Section Notes 1

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Agenda

1. Assumptions on preferences
2. Preferences and utility functions
3. Examples of utility functions and their two-dimensional representation

Before getting started, let's clear up some confusion on notation! Remember what I said about economists and notation.

A very important distinction in notation between the section notes and the notation used by Chris during lecture is that when comparing bundles of goods (in Ec 2020a, we usually deal with two goods), both Sam and I will use $\vec{x}, \vec{y}$, each of which is a two dimensional vector. The first and second elements of these vectors will denote the amount of good 1 and good 2, respectively. So $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ denotes a bundle of goods with x_1 units of good 1 and x_2 units of good 2. During Chris' lectures, he will be looking at bundles of the form $\begin{bmatrix} x \\ y \end{bmatrix}$,[1] which denotes a bundle of goods with x units of good 1 and y units of good 2. When comparing bundles of bundles, Chris will usually use $\begin{bmatrix} x' \\ y' \end{bmatrix}$, or (x', y'), to denote a distinct bundle of goods from $\begin{bmatrix} x \\ y \end{bmatrix}$.

I know it's confusing, but you get used to the bad notation of economists and will be doing the same in no time!

1 Weak Axiom of Revealed Preferences[2]

Definition 1. If an individual chooses A out of a set of options including B, they should never choose B when faced with a choice of a different set of options which also includes A and B. More formally, if A is ever chosen when B is available, then there can be no set containing both alternatives from which B is chosen and A is not.

[1] Sometimes he uses (x, y), or some variant of the two.

[2] We won't cover this in section.

Formally, it must be the case that:

Definition 2. Given two distinct price vectors $\vec{p} \neq \vec{p}'$ and wealth $w \neq w'$, the Walrasian demand functions $\vec{x}(\vec{p}, w)$ satisfies WARP if the following holds:
If $\vec{p} \cdot \vec{x}(\vec{p}', w') \leq w$ and $\vec{x}(\vec{p}, w) \neq \vec{x}(\vec{p}', w)$, then $\vec{p}' \cdot \vec{x}(\vec{p}, w) > w'$[3]

For graphical representations of WARP, see Figure 2.F.1 of MWG (page 30).

1.1 Extension (Lump Sum Principle)

Example: $w = 60$; $p = 2$; $q = 2$. With subsidy, the resulting wealth and prices are $w = 60$; $p = 2$; $q' = 1$. On the other hand, with a lump sum transfer, we'd get $w = 80$; $p = 2$; $q = 2$. Now draw the resulting two budget sets: $B_{subsidy}$ and $B_{lump\,sum}$. Assume that under the subsidy, the consumer chose $(x, y) = (20, 20)$. Therefore, the government opens up new bundles which may be preferred to the original bundle, $(x, y) = (20, 20)$, which was chosen under the subsidized budget set.

2 Assumptions on preferences

We start the analysis of consumer behavior by establishing the preference relationships for all elements—*i.e.* commodity bundles—of the consumption set $X \subseteq R^L$.[4]

2.1 Rationality

We take the preference relationships ("$\succeq$") to be "rational" in the sense that they are:

1. **complete**: $\forall \vec{x}, \vec{y} \in X$, we have $\vec{x} \succeq \vec{y}$ or $\vec{y} \succeq \vec{x}$ (or both).
2. **transitive**: $\forall \vec{x}, \vec{y}, \vec{z} \in X$, if $\vec{x} \succeq \vec{y}$ and $\vec{y} \succeq \vec{z}$, then $\vec{x} \succeq \vec{z}$.

Note that transitivity is a fairly strong assumption to make and for those of you interested in political economy or social welfare, you'll start thinking about ways to weaken this assumption. To the preference relationships we established in the manner above, we add further assumptions.

2.2 Desirability

We assume that the preferences we established above are such that larger amounts of goods are preferred to smaller amounts. We capture this assumption using one of the following three concepts (note that the mathematical definitions use commodity bundles consisting of two goods):

[3]Using Chris' notation from class, we can rewrite as follows: $x(p, q, w)$ and $y(p, q, w)$ satisfies WARP if the following holds $\forall\, [p, q, w]$ and $[p', q', w']$:
If $px + qy \leq w$ AND $px' + qy' \leq w$ AND $[x, y] \neq [x', y']$, then $p'x + q'y > w'$. This means that the consumer chooses bundle $[x', y']$ given the price-wealth vector $[p', q', w']$.

