Section Notes 12

Wonbin Kang

December 3-4, 2009

Agenda

1. Bertrand (Price Competition) vs. Cournot (Quantity Competition)
2. Stackelberg Model (variant of Cournot)
3. Kreps-Sheinkman (1983) (time permitting)

1 Bertrand vs. Cournot Duopoly

Proposition 1.1. *In the Bertrand Duopoly model, there is a unique Nash Equilibrium (p_1^*, p_2^*), where the two identical firms set their prices equal to the marginal cost c.*

- Both firms earn zero profits.
- Since price is equal to marginal cost, we get the perfectly competitive outcome with only two firms.
- Because of Bertrand competition, each firm faces infinitely elastic demand corrrespondences given the price of the other firm.
- Therefore, monopoly is the only form of price distortion that can arise.

Proposition 1.2. *In the Cournot Duopoly model where per unit cost $c > 0$ for both firms and the inverse demand function $p(q)$ is such that $p'(q) < 0$ and $p(0) > c$; any Nash Equilibrium (q_1^*, q_2^*) will result in market prices greater than c and smaller than the monopoly price.*

- Firms earn positive profits and we are not in a perfectly competitive equilibrium.
- Oligopolies arise under the Cournot model.

Let's look at an example with linear inverse demand functions and constant marginal costs. Assume that both firms have marginal costs equal to c, and the inverse demand function is linear: $P(Q) = \alpha - Q$, where $Q = q_1 + q_2$. Further assume that $\alpha > c$. The profit function for each firm can be written as:

$$\begin{aligned} \pi_1(q_1; q_2) &= q_1 P(q_1 + q_2) - cq_1 = (\alpha - c - q_2)q_1 - q_1^2 \\ \pi_2(q_2; q_1) &= q_2 P(q_1 + q_2) - cq_2 = (\alpha - c - q_1)q_2 - q_2^2 \end{aligned}$$

. We can derive the best response functions for each firm by using the FOCs of each firms' profit maximization problem. Therefore, the best response function for firm i is:[1]

$$q_i = b_i(q_j) = \frac{\alpha - c - q_j}{2} \tag{1}$$

. By the definition of Nash Equilibrium, we know that the following two equations must hold at (q_1^*, q_2^*):

$$\begin{aligned} q_1^* &= \frac{\alpha - c - q_2^*}{2} \\ q_2^* &= \frac{\alpha - c - q_1^*}{2} \end{aligned}$$

. Solving the two equations above defines the Nash Equilibrium:

$$(q_1^*, q_2^*) = \left(\frac{\alpha - c}{3}, \frac{\alpha - c}{3} \right) \tag{2}$$

, which results in total output $Q_{Cournot} = \frac{2}{3} \cdot (\alpha - c)$ and equilibrium price $P_{Cournot} = P\left(\frac{2}{3} \cdot (\alpha - c)\right) = \frac{1}{3} \cdot (\alpha + 2c)$.

Three remarks:

1. $P_{Cournot} = \frac{1}{3} \cdot (\alpha + 2c) > c$ ($\because \alpha > c$).
2. $Q_{Cournot} = \frac{2}{3} \cdot (\alpha - c) > Q_{Monopoly} = \frac{1}{2} \cdot (\alpha - c)$.
3. Each firm earns profits equal to: $\frac{1}{9} \cdot (\alpha - c)^2$[2]

[1] Note that the profit functions for both firms are symmetric. This means that the FOCs will be symmetric leading to symmetric best response functions.

[2] The monopolist's profit maximization problem is:

$$\max_Q P(Q) \cdot Q - cQ \Rightarrow \max_Q (\alpha - c)Q - Q^2$$

. FOC identifies the monopolist's optimal production level: $Q_{Monopoly} = \frac{\alpha - c}{2}$

2 Stackelberg Model

One of the assumptions underlying the Cournot model above was that each firm didn't know how much the other firm was producing. However, what happens if firm 2 is now able to observe how much firm 1 produces and produces accordingly?[3] Further, since firm 1 knows that firm 2 is observing its output, will firm 1 change its output from the Cournot equilibrium identified above?

To repeat the set up from Section 1: Assume that both firms have marginal costs equal to c, and the inverse demand function is linear: $P(Q) = \alpha - Q$, where $Q = q_1 + q_2$. Further assume that $\alpha > c$ and that firm 1 produces first and then firm 2 produces.

In equilibrium, let's assume that firm produces q_1^*. The firm 2's profit maximization problem is equal to:

$$\max_{q_2} P(q_1^* + q_2)q_2 - cq_2 = \max_{q_2}(\alpha - c - q_1^*)q_2 - q_2^2$$

, the FOC for the above optimization problem results in:

$$q_2^* = b_2(q_1^*) = \frac{\alpha - c - q_1^*}{2} \tag{3}$$

. Because firm 1 knows that firm 2 will best respond to any output q_1 that firm 1 produces, firm 1's profit maximization problem is:

$$\max_{q_1} P\left(q_1 + \frac{\alpha - c - q_1^*}{2}\right) q_1 - cq_1 \Rightarrow \max_{q_1} \frac{1}{2} \cdot q_1 \cdot (\alpha - c - q_1) \tag{4}$$

. The FOC identifies the optimal level of production for firm 1 as:

$$q_1^* = \frac{\alpha - c}{2} \tag{5}$$

, and equation 3 identifies the optimal production of firm 2 as:

$$q_2^* = \frac{\alpha - c}{4}. \tag{6}$$

Two remarks in relation to the Cournot model above:

1. $Q_{Stackelberg} = \frac{3}{4} \cdot (\alpha - c) > Q_{Cournot} = \frac{2}{3} \cdot (\alpha - c) > Q_{Monopoly} = \frac{1}{2} \cdot (\alpha - c)$, which means that the equilibrium price in the Stackelberg model is less than that of the Cournot model.

