Section Notes 10

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Agenda

1. Moral Hazard Terminology
2. Principal-Agent (PA) with 3 Actions and 2 Outcomes
3. Takeaway Messages from PA Problem

1 Moral Hazard Terminology

There two types of moral hazard:

1. *ex ante* moral hazard: Given that an agent is insured, the agent will not take costly steps to reduce the probability of losses.
2. *ex post* moral hazard: Given that an agent suffers a loss and the agent's insurance company pays a portion of the loss, the agent will over-consume.

In the model we will look at this week, an agent chooses an action (which is costly for the agent) that influences the *probability* of a good outcome. Principal does not observe the action, but does observe the outcome; which is where the information asymmetry comes from. Therefore, the principal can condition payment on the outcome, but this will not get us to the *first best*.[1] Further assume that the principal is risk neutral and the agent is risk averse.

The key to understanding moral hazard models is to note that there is a trade-off from the perspective of the risk neutral principal: to incentivize the agent to put in more effort by varying the wages according to an observable (or contractible; or verifiable) variable which is *not perfectly correlated* with effort; and to provide some insurance such that the risk averse agent's individual rationality constraint is satisfied. In other words, if an agent bears risk because there exists noise with regards to the observable variable (quantity below), the agent will have to be paid a higher (expected) wage to achieve its reservation utility (in other words, for the Individual Rationality Constraint to hold).

[1] "First best" as defined in Section Note 9, Full Information (First Degree Price Discrimination). Recall my explanation for why we call the full information case as the first best. If this is unclear, review Lecture Note 15.

2 Principal-Agent with 3 Actions and 2 Outcomes

Example 2.1. A firm hires a scientist to develop a new product. The scientist can choose a level of $e \in \{e_l, e_m, e_h\}$, where $e_l < e_m < e_h$[2] and gets a wage w. The scientist's utility is additive and is written as:

$$u(e, w) = \ln w - c(e),$$

where $c(e_l) = 0$; $c(e_m) = 1$; and $c(e_h) = 2$. The reservation utility $\bar{u}_0$ is normalized and equal to zero (0) and known by both the principal and the agent.

There are two possible outcomes:[3] a good outcome (the scientist invents a widget) denoted x_G and a bad outcome (the scientist fails to invent a widget) denoted x_B. Assume that the probability of a good outcome given the level of effort are as follows:

$$\begin{aligned}
\Pr(x_G|e_l) &\equiv p_l = \frac{1}{10} \\
\Pr(x_G|e_m) &\equiv p_m = \frac{1}{2} \\
\Pr(x_G|e_h) &\equiv p_h = \frac{2}{3}
\end{aligned}$$

Note that the probability of the good outcome is increasing in the level of effort, or higher effort levels first order stochastically dominate (FOSD) lower effort levels.[4]

Revenue to the firm is equal to 12 if the outcome is x_G and 0 otherwise. With a slight abuse of notation, let's say that $x_G = 12$ and $x_B = 0$. *The principal has bargaining power and so can set the contract.* Recall that if the principal has full bargaining power, then "the agent always receive her reservation utility level, and Pareto comparison between contracts depends only on the principal's utility that is induced by those contracts."[5]

Who is the principal and who is the agent?

2.1 First Best

When you have an adverse selection, signaling, or moral hazard model, always look at the first best equilibria and the first best outcomes.

The firm will offer a contract of the form $[e_i, w(e_i)] = [e_i, w_i]$, which denotes a level of effort for a given wage level. [6] Since we're in the first best, we don't

[2] The model would be more realistic if the scientist's choice set was continuous and we could solve by setting up the optimization problem and the corresponding Lagrangian.

[3] A more general model would have a continuous set of outcomes.

[4] Check that this is in fact the case by drawing the cdfs for each level of effort.

[5] Lecture Note ch 15, p. 469.

[6] For those of you familiar with Mechanism Design, the problem, like many of our previous problems, boils down to the uninformed party finding the optimal allocation given the messages of the informed party, or finding the optimal mechanism.

have to worry about the agent's IC, but we do have to consider the agent's IR (or PC). The agent's IR given the wage menu above is:

$$\ln w_i - c(e_i) \geq \bar{u}_0,$$

which will bind with equality leading to:[7]

$$w_i = \exp\{c(e_i)\}. \tag{1}$$

The firm will maximize its expected profit, which can be written as follows:

$$[e_i, w_i] \in \arg\max_{e,w} \{p_i \cdot x_G + (1-p_i)x_B - w_i\}.$$

Since we're in a 3 action, 2 outcome case, we can solve the firm's expected profit maximization problem by comparing the following three outcomes:

$$\begin{aligned}
\mathbb{E}[\pi_l] &= p_l \cdot x_G - w_l = \frac{1}{10} \cdot 12 - \exp\{0\} = \frac{1}{5} \\
\mathbb{E}[\pi_m] &= p_m \cdot x_G - w_m = \frac{1}{2} \cdot 12 - \exp\{1\} \approx 3.28 \\
\mathbb{E}[\pi_h] &= p_h \cdot x_G - w_h = \frac{2}{3} \cdot 12 - \exp\{2\} \approx 0.61
\end{aligned}$$

Therefore, the optimal complete information contract (or wage menu) is $[e_m, \exp\{1\}]$.[8] The expected profit at the optimal complete info contract is equal to approximately 3.28.

