Summary of Slides

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1 Introduction

General Form. A general minimization problem has the form

$$\min_x f(x) \quad \text{s.t. } x \in X,$$

for a *constraint set* $X \subseteq \mathbb{R}^n$ (often given by some *constraint functions* and an *objective function* $f : X \to \mathbb{R}$. We want to find an optimal value or *minimizer* $x^* \in X$ such that

$$f(x^*) \leq f(x), \quad \forall x \in X.$$

Types of Optimization Problems.

1. (a) Discrete: X is a discrete set, also called *interger programming*.
   (b) Continuous: X is continuous (ie. uncountable)
2. (a) Linear: Objective functions and constraints are all linear:

   $$\min_x c^\top x, \quad \text{s.t. } Ax \leq b,\ x \geq 0.$$

   Constraints describe a polyhedron. Efficiently solvable.
   (b) Quadratic: Objective function is quadratic, constraints linear:

   $$\min_x \frac{1}{2} x^\top Q x + c^\top x, \quad \text{s.t. } Ax \leq b,\ Ex = d.$$

   If Q is positive semidefinite, the objective is convex and the problem is polynomially solvable.
   (c) Nonlinear: no further constraints.
3. (a) Unconstrained: Optimal solution searched in full $\mathbb{R}^n$. Easier to characterize, and usually to solve.
   (b) Constrained: Optimal solution in an admissible region, usually more difficult to setup/characterize.

Convexity. A set X is convex, if for all $x, y \in X$ and $\alpha \in [0, 1]$:

$$\alpha x + (1 - \alpha)y \in X.$$

This means that X contains all convex combinations of points from it.

Convex Functions. If X is a convex set, then $f : \mathbb{R} \to \mathbb{R}$ is called convex if for all $x, y \in X$ and $\alpha \in [0, 1]$:

$$f(\alpha x + (1 - \alpha)y) \leq \alpha f(x) + (1 - \alpha)f(y).$$