Bit bifurcation by cotranscriptional folding

May 1, 2017

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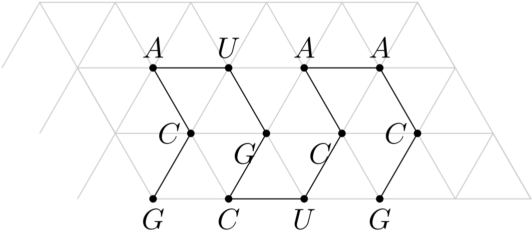

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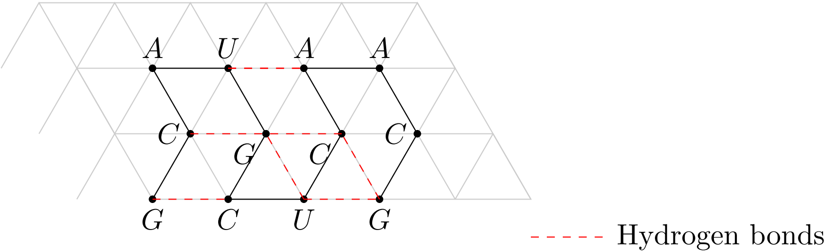
Ex). A primary structure $GCAAGCUCUACG$ may take this secondary structure.



Definition

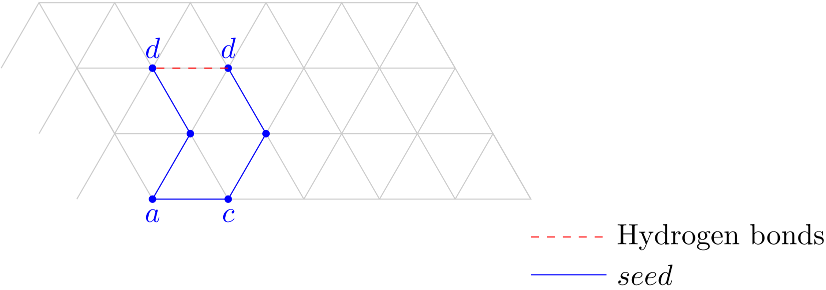
An oritatami system is a 6-tuple $\Xi = (\Sigma, \mathcal{H}, \alpha, \delta, \sigma, w)$, where

- w A primary sequence.
- $\mathcal{H} \subseteq \Sigma \times \Sigma$ Specifying between which types of beads can form a hydrogen-bond-based interaction.
- $\delta \geq 1$ delay time.
- σ An initial conformation called *seed*.
- $\alpha \in \mathbb{N}$ Beads can form at most α bonds.

Definition

Elongation

Ex.) $\mathcal{H} = \{(a,a),(b,b),(c,c),(d,d)\}$, delay time $\delta = 3$, primary structure $w = b \bullet ac \bullet bd \bullet c \cdots$.



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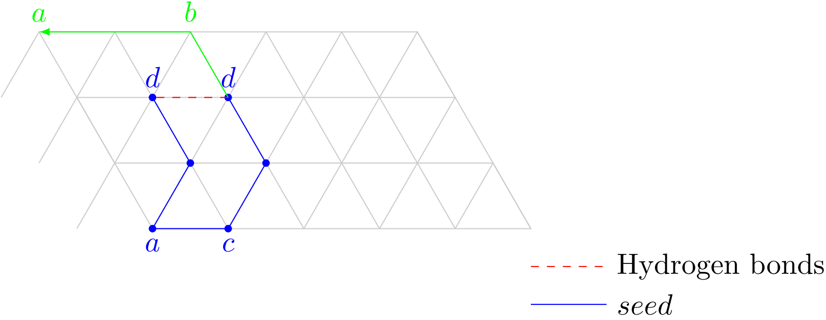

