

### Assignment 3

Due: Wednesday, July 5, 2017, upload before 11:30pm

1) (20 pts.) Do Exercises 40 and 50 of Section 5.1 (page 331).

2) (20 pts.) Do Exercises 38 and 40 of Section 5.4 (page 371).

3) (20 pts.) A student in a class is **popular** if this student is followed by every other student on social media, but follows none of them. There is at most one popular at a class<sup>1</sup>. Your task is to find the popular student, if one exists, at a class by asking only one type of question asking a student if he follows another student. Every student must answer your questions correctly and honestly. If Johan and Smith are two student in a class, you can ask Johan whether he follows Smith; He must answer correctly. Use induction to show that if there are  $n$  students in a class, then you can find the popular student, if there is one, with at most  $3(n - 1)$  questions. *Hint: First, ask a question to eliminate one student as popular. Then use the inductive hypothesis to identify a potential popular student. Finally, ask two more questions to determine whether this student is actually a popular student.*

4) (15 pts.) Use induction on  $n$  to prove that  $\sum_{i=0}^{n-1} \frac{i}{2^i} = 2 - \frac{n+1}{2^{n-1}}$

5) (15 pts.) Consider the sets  $P = (A - B) - C$  and  $Q = (A - C) - (B - C)$ . Determine which relationship ( $\subseteq$ ,  $=$ ,  $\supseteq$ ) holds between the two sets  $P$  and  $Q$ .

Your answer will be either  $P \subseteq Q$ , or  $P = Q$ , or  $P \supseteq Q$ .

Justify your answer three ways by

1. drawing the Venn diagram,
2. constructing the membership table, and
3. proving it (using set identities with set builder notation).

6) (10 pts.) Do Exercises 40 of Section 2.2 (page 137).

**Note:** As you will have read on the course website, you need to submit your answers typed and as one PDF file named LastName.FirstName.3.PDF before the due date stated above. Follow the the template given at <https://www.cs.purdue.edu/homes/amahmoo/cs182summer2017/homeworksolutiontemplate.pdf>.

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<sup>1</sup>If there were two, they would follow each other. A particular class may have no popular students