Which of the following are propositions?

(a) $x + y < 10$.

(b) $x + y < 10$ has 55 nonnegative integer solutions.

(c) There is life on other planets.

Translate these statements into logical expressions.

(a) All birds that are not penguins fly.

(b) There are infinitely many primes.

Which of the following are logically equivalent to $p \oplus q$?

(a) $(q \to p) \to (\neg q \to \neg p)$

(b) $(q \to p) \to \neg(p \to q)$

(c) $(\neg q \to \neg p) \to \neg(\neg p \to \neg q)$

Resolution states that
$$\begin{array}{l} p \vee q \\ p \vee \neg q \\ \hline \therefore\ p \end{array}$$
Prove that resolution is a valid rule of inference.

Let $f : \mathbb{Z}^+ \to \mathbb{Z}$ be defined as $f(n) = \begin{cases} n/2 & n \text{ even} \\ (1-n)/2 & n \text{ odd} \end{cases}$
where $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ is the set of positive integers.

Is f a one-to-one correspondence?
If so, prove that it is and then find the inverse function f^{-1}.
If not, prove that it is not.

A geometric sequence contains the terms $a_3 = 72$ and $a_6 = 243$.
Determine the common ratio r.

Use mathematical induction to prove that

$$\sum_{i=0}^{n-1} \frac{i}{2^i} = 2 - \frac{n+1}{2^{n-1}}$$

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1. $\{8, 6, 7, 5, 3, 0, 9\} \cup \{1, 3, 5, 7, 9\} = \ ?$

2. $\{8, 6, 7, 5, 3, 0, 9\} \cap \{0, 2, 4, 6, 8\} = \ ?$

3. $\{8, 6, 7, 5, 3, 0, 9\} - \{3, 6, 9\} = \ ?$

4. $\{1, 2, 3\} \times \{a, b\} = \ ?$

Give a big-O estimate for $f(x) = \dfrac{(x^4 + 3x^2)(x-1)^2}{2x^5}$.

Order these functions so that each is big-Ω of the next:

$$(\log n)^3 \ , \ n^3/10^6 \ , \ \sqrt{n} \ , \ 100n + 101 \ , \ 3^n \ , \ n! \ , \ 2^n n^2$$

Consider the automaton shown below.

(a) Give an English description of the language accepted by the automaton.

(b) Give a regular expression for the language accepted by the automaton.