Section Notes 10

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Agenda

1. Competitive Screening (Insurance Model Set Up)
2. Main Results
3. Example

1 Competitive Insurance Model Set-Up

Please read Rothschild & Stiglitz (1976)[1]

1.1 Demand Side

We have a population of *risk averse* expected utility maximizing consumers. These consumers move second and they accept or reject a firm's offer. A representative consumer has wealth equal to W; probability of losing L equal to p_i, where $i \in \{G, B\}$, and such that $p_G < p_B$. "G" is for a good type of consumer with low risk of loss and "B" is a bad type of consumer with a high risk of loss. Assume that the probability of the consumer being a good type is equal to λ, which means that the average risk (or pooled level of risk) is equal to $\bar{p} = \lambda p_G + (1-\lambda)p_B$. Further assume that the expected utility of a consumer of type i is equal to:

$$U_i = p_i \cdot u(consumption\, in\, s_2) + (1 - p_i) \cdot u(consumption\, in\, s_1),$$

where s_1 and s_2 denotes the level of consumption in State 1 where there is no loss; and level of consumption in State 2 where there is a loss, respectively. Also, $u'(\cdot) > 0$ and $u''(\cdot) < 0$ for simplicity.

What have we just assumed here?

[1] Rothschild, Michael and Joseph Stiglitz. 1976. *Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information.* The Quarterly Journal of Economics. 90(4): 629-49.

- The level of risk aversion is equal for both good and bad types of consumers.[2]
- We have the same utility function in both the loss state (State 2) and the no loss state (State 1).

1.2 Supply Side

Multiple *risk neutral* profit-maximizing monopolistic firm which move first and offer potentially multiple contracts which can be completely characterized by the following vector: $\vec{\kappa} = [M, R]$,[3] where "M" denotes the premium, and "R" denotes the reimbursement amount.

Taking the demand and supply side together we have the following trivial results:

- A consumer of type $i \in \{G, B\}$ who has purchased insurance will consume, with probability p_i, $s_2 = W - M - L + R$ in State 2 and with probability $1 - p_i$ she will consume $s_1 = W - M$ in State 1. Without insurance, consumption is equal to $s_{2,no\,insurance} = W - L$ and $s_{1,no\,insurance} = W$, respectively. Note that consumption in each state is completely defined by "M" and "R".
- With probability p_i a firm who has sold insurance will have payoff equal to $M - R$ in State 2 and with probability $1 - p_i$ it will have payoff equal to M in State 1.

The firms' expected profit can be written as follows:

$$\begin{aligned}\mathbb{E}[\pi] &= p_i(M - R) + (1 - p_i)(M) \\ &= p_i(W - L - s_2) + (1 - p_i)(W - s_1) \\ &= (W - p_i L) - (p_i s_2 + (1 - p_i) s_1) \qquad (1)\end{aligned}$$

where the first term is equal to the uninsured expected consumption and the second term is equal to the insured expected consumption. This can be rewritten in a more convenient form as follows:

$$s_2 = -\frac{(1 - p_i)}{p_i} s_1 + \frac{1}{p_i} \cdot \{(W - p_i L) - \mathbb{E}[\pi]\} \qquad (2)$$

Do you see why the "sun" is in the lower left hand corner when we draw the $[s_1, s_2]$ Euclidean plane?

[2] Recall our measures of risk aversion from Econ 2020a, and convince yourself.

[3] Rothschild and Stiglitz (1976) use the notation $\vec{\alpha} = (\alpha_1, \alpha_2)$.

1.3 Equilibrium Requirements

1.3.1 Consumer's Problem

Given consumption in the two states, consumers maximize among available contracts:

$$\max_{[M,R]\in\Gamma_{insurer}} U_i = p_i \cdot u(W - M - L + R) + (1 - p_i) \cdot u(W - M)$$

where $\Gamma_{insurer}$ denotes the contracts that the insurer offers.

1.3.2 Insurer's Problem

Given that consumer maximize utility, the insurer provides contracts which satisfy the following conditions:

- No contract offered by a firm makes negative profit.
- No contract not offered would make positive profit.

In short, the firm makes zero profits in equilibrium. Recall from Julian's lecture why this was important.

2 Main Results

1. In a separating equilibrium, the bad types (high risk consumers) are fully insured and the good types (low risk consumers) are not fully insured.
2. No pooling equilibrium exists.
3. There may be no equilibrium in pure strategies.

3 Example[4]

The parameter values are as follows:
$w = 1000 \quad L = 900 \quad p_G = 0.1 \quad p_B = 0.6 \quad \lambda = 0.2 \quad u(s) = \sqrt{s}$

- What does the 45 degree line represent? It represents equal consumption in both states of the world: $s_1 = s_2$. If the consumer is able to purchase an insurance contract which puts her on the 45 degree line, then she has full insurance.
- What if the consumer is uninsured? Then we're at point $A = [W, W - L] = [1000, 100]$

[4] The goal is to derive all values of Rothschild and Stiglitz (1976), Figure III, once we know the parameter values.