[4]For Ec 2020a, we normally deal with situations where $L = 2$.

1. **(weak) monotonicity**: if $\vec{x} \gg \vec{y}$, or $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \gg \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, implies that $\vec{x} \succ \vec{y}$.

2. **strong monotonicity**: if $\vec{x} \geq\geq \vec{y}$, or $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq\geq \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$,[5] and $\vec{x} \neq \vec{y}$ implies that $\vec{x} \succ \vec{y}$.

3. **local non-satiation**[6]: if $\forall \vec{x} \in X$ and for every $\epsilon > 0$, there $\exists \vec{y} \in X$ s.t. $||\vec{x} - \vec{y}|| \leq \epsilon$ and $\vec{y} \succ \vec{x}$.[7]

Note that if preferences are strongly monotone, then it is monotone; and that if preferences are monotone, then it is locally non-satiated.[8]

2.3 Convexity

We also assume that the preferences over the consumption set X are convex.[9] What does this mean? It means that the preferences that we are dealing with follow the rule of *diminishing marginal rates of substitution.* Furthermore, convexity of preferences can be viewed as economic agents' preference for diversification.

Mathematically, convexity of preferences can be written as follows: $\forall \vec{x}, \vec{y}, \vec{z} \in X$, if $\vec{x} \succeq \vec{z}$ and $\vec{y} \succeq \vec{z}$, then $\alpha\vec{x} + (1-\alpha)\vec{y} \succeq \vec{z}$, for all $\alpha \in (0, 1)$.

Furthermore, there is another concept (which you will see below in the context of Leontief and linear preferences) that you may want to familiarize yourself with: preferences are *strictly* convex if: $\forall \vec{x}, \vec{y}, \vec{z} \in X$, if $\vec{x} \succeq \vec{z}$ and $\vec{y} \succeq \vec{z}$, then $\alpha\vec{x} + (1-\alpha)\vec{y} \succ \vec{z}$, for all $\alpha \in (0, 1)$. Compare Figures 3.B.3 and 3.B.4 in MWG to see the intuition behind the mathematicl definitions.

2.4 Continuity

What does continuity mean? As Chris stated in class, the continuity assumption rules out "jumps" in the preferences of the agent.

Definition 3. Mathematically, preferences on the consumption set X are continuous if for any sequence of pairs $\{\vec{x}_n, \vec{y}_n\}_{n=1}^{\infty}$ s.t. $\vec{x}_n \succeq \vec{y}_n \forall n$, we have that $\vec{x} \succeq \vec{y}$, where $\lim_{n\to\infty} \vec{x}_n = \vec{x}$ and $\lim_{n\to\infty} \vec{y}_n = \vec{y}$.

Why do we care? See Section 3 below.

Let's review the continuity restriction on preferences by looking at a preference relationship that is not continuous—*i.e.* the Lexicographic preference relationship. The Lexicographic preference relationship is such that $\vec{x} \succeq \vec{y}$ if $x_1 > y_1$ or if $x_1 = y_1$and $x_2 \geq y_2$. The first commodity determines the preference relationship for all bundles where $x_1 \neq y_1$, but where $x_1 = y_1$, the second

[5] I know that this is a really bad abuse of notation. A better way to write it would be if $x_i \geq y_i$, $\forall i$ and if $x_i > y_i$ for some i, then $\vec{x} \succ \vec{y}$.

[6] Think of this as a weaker form of monotonicity

[7] If you're not familiar with the concept of "norms"—*i.e.* $||\ .\ ||$—think of it as the distance between two vectors in the Euclidean space.

[8] Try Exercise 3.B.1 in MWG using the two good case.

[9] Recall our discussion of convex sets.

commodity determines the preference relationship. If the first and second commodities are the same, then the third commodity determines the preference relationship, and so on.

Example 4. Consider a two commodity world where the sequence of bundles $\vec{x}_n = \begin{bmatrix} \frac{1}{n} \\ 0 \end{bmatrix}$ and $\vec{y}_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Therefore, for every single n, we have that $\vec{x}_n \succ \vec{y}_n$. However, note that at the limits, $\lim_{n\to\infty} \vec{x}_n = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \prec \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \lim_{n\to\infty} \vec{y}_n$. In short, the preference relationship is not preserved at the limit points and therefore, Lexicographic preferences are not continuous.

