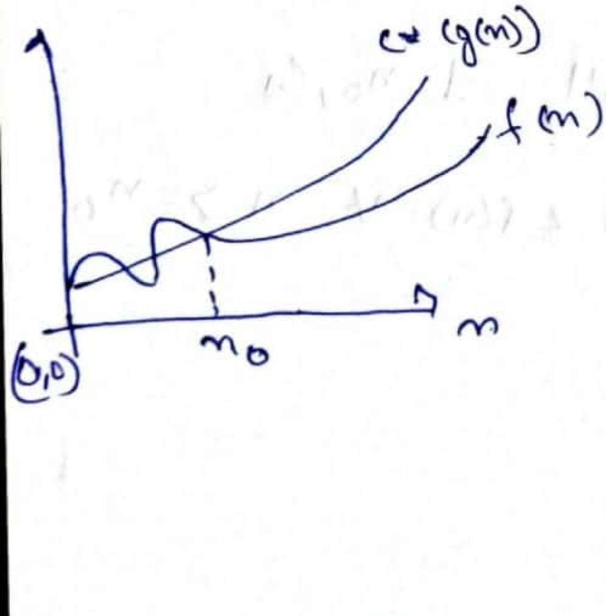
Q.1) Asymptotic Notation: These are language to express the required time & space by an algorithm to solve a given problem.

(1) Big - O Notation: It is notation for the worst case analysis of an algorithm. (Upper bound)

According to it for a two func $f(n)$ & $g(n)$

$f(n) = O(g(n))$, if and only if there exist n_0 & c such that.



$0 \leq f(n) \leq c * g(n)$ for all $n \geq n_0$

Ans ⇒ $n+n^2 = O(n^2)$

here $f(n) = n+n^2$, $g(n) = n^2$.

$n+n^2 \leq n^2+n^2$ ($\because n < n^2, n^2 = n^2$)

$n+n^2 \leq 2n^2$ (here $c=2$) for $n_0=1$.

so $f(n) = O(g(n))$

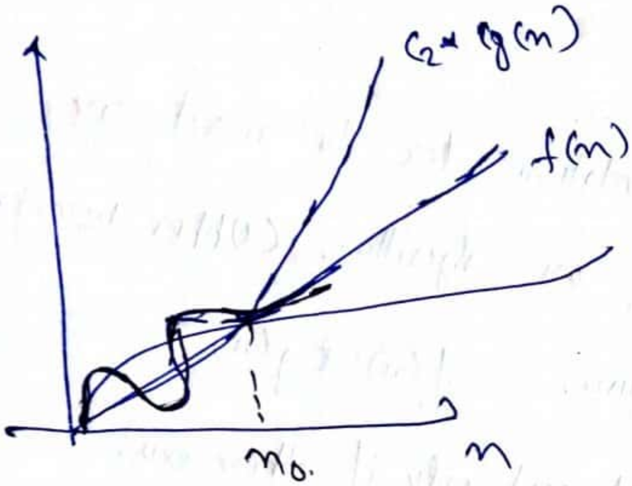
or $n+n^2 = O(n^2)$

• Big theta (Θ) ; For avg case time complexity (~~low~~ tighly bound)

