Homework 1 - Methods in Theoretical Physics (Physics 5350)

I. **DUE DATE**: January 26, 9:00 AM (PLEASE SHOW YOUR WORK)

1. Rewrite the following 32-bit floating point numbers in normalized form and determine the decimal number that they correspond to. Assume that $x = sMB^{e-E}$ where s is the sign bit, M is the 23-bit mantissa, B = 2 is the base, e is the 8-bit exponent and E = 128 is the exponential bias.

(a) 0 00000010 01000000000000000000000

(b) 1 00000110 00101000000000000000000

(c) 0 00100000 01000100000000000000000

2. Conversion from Decimal to Floating Point Representation:

Say we have the decimal number 329.390625 and we want to represent it using floating point numbers. The method is to first convert it to binary scientific notation, and then use what we know about the representation of floating point numbers to show the 32 bits that will represent it.

The first step is to convert what there is to the left of the decimal point to binary. 329 is equivalent to the binary 101001001. Then, leave yourself with what is to the right of the decimal point, in our example 0.390625.

There is an algorithm to convert to different bases that is simple, straightforward, and largely foolproof. Here it is illustrated for base two. Our base is 2, so we multiply this number times 2. We then record whatever is to the left of the decimal place after this operation. We then take this number and discard whatever is to the left of the decimal place, and continue with this progress on the resulting number. This is how it would be done with 0.390625.

0.390625	* 2 = 0.78125	0
0.78125	* 2 = 1.5625	1
0.5625	* 2 = 1.125	1
0.125	* 2 = 0.25	0
0.25	* 2 = 0.5	0
0.5	* 2 = 1	1
0		

Since we've reached zero, we're done with that. The binary representation of the number beyond the decimal point can be read from the right column, from the top number downward. This is 0.011001.

a) Using the above result, determine the 32 bit floating point representation of $(329.390625)_{10}$ in normalized form. Assume that $x = sMB^{e-E}$ where s is the sign bit, M is the 23-bit mantissa, B = 2 is the base, e is the 8-bit exponent and E = 128 is the exponential bias.

b) Use the algorithm outlined above to show that $(0.1)_{10}$ is a repeating binary and therefore cannot be represented accurately in a computer memory.

3. The "Golden Mean" is given by

$$\phi = \frac{\sqrt{5}-1}{2} \approx 0.61803398$$

It turns out that powers ϕ^n satisfy the recursion

$$\phi^{n+1} = \phi^{n-1} - \phi^n$$

Since we know that $\phi^0 = 1$ and $\phi^1 = 0.61803398$, we can easily calculate higher powers of ϕ using this recursion.

Compile and run the program "golden_mean.f90" (found on Canvas) that calculates the first 100 recursions of the *golden mean.*

Compare the results of the recursive calculation with the correct answer. What do you observe? Why do the answers differ?