The key take-away from the first best is that because the agent's action (or effort) is observable to the firm, the firm can condition completely on the observable level of effort (and does not have to rely on the good or bad outcome, which are imperfectly correlated with the effort level). This gets us equation 1, which can be substituted into the objective function of the firm's expected profit maximization problem. This is the first best in terms of being on the Pareto frontier. However, note that the firm gets all of the surplus, which is a consequence of the IR binding with equality for all levels of effort.

Now let's look at the second best, or asymmetric information, case.

2.2 Second Best

Now the firm cannot provide a contract, or wage menu, contingent on the agent's effort because effort is not observable to the firm. This is called the second best

[7] Important to see why this constraint binds with equality. Further, this is the key step which gets us to the first best outcome.

[8] We can write the rest of the contract for low and high levels of effort such that the expected utility of the agent is less than its reservation utility. In the alternative, we could say that the principal makes a take it or leave it offer of this form. Note that at the first best outcome, the optimal contract results in the principal providing complete insurance to the risk averse agent. This won't be the case in the second best (or information asymmetry) case. Recall Proposition 2 from the Lecture Notes Ch 15.

because we add another set of constraints (the ICs) to the principal's maximization problem.

Once we're in the asymmetric information world, we need to consider the following:

1. For any level of effort, what is the least costly way of inducing it for the firm? Formally, this question is equivalent to the Individual Rationality Constraint for each level of effort.

2. What is the utility maximizing effort level for the agent?

3. Given the answers to questions 1. and 2. above, which wage menu maximizes the firm's profit?

To begin, note that the form of the wage menu is $[x_k, w(x_k)] = [x_k, w_k]$ where $k \in \{G, B\}$. Given this wage menu, the firm solves its expected profit maximization problem subject to the agent's IR and IC:

$$\max_{w_G, w_B} \mathbb{E}[\pi] = p_i \cdot (x_G - w_G) + (1 - p_i) \cdot (x_B - w_B) \tag{2}$$

s.t. the agent's participation constraint (IR):

$$p_i \cdot \ln w_G + (1 - p_i) \cdot \ln w_B - c(e_i) \geq \bar{u}_0 \tag{3}$$

and s.t. the agent's incentive compatability constraint (IC):

$$p_i \cdot \ln w_G + (1 - p_i) \cdot \ln w_B - c(e_i) \geq p_j \cdot \ln w_G + (1 - p_j) \cdot \ln w_B - c(e_j) \tag{4}$$

for all $j \neq i$.

Now we need to check each level of effort and see whether all of the constraints are satisfied.

2.2.1 Low effort: e_l

If the principal prefers to induce the lowest level of effort, then a flat wage is optimal; and so, if the agent chooses the lowest effort level, the agent is fully insured.

The IR (which again binds with equality) can be rewritten as:

$$p_l \cdot \ln \underline{w} + (1 - p_l) \cdot \ln \underline{w} - c(e_l) = \ln \underline{w} = 0 \Rightarrow \underline{w} = 1.$$

Since we don't have to consider the IC and we have only one wage level, we can calculate the expected profit of the firm as:

$$\mathbb{E}[\pi | e_l] = \frac{1}{5} = 0.2.$$

A couple questions before moving on:

1. Why is it optimal to fully insure the agent who puts in the lowest level of effort? Note that inducing a "higher" level of effort requires the IC to be satisfied. The result is an inefficient distribution of risk (or risk sharing), which requires the principal to compensate the agent for taking on risk (compared to the first best). In short, it is costly for the principal to condition wage levels on outcomes. Therefore, if the level of effort that the principal wants to induce is the lowest effort level, e_l, then it is optimal for the principal to provide a flat wage rate. The result is low effort because the agent has no incentive to exert a greater level of effort (which is costly for the agent).

2. Why don't we consider the IC? See the answer to 1. above. I noted that with a flat rate, there was no incentive for the agent to exert effort. Therefore, we don't have to consider the IC.

2.2.2 Medium effort: e_m

Given medium effort, the IR of the agent (which binds with equality) will be:

$$\frac{1}{2}\ln w_G + \frac{1}{2}\ln w_B - 1 = 0 \tag{5}$$

and the two ICs are as follows:

$$IC_{ml} : \frac{1}{2}\ln w_G + \frac{1}{2}\ln w_B - 1 \quad \geq \quad \frac{1}{10}\ln w_G + \frac{9}{10}\ln w_B - 0 \tag{6}$$

$$IC_{mh} : \frac{1}{2}\ln w_G + \frac{1}{2}\ln w_B - 1 \geq \frac{2}{3}\ln w_G + \frac{1}{3}\ln w_B - 2 \tag{7}$$

