Algorithms - Assignment 1

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1. Q. Show directly that $f(n) = n^2 + 3n^3 \in O(n^3)$ and $f(n) = n^2 + 3n^3 \in \Omega(n^3)$

$\cancel{n^2 + 3}n^3 \in \Theta(n^3)$

$\Theta(n^3) = O(n^3) \cap \Omega(n^3)$

$\rightarrow \Theta(n^3) \in O(n^3), \Theta(n^3) \in \Omega(n^3)$

$\rightarrow n^2 + 3n^3 \in O(n^3), n^2 + 3n^3 \in \Omega(n^3)$

$\therefore f(n) = n^2 + 3n^3 \in O(n^3)$ and $f(n) = n^2 + 3n^3 \in \Omega(n^3)$ ▨

2. Using the definitions of O and Ω show that $6n^2 + 20n \in O(n^3)$, but $6n^2 + 20n \in \Omega(n^3)$

A. 1) Show $6n^2 + 20n \in O(n^3)$

Let $f(n) = 6n^2 + 20n$, $g(n) = O(n^3)$.

$\exists C, k$ $f(n) \in O(g(n))$ iff $|f(n)| \le C|g(n)|$ where $n \ge k$

We can estimate size of $f(n)$ when $n \ge 1$ because $n^3 \ge n^2$ and $n^3 \ge n$

$0 \le 6n^2 + 20n \le 6n^3 + 20n^3 = 26n^3$

when $n \ge 1$. Thus, we can take $C = 26$, $k = 1$ to show $f(n) = 6n^2 + 20n \in O(n^3)$

2) Show $6n^2 + 20n \notin \Omega(n^3)$

Let $f(n) = 6n^2 + 20n$, $g(n) = n^3$

$\exists C, k$ $f(n) \in \Omega(g(n))$ iff $|f(n)| \ge C|g(n)|$ where $n \ge k$

Because $f(n) \ge 0, g(n) \ge 0$ for all positive n

$$\lim_{n\to\infty} \frac{|f(n)|}{C|g(n)|} = \lim_{n\to\infty} \frac{|6n^2 + 20n|}{|Cn^3|} = \lim_{n\to\infty} \frac{6n^2 + 20n}{Cn^3} = \lim_{n\to\infty} \frac{\frac{6}{n} + \frac{20}{n^2}}{C} = 0 < 1$$

$$\therefore \lim_{n\to\infty} \frac{|f(n)|}{C|g(n)|} = 0 < 1 \quad \text{--- (1)}$$

We can see $|f(n)| < C|g(n)|$ when $n \ge N$; size of n is big enough ※ by (1) $\lim_{n\to\infty} \left|\frac{f(n)}{g(n)}\right| \ne 0$

$\rightarrow \forall C, k > 0$, there is $k' \in (k, \infty)$, that $|f(k')| \ngeq C|g(k')|$

$\therefore 6n^2 + 20n \notin \Omega(n^3)$

3) The function $f(n) = 3n^2 + 10n\log n + 1000n + 4\log n + 9999$ belongs in which of the following complexity categories :

(a) $\Theta(\lg n)$ (b) $\Theta(n^2 \log n)$ (c) $\Theta(n)$ (d) $\Theta(n \lg n)$ (e) $\Theta(n^2)$ (f) None of these

$3n^2 \le 3n^2 + 10n\log n + 1000n + 4\log n + 9999 \le 3n^2 + 10n^2 + 1000n^2 + 4n^2 + 9999n^2 = 11016n^2$

$\therefore k = 1$, $C_1 = 3$, $C_2 = 11016$, $g(n) = n^2$

$\therefore f(n) \in \Theta(n^2)$

A. (e)

4) The function $f(n) = (\log n)^2 + 2n + 4n + \log n + 50$ belongs in which of the following complexity categories:

(a) $\theta(\lg n)$ (b) $\theta((\log n)^2)$ (c) $\theta(n)$ (d) $\theta(n \log n)$ (e) $\theta(n(\lg n)^2)$ (f) None of these

$C_1|g(x)| \le |f(x)| \le C_2|g(x)|$ when $x \ge k$

$\rightarrow \quad 0 < \lim_{x \to \infty} \left| \frac{f(x)}{g(x)} \right| < \infty$

Assume $g(n) = n$

$$0 < \lim_{n \to \infty} \left| \frac{(\log n)^2 + 2n + 4n + \log n + 50}{n} \right|$$

$$= \lim_{n \to \infty} \left| \frac{\frac{(\log n)^2}{n} + \frac{2n}{n} + \frac{4n}{n} + \frac{\log n}{n} + \frac{50}{n}}{1} \right|$$

$$= \frac{2+4}{1} = 6 < \infty$$

$\therefore f(n) \in \theta(n)$

A. (C)

5) The function $f(n) = n + n^2 + 2^n + n^4$ belongs in which of the following complexity categories:

(a) $\theta(n)$ (b) $\theta(n)$ (c) $\theta(n^3)$ (d) $\theta(n \lg n)$ (e) $\theta(n^4)$ (f) None of these

$2^n \le n + n^2 + 2^n + n^4 \le 2^n + 2^n + 2^n + 2^n = 4 \cdot 2^n$

We can choose $C_1 = 1$, $C_2 = 4$, $k = 16$

$\therefore f(n) \in \theta(2^n)$

A. (f)