Purdue University
West Lafaywtte
CS 182

Ahmed Mahmood
Summer 2017

Assignment 6 Model Solutions

Due: Thursday, July 27, 2017, upload before 11:30pm

1) (20 pts.): Which of these relations on the set of all people are equivalence relations?
Determine the properties of an equivalence relation that the others lack.

1. $\{(a, b) \ \mid a$ and b are the same age$\}$
2. $\{(a, b) \ \mid a$ and b have the same parents$\}$
3. $\{(a, b) \ \mid a$ and b share a common parent$\}$
4. $\{(a, b) \ \mid a$ and b have met$\}$

Graded by : Sneha Balasubramanian
Solution:
(1) $\{(a, b) \ \mid a$ and b are the same age$\}$
This relation is
Reflexive
Symmetric :if a is same age as b, b is also same age as a.
Transitive : if a is as old as b and b is as old as c, then a is as old as c.
Thus, it is an equivalence relation.

(2) $\{(a, b) \ \mid a$ and b have the same parents$\}$
This relation is
Reflexive
Symmetric :if a has same parents as b, b also has same parents as a.
Transitive : if a has same parents as b and b has same parents as c, then a has same parents as c.
Thus, it is an equivalence relation.

(3) $\{(a, b) \ \mid a$ and b share a common parent$\}$
This relation is
Reflexive
Symmetric :if a has common parents as b, b also has common parents as a.
Not Transitive : if a has common parents as b and b has common parents as c, then a does not necessarily need to have common parents as c.
Thus, it is not an equivalence relation.

(4) $\{(a, b) \ \mid a$ and b have met$\}$
This relation is

Reflexive

Symmetric :if a has met b, b also has met a.

Not Transitive : if a has met b and b has met c, then a does not necessarily need to have met c.

Thus, it is not an equivalence relation.

2) (10 pts.): Which of these relations on the set of all people are partial orderings?

Determine the properties of a partial ordering that the others lack.

Graded by: Ramya Vulimiri

1. $\{(a, b) \ \mid a$ is no shorter than $b\}$
   Solution:
   This is not a poset as it is not anti-symmetric. Consider that we have a person a and a person b and $a \neq b$, then the order pairs (a, b) and (b, a) can exist in the relation because we can have a and b be the same height.

2. $\{(a, b) \ \mid a$ and b do not have a common friend$\}$
   Solution:
   This is not a poset as it is not reflexive, anti-symmetric or transitive. (a, a) does not belong to the relation since it doesn't make sense to not have common friends with yourself(not reflexive). Consider that we have a person a and a person b, then the order pairs (a, b) and (b, a) can exist in the relation because they don't have common friends but $a \neq b$(not anti-symmetric). If (a, b) and (b, c) belong to the relation, it doesn't necessarily imply that a and c do not have common friends and that (a, c) belongs to the relation(not transitive).

Solution 3) (10 points) Graded by Rashmi Soni

Solve $a_n = 7a_{n-1} - 12a_{n-2}$ with $a_0 = 4$ and $a_1 = 9$

The characteristic equation of the recurrence relation is

$r^2 - 7r + 12 = 0$

$(r - 4)(r - 3) = 0$

$r = 4, 3$

Thus, the roots of the characteristic equation are r_1=4 and r_2=3. Hence the sequence $\{a_n\}$ is a solution to the recurrence relation if and only if

$a_n = \alpha_1(r_1)^n + \alpha_2(r_2)^n$

Substitute the values of r_1 and r_2 :

$a_n = \alpha_1 4^n + \alpha_2 3^n$ ...eq (1)

Now, from the given initial conditions :

Put n=0 in eq(1), we get

$a_0 = \alpha_1 4^0 + \alpha_2 3^0$

$4 = \alpha_1 + \alpha_2$ ...eq (2)

Now, put n=1 in eq(1), we get

$a_1 = \alpha_1 4^1 + \alpha_2 3^1$

$9 = 4\alpha_1 + 3\alpha_2$ ...eq (3)

So, multiplying eq(2) by 4 and then subtracting from eq(3), we get

$\alpha_2 = 7$

Thus, $\alpha_1 = -3$

Substituting in eq(1), we get

$a_n = (-3)4^n + 7.3^n$

Solution 4) (10 points) Graded by Rashmi Soni

Finding sum-of-products expansions of the following Boolean functions :

a) F(x,y,z) = x + y + z

Creating table for F(x,y,z) for all possible values of the variables x,y,z :

x	y	z	x+y+z
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

Thus, the sum-of-products expansions for F(x,y,z) is the sum of the minterms corresponding to the rows of this table that give the value of 1 for the function.Thus,

$F(x, y, z) = xyz + xy\overline{z} + x\overline{y}z + x\overline{y}\overline{z} + \overline{x}yz + \overline{x}y\overline{z} + \overline{x}\overline{y}z$

b) F(x,y,z) = (x + z)y

Creating table for F(x,y,z) for all possible values of the variables x,y,z :

x	y	z	x+z	(x + z)y
1	1	1	1	1
1	1	0	1	1
1	0	1	1	0
1	0	0	1	0
0	1	1	1	1
0	1	0	0	0
0	0	1	1	0
0	0	0	0	0

Thus, the sum-of-products expansions for F(x,y,z) is the sum of the minterms corresponding to the rows of this table that give the value of 1 for the function.Thus,

$F(x, y, z) = xyz + xy\overline{z} + \overline{x}yz$

c) F(x,y,z) = x

Creating table for F(x,y,z) for all possible values of the variables x,y,z :

x	y	z	x
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

Thus, the sum-of-products expansions for F(x,y,z) is the sum of the minterms corresponding to the rows of this table that give the value of 1 for the function.Thus,
$F(x, y, z) = xyz + xy\overline{z} + x\overline{y}z + x\overline{y}\overline{z}$

d) F(x,y,z) = $x\overline{y}$
Creating table for F(x,y,z) for all possible values of the variables x,y,z :

x	y	z	$\overline{y}$	$x\overline{y}$
1	1	1	0	0
1	1	0	0	0
1	0	1	1	1
1	0	0	1	1
0	1	1	0	0
0	1	0	0	0
0	0	1	1	0
0	0	0	1	0

Thus, the sum-of-products expansions for F(x,y,z) is the sum of the minterms corresponding to the rows of this table that give the value of 1 for the function.Thus,
$F(x, y, z) = x\overline{y}z + x\overline{y}\overline{z}$

5) (10 pts.) Which is more likely: rolling a total of 8 when 2 dice are thrown or rolling a total of 8 when 3 dice are thrown?
Graded by: Ramya Vulimiri
Solution: Let us calculate the probability for each case. For two dice, a total of 8 can be reached in 5 cases: (6, 2),(5, 3),(4, 4),(3, 5),(2, 6), so the probability here is $\frac{5}{6^2} = \frac{5}{36}$.
For three dice, a total of 8 can be reached in 21 cases: 6 permutations each of (5, 2, 1), (4, 3, 1), and 3 permutations each of (1, 1, 6), (2, 2, 4), (3, 3, 2). The probability here is $\frac{21}{6^3} = \frac{21}{216}$.
$\frac{5}{36} > \frac{21}{216} \implies$ it is more likely to get 8 with two dice.