for any two function $f(n)$ & $g(n)$

$f(n) = \Theta(g(n))$ if and only if there exists n_0, c_1, c_2

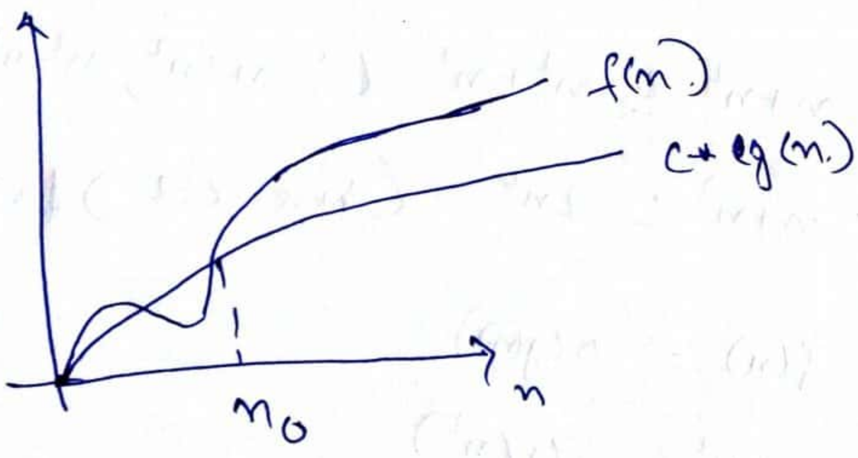
such that $0 \leq c_1 * g(n) \leq f(n) \leq c_2 * g(n)$
[for $n \geq n_0$]



• Big Omega (Ω) : for best case complexity (lower bound)

$f(n) = \Omega(g(n))$ iff $\exists$ n_0, c_1

$\ni$ $0 \leq c_1 * g(n) \leq f(n)$ $\forall$ $n \geq n_0$



Q.2 ⇒ T.C. of for (i=1 to n) { i=i*2 }

Series ⇒ 1, 2, 4, 8, 16 ---- n (G.P.)

$a = 1$, $r = 2$

$t_k = ar^{k-1} \Rightarrow n = a*2^{k-1}$

$\Rightarrow n = 2^{k-1}$

$\Rightarrow 2^k = 2n$

$\Rightarrow k = 2\log_2 n$

so T.C. ⇒ $O(\log_2 n)$

Q.3 ⇒ $T(n) = \{ 3T(n-1) \}$ if $n>0$, otherwise 1 }

$T(n) = 3T(n-1)$ --- (i)

let $n = n-1$, $T(n-1) = 3T(n-2)$

$T(n) = 3^2 T(n-2)$

or $T(n) = 3^3 T(n-3)$

or $T(n) = 3^n T(n-n)$

$T(n) = 3^n T(0) = 3^n$

so T.C. ⇒ $O(3^n)$

Q.4) $T(n) = \{ 2T(n-1) - 1$ if $n > 0$, otherwise $1 \}$

$T(n) = 2T(n-1) - 1$

let $n = n-1$, $T(n-1) = 2T(n-2) - 1$

so $T(n) = 2(2T(n-2) - 1) - 1$

$= 2^2 T(n-2) - 2 - 1$

let $n = n-2$, $T(n-2) = 2T(n-3) - 1$

so $T(n) = 2^2(2T(n-3) - 1) - 2 - 1$

$= 2^3 T(n-3) - 2^2 - 2 - 1$

or

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \dots - 2^1 - 2^0$

$T(0) = 1$, let $n-k = 0$ so $k = n$

$T(n) = 2^n T(n-n) - 2^{n-1} - 2^{n-2} - \dots - 2^1 - 2^0$

$= 2^n - 2^{n-1} - 2^{n-2} - \dots - 2^1 - 2^0$

$= 2^n - (2^{n-1} + 2^{n-2} + \dots + 2^1 + 2^0) \}$ G.P

$T(n) = 2^n - \frac{1(2^n - 1)}{2-1} = \cancel{2^n} - \cancel{2^n} + 1$

so T.C. $\Rightarrow$ $O(1)$

Q.5 $\Rightarrow$

```
int i=1, s=1;
while (s<=n) {
        i++; s=s+i;
        printf("#");
};
```