3 Preferences and utility functions

Why do we care about all of the assumptions that we place on preferences? We care because we want to translate these preference relationships into mathematically tractable functions (or utility functions). Each of the "[r]estrictions on preferences translate into restrictions on the form of utility functions."[10] Let's look at each of the assumptions above.

1. *Desirability*: If we assume that an agent's preferences are monotonic (and so locally non-satiated), then we know that if $\vec{x} \gg \vec{y}$, or $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \gg \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $\vec{x} \succ \vec{y}$, or in terms of mathematically tractable utility functions $u(\vec{x}) > u(\vec{y})$.This means that the function $u : X \to \mathbb{R}$ is an increasing function. Furthermore, if preferences are strongly monotone, this implies that preferences are monotone *and* locally non-satiated. If preferences are locally non-satiated, indifference curves are not thick.

2. *Convexity*: If we assume that an agent's preferences are convex, then we know that the utility function representing these preferences must by quasiconcave. [11]Remember what we liked about quasiconcave utility functions from the math review?

3. *Continuity*: We want the preference relationships we defined above to be continuous because this guarantees the existence of a continous utility function $u(\vec{x})$ that represents these preferences. Why do we want continuous utility functions? It makes the math easier! We can *usually* differentiate continuous utility functions.[12]

[10] MWG p.49.

[11] To see this, we need to know another definition of quasiconcavity, which Chris discussed in class; namely that a function f is quasiconcave iff $f(\alpha\vec{x} + (1-\alpha)\vec{y}) \geq min\{f(\vec{x}), f(\vec{y})\}$. For those interested, see Section 21.3 of Simon and Blume (1994) and MWG (pp.49-50).

[12] An example of where continuity is satisfied but we are unable to use calculus is the Leontief preferences. Aside from continuity, a function must also be *smooth* for it to be differentiable. In this class, we will assume that functions are C^2, or twice differentiable.

4 Examples of utility functions and their two-dimensional representation

Before looking at a few examples, we need to remember a few things. In this class, we'll usually be dealing with two goods and functions with two variables that map these two variables onto the real line—*i.e.* $f : \Re^2_+ \to \Re$. Because we are, in many instances, interested in the shape of these graphs, we might want to draw them on the three-dimensional Euclidean space. However, another way to represent these three dimensional graphs is to use "level curves" (or "level sets") on a two-dimensional space (which most of us are more accustomed to). Note that in the context of utility functions, these level curves are called "indifference curves"; when dealing with production functions, they may be called "isoquants"; or when dealing with profit functions, "iso-profit curves." They all mean the same thing! Now let's look at a few utility functions where the consumption space consists of two goods and a function $u : \Re^2_+ \to \Re$.

4.1 Leontief utility function

1. Leontief utility functions can be written as $u(x_1, x_2) = \min\{ax_1, bx_2\}$. The simplest form would set $a = b = 1$. Therefore, for an arbitrary level of utility $\bar{u}$, the equation representing this indifference curve would be: $\min\{x_1, x_2\} = \bar{u}$. Let's draw the indifference curves.

2. Desirability assumptions:

   (a) Are Leontief preferences strongly monotone? No. What happens when $x_1 > x_2$ and you add $\epsilon > 0$ to only good x_1? $u(x_1, x_2) = \min\{x_1, x_2\} = \bar{u} = \min\{x_1 + \epsilon, x_2\}$. Therefore, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim \begin{bmatrix} x_1+\epsilon \\ x_2 \end{bmatrix}$, when $x_1 > x_2$.

   (b) However, are Leontief preferences monotone? Yes, because it must be the case that $\min\{x_1 + \epsilon, x_2 + \epsilon\} > \min\{x_1, x_2\}$. Which means that $\begin{bmatrix} x_1+\epsilon \\ x_2+\epsilon \end{bmatrix} \succ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

3. Are Leontief preferences convex? We could go through the mathematical steps (and in the instance of Lexicographic preferences, we have to), but let's consider a short cut. What is a consequence of convexity of preferences? Recall that it is the quasiconcavity of utility functions. What do quasiconcave utility functions have? Recall from math review that if a function is quasiconcave, the set C_a^+, or the upper level set, is convex. Look at the indifference curves we drew and find the upper level set. Are preferences convex? Yes. Are they strictly convex? No.

