M2 Trigonometry

含 2012-2025 年与 Trigonometryn 相关的全部题目, 排版舒服, 方便练习

作者: Yau YukMing

组织: 莘睿未来

时间: Aug 27, 2025

版本: 1.2

Level: Senior

No pain, No gain

问题 0.1 2012Q9

(a) Using integration by parts, find $\int x \sin x \, \mathrm{d}x$.

(b)

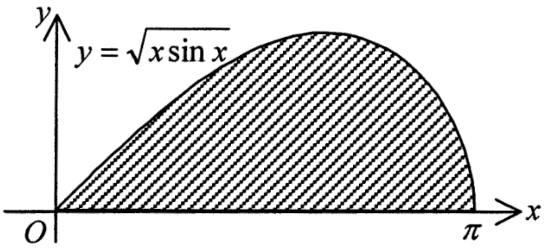

Figure 4

图 1: 2012Q9

Figure 4 shows the shaded region bounded by the curve $y = \sqrt{x \sin x}$ for $0 \leq x \leq \pi$ and the x-axis. Find the volume of the solid generated by revolving the region about the x-axis.

(4 marks)

continue 2012Q9

问题 0.2 2012Q10

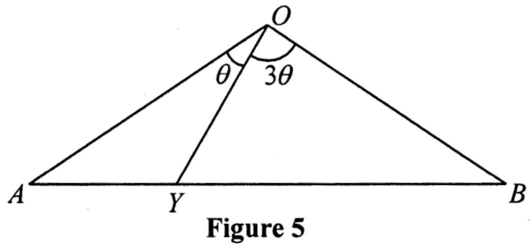

图 2: 2012Q10

In Figure 5, OAB is an isosceles triangle with $OA = OB, AB = 1, AY = y, \angle AOY = \theta$ and $\angle BOY = 3\theta$.

(a) Show that $y = \frac{1}{4}\sec^2\theta$.

(b) Find the range of values of y.

Hint: you may use the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(6 marks)

continue 2012Q10

问题 0.3 2012Q13

(a) (i) Suppose $\tan u = \frac{-1+\cos\frac{2\pi}{5}}{\sin\frac{2\pi}{5}}$, where $\frac{-\pi}{2} < u < \frac{\pi}{2}$.

Show that $u = \frac{-\pi}{5}$.

(ii) Suppose $\tan \nu = \frac{1+\cos\frac{2\pi}{5}}{\sin\frac{2\pi}{5}}$.

Find v, where $\frac{-\pi}{2} < v < \frac{\pi}{2}$.

(4 marks)

(b) (i) Express $x^2 + 2x\cos\frac{2\pi}{5} + 1$ in the form $(x+a)^2 + b^2$, where a and b are constants.

(ii) Evaluate $\int_{-1}^{1} \frac{\sin\frac{2\pi}{5}}{x^2+2x\cos\frac{2\pi}{5}+1}\,\mathrm{d}x$.

(6 marks)

(c) Evaluate $\int_{-1}^{1} \frac{\sin\frac{7\pi}{5}}{x^2+2x\cos\frac{7\pi}{5}+1}\,\mathrm{d}x$.

(3 marks)

continue 2012Q13

问题 0.4 2013Q1

Find $\frac{\mathrm{d}}{\mathrm{d}x}(\sin 2x)$ from first principles.

(4 marks)

问题 0.5 2013Q7

(a) Prove the identity $\tan x = \frac{\sin 2x}{1+\cos 2x}$.

(b) Using (a), prove the identity $\tan y = \frac{\sin 8y \cos 4y \cos 2y}{(1+\cos 8y)(1+\cos 4y)(1+\cos 2y)}$.

(5 marks)

问题 0.6 2013Q11

(a) Let $0 < \theta < \frac{\pi}{2}$. By finding $\frac{\mathrm{d}}{\mathrm{d}\theta} \ln(\sec\theta + \tan\theta)$, or otherwise, show that $\int \sec\theta \mathrm{d}\theta = \ln(\sec\theta + \tan\theta) + C$, where C is any constant.

