Section Notes 11

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Agenda

1. Fundamental Welfare Theorem Failure[1]
2. Externalities
3. Monopoly and Price Discrimination

1 Fundamental Welfare Theorem Failure

What assumptions do we need for the Welfare Theorems to hold?[2]

1. Complete markets: no unpriced goods. This assumption fails when we have externalities.
2. No market power: all economic agents are price-takers (competitive markets for all goods). This assumptions fails when we are dealing with monopolies/monopsonies, oligopolies/oligopsonies.
3. Complete information for all agents. This assumption fails if we have uncertainty in the payoffs or when there is asymmetric information or imperfect information.

2 Bilateral Externalities

2.1 Definition

Externalities exist when the actions of one economic agent *directly* affect the utility of other agents. As Chris stated in class, if the effect of one agent's action is *indirectly* through prices, then it isn't an externality.[3] Don't get too hung up

[1] I won't go through this in class.

[2] Recall from the previous section notes that convexity of preferences was necessary for the Second Fundamental Welfare Theorem to hold. See also Practice Problem 17.3.

[3] Viner (1931) coined the expression "pecuniary externality."

on the definition. It isn't always clear whether some action is an externality or not.

Formally, an externality occurs when we can write the utility function of an agent i as follows:

$$u_i(\vec{x}, \vec{y}) = f(x_i, y_i) + g(x_j, y_j) \tag{1}$$

, where $i \neq j$. Recall that in all of our previous models the utility of agent i was determined solely by his or her consumption bundle.

2.2 Example

Assume that the good/activity of interest is playing the bag pipes in-doors. Let h_1 denote agent 1's time spent playing the bag-pipes, then we can write agent 1's utility function as follows:

$$u_1(m_1, h_1) = m_1 + \phi_1(h_1) \tag{2}$$

, where $\phi_1'(\cdot) > 0$ and $\phi_1''(\cdot) < 0$. If we also assume that agent 1's budget constraint is: $m_1 + ph_1 \leq w_1$, we can plug the binding constraint into the objective function to get to the following optimization problem for agent 1:

$$\max_{h_1} w_1 - ph_1 + \phi_1(h_1) \tag{3}$$

. Problem 3 has the following first order condition:

$$\phi_1'(h_1^*) = p = 0 \tag{4}$$

[4], and assume that $p = 0$. For those interested, why we assume that $p = 0$ can be found in 2.5.

Agent 2 is agent 1's neighbor and her utility function[5] is:

$$u_2(m_2, h_1) = m_2 + \phi_2(h_1)$$

, where $\phi_2'(\cdot) < 0$ and $\phi_2''(\cdot) < 0$, which means that playing the bag pipes is a negative externality.

If we let agent 2 decide the level of h_1, we would have $h_1^* = 0$. Can you see why?

The socially optimal level of playing bag pipes indoors, which we denote as h_1^{FB} will solve:

$$\max_{h_1} \{u_1(m_1, h_1) + u_2(m_2, h_1)\} \Leftrightarrow \max_{h_1} \{\phi_1(h_1) + \phi_2(h_1)\} \tag{5}$$

. The first order conditions for the optimization problem 5 is:

[4] Assuming that there exists an interior solution. For this part of the class, we normally assume that an interior solution exists and don't deal with the K-T conditions.

[5] Notice that agent 2 doesn't get to choose h_1, but it enters her utility function.

$$\phi_1'(h_1^{FB}) + \phi_2'(h_1^{FB}) = 0. \tag{6}$$

Now compare FOCs 4 and 6:

$$\phi_1'(h_1^{FB}) = -\phi_2'(h_1^{FB}) > 0 = \phi_1'(h_1^*) \Leftrightarrow \phi_1'(h_1^{FB}) > \phi_1'(h_1^*)$$

, therefore,

$$h_1^{FB} < h_1^*. \tag{7}$$

Inequality 7 tells us that when we're dealing with a negative externality, we have too much of the externality than is socially optimal. See Figure 1 at the end of the notes.

Try this for when we have a positive externality (public good).

2.3 Pareto Optimality of h_1^*

Is the level of bag pipe playing defined by 4 above Pareto optimal? It would seem as though that you can't improve agent 2's utility without decreasing agent 1's utility (decreasing $h_1$would benefit agent 2 but would hurt agent 1, and vice-versa).

However, $h_1^*$is not Pareto optimal. See Figure 2 at the end of the notes. By decreasing $h_1^*$by $\Delta h > 0$, the increase in utility for agent 2 is relatively larger than the decrease in utility for agent 1. Therefore, agent 2 can compensate agent 1, using transfers of the numeraire good, to make agent 1 as well off as when he was playing bag pipes to his heart's content.

In short, efficiency is determined by the quantity of h_1 and not by how the numeraire good is distributed. This leads us to the Coase Theorem, which in a nut shell states:

Theorem. *If property rights are clearly defined, transactions costs are equal to zero, contracts are costlessly and perfectly enforced, parties have complete and perfect information, then the initial allocation of property rights will not matter because bargaining amongst the parties will lead to the Pareto optimal (or efficient) outcome.*

2.4 Solutions to Externalities

Possible solutions include:

1. Quotas

2. Pigouvian taxation / subsidies (which forces the party with the property right to internalize the externality)[6]

[6] In the example above, we could force agent 1 to solve the following optimization problem, which leads to the optimal taxation that results in the Pareto optimal level of bag pipe playing:

$$\max_{h_1} \{\phi_1(h_1) - t \cdot h_1\} \Rightarrow \phi_1'(h_1^{**}) = t \Rightarrow t^* = -\phi_2'(h_1^{FB}) > 0$$

