Purdue University
West Lafayette
CS 182

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Summer 2017

Assignment 1 Solution

Due: Tuesday, June 20, 2017, upload before 11:30pm

1) (5 pts.) Graded by : Sneha Balasubramanian

Housekeeping tasks

2) (20 pts.) Graded by : Sneha Balasubramanian

Do Exercise 31 of Chapter 1.1 (page 15).

Solution:

A) P$\wedge$ $\neg$P

p	$\neg$p	p$\wedge$ $\neg$p
T	F	F
F	T	F

B) p$\vee$ $\neg$p

p	$\neg$p	p$\wedge$ $\neg$p
T	F	T
F	T	T

C) (p$\vee$ $\neg$q) $\rightarrow$ q

p	q	$\neg$q	(p$\vee$ $\neg$q)	(p$\vee$ $\neg$q) $\rightarrow$ q
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

D) (p$\vee$q) $\rightarrow$ (p$\wedge$q)

p	q	p$\vee$q	(p$\wedge$q)	(p$\vee$q) $\rightarrow$ (p$\wedge$q)
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

E) (p$\rightarrow$q) $\leftrightarrow$ ($\neg$q $\rightarrow$ $\neg$p)

p	q	¬p	¬q	(p→q)	(¬q → ¬p)	(p→q) ↔ (¬q → ¬p)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

F) (p→q) → (q→p)

p	q	(p→q)	(q→p)	(p→q) → (q→p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

3) (25 pts.) Do Exercises 19, 21, and 23 of Chapter 1.2 (page 23).

Graded by : Ramya Vulimiri

Let p and q be the statements such that

p: A is a knight q: B is a knight

Then, $\neg p$: A is a knave $\neg q$: B is a knave

19) A: At least one of us is a knave   B: Nothing

Let us first consider the possibility that A is a knight; this is the statement that p is true. If A is a knight, then he is telling the truth when he says at least one of them is a knave, so the statement $(p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ is true. The only possibility consistent with our assumption p is true is that $\neg q$ is true. So one conclusion we can draw is that **A is a knight and B is a knave**.

If A is a knave($\neg p$ is true), then because everything a knave says is false, A's statement that at least one of them is a knave is false, which means that neither of them are knaves(the statement $(p \wedge q)$ is true). This is not consistent with our assumption.

21) A: $\neg p \vee q$ B: Nothing

Let us first consider the possibility that A is a knight; this is the statement that p is true. If A is a knight, then the statement $(\neg p \vee q)$ is true. That leads to 3 possibilities: (i) $\neg p$ is true, meaning A is a knave; inconsistent with assumption. (ii) q is true, meaning B is a knight. (iii) $\neg p$ is true and q is true: inconsistent from reasoning in (i).

If A is a knave, he lies and $\neg(\neg p \vee q)$ is true, implying $(p \wedge \neg q)$ is true, which means A is a knight and B is a knave, again inconsistent with our assumption that A is a knave.

From (ii), A is a knight and B is a knight.

23) A: $\neg p \wedge \neg q$ B: Nothing

Let us first consider the possibility that A is a knight; this is the statement that p is true. If A is a knight, then the statement $(\neg p \wedge \neg q)$ is true, which means $\neg p$ should be true. Inconsistent.

If A is a knave($\neg p$ is true), the negation of his statement is true, $\neg(\neg p \wedge \neg q)$ is true, that is, $(p \vee q)$ is true. Again we have 3 possibilities, (i) p is true: Inconsistent. (ii) q is true: A is a knave and B is a knight. (iii) Both p and q are true: Inconsistent as in (i).

Hence, A is a knave and B is a knight.

4) (25 pts.) Graded by Rashmi Soni

Do Exercises 50 and 51 of Chapter 1.3 (page 36).

Solution:

50)

a) $(p \downarrow p)$

p	$p \downarrow p$
T	F
F	T

And, $(\neg p)$

p	$\neg p$
T	F
F	T

Since, the truth values of the compound propositions $(p \downarrow p)$ and $(\neg p)$ agree for all possible combinations of the truth values of p, they are logically equivalent.

b) $(p \downarrow q) \downarrow (p \downarrow q)$

p	q	$p \downarrow q$	$(p \downarrow q) \downarrow (p \downarrow q)$
T	T	F	T
T	F	F	T
F	T	F	T
F	F	T	F

And, $(p \vee q)$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Since, the truth values of the compound propositions $((p \downarrow q) \downarrow (p \downarrow q))$ and $(p \vee q)$ agree for all possible combinations of the truth values of p and q, they are logically equivalent.

c) The set $\{\neg, \wedge\}$ is a functionally complete collection of logical operators. Similarly, the set $(\neg, \vee)$ is a functionally complete collection because $\wedge$ can be defined in terms of $\vee$ as:

$p \wedge q \equiv \neg (\neg p \vee \neg q)$

From the results of parts (a), (b), and Exercise 49, both $\neg$ and $\vee$ can be represented by $\downarrow$ because :

$\neg p \equiv p \downarrow p$

$p \vee q \equiv (p \downarrow q) \downarrow (p \downarrow q)$

Thus, we can conclude that the set $\{\downarrow\}$ is a functionally complete collection of logical operators.

51)

p	q	p → q	p ↓ p	(p ↓ p) ↓ q	((p ↓ p) ↓ q) ↓ ((p ↓ p) ↓ q)
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	F	T

Thus, the truth values of the compound propositions $(p \rightarrow q)$ and $((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$ agree for all possible combinations of the truth values of p and q, they are logically equivalent.

Also using logical equivalences, we know that $(p \rightarrow q) \equiv (\neg p \vee q) \equiv ((p \downarrow p) \vee q) \equiv ((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$