Series $\Rightarrow$ 1, 3, 6, 10, 15, 21, 28 ----- n

1st iteration $\Rightarrow$ $s = s+1$

2nd iteration $\Rightarrow$ $s = s+1+2$

till $\Rightarrow$ $1+2+3+ \dots + k <= n$

$$\frac{k*(k+1)}{2} <= n$$

or $O(k^2) <= n$

or $k = O(\sqrt{n})$

so T.C. = $O(\sqrt{n})$

Q.6 ⇒ for (i=1 ; i*i <= n ; i++)

count ++

let loop run till k i=k

$k^2 <= n$

$k <= \sqrt{n}$

so T.C ⇒ $O(\sqrt{n})$

Q.7 ⇒ for (i=n/2 ; i<=n ; i++) $O(n)$

for (j=1 ; j<=n ; j=j*2) $O(\log n)$

for (k=1 ; k<=n ; k=k*2) $O(\log n)$

so T.C ⇒ $O(n \log^2 n)$

Q.8 ⇒

```
function (int n) {
    if (n==1) return;
    for (i=1 to n) {
        for (j=1 to n) {
            print ("*");
        }
    }
    function (n-3);
}
```

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Recurrence Relation $\Rightarrow$ $T(n) = T(n-3) + n^2$

or $T(n) = T(n-6) + 2*n^2$

$T(n) = T(n-9) + 3n^2$

or $T(n) = T(n-3k) + kn^2$

$T(1) = 0$, $n - 3k = 1$ $\Rightarrow k = \frac{n-1}{3}$

so $T(n) = T(1) + \frac{(n-1)}{3} n^2$

so T.C. $\Rightarrow$ $O(n^3)$

Q.9 $\Rightarrow$

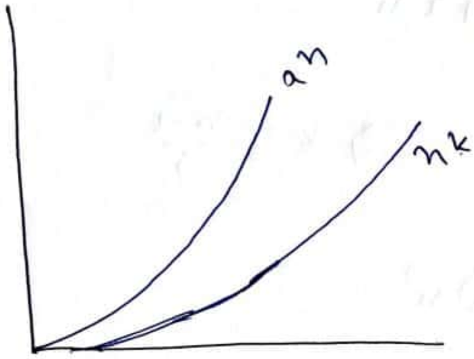
```
for (i=1 to n){
    for (j=1 ; j<=n ; j=j+i)
        printf ("*");
}
```

i	j	times
1	$1 \to n$	n
2	$1 \to n$	$\frac{n}{3}$
3	$1 \to n$	$n/3$
⋮	⋮	⋮
n	$1 \to n$	1
		$n \log n$

T.C. = $\cancel{O(n^2)}$ $O(n \log n)$

Q.10) Find asymptotic relation btw n^k & a^n, $k>=1$ & $a>1$ are constants, find c & n_0 for which relations holds.

Sol →



$$n^k = O(a^n)$$

$$n^k \le a^n, \quad c \;\forall\; c>0 \;\&\; n \geq n_0$$

let $n = n_0$

$$n_0^k \le c \cdot a^{n_0}$$

[so let $k = a = 3$]

[$n_0^3 \le c\, 3^{n_0}$ so $c \geq 1$ & $n_0 \geq 1$]

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Q.11 ⇒

```
void fun (int n){
    int i=0, j=1;
    while (i<n){
        i = i+j;
        j++;
    }}
```

Series ⇒ 0, 1, 3, 6, 10, 15 --

let at last iteration :