2. Comparing the profits of each firm to that of the Cournot model:

$$\begin{aligned} \pi_1^{Stackelberg} &= \frac{1}{8} \cdot (\alpha - c)^2 > \frac{1}{9} \cdot (\alpha - c)^2 = \pi_1^{Cournot} \\ \pi_2^{Stackelberg} &= \frac{1}{16} \cdot (\alpha - c)^2 < \frac{1}{9} \cdot (\alpha - c)^2 = \pi_2^{Cournot}. \end{aligned}$$

[3] In game theory, this is sequential action, and the relevant equilibrium concept is subgame perfect equilibrium.

3 Kreps-Scheinkman (1983)

In the Cournot model, we assume that firms compete on the basis of quantity produced. We can interpret this as a firm choosing its production capacity. If that is the case, then what is the equilibrium when firms first choose their capacity levels and then engage in Bertrand price competition? This is the question addressed by David M. Kreps and Jose A. Scheinkman, 1983, *Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes*, Bell Journal of Economics, 14(2): 326-337. The result of the paper is clearly stated in the title of the article.

Let's examine this result using MWG 12.C.11(a).[4]

Consider a capacity constrained duopoly pricing game. Firm j's capacity is q_j for $j = 1, 2$, and it has a constant cost per unit of output of $c \geq 0$ up to this capacity limit. Assume that the market demand function $x(p)$ is continous and strictly decreasing at all p such that $x(p) > 0$ and that there exists a price $\tilde{p}$ such that $x(\tilde{p}) = q_1 + q_2$. Suppose also that $x(p)$ is concave and let $p(\cdot) = x^{-1}(\cdot)$ denote the inverse demand function.

The rationing scheme if as follows:

If $p_j > p_i$:

$$\begin{aligned} x_i(p_1, p_2) &= \min\{q_i,\, x(p_i)\} && (7) \\ x_j(p_1, p_2) &= \min\{q_j,\, \max\{x(p_j) - q_i,\, 0\}\} && (8) \end{aligned}$$

, and if $p_j = p_i = p$:

$$x_i(p_1, p_2) = \min\left\{q_i, \max\left\{\frac{x(p)}{2},\, x(p) - q_j\right\}\right\},\ \forall i \in \{1, 2\}.$$

Further assume that the higher value consumers (those consumers with a higher willingness to pay) are served first.

Suppose that $q_1 < b_1(q_2)$ and $q_2 < b_2(q_1)$, where $b_i(q_j)$ is the best response function for a firm i given production level q_j by firm j. Show that $p_1^* = p_2^* = p(q_1 + q_2)$ is a Nash Equilibrium.

Assume that firm 2 charges $p_2^* = p(q_1 + q_2)$, the price at which both firms are producing at their respective capacities. If firm 1 charges $p_1 \leq p(q_1 + q_2)$, firm 1 will sell its full capacity level q_1, resulting in profits equal to $(p_1 - c)q_1$, which is less than the profits that firm 1 would get if it charged $p_1^* = p_2^* = p(q_1 + q_2)$, $(p_1^* - c)q_1$.[5] Therefore, there is no profitable deviation for firm 1 to charge below p_2^*.

Is there a profitable deviation for firm 1 to charge above $p_2^* = p(q_1 + q_2)$? Note that from the rationing scheme set out above in equations 7 and 8, the residual demand for firm 1 can be rewritten as follows:

[4] I won't cover 12.C.11(b), since you need to know some game theory concepts.

[5] Notice how important the capacity constraints are. Unlike in our previous Bertrand example, firm 1 does *not* produce goods to satisfy the entire demand.

$$x_1(p_1, p_2) = \min\{q_1,\, x(p_1) - q_2\}$$

. Note that $x(p_1) - q_2$ is the solution to firm 1's profit maximization problem:

$$\max_{x_1} \{p(x_1 + q_2) - c\}\, x_1 \; s.t. \; x_1 \leq q_1$$

. Solving the maximization problem results in a best response function: $x_1^* = b_1(q_2)$, which by assumption is greater than firm 1's capacity: $b_1(q_2) > q_1$, meaning that firm 1 can't produce $b_1(q_2)$. Therefore, we have a corner solution to the problem, where $x_1^* = q_1$. Therefore, $p_1 = p(q_1 + q_2) = p_2^*$ and we have contradiction.

Therefore, it is optimal for firm 1 to charge $p(q_1 + q_2)$ given that firm 2 does the same.

By symmetry, the same argument applies to firm 2 and so we are at an equilibrium.

Since we know the result of the Bertrand price competition given capacity constraints, firms will now maximize profits by solving for the optimal capacities (q_1^*, q_2^*), given prices will equal $p(q_1 + q_2)$. Sound familiar? It should, because this is the set up of the Cournot model.