1 Heighway dragon

The Heighway dragon is one of fractals. It can be described by a binary sequence that is called "paperfolding sequence" [1].

1.1 Paperfolding sequence

The paperfolding sequence is named after the following operation to generate it. First, take a piece of paper and fold it in half lengthwise, then fold the result in half again, and so on. Next, unfold the paper. The resulting sequence $(P_i)_{i\geq 0}$ of "hills" (1) and "valleys" (0) is a paperfolding sequence. For example, after one fold, we get the pattern in Figure 1 (Left). After two folds and ten folds, we get the respective patterns in Figure 1 (Center) and (Right).

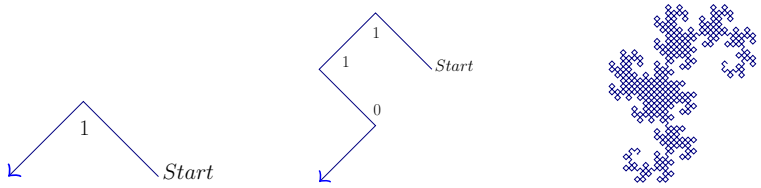

Fig. 1. (Left) Heighway Dragon: One Fold. (Center) Heighway Dragon: Two Folds. (Right) Paperfolding Sequence: Ten Folds [1].

The paperfolding sequence is a sequence that can be generated by the deterministic finite automaton with output (DFAO) in Figure 2 (automatic sequences). Each state of a DFAO is assigned with a letter of an alphabet as an output. Let "n" be index of P and, n is non-negative integer. We compute P_n by feeding the DAFO with the base-2 representation of n, starting with least significant bit, and then applying an output alphabet of the last state reached. Here are the first few terms of the limiting sequence.

$$n = \ 0\,1\,2\,3\,4\,5\,6\,7\,8\ldots$$
$$P_n = \ 1\,1\,0\,1\,1\,0\,0\,1\,1\ldots$$

P_n describes the Heighway Dragon as shown the exemplification in Figure 1. For example, let "0" of P_n be "right", and let "1" of P_n be "left." First, draw a line. Next, P_i is "1", so turn left, and draw the line, then the pattern in Figure 1 (Center) is obtained. P_2 is also 1, so turn left, and draw a line. P_3 is 0, so turn right, and draw a line, then the pattern in Figure 1 is obtained. We can elongate Heighway Dragon by repeating these process.

References

1. Jean-Paul Allouche and Jeffrey Shallit. *Automatic Sequences: Theory, Applications, Generalizations.* Cambridge University Press, 2003.

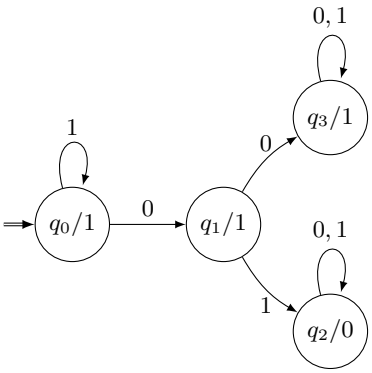

Fig. 2. DFAO for Paper folding sequence[1]