(2 marks)

(b) (i) Using (a) and a suitable substitution, show that $\int \frac{\mathrm{d}u}{\sqrt{u^2-1}} = \ln\left(u + \sqrt{u^2-1}\right) + C$ for $u > 1$.
(ii) Using (b)(i), show that $\int_0^1 \frac{2x}{\sqrt{x^4+4x^2+3}}\,\mathrm{d}x = \ln(6 + 4\sqrt{2} - 3\sqrt{3} - 2\sqrt{6})$.

(5 marks)

(c) Let $t = \tan\phi$. Show that $\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{1}{1+t^2}$.
Hence evaluate $\int_0^{\frac{\pi}{4}} \frac{\tan\phi}{\sqrt{1+2\cos^2\phi}}\,\mathrm{d}\phi$.

(5 marks)

continue 2013Q11

问题 0.7 2013Q12

图 3: 2013Q12

In Figure 3, the distance between two houses A and B lying on a straight river bank is 40 m . The width of the river is always 30 m . In the beginning, Mike stands at the starting point P in the opposite bank which is 30 m from A. Mike's wife, situated at A, is watching him running along the bank for x m at a constant speed of $7 \mathrm{~m} \mathrm{~s}^{-1}$ to point Q and then swimming at a constant speed of $1.4 \mathrm{~m} \mathrm{~s}^{-1}$ along a straight path to reach B.

(a) Let T seconds be the time that Mike travels from P to B.

(i) Express T in terms of x.

(ii) When T is minimum, show that x satisfies the equation $2x^2 - 160x + 3125 = 0$. Hence show that $QB = \frac{25\sqrt{6}}{2}$ m.

(6 marks)

(b) In Figure 4, Mike is swimming from Q to B with QB equal to the value mentioned in (a)(ii). Let $\angle MAB = \alpha$ and $\angle ABM = \beta$, where M is the position of Mike.

(i) By finding $\sin\beta$ and $\cos\beta$, show that $MB = \frac{200\tan\alpha}{\tan\alpha + 2\sqrt{6}}$.

(ii) Find the rate of change of α when $\alpha = 0.2$ radian. Correct your answer to 4 decimal places.

(7 marks)

continue 2013Q12

问题 0.8 2014Q4

Let $x = 2y + \sin y$. Find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ in terms of y.

(3 marks)

问题 0.9 2014Q13

(a) Prove that $1 - \cos 4\theta - 2\cos 2\theta \sin^2 2\theta = 16\cos^2\theta \sin^4\theta$.

(2 marks)

(b) Show that $\int_0^{n\pi} \cos^2 x \sin^4 x \, \mathrm{d}x = \frac{n\pi}{16}$, where n is a positive integer.

(4 marks)

(c) Let $\mathrm{f}(x)$ be a continuous function such that $\mathrm{f}(k-x)=\mathrm{f}(x)$, where k is a constant. Show that $\int_0^k x\mathrm{f}(x)\mathrm{d}x = \frac{k}{2}\int_0^k \mathrm{f}(x)\mathrm{d}x$.

(4 marks)

(d)

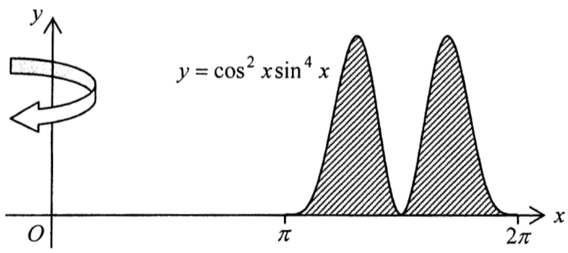

Figure 5

图 4: 2014Q13

Figure 5 shows the shaded region bounded by the curve $y=\cos^2 x\sin^4 \dot{x}$ and the x-axis, where $\pi \le x \le 2\pi$. Find the volume of the solid of revolution when the shaded region is revolved about the y-axis.

(4 marks)

continue 2014Q13

问题 0.10 2015Q2

Let $y = x\sin x + \cos x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

(b) Let k be a constant such that $x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + k\frac{\mathrm{d}y}{\mathrm{d}x} + xy = 0$ for all real values of x. Find the value of k.

(5 marks)

问题 0.11 2015Q7

(a) Prove that $\sin^2 x \cos^2 x = \frac{1-\cos 4x}{8}$.

(b) Let $\mathrm{f}(x) = \sin^4 x + \cos^4 x$.

