1 Heighway dragon

The Heighway dragon is one of fractals. It can be described by a binary sequence that is called "paperfolding sequence" [**?**].

1.1 Paperfolding sequence

The paperfolding sequence is named after the following operation to generate it. First, take a piece of paper and fold it in half lengthwise, then fold the result in half again, and so on. Next, unfold the paper. The resulting sequence $(P_i)_{i\geq 0}$ of "hills" (1) and "valleys" (0) is a paperfolding sequence. For example, after one fold, we get the pattern in Figure 1 (Left). After two folds and ten folds, we get the respective patterns in Figure 1 (Center) and (Right).

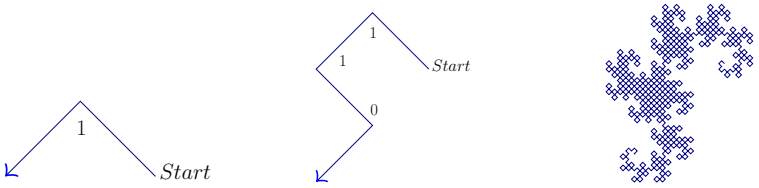

Fig. 1. (Left) Heighway dragon: One Fold. (Center) Heighway dragon: Two Folds. (Right) Paperfolding Sequence: Ten Folds [**?**].

The paperfolding sequence can be generated by the deterministic finite automaton with output (DFAO) in Figure 2 (automatic sequences). Each state of a DFAO is assigned with a letter of an alphabet as an output. For $n \geq 0$, n-th element of the sequence, i.e, P_n can be computed by feeding the DAFO with the base-2 representation of n, starting with the least significant bit, and then applying an output letter of the last state reached. Here are the first few terms of the limiting sequence. We can obtain the Heighway dragon by P_n, as shown in Figure 1.

$$\begin{aligned} n &= 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\cdots \\ P_n &= L\ L\ R\ L\ L\ R\ R\ L\ L\cdots \end{aligned}$$

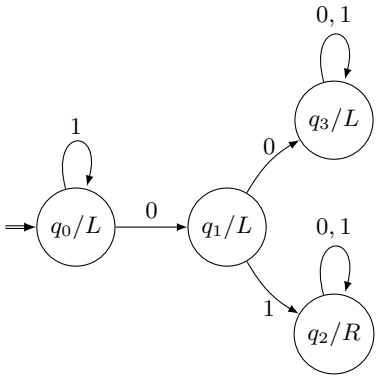

Fig. 2. DFAO for Paper folding sequence[**?**]