4. If we solve a utility maximization problem (hereinafter "UMP") with Leontief utility functions, intuitively the optimal point will be at one of the kinks (in our example, along the locus $x_1 = x_2$) . We'd be wasting our wealth if we weren't on the locus. Note that when solving this particular

UMP, we can't use differentiation and first order conditions to find critical points, despite the fact that preferences and the utility function are continous.[13]

4.2 Linear utility function (1)

1. We can rewrite linear utility functions of the form $v(x_1, x_2) = \alpha x_1 + \beta x_2$, where $\alpha, \beta > 0$, as $u(x_1, x_2) = ax_1 + x_2$, where $a = \frac{\alpha}{\beta} > 0$. Do you remember why? Hint: it has to do with the ordinality of utility. Therefore, for a given level of utility $\bar{u}$, the equation representing this indifference curve is $ax_1 + x_2 = \bar{u} \Rightarrow x_2 = -ax_1 + \bar{u}$. Let's draw these indifference curves, but with a twist. Add a budget line where the price of good 1 and good 2 is equal to p, q, respectively, and wealth is equal to w. You'll need to consider two different cases. Can you see this?

2. Are the preferences represented by the linear utility function above strongly monotonic? Note that $u(x_1+\epsilon, x_2) = a(x_1+\epsilon)+x_2 > ax_1+x_2 = u(x_1, x_2)$, which means that $\begin{bmatrix} x_1+\epsilon \\ x_2 \end{bmatrix} \succ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Therefore, preferences which can be represented by linear utility functions of the form above are strongly monotonic. This means that linear utility functions represent preferences which are monotonic and locally non-satiated.

3. Are preferences represented by linear utility functions convex? Again, instead of going through the math, let's look at the upper level sets using the graph we drew. Are preferences convex? Yes. Are they strictly convex? No.

4. If we solve the UMP with linear utility, the optimizing agent will consume all in good 1 or all in good 2 depending on whether $|-a| > |-\frac{p}{q}|$or whether $|-a| < |-\frac{p}{q}|$. Does this make sense? Think in terms of the ratio of marginal utilities of consuming good 1 and good 2. If this doesn't seem clear to you, don't worry. We're going to go through it next week.

4.3 Linear utility function (2)

1. Now let's look at an extension of the linear utility function we saw above where $u(x_1, x_2) = ax_1 - x_2$, where $a > 0$. We can write the equation for a given level of utility $\bar{u}$ as $ax_1 - x_2 = \bar{u} \Rightarrow x_2 = ax_1 - \bar{u}$. Let's draw these indifference curves with the same budget line as above. Note that the slope on the indifference curves is positive.

2. Desirability assumptions:

   (a) Are the preferences represented by this particular linear utility function strongly monotonic? No. Note that $u(x_1 + \epsilon, x_2) = a(x_1 + \epsilon) -$

[13] Note that I said "usually" in 2.3 above.

$x_2 > ax_1 - x_2 = u(x_1, x_2)$, which means that $\begin{bmatrix} x_1+\epsilon \\ x_2 \end{bmatrix} \succ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. However, note also that $u(x_1, x_2+\epsilon) = ax_1 - x_2 - \epsilon < ax_1 - x_2 = u(x_1, x_2)$, which means that $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \succ \begin{bmatrix} x_1 \\ x_2+\epsilon \end{bmatrix}$.

(b) Are preferences monotonic? No. Consider the a bundle of goods such that $u(x_1 + \epsilon_1, x_2 + \epsilon_2) = ax_1 + a\epsilon_1 - x_2 - \epsilon_2$, where $\epsilon_2 > a\epsilon_1$. Then $u(x_1 + \epsilon_1, x_2 + \epsilon_2) = ax_1 + a\epsilon_1 - x_2 - \epsilon_2 < ax_1 - x_2 = u(x_1, x_2)$ which means that $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \succ \begin{bmatrix} x_1+\epsilon_1 \\ x_2+\epsilon_2 \end{bmatrix}$.