(i) Express $\mathrm{f}(x)$ in the form $A \cos Bx + C$, where A, B and C are constants.

(ii) Solve the equation $8\mathrm{f}(x) = 7$, where $0 \leq x \leq \frac{\pi}{2}$.

(7 marks)

问题 0.12 2015Q8

(a) Using mathematical induction, prove that $\sin\frac{x}{2}\sum_{k=1}^{n}\cos kx=\sin\frac{nx}{2}\cos\frac{(n+1)x}{2}$ for all positive integers n.

(b) Using (a), evaluate $\sum_{k=1}^{567}\cos\frac{k\pi}{7}$.

(8 marks)

问题 0.13 2016Q6

(a) Prove that $x+1$ is a factor of $4x^3+2x^2-3x-1$.

(b) Express $\cos 3\theta$ in terms of $\cos\theta$.

(c) Using the results of (a) and (b), prove that $\cos\frac{3\pi}{5}=\frac{1-\sqrt{5}}{4}$.

(6 marks)

问题 0.14 2016Q10

(a) Let $\mathrm{f}(x)$ be a continuous function defined on the interval $[0, a]$, where a is a positive constant. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

(3 marks)

(b) Prove that $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\mathrm{d}x = \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan x}\right)\mathrm{d}x$.

(3 marks)

(c) Using (b), prove that $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\mathrm{d}x = \frac{\pi \ln 2}{8}$.

(3 marks)

(d) Using integration by parts, evaluate $\int_0^{\frac{\pi}{4}} \frac{x\sec^2 x}{1+\tan x}\,\mathrm{d}x$.

(3 marks)

continue 2016Q10

问题 0.15 2017Q1

Find $\frac{\mathrm{d}}{\mathrm{d}\theta} \sec 6\theta$ from first principles.

(5 marks)

问题 0.16 2017Q7

(a) Prove that $\sin 3x = 3\sin x - 4\sin^3 x$.

(b) Let $\frac{\pi}{4} < x < \frac{\pi}{2}$.

(i) Prove that $\frac{\sin 3\left(x-\frac{\pi}{4}\right)}{\sin\left(x-\frac{\pi}{4}\right)} = \frac{\cos 3x+\sin 3x}{\cos x-\sin x}$.

(ii) Solve the equation $\frac{\cos 3x+\sin 3x}{\cos x-\sin x} = 2$.

(8 marks)

问题 0.17 2017Q11

(a) Using $\tan^{-1}\sqrt{2}-\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)=\tan^{-1}\left(\frac{\sqrt{2}}{4}\right)$, evaluate $\int_0^1 \frac{1}{x^2+2x+3}\,\mathrm{d}x$.

(3 marks)

(b) (i) Let $0\le\theta\le\frac{\pi}{4}$. Prove that $\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$ and $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$. (ii) Using the substitution $t=\tan\theta$, evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{\sin 2\theta+\cos 2\theta+2}\,\mathrm{d}\theta$.

(5 marks)

(c) Prove that $\int_0^{\frac{\pi}{4}} \frac{\sin 2\theta+1}{\sin 2\theta+\cos 2\theta+2}d\theta=\int_0^{\frac{\pi}{4}} \frac{\cos 2\theta+1}{\sin 2\theta+\cos 2\theta+2}d\theta$.

(2 marks)

(d) Evaluate $\int_0^{\frac{\pi}{4}} \frac{8\sin 2\theta+9}{\sin 2\theta+\cos 2\theta+2}\,\mathrm{d}\theta$.

(3 marks)

continue 2017Q11

问题 0.18 2018Q3

(a) If $\cot A = 3\cot B$, prove that $\sin(A+B) = 2\sin(B-A)$.

(b) Using (a), solve the equation $\cot\left(x+\frac{4\pi}{9}\right) = 3\cot\left(x+\frac{5\pi}{18}\right)$, where $0 \le x \le \frac{\pi}{2}$.

(5 marks)

问题 0.19 2018Q10

(a) (i) Prove that $\int \sin^4 x \, \mathrm{d}x = \frac{-\cos x \sin^3 x}{4} + \frac{3}{4} \int \sin^2 x \, \mathrm{d}x$. (ii) Evaluate $\int_0^\pi \sin^4 x \, \mathrm{d}x$.