3. Coasian Bargaining (or as Aghion would say: "TIOLI"[7])
4. Market-making
5. Stigma/social recognition[8]

2.5 Short Aside

Assume that the numeraire good is good 1. Define the indirect utility function of agent i who has quasi linear preferences as follows:

$$\begin{aligned} v_i(\vec{p}, w_i, h) &= \max_{x_i \geq \geq 0} u_i(\vec{x}_i, h) = \max_{x_i \geq \geq 0} g_i(\vec{x}_{-1i}, h) + x_{1i} \\ & s.t. \quad \vec{p} \cdot \vec{x}_i \leq w_i \end{aligned}$$

, which has the following solutions (or demand correspondences):

$$\vec{x}^*_{-1i}(\vec{p}, h)$$

, and

$$x^*_{1i}(\vec{p}, h) = w_1 - \vec{p} \cdot \vec{x}^*_{-1i}(\vec{p}, h)$$

. Therefore,

$$v_i(\vec{p}, w_i, h) = g_i(\vec{x}_{-1i}(\vec{p}, h), h) + w_i - \vec{p} \cdot \vec{x}^*_{-1i}(\vec{p}, h) = w_i + \phi_i(\vec{p}, h) = w_i + \phi_i(h) \tag{8}$$

, and we have the desired form, where the final equality comes from the assumption that prices are fixed.

3 Price Discrimination and Monopoly

Types of price discrimination[9]

1. First Degree Price Discrimination: Monopolist observes the type of the consumer and is able to calculate each consumer's demand correspondences (or their willingness to pay). The monopolist uses this to charge each individual his or her willingness to pay. AKA perfect price discrimination or individual pricing. Generally, the FB outcome can be implemented, but the monopolist seller gets all of the surplus. See Figure 3.

[7] "Take-it-or-leave-it" offers by the person *with the property right/ex ante bargaining power.*
[8] Isn't this just another form of internalization?
[9] This taxonomy isn't really helpful in understanding price discrimination, but almost all of the text books have it.

2. Second Degree Price Discrimination: Monopolist is unable to observe the types of each consumer, and so proposes a menu of prices and quantities. Consumers self-select themselves into their optimal quantity and price, which can derived using the Revelation Principle (see MWG 14.C.2 for more information). Screening models. Ex) Intel's 486 DX and SX, and 487 micro processors, where SX is just a damaged version of the DX, and 487 was an upgrade of the SX to the DX.

3. Third Degree Price Discrimination: Monopolist is uable to observe types, but is able to observe something immutable that is related the a consumer's willingness to pay. Monopolist charges based on this characteristic.

3.1 Basic Monopolist's Problem

Recall that a producer/seller of goods in a competitive market must solve its profit maximization problem which can be written as follows:

$$\max_q pq - c(q) \Rightarrow p = c'(q^C)$$

, or the price (which is exogeneous) is equal to the marginal cost.

The monopolist's problem is slightly different:
[10]

$$\max_q p(q)q - c(q) \Rightarrow p(q^M) + p'(q^M)q^M = c'(q^M) \tag{9}$$

, where the LHS of equation 9 is the marginal revenue $\left(\frac{d[p(q)q]}{dq}\right)$ of the monopolist and the RHS is the marginal cost. We see that $p(q^M) > c'(q^M)$ because $p'(\cdot) < 0$, which means that the price under a monopoly exceeds the marginal cost of production. This in turn means that $q^M < q^C$.

3.2 Harvard Football Example

Assume that the good of interest is The Game football tickets, which are being supplied by a monopolistic seller with cost function equal to: $c(Q) = \frac{Q^2}{20}$, where Q is the total number of tickets supplied to the market, including both those supplied to alumni and to students: $Q = q_a + q_s$.

Further assume that there are two types of consumers: alumni and students. The demand function for alumni can be written as: $q_a(p_a) = 100 - p_a$ and $q_s(p_s) = 200 - 10p_s$. The monopolist has information on whether a consumer is a student or an alumni and so is able to engage in Third Degree Price Discrimination.

[10] Notice that we use the inverse demand function $p(\cdot)$ in the objective function.

3.2.1 No Students

If we assume that students are in reading period and unable to attend, the monopolist's problems is:

$$\max_{q_a=Q} \{p_a(Q) \cdot Q - c(Q)\} = \max_{Q} \left\{(100 - Q) \cdot Q - \frac{Q^2}{20}\right\} \tag{10}$$

, the FOCs are (assuming an interior solution):

$$100 - 2Q - \frac{Q}{10} = 0 \Rightarrow Q^* = q_a^* \approx 47.6$$

, and the resulting optimal price and profit are: $p_a^* \approx 52.4$ and $\pi_{npd}^* = q_a^* \cdot p_a^* - c(q_a^*) \approx 2381$.

3.2.2 Third Degree Price Discrimination

Now the monopolist solve the following problem:

$$\max_{q_a,q_s} \{p_a(q_a) \cdot q_a + p_s(q_s) \cdot q_s - c(q_a + q_s)\} \quad \Rightarrow$$
$$\max_{q_a,q_s} \left\{(100 - q_a) \cdot q_a + \left(20 - \frac{q_s}{10}\right) \cdot q_s - \frac{(q_a + q_s)^2}{20}\right\}$$

, which has the following FOCs:

$$100 - 2q_a = \frac{(q_a + q_s)}{10} \tag{11}$$
$$20 - \frac{q_s}{5} = \frac{(q_a + q_s)}{10} \tag{12}$$

. Solving the system of linear equations 11 and 12 results in $q_a^* \approx 45.2$, $p_a^* \approx 54.8$, $q_s^* \approx 51.6$, $p_s^* \approx 14.84$, and $\pi_{pd}^* = q_a^* \cdot p_a^* + q_s^* \cdot p_s^* - c(q_a^* + q_s^*) \approx 2774 > \pi_{npd}^*$.