Section Notes 6

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Agenda

1. Midterm Studying Tips
2. Midterm Review Question: the Price Fixing Game

1 Midterm Studying Tips:

- Expect this class to take a lot of your time between now and the midterm. Materials up to PBE (signaling games) on next week's problem set (BNE) will be on the midterm. Prioritize your time: if you're stuck on the math on a problem set, move on.
- Do old midterm exams and time yourself. Anything other than PBE is fair game.
- Make sure you understand how to do everything on the "systematic approach handout".
- Review problem set and practice problem set solutions.

2 Midterm Review Question: the Price Fixing Game

Questions (a) through (c): The extensive form game is found in Appendix A-1.

Subpart (a) At how many information sets does each player move?

$$S_1 \equiv \{\{C, F\}, q_1 = 30, q_1 \in (0, \infty)\}$$

$$S_2 \equiv \{\{C, F\}, q_2 = 30, q_2 \in (0, \infty)\}$$

Subpart (b) The problem is information set 1.3. The game as represented in the extensive form fails the assumption of "perfect recall."[1] The players don't know what he or she played in the previous round. However, for purposes of this game, this assumption is not really necessary.

Subpart (c) Note that we can rewrite the game above in a 2×2 matrix:

	C, 30, q_1	F, 30, q_2
C, 30, q_1	1800, 1800	$q_1(120 - q_1 - q_2)$, $q_2(120 - q_1 - q_2)$
F, 30, q_1	$q_1(120 - q_1 - q_2)$, $q_2(120 - q_1 - q_2)$	$q_1(120 - q_1 - q_2)$, $q_2(120 - q_1 - q_2)$

1. Let's first consider what happens in the Cournot part of the game. We see that the NE of the Cournot part (note that it isn't a subgame) of the game is $q_1^* = q_2^* = 40$, for a payoff of $\pi_1^* = \pi_2^* = 1,600$. Therefore, a candidate equilibrium strategy profile would be as follows:

$$\sigma^* = \left[(F,\, q_1 = 30,\, q_1 = 40)\,,\, (F,\, q_2 = 30,\, q_2 = 40)\right] \tag{1}$$

   (a) Given the strategies of the other player, note that each player has no incentive to deviate from the Cournot equilibrium production: it is by definition a best response.

   (b) Further, a player does not have an incentive to deviate from F to C since that would merely get her the same payoff.

   (c) Therefore, this is a NE. However, is it the only NE? No.

2. Let's consider when it would be a best response for player 1 (and since this is a symmetric problem, player 2) to play (C, 30, q_1)[2].

$$1800 \geq u_1(F,\, 30,\, q_1 | C,\, 30,\, q_2)\, \forall q_1 \Rightarrow 1800 \geq u_1(F,\, 30,\, q_1^* | C,\, 30,\, q_2)\, \forall q_1 \tag{2}$$

$$q_1^* = \arg\max_{q_1} q_1(120 - q_1 - q_2) \Rightarrow q_1^* = \frac{120 - q_2}{2}$$

   Now we substitution our equation for q_1^*into inequality 2:

$$1800 \geq \left(\frac{120 - q_2}{2}\right)^2 \Rightarrow q_2 \geq 35.15$$

   Since this is a symmetric problem, the same will apply for player 2. Therefore the NE we're looking for is:

$$\sigma^* = \left[(C,\, q_1 = 30,\, q_2 \in [35.15, \infty))\,,\, (C,\, q_2 = 30,\, q_2 \in [35.15, \infty))\right] \tag{3}$$

   What's going on in this game? Why are each of the players indifferent between producing an amount greater than 35.15 in the Cournot game and not an amount greater than 30?

[1] This is not the same as perfect information.

[2] Do you see that the choice of q_1 is off the equilibrium path? Since we're looking for NE and not SPNE, we know that non-credible threats can be sustained in equilibrium.

3. Now let's refine the set of NE identified in 1 and 3 above. Let's first use the THP refinement. Recall that THP excludes equilibrium strategy profiles which include a weakly dominated strategy.

   (a) $\sigma^* = [(F,\, q_1 = 30,\, q_1 = 40)\,,\, (F,\, q_2 = 30,\, q_2 = 40)]$ is not a THPNE since it is weakly dominated by $\sigma^* = [(C,\, q_1 = 30,\, q_1 = 40)\,,\, (C,\, q_2 = 30,\, q_2 = 40)]$. Remember that you only need one strategy to weakly dominate another for the latter to be weakly dominated.

   (b) Now consider the second equilibria in 3. The strategy $\sigma_1 = (C,\, q_1 = 30,\, q_2 \in (60, \infty))$ is weakly dominated by $\sigma_1^* = (C,\, q_1 = 30,\, q_2 \in [35.15, 60))$. Do you see why? Check the first-order conditions for player 1 and note that player 1 would never produce more than 60, even if the action is off the equilibrium path.

   (c) Therefore, the THPNE of this game is:

$$\sigma^* = [(C,\, q_1 = 30,\, q_2 \in [35.15,\, 60))\,,\, (C,\, q_2 = 30,\, q_2 \in [35.15,\, 60))]$$

4. Now let's refine the set of NE using SPNE. How many subgames? Since there is only one subgame, SPNE is equivalent to NE.