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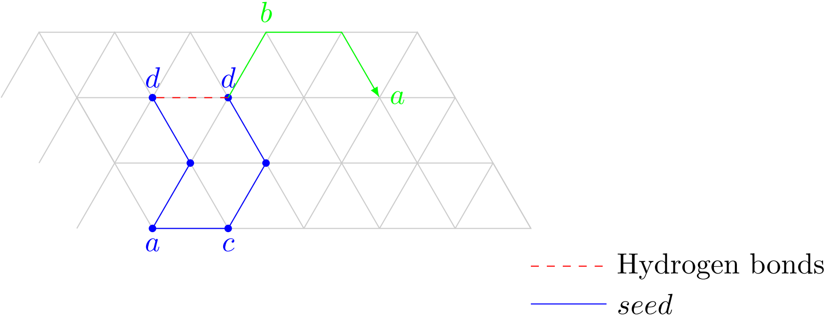

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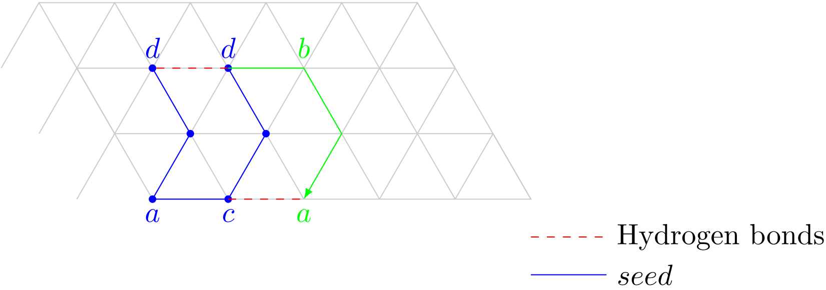

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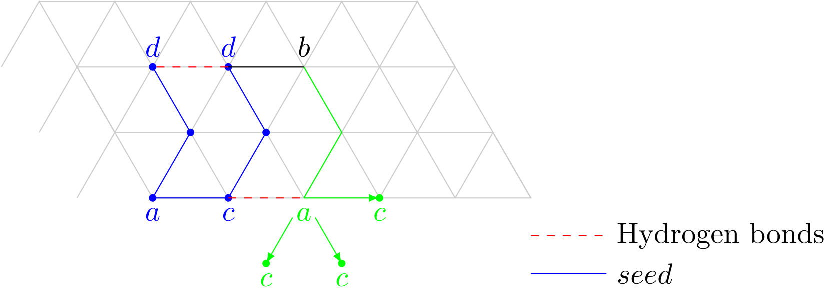

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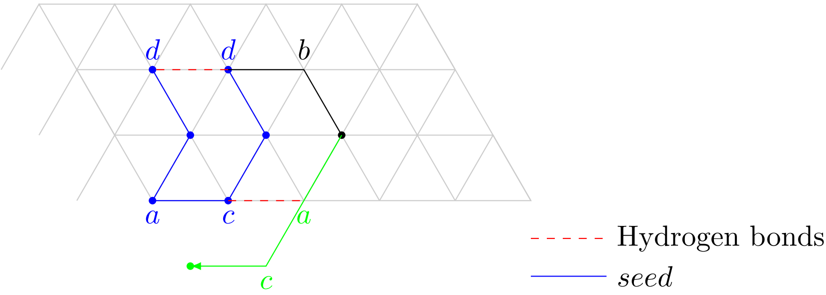

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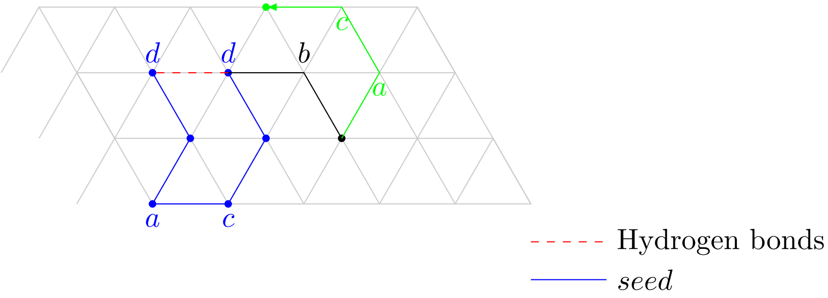