(5 marks)

(b) (i) Let $\mathrm{f}(x)$ be a continuous function such that $\mathrm{f}(\beta - x) = \mathrm{f}(x)$ for all real numbers x, where β is a constant. Prove that $\int_0^\beta x\mathrm{f}(x)\mathrm{d}x = \frac{\beta}{2} \int_0^\beta \mathrm{f}(x)\mathrm{d}x$.

(ii) Evaluate $\int_0^\pi x \sin^4 x \, \mathrm{d}x$.

(5 marks)

(c) Consider the curve $G : y = \sqrt{x} \sin^2 x$, where $\pi \leq x \leq 2\pi$. Let R be the region bounded by G and the x-axis. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(3 marks)

continue 2018Q10

问题 0.20 2019Q7

(a) Using integration by parts, find $\int e^x \sin \pi x \, \mathrm{d}x$.

(b) Using integration by substitution, evaluate $\int_0^3 e^{3-x} \sin \pi x \, \mathrm{d}x$.

(7 marks)

问题 0.21 2019Q10

(a) Let $0 \le x \le \frac{\pi}{4}$. Prove that $\frac{1}{2+\cos 2x} = \frac{\sec^2 x}{2+\sec^2 x}$.

(1 mark)

(b) Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{2+\cos 2x}\,\mathrm{d}x$.

(3 marks)

(c) Let $\mathrm{f}(x)$ be a continuous function defined on $\mathbf{R}$ such that $\mathrm{f}(-x) = -\mathrm{f}(x)$ for all $x \in \mathbf{R}$.

Prove that $\int_{-a}^{a} \mathrm{f}(x) \ln\left(1+e^x\right) \mathrm{d}x = \int_0^a x\mathrm{f}(x)\mathrm{d}x$ for any $a \in \mathbb{R}$.

(4 marks)

(d) Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{\sin 2x}{(2+\cos 2x)^2} \ln\left(1+e^x\right) \mathrm{d}x$.

(5 marks)

continue 2019Q10

问题 0.22 2020Q3

(a) Let x be an angle which is not a multiple of $30°$. Prove that

(i) $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$,

(ii) $\tan x \tan(60° - x)\tan(60° + x) = \tan 3x$.

(b) Using (a)(ii), prove that $\tan 55° \tan 65° \tan 75° = \tan 85°$.

(6 marks)

问题 0.23 2020Q4

(a) Find $\int \sin^2 \theta \, \mathrm{d}\theta$.

(b) Define $\mathrm{f}(x) = 4x\left(1 - x^2\right)^{\frac{1}{4}}$ for all $x \in [0, 1]$. Denote the graph of $y = \mathrm{f}(x)$ by G. Let R be the region bounded by G and the x-axis. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(6 marks)

问题 0.24 2020Q10

(a) Using integration by substitution, prove that $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \ln\left(\sin\left(\frac{\pi}{4}-x\right)\right)\mathrm{d}x = \int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \ln(\sin x)\mathrm{d}x$.

(b) Using (a), evaluate $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \ln(\cot x - 1)\mathrm{d}x$.

(3 marks)

(c) (i) Using $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$, or otherwise, prove that $\cot\frac{\pi}{12} = 2+\sqrt{3}$. (ii) Using integration by parts, prove that $\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{x\csc^2 x}{\cot x - 1}\,\mathrm{d}x = \frac{\pi}{8}\ln(2+\sqrt{3})$.

(7 marks)

continue 2020Q10

问题 0.25 2021Q4

(a) Prove that $\cos 2x + \cos 4x + \cos 6x = 4 \cos x \cos 2x \cos 3x - 1$.

(b) Solve the equation $\cos 4\theta + \cos 8\theta + \cos 12\theta = -1$, where $0 \leq \theta \leq \frac{\pi}{2}$.

(6 marks)

问题 0.26 2021Q9

(a) Let $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$.

(i) Find $\frac{\mathrm{d}}{\mathrm{d}\theta} \ln(\sec\theta + \tan\theta)$.

(ii) Using the result of (a)(i), find $\int \sec\theta d\theta$. Hence, find $\int \sec^3\theta d\theta$.

