1 Heighway Dragon

The Heighway Dragon is one of fractals, and we can describe it by a binary sequence [1]. This sequence is called "paperfolding sequence".

1.1 Paperfolding sequence

First, take a rectangular piece of paper and fold it in half lengthwise, then fold the result in half again, etc. Next, unfold the paper. The resulting sequence $(P_i)_{i\geq 0}$ of "hills" (1) and "valleys" (0) is a paperfolding sequence. For example, after one fold, we get the pattern in Figure. 1. After two folds, we get the pattern in Figure. 2. After ten folds, we get the pattern in Figure. 3.

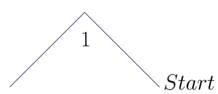

Fig. 1. Heighway Dragon: One Fold [1]

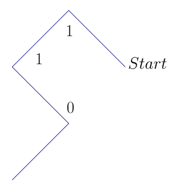

Fig. 2. Heighway Dragon: Two Folds [1]

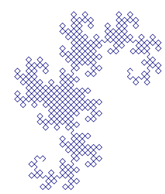

Fig. 3. Paperfolding Sequence: Ten Folds [1]

The paperfolding sequence is one of the automatic sequences. Automatic sequences are a sequence that can be generated by a deterministic finite automaton with output (DFAO). The Figure. 4 is a DFAO for paperfolding sequence. Each state on the DFAO has an alphabet of output. Let be n is a sequence of input. We compute P_n by feeding a finite-state automaton with the base-2 representation of n, starting with least significant bit, and then applying an output alphabet of the last state reached. Here are the first few terms of the limiting sequence P:

$$\begin{aligned} n &= \ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ldots \\ P_n &= \ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ldots \end{aligned}$$

We describe Heighway Dragon by using of P_n. For example, we denote "0" of P_n is "right", and "1" is "left." First, draw a line. Next, P_i is "1", so turn left, and draw the line. We get the pattern in Fig.1. P_2 is also 1, so turn left, and draw a line. P_3 is 0, so turn right, and draw a line. We get the pattern in Fig.2. We can elongate Heighway Dragon by repeating these process.

References

1. Jean-Paul Allouche and Jeffrey Shallit. *Automatic Sequences: Theory, Applications, Generalizations.* Cambridge University Press, 2003.

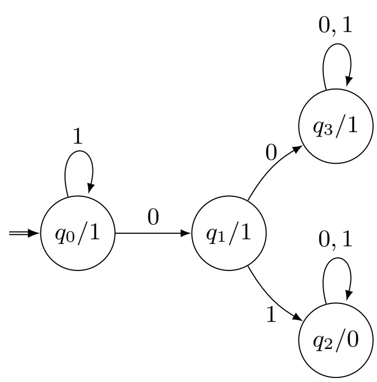

Fig. 4. DFAO for Paper folding sequence[1]