(c) Are these preferences locally non-satiated? Yes. For a proof of this we need to understand the concept of norms in a metric space. However, consider whether there exists a bundle $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ very close to the original bundle $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ such that $||\vec{x} - \vec{y}|| < \epsilon$. An example might be $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1+\epsilon \\ x_2 \end{bmatrix}$.

3. Are the preferences convex? Look at the upper level sets of the graph we drew. Are these sets convex? Yes, weakly.

4. If we solve the UMP, intuitively, an agent will never consume good 2 since this reduces utility. The agent will consume only good 1.

4.4 Quadratic utility function

1. When the functional form of the utility function is quadratic, it may take the following form: $u(x_1, x_2) = -(x_1 - a)^2 - (x_2 - b)^2$. Trivially, for a given level of utility $\bar{u}$, the equation representing the indifference curve on the two-dimensional space is: $-(x_1 - a)^2 - (x_2 - b)^2 = \bar{u} \Rightarrow (x_1 - a)^2 + (x_2 - b)^2 = -\bar{u}$. The indifference curves for preferences represented by the above quadratic function will be concentric circles (the center of these circles being (a, b)) and the level of utility depicted by the circles is decreasing as the radius of the circle increases.

2. Before considering the weak/strong monotonicity of preferences represented by a quadratic utility function of the form above, note that: $u : \mathbb{R}^2 \to 0 \cup \mathbb{R}^-$, or $\bar{u} \leq 0$.

   (a) Now on the issue of monotonicity, you can go through the math, but let's try to get the intuition here. Recall that monotonicity of preferences meant that "more is better." Without loss of generality (hereinafter, "WLG"), assume that $a, b > 0$. Consider a bundle such that $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \gg \begin{bmatrix} a \\ b \end{bmatrix}$, if you increase consumption of good 1 and/or good 2, your utility decreases.

   (b) Since, these quadratic preferences are not monotonic, they can't be strongly monotonic.

(c) Are these preferences locally non-satiated? No. At the bliss point, or $[a, b]$, there does NOT exist a point arbitrarily close which is preferred by the consumer. If you aren't getting it, check out the definition of local non-satiation.

3. Are the preferences convex? Look at the upper level sets of the picture we drew. What do the upper level sets look like? Do you see that they are circles? Circular sets (in two-dimensions) are convex sets.

4. What is the solution of the UMP going to look like? Given a budget set, $B_{p,q,w} \equiv \left\{(x_1, x_2) \in \Re^2 : px_1 + qx_2 \leq w\right\}$, we're going to look for the point in the budget set that is closest to (a, b). Do you see anything else? Do you see that with quadratic utility functions, Walras' law might not hold and that you might choose a bundle of goods in the interior of the budget set?

4.5 Quasi-linear utility function

Before going through the same exercises above, I'd like to point out that quasi-linear utility functions have certain attractive properties, which we will cover in the context of solving UMPs in the next section, such as additivity of utility with regards to the numeraire good and no wealth effects. Here is the definition of quasi-linear preferences.

Definition. Preferences are quasi linear with regards to a certain good (usually good 1, and usually referred to as the numeraire good) 1) if all indifference curves are parallel displacements of each other along the numeraire good's axis; and 2) if the numeraire is in fact a good. Quasi-linear preferences with regards to good 1 (or the numeraire) over L goods can be represented by a quasi-linear utility function of the form $u(\vec{x}) = ax_1 + \phi(\vec{x}_{-1})$, where $a > 0$.[14]

1. For ease of exposition, let's consider a specific quasi-linear utility function, $u(x_1, x_2) = x_1 + \sqrt{x_2}$. For a given level of utility $\bar{u}$, the indifference curve for the equation is $x_1 + \sqrt{x_2} = \bar{u} \Rightarrow x_2 = (\bar{u} - x_1)^2$, where $x_1 < \bar{u}$ and $x_2 > 0$. Draw the indifference curves.

2. Since $u(x_1+\epsilon, x_2) = x_1+\epsilon+\sqrt{x_2} > x_1+\sqrt{x_2} = u(x_1, x_2)$, which means that $\begin{bmatrix} x_1+\epsilon \\ x_2 \end{bmatrix} \succ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, we know that the preferences represented by the quasi-linear utility function above must be strongly monotonic. This implies weak monotonicity, which in turn implies local non-satiation.