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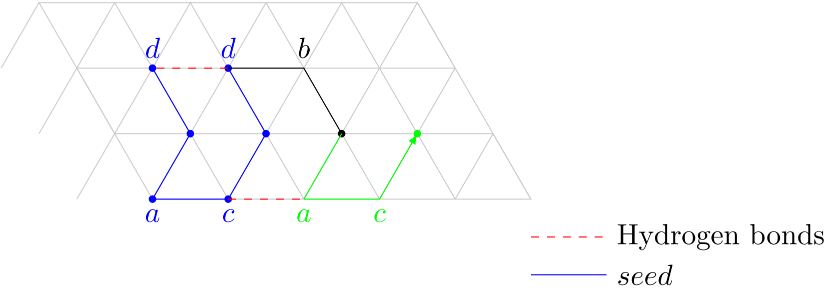

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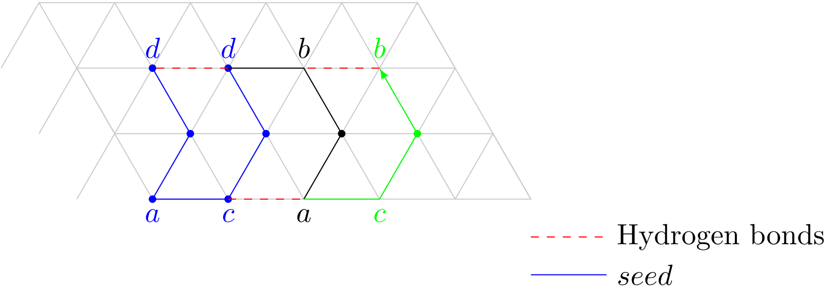

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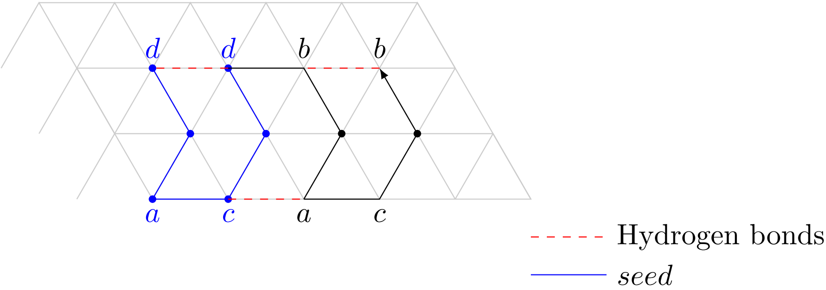


Binary counter

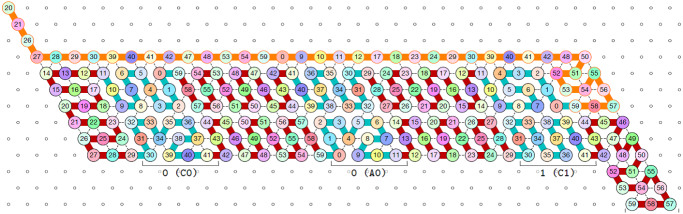


Figure: Oritatami binary counter.

Theme

We design an oritatami system that self-assembles an n-bit fraction of the Heighway dragon.

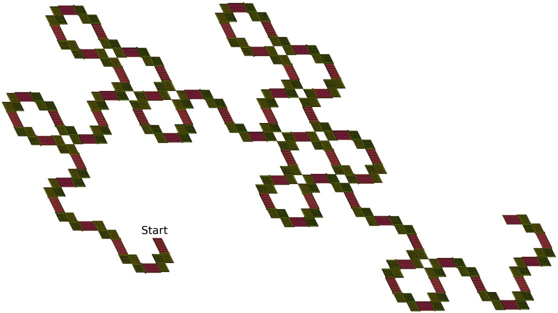


Figure: 6-bit Heighway dragon by the proposed oritatami system.

What is Heighway dragon?

Heighway dragon

The heighway dragon is a kind of self-similar fractal dragon curve. It can be described by a binary sequence that is called "paperfolding sequence".

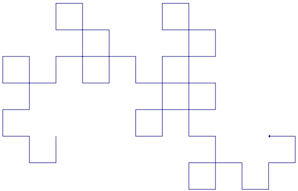


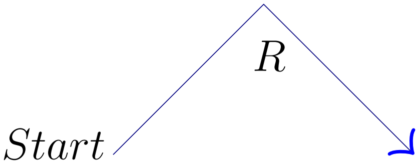
Figure: 6-bit Heighway dragon.