$n = 0 + 1 + 2 + 3 + 4 + 5 + --- + k$

$n = \frac{k(k+1)}{2}$

$n = \frac{k^2+1}{2}$

$n \cong k^2$

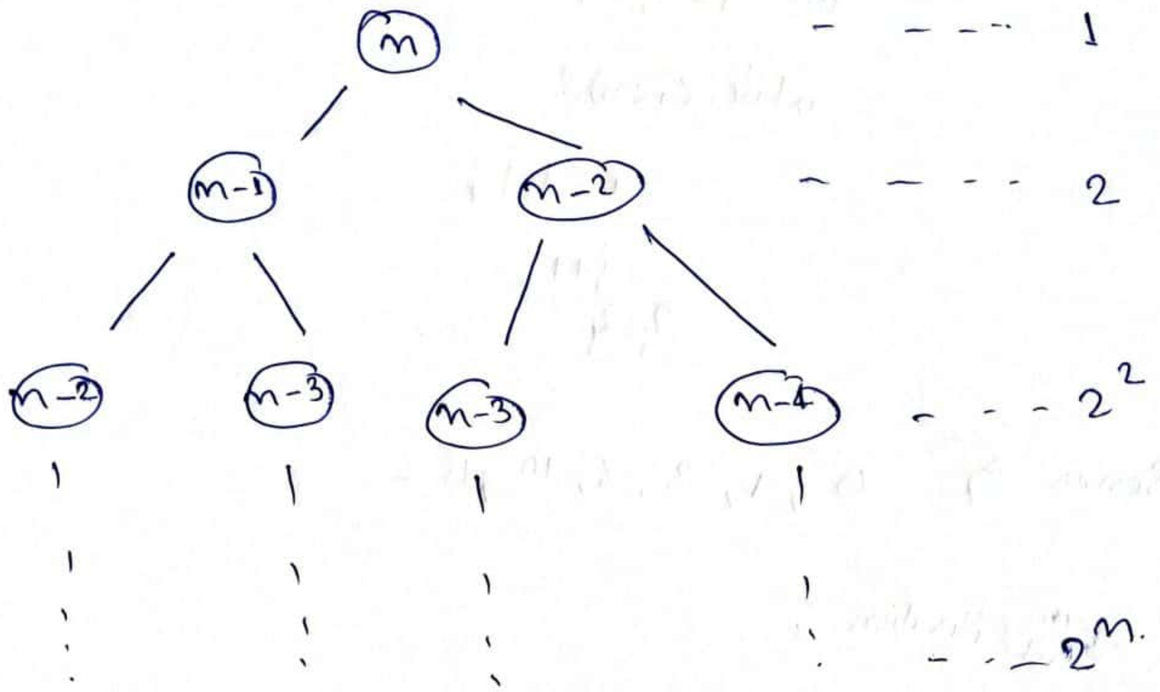
$k \cong \sqrt{n}$

so T.C. ⇒ $O(\sqrt{n})$.

Q.12 ⇒ Recurrence relation for fibonacci series..

$T(n) = T(n-1) + T(n-2) + 1$

using Recurrence tree method :



$$T.C. = 1 + 2 + 4 + \cdots + 2^n = \frac{1\,(2^{n+1} - 1)}{2-1} = 2^{n+1} - 1$$

so $T.C. = O(\cancel{n}\, 2^n)$

Space Complexity : Space complexity of fibonacci series using recursion is proportional to height of recurrence tree.

so $S.C. \rightarrow O(n)$

Q.13 ⇒ Write code for complexity.

(i) $n \log n$

```
for (i to n)
{
        for (j=1 , j<=n, j*=2)
                O(1) statements
}
```

(ii) n^3

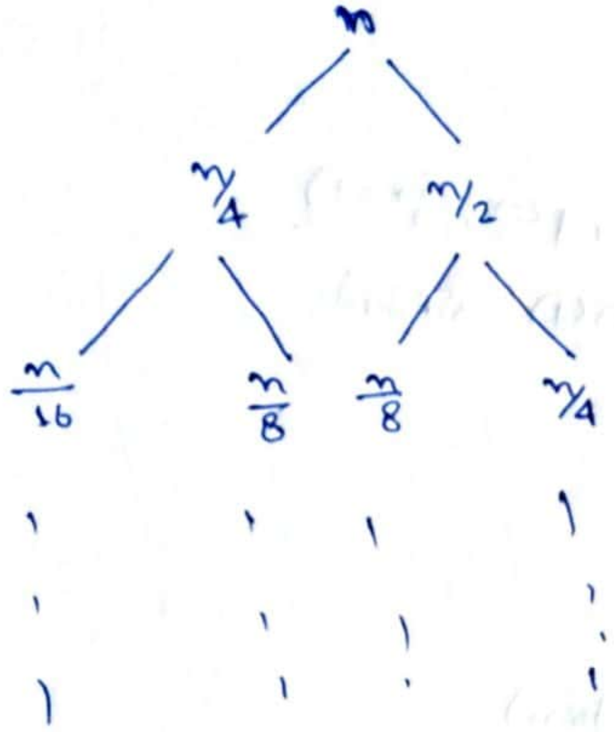
```
for (i to n)
    for (j to n)
        for (k to n)
            O(1) statements
```