Questions (d) through (f): The extensive form game is found in Appendix A-2.

Subpart (d) Since this is a symmetric game, $\forall i \in \{1, 2\}$:

$$S_i = \{\{C, F\}\,, q_i = 30, q_{i3} \in (0, \infty), q_{i4} \in (0, \infty), q_{i5} \in (0, \infty)\}$$

Subpart (e) The extensive form game merely gets rid of the information sets after the initial choice of "C" or "F". See Appendix A-2

Subpart (f) The game is sequential, we don't have perfect information, and we don't have multiple types: Therefore, the equilibrium concept is SPNE.

1. How many subgames do we have? We have 6 subgames starting at decisions nodes labeled 1.1; 1.2; 1.3; 1.4; 1.5; and 2.2. Notice that decision nodes 1.2 and 2.2 are not only singletons but the action set is also a singleton (both players can only play $q_1 = q_2 = 30$), and so this is the NE strategy of these subgames.

2. The subgames starting at 1.3; 1.4; and 1.5 are Cournot with the unique NE equal to $q_1^* = q_2^* = 40$ in each subgame for a payoff of 1,600 for each player.

3. Moving up the game tree (see Appendix A-3), we can rewrite the game in the following form:

	C	F
C	$\underline{1800}$, $\underline{1800}$	$\underline{1600}$, 1600
F	1600, $\underline{1600}$	$\underline{1600}$, $\underline{1600}$

Where we find that the pure strategy SPNEs of the entire game are:

$$\sigma^* = \left[(C,\, q_1 = 30,\, q_{13} = 40,\, q_{14} = 40,\, q_{15} = 40)\,,\, (C,\, q_2 = 30,\, q_{23} = 40,\, q_{24} = 40,\, q_{25} = 40) \right] \tag{4}$$

$$\sigma'^* = \left[(D,\, q_1 = 30,\, q_{13} = 40,\, q_{14} = 40,\, q_{15} = 40)\,,\, (D,\, q_2 = 30,\, q_{23} = 40,\, q_{24} = 40,\, q_{25} = 40) \right]. \tag{5}$$

4. Are there any refinements? Do you see that THP would eliminate 5 as a SPNE?

Questions (g) through (k): Without considering the extensive form of the game, you should be able to tell that this is akin to a signaling game where the Regulator is the Receiver who chooses action I and the firms are the Senders who send a message which is equal to the price. Here there, are two firms (unlike in the signaling models where there was only one Sender), but in reality, we only care about whether the firms are colluding (type 1) or not colluding (type 2).

Subpart (g) Remember in the signaling models from class and section, we only had two possible messages that could be sent by the Sender. This resulted in two information sets for the Receiver. By the same logic, since there are an infinite number of prices (messages), we have an infinite number of information sets for which we must set the regulator's level of action:

$$S_{Regulator} \equiv \{ I(p) \in [0,1] : p \in P \},$$

where P is the set of all possible equilibrium prices. This won't be that big of a problem.

Subpart (h) Now the regulator's info set spreads across the terminal nodes from before and therefore, there is only one subgame, (the entire game). The equilibrium concept is PBE since we have to deal with the regulator's belifes.

Subpart (i) Consider the case where both firms choose to collude (play "C"). Then the equilibrium price will equal 60. If the players are not colluding, the the price is not equal to 60. Therefore,

1. if the regulator sees the message $p \neq 60$, then the regulator believes that firms are not colluding $\Pr(C, C) = 0$ and so regulator's payoff is 1-I. And so the regulator best responds to the message of $p \neq 60$ by setting $I^* = 0$;

if the regulator sees the message $p = 60$, then the regulator believes that $\Pr(C, C) = 1$ and solves for I^* s.t.:

$$I^* = \arg\max_I \left\{ \sqrt{I}\,(1 - I) + \left(1 - \sqrt{I}\right)(-I) \right\} = \arg\max_I \left\{ \sqrt{I} - I \right\}$$

$$\Rightarrow^{FOC} \frac{1}{2} \cdot \frac{1}{\sqrt{I}} - 1 = 0 \Rightarrow I^* = \frac{1}{4}$$

Subpart (j)

1. The equilibrium outcomes when both firm collude is: C_1, C_2, $q_1 = 30$, $q_2 = 30$, $I^* = \frac{1}{4}$. If we include a fine equally distributed amongst the two firms, the total payoffs for both firms and the regulator are: $\pi_1 = 1800 - \frac{1}{2}f$, $\pi_2 = 1800 - \frac{1}{2}f$, and $\pi_{regulator} = \frac{1}{4}$.
2. If there is no collusion, the outcome is: F_1, F_2, $q_1 = 40$, $q_2 = 40$, $I^* = 0$ and the total payoffs are: $\pi_1 = 1600$, $\pi_2 = 1600$, and $\pi_{regulator} = 1$.
3. Therefore, if $f \leq 400$, firms are better off colluding and if $f > 400$, the equilibrium has firms not colluding (though one of the firms could play "C" in equilibrium).

Subpart (k) Skip.