Paperfolding sequence

1-bit Heighway Dragon

Paperfolding sequence : P

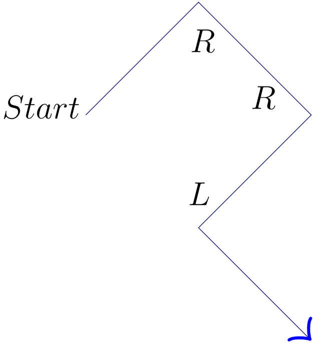
$$P = R$$



2-bit Heighway Dragon

Paperfolding sequence : P

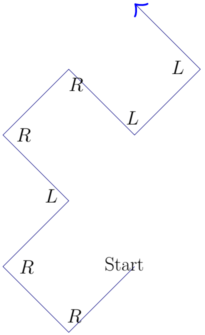
$$P \;=\; R\,R\,L$$



3-bit Heighway Dragon

Paperfolding sequence : P

$$P = R\,R\,L\,R\,R\,L\,L$$



6-bit Heighway Dragon

Paperfolding sequence : P

$$P = R\,R\,L\,R\,R\,L\,L\,\ldots$$

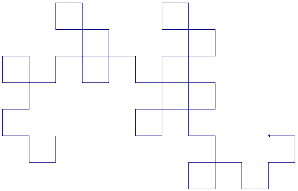


Figure: 6-bit Heighway dragon.

DFAO

The paperfolding sequence can be generated by the deterministic finite automaton with output (DFAO).

DFAO

Input : $i\ =\ 0\ 1\ 2\ 3\ 4\ 5\ \ldots$
Output : P_i (Paperfolding sequence)

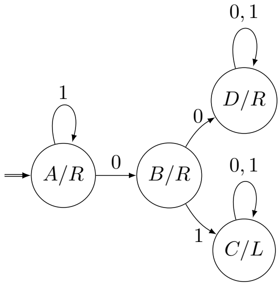


Figure: DFAO for Paperfolding sequence.

DFAO

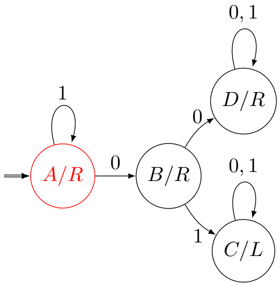


Figure: DFAO for Paperfolding sequence.

Paperfolding sequence

$$
\begin{aligned}
i &= \ 0\ 1\ 2\ 3\ 4\ 5\ \ldots \\
P_i &=
\end{aligned}
$$

$\ast$ Input the base-2 representation of i from its LSB. $0 \to 0$

DFAO

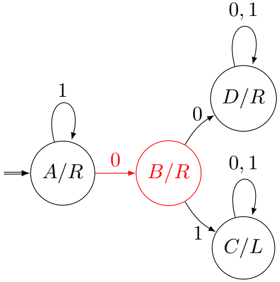


Figure: DFAO for Paperfolding sequence.

Paperfolding sequence

$$
\begin{array}{rcl}
i & = & {\color{red}0}\ 1\ 2\ 3\ 4\ 5\ \ldots \\
P_i & = & {\color{red}R}
\end{array}
$$

∗ Input the base-2 representation of i from its LSB. $0 \to 0$

DFAO

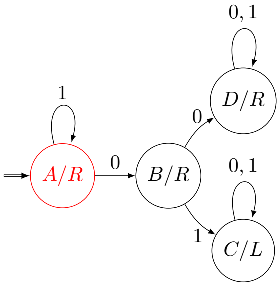


Figure: DFAO for Paperfolding sequence.

Paperfolding sequence

$$
\begin{aligned}
i &= 0\ 1\ 2\ 3\ 4\ 5\ \ldots \\
P_i &= R
\end{aligned}
$$

∗ Input the base-2 representation of i from its LSB. $1 \rightarrow 1$

DFAO

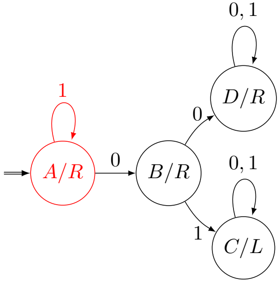


Figure: DFAO for Paperfolding sequence.

