Digital Signal Processing Tutorial (382.047/64/66/67/68/69/70), 2024S

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ISP

Assignment 1

Analogue Signals and Systems

Information about assignments and in particular on Moodle submission modalities can be found in DSP-Tutorial_2024S_CourseInfo.pdf.
Submission deadline is Tue **16. April 2024, 08:00.**

Exercise 1 *Complex Numbers (20%)*

These complex numbers are given:

$$c_1 = -5 + j3$$
$$c_2 = \frac{\sqrt{2}}{2} e^{-\frac{j3\pi}{4}}$$
$$c_3 = \frac{1}{\sqrt{2}} + \frac{j1}{\sqrt{2}}$$

Calculate the following numbers and show the calculations/derivations in the report.

- $c_4 = c_1 + c_2$
- $c_5 = c_1 \cdot c_2$
- $c_6 = |c_3|^2$
- $c_7 = \arg(c_3)$
- $c_8 = \frac{c_1}{c_2}$
- $c_9 = c_1 \cdot c_1^*$

Subsequently check your results with Matlab. Useful functions in this context are `abs` and `angle`. You do not need to add those checks to your report, or show the corresponding code.

Exercise 2 *Fourier Transform (25%)*

The lecture notes show the following Fourier transform pair for the cosine wave (DSP_2.pdf, page 37):

$$x(t) = \hat{X}\cos(2\pi f_0 t) \leftrightarrow X(f) = \frac{\hat{X}}{2}\delta(f - f_0) + \frac{\hat{X}}{2}\delta(f + f_0)$$

Mathematically proof this relation. To do so, use Eulers formula to express the cosine in the time domain as a sum of complex exponentials and the Fourier transform of a complex exponential function from DSP_2.pdf, page 38.

Add a diagram of $X(f)$ in the report (draw the real and the imaginary part of $X(f)$ in the same diagram).

Exercise 3 *Time Shift and Phase (20%)*

Given are two sines according to the following formula

$$x_i(t) = \sin(2\pi f_i t)\,, with\ i \in \{1,2\}$$

with $f_1 = 1\text{Hz}$ and $f_2 = 3\text{Hz}$.

All two sines are time delayed by $\tau = 0.1\text{s}$ to yield

$$y_i(t) = \sin\bigl(2\pi f_i(t - 0.1)\bigr).$$

This corresponds to a phase shift. Thus, the delayed sines may also be written as

$$y_i(t) = \sin(2\pi f_i t + \phi_i).$$

a) Calculate the phase shifts ϕ_i for each sine, and verify that this corresponds to the "Shift Theorem" of the Fourier Transform (DSP_2.pdf, page 41).

b) For both sines in separated plots: Plot the original signal, the time delayed signal and the phase shifted signal. Since the latter two are identical, show this by plotting the first with a solid line and the overlaid one in a different colour with a dashed line. Plot each signal from 0 to 1s. In Matlab use the following time-vector: `t= 0:0.001:1`.

Exercise 4 *Linearity and Time Invariance (35%)*

Examine the following systems (input $x(t)$ and output $y(t)$) for linearity and time invariance. Clearly show the mathematical derivations and state if the systems are linear and/or time-invariant.

- $y(t) = \bigl(x(t)\bigr)^2$
- $y(t) = x(t) \cdot \sin(\Omega_0 t)$