(4 marks)

(b) Let $\mathrm{g}(x)$ and $\mathrm{h}(x)$ be continuous functions defined on $\mathbf{R}$ such that $\mathrm{g}(x)+\mathrm{g}(-x) = 1$ and $\mathrm{h}(x) = \mathrm{h}(-x)$ for all $x \in \mathbf{R}$. Using integration by substitution, prove that $\int_{-a}^{a} \mathrm{g}(x)\mathrm{h}(x)\mathrm{d}x = \int_{0}^{a} \mathrm{h}(x)\mathrm{d}x$ for any $a \in \mathbf{R}$.

(3 marks)

(c) Evaluate $\int_{-1}^{1} \frac{3^x x^2}{(3^x+3^{-x})\sqrt{x^2+1}} \mathrm{d}x$.

(5 marks)

continue 2021Q9

问题 0.27 2021Q11

Define $P = \begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$, where $\frac{\pi}{2} < \theta < \pi$.

(a) Let $A = \begin{pmatrix} \alpha & \beta \\ \beta & -\alpha \end{pmatrix}$, where $\alpha, \beta \in \mathbf{R}$.

Prove that $PAP^{-1} = \begin{pmatrix} -\alpha\cos 2\theta + \beta\sin 2\theta & -\beta\cos 2\theta - \alpha\sin 2\theta \\ -\beta\cos 2\theta - \alpha\sin 2\theta & \alpha\cos 2\theta - \beta\sin 2\theta \end{pmatrix}$.

(3 marks)

(b) Let $B = \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$.

(i) Find θ such that $PBP^{-1} = \begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix}$, where $\lambda, \mu \in \mathbf{R}$.

(ii) Using the result of (b)(i), prove that $B^n = 2^{n-2}\begin{pmatrix} (-1)^n + 3 & \sqrt{3}(-1)^{n+1} + \sqrt{3} \\ \sqrt{3}(-1)^{n+1} + \sqrt{3} & 3(-1)^n + 1 \end{pmatrix}$ for any positive integer n.

(iii) Evaluate $\left(B^{-1}\right)^{555}$.

(9 marks)

continue 2021Q11

问题 0.28 Let $\frac{\pi}{4} < \theta < \frac{\pi}{2}$.
(a) Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$. (b) Solve the equation $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 5$.

(5 marks)

问题 0.29 2022Q3

Let $\frac{\pi}{4} < \theta < \frac{\pi}{2}$.

(a) Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$.

(b) Solve the equation $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 5$.

(5 marks)

问题 0.30 2022Q10

Let $\mathrm{g}(x) = \cos^2 x \cos 2x$.

(a) Prove that $\int \mathrm{g}(x)\mathrm{d}x = \frac{\sin 2x \cos^2 x}{2} + \frac{1}{2}\int \sin^2 2x \, \mathrm{d}x$.

(2 marks)

(b) Evaluate $\int_0^{\pi} \mathrm{g}(x)\mathrm{d}x$.

(2 marks)

(c) Using integration by substitution, evaluate $\int_0^{\pi} x \, \mathrm{g}(x)\mathrm{d}x$.

(4 marks)

(d) Evaluate $\int_{-\pi}^{2\pi} x \, \mathrm{g}(x)\mathrm{d}x$.

(4 marks)

continue 2022Q10

问题 0.31 2022Q11

(a) Let n be a positive integer. Denote the 2×2 identity matrix by I.

(i) Let A be a 2×2 matrix. Simplify $(I - A)\left(I + A + A^2 + \cdots + A^n\right)$.

(ii) Let $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, where θ is not a multiple of 2π.

It is given that $A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$.

(1) Prove that $(I - A)^{-1} = \frac{1}{2\sin\frac{\theta}{2}} \begin{pmatrix} \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \end{pmatrix}$.

(2) Using the result of (a)(i) and (a)(ii)(1), prove that $I + A + A^2 + \cdots + A^n = \frac{\sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}} \begin{pmatrix} \cos\frac{n\theta}{2} & -\sin\frac{n\theta}{2} \\ \sin\frac{n\theta}{2} & \cos\frac{n\theta}{2} \end{pmatrix}$.

