Economics 2020a / HBS 4010 / Kennedy School API-111
Midterm Examination
Fall 2010
October 18, 2010

Instructions: You have 90 minutes to complete the following examination. The exam has three questions, each with multiple parts. Answer all parts of all questions in the exam books provided. Partial credit will be awarded on the basis of (partially correct) work shown, so **write legibly** and **show your work.** If you believe a question is ambiguous, clearly state any assumptions you are making.

The examination is open book – you may use any books or notes you desire.
You may also use a calculator on this exam.

This exam has 3 pages, including this one. The exam has 80 points, with point distribution shown below.

Question	Points
1	30
2	30
3	20

1. [30 points] Comparative Statics with Three Goods

Consider a three-good world and a consumer with utility function

$u(x, y, z) = 2\ SQRT(xy) + f(z)$, where f is strictly increasing and strictly concave.

You can assume throughout the problem that this is a quasiconcave utility function and that all three goods are normal goods without verifying these facts.

Denote the price of good x by p, the price of good y by q, and the price of good z by r.

(a) (6 points) Suppose that $f'(9) = 1$. Verify that the bundle (1, 4, 9) satisfies the first-order conditions for the Consumer Problem with $W = 52$, $p = 8$, $q = 2$, $r = 4$.

(b) (6 points) Demonstrate that (1, 4, 9) is the unique bundle satisfying the first-order conditions for the Consumer Problem under the conditions given in (a).

For parts (c), (d) and (e), consider an uncompensated price change from initial conditions ($p^0 = 8$, $q^0 = 2$, $r^0 = 4$, $W^0 = 52$) to new prices ($p^1 = 8$, $q^1 > 2$, $r^1 = 4$, $W^1 = 52$). Denote the optimal bundle at new conditions by (x^1, y^1, z^1). You can assume that

(1) (1, 4, 9) is the optimal bundle given $W = 52$, $p = 8$, $q = 2$, $r = 4$,

(2) there is an interior optimal solution to the Consumer Problem under new conditions.

If you cannot solve (c) and (d) , you can answer these questions assuming the specific functional form $f(z) = 6\ SQRT(z)$ for partial credit.

(c) (6 points) Is $z^1 > 9$, $z^1 < 9$, $z^1 = 9$, or is it impossible to tell without knowing the specific value of q^1? Explain your answer.

(d) (6 points) Is $x^1 > 1$, $x^1 < 1$, $x^1 = 1$, or is it impossible to tell without knowing the specific value of q^1? Explain your answer.

(e) (6 points)) Provide the economic intuition for your results in (c) and (d) – explaining why the sign of the comparative static results for x and z are the same or are different (in sign) in response to a price increases for y.

2. [30 points] Quasilinear Utility and Hicksian Demand

Suppose that a consumer has quasilinear utility function u(x, y) = x + 36 SQRT (y).

On Problem Set 3, you showed that with w = 200, price p = 1 for the numeraire good (x) and price q for the non-numeraire good (y).

(1) the optimal bundle with price q = 6 is (146, 9), for utility $\bar{u} = 254$.

(2) the optimal bundle with price q = 9 is (164, 4), for utility $\bar{u} = 236$.

(a) (6 points) Explain how you know that the optimal bundle with wealth 54 and prices p = 1, q = 6 is (0, 9) (corresponding to utility 108).

Assume for the remainder of the problem that the price of the numeraire good is 1 (p=1).

(b) (8 points) Solve for Hicksian demand function for good y: $h_y(q, \bar{u} = 108)$.

(c) (8 points) Solve for Hicksian demand function for good y: $h_y(q, \bar{u} = 72)$.
HINT: Bundle (0, 4) produces utility 72 and is optimal with prices (p = 1, q = 9) and wealth 36.

(d) (8 points) Suppose that you want to find EV and CV for a price change:

Original Conditions (Time 0): $W_0 = 54$, $p^0 = 1$, $q^0 = 6$
New Conditions (Time 1): $W_1 = 54$, $p^1 = 1$, $q^1 = 9$

Is EV = CV for this price change? Explain your answer.

3. [20 points] Arrow-Pratt Measures and Utility Functions

Suppose that person 1 has Bernoulli utility function u(w), that person 2 has Bernoulli utility function v(w) for money and that both consumers are strictly risk averse. Suppose further that both utility functions are twice continuously differentiable, that u(0) = 0, u(100) = 1, v(0) = 0, v(100) = 1 and that a comparison of Arrow-Pratt coefficients shows that person 1 is more risk averse than person 2: $r_{A1}(w) > r_{A2}(w)$ at all values of w.

(a) (8 points) Define Lottery L to have probability 0.5 of outcome 100 and probability 0.5 of outcome 0 so that E[u(L)] = E[v(L)] = ½. How is the fact that E[u(L)] = E[v(L)] consistent with the assumption that person 1 is strictly more risk averse than person 2?

(b) (6 points) What do you know, if anything, about whether u(200) and v(200) are greater than, less than, or equal to 2?

(c) (6 points) What do you know, if anything, about the comparison between u(200) and v(200)?