- The zero profit lines, $\overline{AB}$ and $\overline{AG}$, denote actuarily fair contracts.[5] In other words, all points on the $\overline{AB}$ and $\overline{AG}$ describe consumption in state 1 and state 2 such that insurance is priced to be actuarily fair. To see this, substitute $s_1 = W - M$ and $s_2 = W - M - L + R$ into equation 2 after setting $\mathbb{E}[\pi]$ =0.

3.1 Separating Equilibrium

3.1.1 Actuarily Fair Line for Bad Types and Full Insurance

Using equation 1 (or equation 2), we can write the actuarily fair line for bad types ($\overline{AB}$) as follows:

$$\begin{aligned}(W - p_B L) - (p_B s_2 + (1 - p_B) \cdot s_1) &= 0 \\ \Rightarrow (1000 - (0.6 \cdot 900)) - (0.6 s_2 + (1 - 0.6) \cdot s_1) &= 0 \\ \Rightarrow s_2 = 766.7 - \frac{2}{3} s_1 & \qquad (3)\end{aligned}$$

Note that the slope of the actuarily fair line for the bad type is equal to $-\frac{1-p_B}{p_B} = -\frac{0.4}{0.6}$

Now using equation 4, we can solve for the full insurance, zero-profit point for the bad types, or point α_B in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 766.7 - \frac{2}{3} s \Rightarrow s = s_1 = s_2 = 460$$

3.1.2 Actuarily Fair Line for Good Types and Full Insurance

Using equation 1 (or equation 2), we can write the actuarily fair line for good types ($\overline{AG}$) as follows:

$$\begin{aligned}(W - p_G L) - (p_G \cdot s_2 + (1 - p_G) \cdot s_1) &= 0 \\ \Rightarrow (1000 - (0.1 \cdot 900)) - (0.1 \cdot s_2 + (1 - 0.1) \cdot s_1) &= 0 \\ \Rightarrow s_2 = 9100 - 9 \cdot s_1 & \qquad (4)\end{aligned}$$

Note that the slope of the actuarily fair line for the good type is equal to $-\frac{1-p_G}{p_G} = -\frac{0.9}{0.1}$

Now using equation 4, we can solve for the full insurance, zero-profit point for the good types, or point β in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 9100 - 9s \Rightarrow s = s_1 = s_2 = 910$$

[5] Recall that if an insurance contract is actuarily fair, the premium should equal to the expected payout, or $M = p_i R$.

3.1.3 Indifference Curve $\bar{U}_B$ and Solving for α_G

How do we pin down the consumption in each state for α_G? We use the fact that the indifference curve through the point α_B also passes through α_G and the fact that α_G lies on the good types actuarily fair line.

1. Solve for the level of $\bar{U}_B$:

$$\bar{U}_B = (1-p_B)\cdot u(460)+p_B\cdot u(460) = (1-p_B)\sqrt{460}+p_B\sqrt{460} = \sqrt{460} \approx 21.4$$

2. The bad type is indifferent between α_B and α_G. Therefore, if we let $\alpha_G = [s_1, s_2]$, we have:

$$U_B\left(\alpha_G = [s_1, s_2]\right) = p_B\sqrt{s_2} + (1-p_B)\sqrt{s_1} = 21.4. \tag{5}$$

3. Further, the point α_G lies on the good types actuarily fair line which we solved for in equation 4:

$$s_2 = 9100 - 9 \cdot s_1 \tag{6}$$

4. Solve the system of equations involving equations 5 and 6, which results (after some very laborious algebra) in:

$$\alpha_G = [s_1, s_2] = [987,\ 217] \tag{7}$$

3.1.4 Separating Equilibrium?

The separating equilibrium results in the bad type consuming 460 in each period and the good type consuming 987 in state 1 and 217 in state 2. We can write this as:460

$$\alpha_B = [460,\ 460]\,;\ \alpha_G = [987,\ 217]\,.$$

Further, since we know the values of W and L, we can solve for $\vec{\kappa}_B = [M_B, R_B]$ and $\vec{\kappa}_G = [M_G, R_G]$. I won't bother going through the algebra here or below.

Is this result consistent with Main Result 1. above?

3.1.5 Certainty Equivalent for Good Types

To put the good type's consumption α_G into perspective, we can also ask: Give the probability of loss for the good type and her consumption α_G in the separating equilibrium, what is the good type's certainty equivalent?[6]

We can answer this question using the same methodology we used to solve for the separating equilibrium.