(7 marks)

(b) Using (a)(ii), evaluate

(i) $\cos\frac{5\pi}{18}+\cos\frac{5\pi}{9}+\cos\frac{5\pi}{6}+\cdots+\cos 25\pi$;

(ii) $\cos^2\frac{\pi}{7}+\cos^2\frac{2\pi}{7}+\cos^2\frac{3\pi}{7}+\cdots+\cos^2 7\pi$.

(6 marks)

continue 2022Q11

问题 0.32 Let $\mathrm{f}(x) = -x \sin x$.
(a) Prove that $\mathrm{f}\left(\frac{\pi}{2} + h\right) - \mathrm{f}\left(\frac{\pi}{2}\right) = \pi \sin^2\left(\frac{h}{2}\right) - h \cos h$.
(b) Using (a), find $\mathrm{f}'\left(\frac{\pi}{2}\right)$ from first principles.

(5 marks)

问题 0.33 2023Q3

(a) Find a pair of constants p and q such that $11\sin x + 7\cos x \equiv p(3\sin x + \cos x) + q(3\cos x - \sin x)$.

(b) Evaluate $\int_0^{\frac{\pi}{4}} \frac{11\sin x+7\cos x}{3\sin x+\cos x}\,\mathrm{d}x$.

(6 marks)

问题 0.34 2023Q4

(a) Prove that $\cos 3x = 4\cos^3 x - 3\cos x$.

(b) Using (a), solve the equation $\sec^3 x - 6\sec^2 x + 8 = 0$, where $\frac{\pi}{2} < x < \frac{3\pi}{2}$.

(5 marks)

问题 0.35 2023Q8

(a) Let $\theta \in \mathbf{R}$. Using mathematical induction, prove that $\sin\theta \sum_{k=1}^{n} \sin 2k\theta = \sin n\theta \sin(n+1)\theta$ for all positive integers n.

(b) Using (a), find a pair of rational numbers a and b such that $\sum_{k=1}^{111} \sin\frac{k\pi}{11}\cos\frac{k\pi}{11} = a\sin b\pi$, where $0 < b < \frac{1}{2}$.

(8 marks)

问题 0.36 2023Q12

(a) Let a be a non-zero constant. Prove that $\int_0^1 x^2 e^{ax}\,\mathrm{d}x = \frac{(a^2-2a+2)e^a-2}{a^3}$.

(3 marks)

(b) Using (a) and integration by substitution, evaluate $\int_0^{e-1} x(\ln(1+x))^2\,\mathrm{d}x$.

(4 marks)

(c) Evaluate $\int_0^{\frac{\pi}{2}} (\ln(1+(e-1)\cos x))^2 \sin 2x\,\mathrm{d}x$.

(3 marks)

(d) Evaluate $\int_{\frac{\pi}{2}}^{\pi} (\ln(1+(e-1)\sin x))^2 \sin 2x\,\mathrm{d}x$.

(3 marks)

问题 0.37 2024Q4

(a) Let $0 < x < \frac{\pi}{2}$. Prove that $\csc 2x - \cot 2x = \tan x$.

(b) Solve the equation $(\csc 3\theta - \cot 3\theta)(\csc \theta - \cot \theta) = 1$, where $\frac{\pi}{6} < \theta < \frac{\pi}{3}$.

(5 marks)

问题 0.38 2024Q5

(a) Let k be a constant.

Using integration by parts, prove that $\int \cos(k \ln x) \mathrm{d}x = \frac{x}{1+k^2}(\cos(k \ln x) + k \sin(k \ln x))+$ constant .

(b) Using (a), or otherwise, evaluate $\int_1^e \sin^2(\pi \ln x) \mathrm{d}x$.

(6 marks)

问题 0.39 2024Q6

Consider the system of linear equations in real variables x, y, z

$$(E):\begin{cases} 3x+y-9z=0 \\ 2x+y-7z=0 \end{cases}$$

(a) Solve (E).

(b) Someone claims that (E) has a unique solution (x, y, z) satisfying $\sin x+\cos y-\cos z=0$, where $0<z<\frac{\pi}{2}$. Do you agree? Explain your answer.

(6 marks)

问题 0.40 2024Q9

Two toy cars, A and B, move along a straight line towards east with constant speeds 4 metres per second and 1 metre per second respectively. They start at the point O at the same time. The fixed point C is 10 metres due north of O. The following figure shows the positions of A and B after t seconds. Let $\angle ACB = \theta$ radians .

