EQ2401 Project 1

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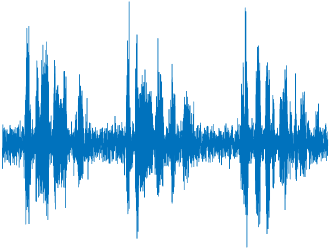
System model

- Noisy signal
- Sequences with only noise



$$y(n) = x(n) + e(n)$$
$$y(n) = e(n)$$

$$\begin{aligned} \mathrm{E}\{x(n)y(n-k)^*\} &= \mathrm{E}\{x(n)x(n-k)^*\} + \mathrm{E}\{x(n)e(n-k)^*\} \\ &= r_{xx}(k) \end{aligned}$$

- Use information in $e(n)$ to estimate $x(n)$ from $y(n)$

Wiener Filtering

FIR
$$\hat{x}_{FIR}(n) = \theta_{opt}^T Y$$
$$\theta_{opt} = \Sigma_{YY}^{-1} \Sigma_{XY}$$

Non-causal
$$H(z) = \frac{\Phi_{XY}(z)}{\Phi_{YY}(z)}$$

Causal
$$H(z) = \frac{1}{\Phi_{YY}^{+}(z)} \left\{ \frac{\Phi_{XY}(z)}{\Phi_{YY}^{-}(z)} \right\}_{+}$$

How do we get $\Sigma_{XY}, \Phi_{XY}(z)$?

AR-model

- $x(n)$ not directly observable
- Assume $e(n)$ wide-sense stationary
- Idea: Model $y(n)$, $e(n)$ as AR-processes

$$y(n) = \frac{1}{A(q)} w(n), \quad \mathrm{E}\{w(n)w(n)^*\} = \sigma^2$$

- Calculate autocorrelations r_{yy}, r_{ee}
- Estimate AR-parameters and variances

$$\hat{r}_{xx} = r_{yy} - r_{ee}$$

$$\hat{\sigma}^2 = r(0) - \sum_{i=1}^{N} a_i r(i)$$

Spectrum Estimation

- Calculate spectra from AR-models

$$\Phi(z) = \frac{\sigma^2}{A(z)A(z^{-1})}$$

- Straightforward implementation for Non-causal Wiener
- Spectral factor in Causal Wiener

$$\Phi^+(z) = \frac{\sigma}{A(z)}$$