1. We can solve for $\bar{U}_G$ by solving the following:

$$\bar{U}_G = p_G\cdot u(217)+(1-p_G)\cdot u(987) = 0.1\sqrt{217}+(1-0.1)\sqrt{987} = 29.75 \tag{8}$$

[6] We defined "CE" in Econ 2020a.

2. At the CE, the good type's utility, which I denote as $\bar{U}_G^{CE}$, is as follows:

$$\bar{U}_G^{CE} = p_G \cdot u(s) + (1 - p_G) \cdot u(s) = \sqrt{s} \tag{9}$$

3. Setting equation 8 to equal equation 9, we can solve for the CE consumption for the good type, s:

$$\sqrt{s} = 29.75 \Rightarrow s = 885.$$

3.2 Pooling Equilibrium

At the pooling equilibrium (if it exists), both the good types and the bad types will choose the same contract. Keeping this in mind, we follow the same procedures that we used to find the separating equilibrium above.

Two points worth re-emphasize are the following:

- No contract offered by a firm makes negative profit.
- No contract not offered would make positive profit.

I've written this twice because this tells us that any contract that is offered by the insurer must lie on a zero profit/actuarily fair line.

3.2.1 Actuarily Fair Line and the Pooling Equilibrium

We need to first consider what the probability of state 2 occurring given that both good and bad types will buy the same contract. Denote the pooled probability of state 2:

$$\bar{p} = \lambda p_G + (1 - \lambda) p_B = (0.2 \times 0.1) + (0.8 \times 0.6) = 0.5$$

Using equation 1 (or equation 2), we can write the actuarily fair line for pooled contracts ($\overline{AP}$) as follows:

$$\begin{aligned} (W - \bar{p}L) - (\bar{p} \cdot s_2 + (1 - \bar{p}) \cdot s_1) &= 0 \\ \Rightarrow (1000 - (0.5 \cdot 900)) - (0.5 \cdot s_2 + (1 - 0.5) \cdot s_1) &= 0 \\ \Rightarrow s_2 = 1100 - s_1 & \end{aligned} \tag{10}$$

Note that the slope of the actuarily fair line for the bad type is equal to $-\frac{1-\bar{p}}{\bar{p}} = -\frac{0.5}{0.5}$

Now using equation 11, we can solve for the full insurance, zero-profit point for the pooled contract, or point P in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 1100 - s \Rightarrow s = s_1 = s_2 = 750$$

Again, we can use $P = \vec{s}_p = [750, 750]$ to solve for the premium and the redistribution (since we know W and L). This in turn fully identifies the pooling insurance contract $\vec{\kappa}_p = [M_p, R_p]$.

3.2.2 Non-Existence of Pooling Equilibrium

I use a graphical argument to show that $P = \vec{s}_p = [750,\ 750]$ is not a pooling equilibrium despite satisfying the conditions above. Consider the point ε, which we can further write as $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ in Figure III. If an insurer were to offer this consumption bundle, the good type consumer would prefer $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ to $\vec{s}_p = [750,\ 750]$ because of the single crossing principle. Furthermore, $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ is below the pooled actuarily fair/zero profit line $\overline{AP}$, which means that the insurer can make positive profits if they were to offer this contract. Because of what I (re)emphasized at the beginning of this section on pooling equilibrium, the immediate result is that there does not exist a pooling equilibrium. This is Main Result 2.

Also note that because $\vec{s}_p = [750,\ 750]$ lies below the indifference curve $\bar{U}_G$, which passes through the separating equilibrium α_G, the good types do not prefer the pooling consumption bundle to the separating equilibrium.

3.3 Pooling Equilibrium (with high λ)

3.3.1 Actuarily Fair Line and the Pooling Equilibrium with high λ

Assuming that there are a lot of good types and $\lambda = 0.95$, we can solve for the pooled probability of state 2 occurring:

$$\bar{p}' = \lambda p_G + (1-\lambda)p_B = (0.95 \times 0.1) + (0.05 \times 0.6) = \frac{1}{8}$$

Using equation 1 (or equation 2), we can write the actuarily fair line for pooled contracts ($\overline{AP'}$) as follows:

$$\begin{aligned} (W - \bar{p}'L) - (\bar{p}'s_2 + (1-\bar{p}')\cdot s_1) &= 0 \\ \Rightarrow \left(1000 - \left(\frac{1}{8}\cdot 900\right)\right) - \left(\frac{1}{8}\cdot s_2 + (1-0.5)\cdot s_1\right) &= 0 \\ \Rightarrow s_2 = 7100 - 7s_1 & \qquad (11) \end{aligned}$$

Note that the slope of the actuarily fair line for the bad type is equal to $-\frac{1-\bar{p}'}{\bar{p}'} = -\frac{\frac{7}{8}}{\frac{1}{8}}$

Now using equation 11, we can solve for the full insurance, zero-profit point for the pooled contract, or point P' in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 7100 - 8s \Rightarrow s = s_1 = s_2 = 887.5$$

Again, we can use $P' = \vec{s}_{p'} = [887.5,\ 887.5]$ to solve for the premium and the redistribution (since we know W and L). This in turn fully identifies the pooling insurance contract $\vec{\kappa}_{p'} = [M_p,\ R_p]$, where $\lambda = 0.95$.