Which of the two ICs will bind with equality? In many instances where there are multiple ICs, one or more of the ICs will bind with equality. For purposes of this class,[9] note that the IC for the induced level of effort usually binds with the level of effort one unit lower than the level of effort under consideration. Therefore, from inequality (6) above, we get the following equality:

$$\frac{2}{5}\ln w_G - \frac{2}{5}\ln w_B = 1. \tag{8}$$

Solving equations (5) and (8), we get: $\ln w_G = 2.25$ and $\ln w_B = -0.25$. Of course, we need to check that these values satisfy inequality (7), or the IC_{mh}. Since the IC for the medium and high levels of effort is satisfied,

$$\begin{aligned} w_G &\approx 9.49 \\ w_B &\approx 0.78 \end{aligned}$$

[9] Choosing which ICs will bind is tricky and is sometimes substituted using what is called the IC FOC, which we won't consider here.

Note that there was solution to this system of equations and inequalities. However, there may be instances where a solution does not exist, in which case the given effort level is not implementable.[10]

The expected profit of the firm if it makes a take it or leave it offer as identified above is:

$$\mathbb{E}\left[\pi|e_m\right] = \frac{1}{2}\cdot(12-9.49)+\frac{1}{2}\cdot(0-0.78)\approx 0.87.$$

2.2.3 High effort: e_h

Given high effort, the IR of the agent (which binds with equality) will be:

$$\frac{2}{3}\ln w_G + \frac{1}{3}\ln w_B - 2 = 0 \tag{9}$$

and the two ICs are as follows:

$$IC_{hl}: \frac{2}{3}\ln w_G + \frac{1}{3}\ln w_B - 2 \quad \geq \quad \frac{1}{10}\ln w_G + \frac{9}{10}\ln w_B - 0 \tag{10}$$

$$IC_{hm}: \frac{2}{3}\ln w_G + \frac{1}{3}\ln w_B - 2 \geq \frac{1}{2}\ln w_G + \frac{1}{2}\ln w_B - 1 \tag{11}$$

Again the question is which of the two IC constraints will bind with equality. According to the rule of thumb above, it should be inequality (11), or the IC_{hm}, above. We can rewrite as:

$$\frac{1}{6}\ln w_G - \frac{1}{6}\ln w_B = 1. \tag{12}$$

Solving equations (9) and (12), we get: $\ln w_G = 4$ and $\ln w_B = -2$. Of course, we need to check that these values satisfy inequality (10), and we find that $0 \geq 0.4 - 1.8$ which is a contradiction. Therefore, the contract with wages equal to $w_G \approx 54.6$, $w_B \approx 0.135$, and $\mathbb{E}\left[\pi|e_h\right] = \frac{2}{3}\cdot(12-54.6)+\frac{1}{3}\cdot(0-0.135)$ is not implementable.

2.2.4 Conclusion

The principal maximizes its expected profit by making a take it or leave it offer where

$$\left[w_G, w_B\right] = \left[9.49,\, 0.78\right].$$

This second best contract results in an expected profit equal to $0.87 > 0.2$ and the agent/scientist puts in medium effort.

[10] One possible sufficiency test for implementability is: 1) adjacency and 2) monotonicity. You can learn about these concepts in Contract Theory.

3 Takeaway Messages from PA Problem

- What are the welfare implications of non-contractible (or unobservable) effort?
  - The scientist (or agent) is equally well off, in that in expectation, the agent's utility is equal to the reservation utility, which is zero. This has to be true since the IR always binds in the example above.
  - The firm (or principal) is worse off: full info profit (or first best) is equal to 3.28 and the asymmetric info profit (or second best) is equal to 0.87.
  - The asymmetric information leads to a welfare loss and a Pareto sub optimal outcome (or a second best outcome).
- When contracting for high effort pay more for outcomes that have a greater correlation with high effort (which results in wage structures which may not be monotonic).[11]
- If sabotage is possible, wage menu should be weakly increasing in outcome. The idea is that if destruction of output is possible, then the agent can destroy output to maximize its payoff.
- All relevant information should be used in determining the wage structure.[12]

4 FOSD vs MLRP

PDF

	x_1	x_2	x_3
a'	0.55	0.2	0.25
a^*	0.05	0.65	0.3
LR	11	0.31	0.83

The point of this exercise is to see that the pdf leads to a^* FOSD a', where a^* is the high effort level and a' is the low effort level. However, we don't get the monotonically increasing wage menu (increasing wage as outcome increases), because the high effort level is highly correlated with the medium outcome. Therefore, the optimal wage contract will likely have:

$$w_2^* > w_3^* > w_1^*$$

to implement the high level of effort a^* instead of a'. Keep the following proposition in mind:

[11] Consider the example in Lecture Note 15 where higher effort FOSDs lower effort, but the likelihood ratios are not monotonically decreasing. The pdf can be found below.

[12] For a review of the final three bullet points, review Lecture Note 15. There are a few major typos, but you should be able to get through it.

Proposition 4.1. *The optimal incentive scheme is monotonically increasing in the outcome if and only if MLRP holds.*