

Homework 4

M1399.000100, Seoul National University, Spring 2021

Due 23:59 Sunday, 2021-06-13

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No late submission is accepted.

Q1. Textbook problems

1. 심송용 연습문제 8.2, 8.3, 8.4.

Q2. Acceptance-rejection sampling

The Gamma distribution with shape parameter α and scale parameter 1 has density proportional to

$$f(x) = \begin{cases} x^{\alpha-1} e^{-x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

1. \item (30 pts) Show that, for $\alpha > 1$,

$$g(x) = \frac{(\alpha - 1)^{\alpha-1} e^{-(\alpha-1)}}{1 + [x - (\alpha - 1)]^2 / (2\alpha - 1)}$$

dominates $f(x)$, i.e., $g(x) \geq f(x)$ for all x . (.

$$\frac{d}{dx} (x^{\alpha-1} e^{-x} (2\alpha - 1 + [x - (\alpha - 1)]^2)) = -x^{\alpha-2} e^{-x} (x - \alpha)^2 [x - (\alpha - 1)].)$$

2. Propose an algorithm that generate a $\text{Gamma}(\alpha, 1)$ random number for a given $\alpha > 1$.
3. Implement your algorithm and test.

Q3. Importance sampling

Consider testing the hypotheses $H_0 : \lambda = 2$ versus $H_a : \lambda > 2$ using 25 observations from a $\text{Poisson}(\lambda)$ model. Rote application of the central limit theorem would suggest rejecting H_0 at $\alpha = 0.05$ when

$$Z > 1.645, \text{ where } Z = \frac{\bar{X} - 2}{\sqrt{2/25}}.$$

1. Estimate the size of this test (i.e., the type I error rate) using three Monte Carlo approaches: standard, antithetic, (unstandardized) importance sampling. Provide a confidence interval for each estimate. Discuss the relative merits of each variance reduction technique.

For the importance sampling approach, use a Poisson envelope (h in the course notes) with mean equal to the H_0 rejection threshold, namely $\lambda = 2.4653$.

2. Draw the power curve for this test for $\lambda \in [2.2, 4]$, using the same three techniques. Provide pointwise confidence bands in each case. Discuss the relative merits of each technique in this setting.