3.3.2 Non-Existence of Equilibrium

The consumption bundle $P' = \vec{s}_{p'} = [887.5, 887.5]$ is preferred by the good type to α_G, so if we had contracts α_G and α_B, a firm could make positive profits by offering a contract like γ. The result is that there is no equilibrium, which is equivalent to Main Result 3.

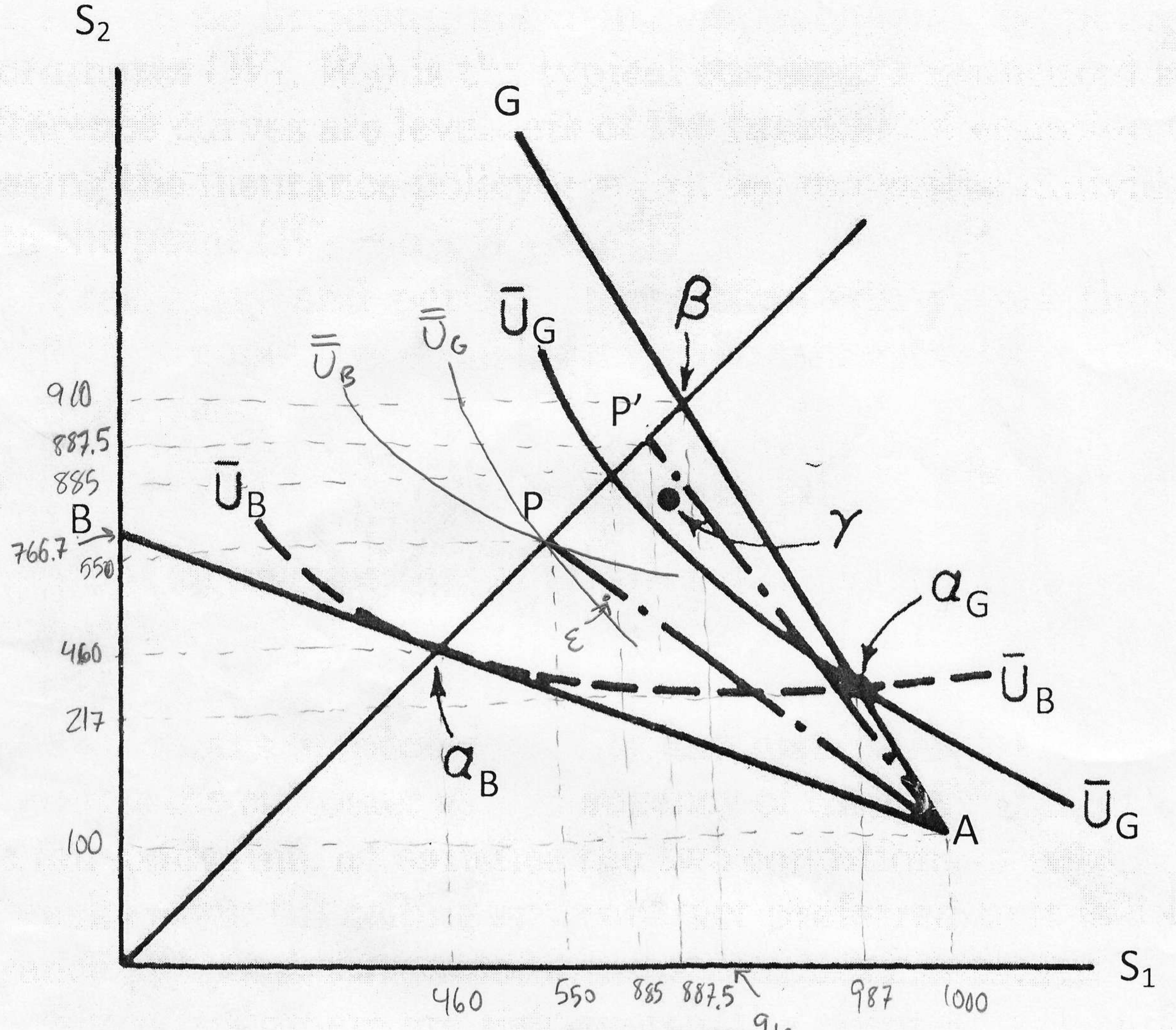

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FIGURE III