Paperfolding sequence

$$i \;=\; 0\ {\color{red}1}\ 2\ 3\ 4\ 5\ \ldots$$
$$P_i \;=\; R\ {\color{red}R}$$

∗ Input the base-2 representation of i from its LSB. $1 \to 1$

DFAO

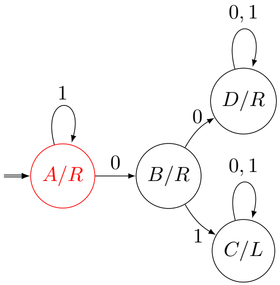


Figure: DFAO for Paperfolding sequence

Paperfolding sequence

$$
\begin{aligned}
i &= 0\ 1\ {\color{red}2}\ 3\ 4\ 5\ \ldots \\
P_i &= R\ R
\end{aligned}
$$

∗ Input the base-2 representation of i from its LSB. $2 \rightarrow 10$

DFAO

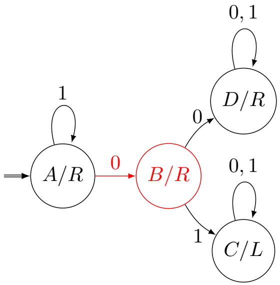


Figure: DFAO for Paperfolding sequence

Paperfolding sequence

$$i = 0\ 1\ 2\ 3\ 4\ 5\ \ldots$$
$$P_i = R\ R$$

∗ Input the base-2 representation of i from its LSB. $2 \to 10$

DFAO

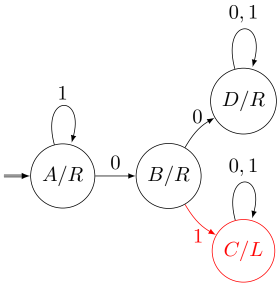


Figure: DFAO for Paperfolding sequence.

Paperfolding sequence

$$
\begin{aligned}
i &= 0\ 1\ 2\ 3\ 4\ 5\ \ldots \\
P_i &= R\ R\ L
\end{aligned}
$$

$*$ Input the base-2 representation of i from its LSB. $2 \to 10$

DFAO

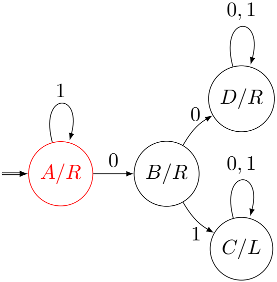


Figure: DFAO for Paperfolding sequence.

Paperfolding sequence

$$\begin{aligned} i &= 0\ 1\ 2\ \color{red}{3}\ 4\ 5\ \ldots \\ P_i &= R\ R\ L \end{aligned}$$

* Input the base-2 representation of i from its LSB. $3 \to 11$

DFAO

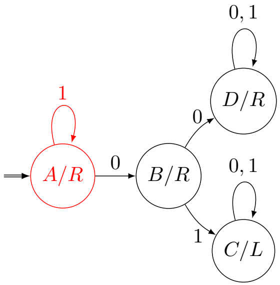


Figure: DFAO for Paperfolding sequence.

Paperfolding sequence

$$\begin{array}{rcl} i & = & 0\ 1\ 2\ 3\ 4\ 5\ \ldots \\ P_i & = & R\ R\ L\ R \end{array}$$

$\ast$ Input the base-2 representation of i from its LSB. $3 \rightarrow 11$

DFAO

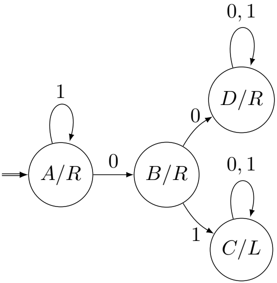


Figure: DFAO for Paperfolding sequence.

Paperfolding sequence

$$\begin{array}{rcl} i & = & 0\ 1\ 2\ 3\ 4\ 5\ 6 \ldots \\ P_i & = & R\ R\ L\ R\ R\ L\ L \ldots \end{array}$$

Implementation

The paperfolding sequence can be generated by the deterministic finite automaton with output (DFAO).