Section Notes 7

Abby Friedman, Wonbin Kang

October 22, 2010

Agenda

1. Stochastic Dominance
2. Introduction to Game Theory (basic definitions, assumptions, and representations)

1 Stochastic Dominance[1]

1.1 First-Order Stochastic Dominance (FOSD)

By way of background, for those of you who have taken statistics, you know that there are multiple ways to represent the distribution of a random variable. For example, you can use the method of moments (the first, second, third, fourth,...moment), maximum likelihood estimators, etc. The most frequently used is the expected value and the variance of a distribution. Then for any two distributions (or lotteries) over money outcomes, $F(\cdot)$ and $G(\cdot)$,[2]we can define the preferences of an economic agent over these two money outcome distributions based on their respective expected values and variances. The former is first-order stochastic dominance and the latter is second-order stochastic dominance.

For any two distributions over money outcomes, $F(\cdot)$ and $G(\cdot)$, if $F(\cdot)$ first-order stochastically dominates $G(\cdot)$, then all agents with weakly increasing utility functions will weakly prefer $F(\cdot)$ to $G(\cdot)$. Note that first-order stochastic dominance is a very strict condition and typically we will not have either distribution first-order stochastically dominating another.

More formally, we can define FOSD as follows:

[1] The handout on stochastic dominance on the course website is a great source to review the key concepts.

[2] The random variable X is the money outcome and $F(\cdot)$ and $G(\cdot)$ are the cdfs which represent the lotteries over which we defined an economic agent's preferences in the previous section. We can think of these cdfs as lotteries because the lotteries we considered in the previous section was merely a vector of probabilities over a discrete number of events.

Definition 1. The distribution over money outcomes $F(\cdot)$ first-order stochastically dominates $G(\cdot)$ if, for $\forall u : \mathbb{R} \to \mathbb{R}$, s.t. $u(\cdot)$ is a non-decreasing function, we have that:

$$\int u(x)dF(x) \geq \int u(x)dG(x),$$

or

$$\int u(x)f(x)dx \geq \int u(x)g(x)dx,$$

where $f(x)$ and $g(x)$ are the respective pdfs of $F(x)$ and $G(x)$. This implies that the distribution of monetary payoffs $F(\cdot)$ first-order stochastically dominates the distribution $G(\cdot)$ if and only if $F(x) \leq G(x)$, $\forall x$.

Try drawing a picture. Draw two cdfs, s.t. $F(x) \leq G(x)$, $\forall x$, and reverse the axis to see that for a given probability you want to have a greater value of the random variable X.

1.2 Second-Order Stochastic Dominance (SOSD)

SOSD uses the variance of a distribution to determine whether an economic agent prefers one money lottery, represented again by a cdf, over another.

If $F(\cdot)$ second-order stochastically dominates $G(\cdot)$, then the 2 distributions over money (or money lotteries) have the same expected value, but all risk-averse economic agents prefer $F(\cdot)$ to $G(\cdot)$, which implies that $F(\cdot)$ is less risky than $G(\cdot)$. In short, because the two distributions over money outcomes have the same expected value, $G(\cdot)$ can be represented as a mean-preserving spread of $F(\cdot)$.

More formally, we can define SOSD as follows:

Definition 2. For any two distributions $F(\cdot)$ and $G(\cdot)$ with the same mean, $F(\cdot)$ second-order stochastically dominates (or is less risky than) $G(\cdot)$ if for every non-decreasing *concave* function $u : \mathbb{R} \to \mathbb{R}$, we have that:

$$\int u(x)dF(x) \geq \int u(x)dG(x).$$

This implies that the distribution of monetary payoffs $F(\cdot)$ second-order stochastically dominates the distribution $G(\cdot)$ if and only if:

$$\int_{-\infty}^{x} G(t)dt \geq \int_{-\infty}^{x} F(t)dt, \forall x.$$

Again, drawing a picture may provide you with some intuition.

1.3 Combining FOSD and SOSD

If L_1 FOSDs L_2, and L_2 SOSDs L_3, then all risk-averse economic agents will prefer L_1 to L_3 due to transitivity. However, we don't say that L_1 SOSDs L_3.