(iii) $\log(\log n)$

~~for (int i=0; i<n;~~

```
int i=n;
while (i>0)
{   ---}
    ---}
    i = √i ;
}
```

Q.14) $T(n) = T(n/4) + T(n/2) + cn^2$



$\longrightarrow cn^2$

$\longrightarrow \frac{cn^2}{16} + \frac{cn^2}{4} = \frac{5cn^2}{16}$

$\longrightarrow \frac{cn^2}{256} + \frac{cn^2}{64} + \frac{cn^2}{64} + \frac{cn^2}{16}$

$= \frac{25cn^2}{256}$

so $T(n) = c(n^2 + \frac{5n^2}{16} + \frac{25n^2}{256} + \dots)$

here $r = \frac{5}{16}$ so $S_n = \frac{1}{1-r}$

$T(n) = cn^2(1 + \frac{5}{16} + \frac{25}{256} + \dots)$

$= cn^2 \left(\frac{1}{1-\frac{5}{16}}\right) = cn^2 \times \frac{16}{11}$

so T.C $\Rightarrow \Theta(n^2)$

Q.15 ⇒

```
int fun (int n)
{
    for (i to n)
        for (j=1 ; j<n ; j+=i) {
            O(1) task
}       }
```

i	j	times
1	$1 \to n$	$n-1$
2	$1 \to n$	$(n-1)/2$
3	$1 \to n$	$(n-1)/3$
⋮	⋮	⋮
n	$1 \to n$	$n-1/n$
		$n \log n$

$$\left[T.C. \Rightarrow O(n \log n) \right]$$

Q.16 ⇒

```
for (i=2 ; i<=n ; j=pow(i,k))
{
    O(1)
}
```

sol ⇒ Series = $2, 2^k, 2^{2k}, 2^{3k} \ldots,$

let last term be 2^{x*k}

$n = 2^{kx}$

$\log n = x \log 2^k$

Q.16 ⇒ for (int i = 2; i <= n; i = pow (i, k))

{ O(1);

}

$i = 2, 2^{k}, 2^{k^2}, 2^{k^3} \text{ ---- }, \cancel{2^k}\ 2^{k^x}$

$$n = 2^{k^x}$$

$$\log n = k^x \log 2$$

$$\frac{\log \log n}{\log 2} = x \log k$$

$$x = \frac{\log \log n}{\log 2 + \log k}$$

So T.C. ⇒ $O(\log \log n)$

Q.17 ⇒ ~~T(n) = T(n-1) + n~~ $T(n) = T\left(\frac{99n}{100}\right) + T\left(\frac{n}{100}\right)$

$$n$$

$$\frac{99n}{100} \qquad \frac{n}{100}$$

$$\frac{99^2 n}{100^2} \qquad \frac{99n}{100^2} \qquad \frac{99n}{100n^2} \qquad \frac{n}{100^2}$$

If we take longer branch i.e. $\frac{99n}{100}$

T.C. $\Rightarrow \log_{\frac{100}{99}} n \cong \log n$

$n = \left(\frac{99}{100}\right)^k$ $\qquad k = \log_{\frac{100}{99}} n$

$T(n) = n \left(\frac{\log \frac{100}{99}}{100}\right)^n = O(n \log_{99} n)$

Q18 ⇒ Increasing of growth.