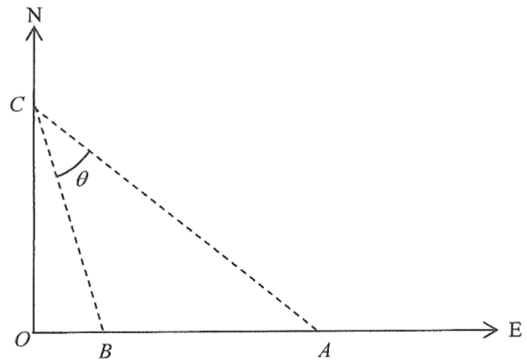

图 5: Caption

(a) Prove that $\tan\theta = \frac{15t}{2(t^2+25)}$.

(b) It is given that $\angle BAC = \angle ACB$ when $t = T$. Find

(i) T,

(ii) the rate of change of θ when $t = T$.

(7 marks)

continue 2024Q9

问题 0.41 2024Q11

(a) Let a be a non-zero constant. Using integration by substitution, find $\int \frac{1}{x^2+a^2}\,\mathrm{d}x$ in terms of a.

(3 marks)

(b) Let $\mathrm{g}(x)$ and $\mathrm{h}(x)$ be continuous functions defined on $\mathbf{R}$. It is given that $\mathrm{g}(x)$ is an even function and $\mathrm{h}(x)$ is an odd function. Prove that $\int_{-c}^{c} \frac{\mathrm{g}(x)}{1+e^{\mathrm{h}(x)}}\mathrm{d}x = \int_0^c \mathrm{g}(x)\mathrm{d}x$, where c is a constant.

(4 marks)

(c) Evaluate $\int_{-1}^{1} \frac{3^x+3^{-x}}{\left(1+e^{\sin^3 x}\right)(9^x+9^{-x}+7)}\mathrm{d}x$.

(5 marks)

continue 2024Q11

问题 0.42 2025Q2

Let $\mathrm{f}(x) = \frac{2x}{\tan x}$, where $0 < x < \frac{\pi}{2}$.

(a) Prove that $\mathrm{f}\left(\frac{\pi}{4} + h\right) - \mathrm{f}\left(\frac{\pi}{4}\right) = \frac{2h - 2h\tan h - \pi \tan h}{1 + \tan h}$.

(b) Find $\mathrm{f}'\left(\frac{\pi}{4}\right)$ from first principles.

(5 marks)

问题 0.43 2025Q6

Let $\frac{\pi}{2} < x < \frac{3\pi}{4}$.

(a) Find a pair of constants h and k such that $\frac{\sec x + \tan x}{\csc x - \cot x} + \frac{\sec x - \tan x}{\csc x + \cot x} \equiv h \sec x \csc x + k$.

(b) Solve the equation $\frac{\sec x + \tan x}{\csc x - \cot x} + \frac{\sec x - \tan x}{\csc x + \cot x} + 6 = 0$.

(5 marks)

问题 0.44 2025Q9

(a) Using integration by substitution, prove that $\int \frac{1}{(1+x^2)^2}\,\mathrm{d}x = \frac{1}{2}\tan^{-1}x + \frac{x}{2(1+x^2)} +$ constant.

(b) Consider the curve $C : y = \frac{1}{1+3x^2}$. Let R be the region bounded by C, the straight line $x = 1$ and the two axes. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(8 marks)

问题 0.45 2025Q11

(a) Let m be a positive integer. Express $\int_0^\pi e^{-x} \sin 2mx \, \mathrm{d}x$ in terms of m.

(3 marks)

(b) (i) Evaluate $\int_0^\pi e^{-x} \sin 5x \cos 3x dx$.

(ii) Evaluate $\int_0^{2\pi} e^{-x} \sin 5x \cos 3x \, \mathrm{d}x$.

(6 marks)

(c) Evaluate $\int_{e^{\frac{-3\pi}{2}}}^{e^{\frac{\pi}{2}}} \cos\left(\ln x^5\right) \sin\left(\ln x^3\right) \mathrm{d}x$.

(4 marks)

continue 2025Q11