **We will** … discuss specific examples of modeling in detail!