(a) $100 < \log\log n < \log n < \sqrt{n} < n < n\log n < n^2 < 2^n < 2^{2n} < 4^n < n!$

(b) $1 < \log\log n < \sqrt{\log(n)} < \log n$ $\cancel{< 2n < 4n < 2(2^n) < \log}$ $< \log 2n < 2\log n < n < 2n < 4n < n^2\log n < n^2 < \log(n!) < 2^{2n} < n!$

(c) $96 < \log_8 n <$ $\cancel{n\log}$ $\log_2 n < 5n < n\log_8(n) < n\log_2 n < 8n^2 < 7n^3 <$ $\boxed{\cancel{8^n(2n) < \log(n!)} < n}$ $\log n! < 8^{2n} < n!$

Q.19 ⇒ Linear Search :

```
for (i=0 to k-1)
{   if (ar[i] = key)
        { return i;
        }
    return -1;
}
```

Q.20 ⇒ Iterative Insertion Sort :

~~void insertion_sort (arr, n)~~
~~Loop from i=1 to n-1~~
~~pick element ar[i] & insert it into sorted into sorted sequence.~~

```
void insertion_sort (int arr[], int n)
{   int i, temp, j;
    for i ← 1 to n
    {   temp ← arr[i];
        j ← i-1;
        while (j>=0 AND arr[j] > temp)
        {   arr[j+1] ← arr[j];
```

```
            j ← j-1;
        }
        arr[j+1] ← temp;
    }
}
```

Recursive Insertion sort →

```
void recursive_insertion_sort (int arr[], int n)
{       if (n<=1)
            return
        recursive_insertion_sort (arr, n-1)
        val = arr[n-1]
        pos = n-2
        while ( pos>=0 && arr[pos] > val) {
                arr[pos+1] = arr[pos]
                pos = pos-1
        }
        arr[pos+1] = val
}.
```

It is called online sorting because it provided ~~one~~ one sorted element at a time & ~~sequence of sorted as consider~~ produces a partial solution without considering future elements.

Q.21 =>

Algorithm	Time complexity: Best case	Average Case	Worst Case
(1) Bubble sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
(2) Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
(3) Merge sort	$O(n\log n)$	$O(n\log n)$	$O(n\log n)$
(4) Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
(5) Quick sort	$O(n\log n)$	$O(n\log n)$	$O(n^2)$
(6) Heap sort	$O(n\log n)$	$O(n\log n)$	$O(n\log n)$

Q.22 =>

Algorithm	Inplace	Stable	Online Sorting
Bubble Sort	✓	✓	✗
Selection Sort	✓	✗	✗
Merge Sort	✗	✓	✗
Insertion Sort	✓	✓	✓
Quick Sort	✗	✗	✗
Heap Sort	✓	✗	✗

Q.23 ⇒ Recursive Binary Search :

```
int b_search ( int arr [], int l, int r , int x ).

{   if (l > r)
        return -1;
    int m = (l+r) /2
    if (arr[m] = x)
        return m;
    else if (arr [m] < x)
        ~~return~~ b_search (arr, m+1, r, x);

    else
        b_search (arr, l, m-1, x

}
```

Iterative Binary Search :

```
int binarysearch (int arr [], int n, int x )
{
    l=0 , r= n-1;
    while ( l<r)
    {   m = (l+r) /2
        if (arr [m] = x)         return m;
        else if ( arr [m] < x)   l = m+1;
        else    r = m-1;
    }
```

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return -1 ;

}

Time & Space Complexity of Iterative Binary search ⇒ $O(\log n)$ & $O(1)$

Time & Space Complexity of Recursive Binary search ⇒ $O(\log n)$, $O(\log n)$

Q24⇒ Recurrence Relation for Binary Search ⇒

$$T(n) = T(n/2) + 1$$