3. Are these preferences convex? Look at the indifference curves we drew above. If we assume that the consumption set $X \equiv \Re^2_+$, then we see that the upper level sets are convex.

[14] See also MWG Definition 3.B.7.

4. What is the solution to the UMP? Whether the agent is going to consume both good 1 and good 2 or whether the agent will consume only good 2 will depend on the relative values of good 1 and good 2 and the agent's wealth. We're going to solve this using Kuhn-Tucker in the next section. So let's skip for now. But look at the picture you drew and see if you can glean some ideas.

4.6 Lexicographic utility function

1. What is the equation for an indifference curve? Recall that with lexicographic preferences, utility functions are not continuous. Note that for all bundles $\vec{x} \neq \vec{y}$, or $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \neq \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $u(x_1, x_2) \neq u(y_1, y_2)$. Therefore, we are unable to draw indifference curves. Each distinct bundle has its own level of utility.

2. Are lexicographic preferences strongly monotone? Consider the bundles $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} x_1+\epsilon \\ x_2 \end{bmatrix}$, and $\begin{bmatrix} x_1 \\ x_2+\epsilon \end{bmatrix}$. Note that an agent with lexicographic preferences would $\begin{bmatrix} x_1 \\ x_2+\epsilon \end{bmatrix} \succ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\begin{bmatrix} x_1+\epsilon \\ x_2 \end{bmatrix} \succ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Therefore, lexicographic preferences are strongly monotone.

3. Are lexicographic preferences convex? We can't rely on a graph since we couldn't draw one. Suppose that $\vec{x} \succeq \vec{z}$ and $\vec{y} \succeq \vec{z}$, and that $\vec{x} \neq \vec{y}$. Suppose also that $\alpha \in (0, 1)$. If the agent has lexicographic preferences, then $x_1 > z_1$ or $x_1 = z_1$ and $x_2 \geq z_2$. Further, we know that $y_1 > z_1$ or $y_1 = z_1$ and $y_2 \geq z_2$. Therefore, we have that $\alpha x_1 + (1 - \alpha)y_1 > z_1$ (for three of the four possible cases) or $\alpha x_1 + (1 - \alpha)y_1 = z_1$ and $\alpha x_2 + (1 - \alpha)y_2 \geq z_2$. Which shows that preferences are convex.

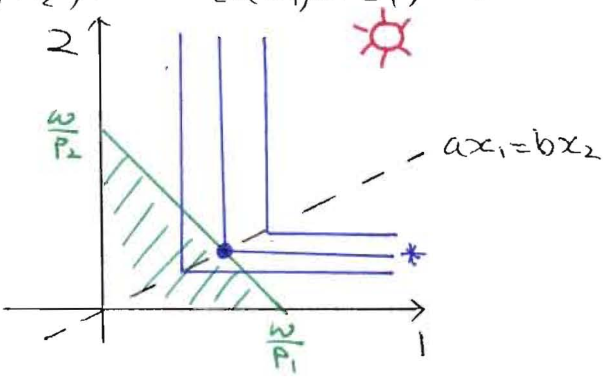
4. What is the solution to the utility maximization problem? Intuitively, you'd want to maximize consumption of good 1. Therefore, if p is the price of good 1 and q is the price of good 2, then $x_1^* = \frac{w}{p}$ and $x_2^* = 0$.

4.7 Exercises

As a test of your understanding of the assumptions we place on preferences, try the above exercises for Cobb-Douglas utility, $u(x_1, x_2) = ax_1^{\alpha}x_2^{1-\alpha}$ and for preferences that are represented by the utility function of the form: $u(x_1, x_2) = \max\{x_1, x_2\}$. If you need help drawing the indifference curves, see the Miller Notes pp.37-41 for the Cobb-Douglas utility functions. Are preferences represented by the two utility functions strongly monotonic? Monotonic? Locally non-satiated? Are preferences convex? What can we say about the solution to the UMP?