Game Theory Textbooks

Of course, you can rely on MWG for the readings and you can also try the bible of game theory: Fudenberg and Tirole. But these two books are probably better for more advanced classes in Game Theory and as references in the future (especially FT). Two great sources for this class and a firm grounding in Game Theory are:

1. Robert Gibbons, 1992, *Game Theory for Applied Economists*, Princeton University Press.

2. Martin J. Osborne, 2004, *An Introduction to Game Theory*, Oxford University Press.

I especially recommend Gibbons (1992).

2 Introduction to Game Theory

What is a game? You might see the following types of notation when dealing with games:

$$
\begin{aligned}
\Gamma_n &= [I, \{S_i\}, \{u_i(\cdot)\}] \\
\Gamma_e &= [\mathcal{I}, I, \mathfrak{A}, p(\cdot), \alpha(\cdot), \mathcal{H}, \{\Delta(S_i)\}, \{u_i(\cdot)\}].
\end{aligned}
$$

All the above is saying is that, at its core, a game has a set of players, for each player a set of actions, a strategy space for each player, and payoff functions (or outcomes; or preferences over the set of action profiles). Don't worry about these types of representations.

2.1 Common Definitions (less formal than MWG or FT)

Here are a few terms that you'll come across in the next few weeks:

- Players: This is easy. Only one thing to add: you might see the expression "in a finite game" in various propositions or definitions within game theory, in which case, $I < \infty$.

- Actions: When it is a player's turn to move, what will she do? Will she choose UP or DOWN; COOPERATE or DEFECT; LEFT, RIGHT, or CENTER; q or q', where $q, q' \in \mathbb{R}_+$. The actions available to a player can be finite or infinite.

- Information Sets: Concept used to represent a game where we assume that the players DO NOT have *perfect information*. In a sequential (or dynamic) game, which is represented in an *extensive form*,[3] an information set is represented by drawing a circle around the relevant nodes.

[3] I won't cover "extensive form" vs "normal (reduced) form" games.

- Strategy: "A strategy is a *complete contingent plan*, or *decision rule*, that specifies how the player will act in *every possible distinguishable circumstance* in which she might be called upon to move. ... Thus, a player's strategy amounts to a specification of how she plans to move at each one of her information sets, should it be reached during play of the game."[4] You should always try to keep the concepts of "strategy" and "action" separate and know the difference between the two. In a static, simultaneous move game, action and strategy may be the same. However, in more complicated (and so interesting) games, this will not be the case.

- Outcomes/utilities/payoffs: This should be self explanatory. Note that the arguments of player i's utility function is not only her own strategy, but the strategy of the other players: $u_i(s_i, \vec{s}_{-i})$.

- Histories: Don't worry about this now, but the history of player i, denoted H_i, is the set of sequences after which player i is called upon to move or select an action, $a \in \mathcal{A}(h)$, or her action set. We'll discuss histories when we get into dynamic, extensive form games.

- Pure Strategies: Although we tend to think in terms of pure strategies and much of applied game theory focuses on pure strategies, it may be helpful to think of pure strategies as mixed strategies with a degenerate probability distribution. In short, a pure strategy is a mixed strategy that "assigns probability 1 to the action a_i [which is] equivalent to her simply choosing the action a_i...."[5]

- Mixed Strategies: This is covered in more detail below.

- Static vs. Dynamic / Extensive-Form vs Normal-Form / Simultaneous vs Sequential: See the example below.

Example. Market Entry Game.

[4] MWG p.228, emphasis in original.
[5] Osborne (2004, p.108).

2.2 Common Assumptions

Note that all games will not have all of these assumptions. The point is that in more advanced games, we're going to try to relax some of these assumptions.

- Complete Information: Each of the players' payoff functions, which are dependent on the strategies of each player, is *common knowledge* to all of the other players.

- Perfect Information: "A game is one of *perfect information* if each information set contains a single decision node. Otherwise, it is a game of *imperfect information*."[6] If we assume perfect information, then each player knows exactly where they are on the game-tree.

- Common Knowledge: "[A] basic postulate of game theory that all players know the structure of the game, know that their rivals know it, know that their rivals know that they know it, and so on."[7] One aspect of "common knowledge" that we assume in most of our games is that each of the players is rational, knows that the other players know she is rational, knows that she knows it, and so on.

[6] MWG Definition 7.C.1 (emphasis in original)
[7] MWG p.226