CS 18200

MIDTERM EXAM MODEL SOLUTIONS

Monday, July 10, 2017

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Solution 1) a) (7 points)Graded by: Rashmi Soni

Let's consider this proof by cases:

Let n be a positive integer. All the perfect cubes of n which are less than equal to 100 are 1, 8, 27, 64 such that:

when n = 1, n3 = 1, n2 = 1

when n = 2, n3 = 8, n2 = 4

when n = 3, n3 = 27, n2 = 9

when n = 4, n3 = 64, n2 = 16

So,

for n = 1, n2 + n3 = 2

for n = 2, n2 + n3 = 12

for n = 3, n2 + n3 = 36

for n = 4, n2 + n3 = 80

Thus, there is no positive integer n such that n2 + n3 = 100.

Solution 1) b)(5 points) Graded by: Rashmi Soni

Given, $A(x, y) := x$ is angry at y, $F(x) := x$ is friendly, $T(x) := x$ is tall, and the domain for all quantifiers consists of all people

So, $\forall x((T(x) \wedge \neg F(x) \rightarrow \exists y(A(x, y) \wedge T(y) \wedge (x \neq y)))$ translates as

For all people, if a person is tall and not friendly, then there exists another tall person on whom he is angry on.

Or, If a tall person is not friendly, then he/she is angry on another tall person.

Solution 1) c) (7 points) Graded by: Rashmi Soni

Solve

a_n=$a_{n-1} + 2n + 4$, $a_0 = 4$ ...$eq(1)$

Substituting n with (n-1) in equation 1, we get

a_{n-1}=$a_{n-2} + 2(n-1) + 4$ ...$eq(2)$

Now, Substituting n with (n-2) in equation 1, we get

a_{n-2}=$a_{n-3} + 2(n-2) + 4$ ...$eq(3)$

Similarly,

a_{n-3}=$a_{n-4} + 2(n-3) + 4$ ...$eq(4)$

a_{n-4}=$a_{n-5} + 2(n-4) + 4$ ...$eq(5)$

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a_2=$a_1 + 2(2) + 4$ ...$eq(n-1)$

a_1=$a_0 + 2(1) + 4$ ...$eq(n)$

Adding up all the above equations, we get

a_n=a_0+2[n + (n-1) + (n-2) + ... + 1] + 4n

a_n=$a_0 + 2\left(\frac{n(n+1)}{2}\right)$ + 4n

a_n=4+2 $\left(\frac{n(n+1)}{2}\right) + 4n$ $\{Given, a_0 = 4\}$

$a_n = n^2 + 5n + 4$

(d) (7 points) Use the rules of inference to show that the following argument is valid:

$$\begin{array}{ll} & (w \wedge a) \rightarrow p \\ & \neg w \rightarrow m \\ & \neg a \rightarrow i \\ & e \rightarrow (\neg m \wedge \neg i) \\ & \neg p \\ \hline \therefore & \neg e \end{array}$$

Graded by: Ramya Vulimiri

Solution:

$$\begin{array}{rll} 1. & (w \wedge a) \rightarrow p & \textit{Premise} \\ 2. & \neg w \rightarrow m & \textit{Premise} \\ 3. & \neg a \rightarrow i & \textit{Premise} \\ 4. & e \rightarrow (\neg m \wedge \neg i) & \textit{Premise} \\ 5. & \neg p & \textit{Premise} \\ 6. & \neg(w \wedge a) & \textit{Modus Tollens (1,5)} \\ 7. & \neg w \vee \neg a & \textit{DeMorgan's law (6)} \\ 8. & w \vee m & \textit{Equivalent to 2} \\ 9. & a \vee i & \textit{Equivalent to 3} \\ 10. & \neg a \vee m & \textit{Resolution (7,8)} \\ 11. & m \vee i & \textit{Resolution (9,10)} \\ 12. & \neg e \vee (\neg m \wedge \neg i) & \textit{Equivalent to 4} \\ \hline \therefore & \neg e & \textit{Disjunctive Syllogism (11,12)} \end{array}$$

(e) (7 points) Prove by structural induction that $n(T) \geq 2h(T) + 1$ for any full binary tree T, where $n(T)$ is the number of nodes (vertices) in the full binary tree T and $h(T)$ is the height of the full binary tree T.

Graded by: Ramya Vulimiri

Solution:

BASIS STEP: For the full binary tree consisting of just the root r, the result is true because $n(T) = 1$ and $h(T) = 0$, so that $n(T) = 1 \geq 2h(T) + 1 = 2(0) + 1 = 1$.

RECURSIVE STEP: For the inductive hypothesis, we assume that $n(T_1) \geq 2h(T_1) + 1$ and $n(T_2) \geq 2h(T_2) + 1$. We have to prove that $n(T) \geq 2h(T) + 1$, where T is a new full binary tree formed by a new root node r, left subtree T_1 and right subtree T_2.

We know $h(T) = max(h(T_1), h(T_2)) + 1$ and
$n(T) = n(T_1) + n(T_2) + 1$
We find that

$$\begin{aligned}
n(T) &= n(T_1) + n(T_2) + 1 && (1)\\
&\geq (2h(T_1) + 1) + (2h(T_2) + 1) + 1 && (2)\\
&= 2(h(T_1) + h(T_2)) + 3 && (3)\\
&\geq 2(max(h(T_1), h(T_2))) + 3 && (4)\\
&= 2(max(h(T_1), h(T_2)) + 1) + 1 && (5)\\
&= 2(h(T) + 1) && (6)
\end{aligned}$$

$\therefore n(T) \geq 2h(T) + 1$

Solution 2) a) (5 points)Graded by: Rashmi Soni

Final order so that each function is big-O of next function :

$log\sqrt{n^3},\ 3n+5,\ 3nlogn,\ 6n(logn)^2+5n\sqrt{n},\ 20n^2,\ (\frac{n}{logn})^3$

(b) (5 points) Give an efficient algorithm to find the second largest element in an unsorted list.