3.1 Leontief utility function

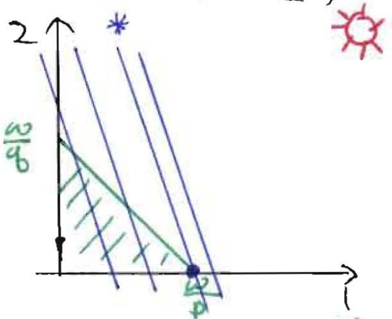
$u(x_1, x_2) = \min\{a x_1, b x_2\}$, $a, b > 0$



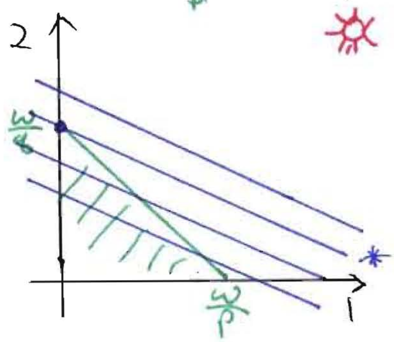
3.2 Linear utility function (1)

$u(x_1, x_2) = a x_1 + x_2$, $a > 0$

$a > \frac{p}{q}$

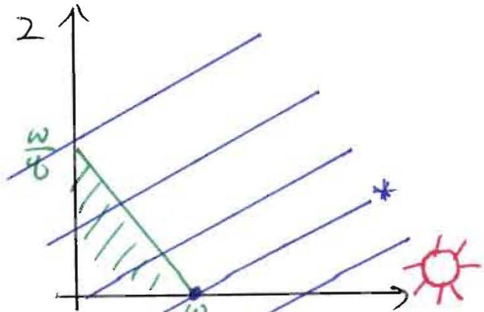


$a < \frac{p}{q}$



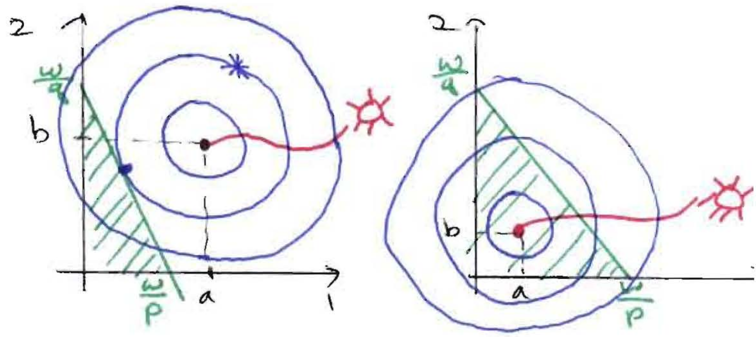
3.3 Linear utility function (2)

$u(x_1, x_2) = a x_1 - x_2$, $a > 0$



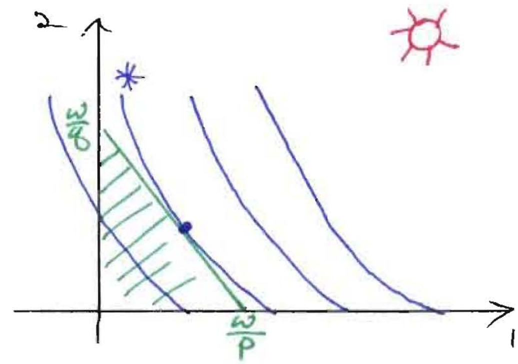
3.4 Quadratic utility func.

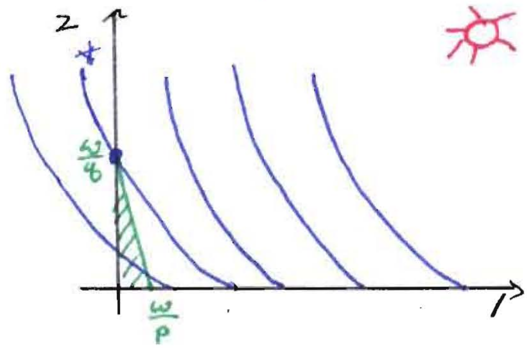
$u(x_1, x_2) = -(x_1 - a)^2 - (x_2 - b)^2$, $a, b > 0$



3.5 Quasi linear utility func.

$u(x, x_2) = x_1 + \sqrt{x_2}$.





3.6 Lexicographic utility func

Can't draw!!

3.7 Cobb-Douglas

TRY!!