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Solution:

```
procedure FindSecondMax(Array A)                    ▷ Array indices 0 .. n-1
    first = second = A[0]
    for i = 1 .. n − 1 do
        if A[i] > first then
            second = first
            first = A[i]
        else if A[i] < first && A[i] > second then
            second = A[i]
    return second
```

(c) (5 points) Show that if A, B, and C are sets, then $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$. Graded by: Ramya Vulimiri

Solution:

We can use the fact that set union is associative and the result that $|A \cup B| = |A| + |B| - |A \cap B|$.

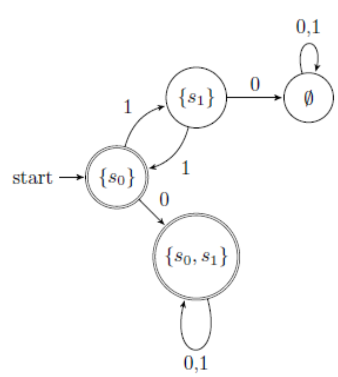
$$\begin{aligned} |A \cup B \cup C| &= |A \cup (B \cup C)| && (7)\\ &= |A| + |(B \cup C)| - |A \cap (B \cup C)| && (8)\\ &= |A| + |B| + |C| - |(B \cap C)| - |(A \cap B) \cup (A \cap C)| && (9)\\ &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|. && (10) \end{aligned}$$

(d) (8 points)

Consider this nondeterministic finite-state automaton:

1. Construct a deterministic finite-state automaton that recognizes the same language.

   Solution:



2. Represent the language that the automaton recognizes using regular expressions.
   Solution: $(11)^* \cup ((11)^*0(0 \cup 1)^*)$

(e) (5 points) Express -88_{10} in signed 8-bit 2's complement binary

Solution:

Converting to Binary:

$88/2 = 44$, remainder is 0

$44/2 = 22$, remainder is 0

$22/2 = 11$, remainder is 0

$11/2 = 5$, remainder is 1

$5/2 = 2$, remainder is 1

$2/2 = 1$, remainder is 0

$1/2 = 0$, remainder is 1

Thus, 88 = 0101 1000

ones complement = 1010 0111

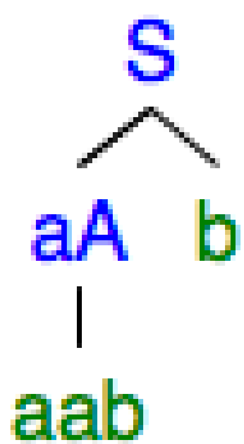
two's complement = 1010 1000

Ans: 1010 1000

(f) (5 points) Give the derivation tree and the Backus-Naur Form of the grammar in MCQ [7].

Solution:

Derivation tree:



The BNF form is:

S:=a

S:= aA— aab

S:= b

Note: The bubble in section of the midterm has 17 questions to bubble in (each is worth 2 points).

1. I have spelled out and bubbled in correctly my first name, last name, and my Purdue ID.

   A True

   B False

2. NFAs are more powerful than DFAs.

   A True

   B False

   Solution: B

3. The halting problem has an exponential time algorithm

   A True

   B False

   Solution: B

4. The least big-O estimate for $f(n) = \dfrac{(3n^3 + n^2)(n+4)}{5n^3 + 10n^2}$ is .

   A $O(n^3)$

   B $O(nlogn)$

   C $O(n^n)$

   D $O(n^2)$

   E None of the above

   Solution: E

5. Which problem can be solved using a finite state automata.

   A Sorting

   B Finding the nth Fibonacci number

   C Halting

   D Matching regular expression

   E None of the above

   Solution: D

6. Finite state machines have infinite memory

   A True

   B False

   Solution: B

7. The language of this grammar is $G = (V, T, S, P)$, $V = \{S, A, a, b\}$, $T = \{a, b\}$, $P = \{S \rightarrow aA, S \rightarrow b, A \rightarrow ab\}$

   A $\{b, aaa\}$

   B $\{b, aab\}$

   C $\{ab\}$

   D None of the above

   Solution: B

8. There are infinite number systems

   A True

   B False

   Solution: A

9. All integers (positive and negative) can be represented uniquely using a sufficient number of digits under which binary representation?

   A Signed magnitude

   B One's complement

   C Two's complement

   D Octal system

   E None of the above

   Solution: C

10. There are infinite prime numbers

    A True

    B False

    Solution: A

11. The value of gcd(91,14) is

A 14

B 7

C 6

D 91

E None of the above

Solution: B

12. Which of the following numbers is congruent to 5 modulo 6

A 0

B 2

C 6

D 17

E None of the above

Solution: D

13. Array A contains n integers. You know that the array contains only two numbers that are not in proper order. Which sorting algorithm achieves $O(n)$ time on all input sequences with two numbers that are not in proper order?

A Bubble sort

B Mergesort

C Insertion sort

D None of the above

Solution: C

14. $\sum_{i=0}^{n-1}(2i+1)$ equals

A n

B n^2

C $n^2 - 1$

D None of the above

Solution: B

15. The range of a function is equal to its co-domain in which function type

    A Injective

    B Surjective

    C Bijective

    D Partial

    Solution: B

16. The function $f(x)= x^2 - x$,from N to N, is an injection.

    A True

    B False

    Solution: B

17. Recursive algorithms are always faster than iterative algorithms.

    A True

    B False

    Solution: B