Section Notes 8

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Agenda

1. Fixed Costs (sunk and non-sunk)
2. Partial Equilibrium

1 Fixed Costs[1]

Consider a production technology that produces one output, one input, and a production function such that $f'(\cdot) > 0$ and $f''(\cdot) < 0$.[2] Assume further that the cost function associated with this production technology is as follows:

$$c(q) = \frac{1}{2}q^2 + q + 3. \tag{1}$$

1. Do you see that there is fixed cost component to this production technology? What is the fixed cost?
2. Is the fixed cost sunk? Note that this is an assumption that you will have to make.
3. Recall that we were able to derive the cost function $c(\vec{\omega}, q)$ from plugging the solutions to the cost minimization problem ("CMP"), $\vec{z}^* = \vec{z}(\vec{\omega}, q)$, back into the objective function, $\vec{\omega} \cdot \vec{z}$. Therefore, $c^* = c(\vec{\omega}, q) = \vec{\omega} \cdot \vec{z}(\vec{\omega}, q)$. When working with cost functions of the form found in equation 1, we are assuming that the input prices, $\vec{\omega}$, are fixed, or that $c(q) = c(\bar{\vec{\omega}}, q)$.

Using the cost function in equation 1, we can calculate the marginal cost of production at q units:

$$MC(q) = c'(q) = q + 1$$

[1] Refer to the graphs at the end of the Section Notes.

[2] Recall that a production function will have an explicit form, while the transformation function will have an implicit form.

, and the average cost of producing q units:

$$AC(q) = \frac{c(q)}{q} = \frac{1}{2}q + 1 + \frac{3}{q}.$$

Using the graphical representation in MWG 5.D, we can draw the following pictures for non-sunk and sunk fixed costs.

2 Partial Equilibrium[3]

2.1 The Model (and Notation)

2.1.1 The Consumers

There are a total of I consumers each with the following utility function:

$$u_i(m_i, x_i) = m_i + \phi_i(x_i)$$

, where $\phi'(\cdot) > 0$ and $\phi''(\cdot) < 0$. Each individual has endowment of the numeraire good m equal to w_i and no endowment of good x. The price vector for the two goods $\vec{p} = [p_m, p_x] = [1, p]$.

In an equilibrium, all consumers will be maximizing their utilities subject to their respective budget constraints.

2.1.2 The Producers (Firms)

There are a total of J producers each of which has a cost function, $c_j(q_j)$,[4] which is measured in terms of units of the numeraire good. Producers (or firms) use the numeraire good, which is supplied by the consumers (recall that consumers are endowed with the numeraire good) in exchange for good x, and transforms them into the output good x which is denoted in the context of the firm as q.

Note also that the firms themselves are owned by the consumers. The ownership share of consumer i in firm j is equal to θ_{ij}, and so $\sum_{i=1}^{I} \theta_{ij} = 1$. It is also important to remember the budget constraint of the consumer's utility maximization problem must also take into consideration this ownership share of firms. Therefore, the total wealth available to consumer i is equal to:

$$w_i + \sum_{j=1}^{J} \theta_{ij} \Pi_j(p^*)$$

, where $\Pi_j(p^*)$ is equal to the profit of firm j at price p^*.

[3] We call it partial equilibrium because we focus on one good and a composite commodity (all other goods).

[4] Note that MWG may use the more general notation y to be consistent with the production plan vector $\vec{y}$.

2.1.3 Market Clears at equilibrium price p^*

It must be the case that at the equilibrium price, p^*, the total amount of good x demanded and supplied must be equal:

$$\sum_{i=1}^{I} x_i = \sum_{j=1}^{J} q_j$$

2.2 Partial Equilibrium Example

What is themarket equi. in the following case: 100 consumers, all with $u_i(m_i, x_i) = m_i + \ln x_i$ and endowment $w_i = \frac{1}{100} w_m$; $\vec{p} = [p_m, p_x] = [1, p]$; and one firm, with cost function $c(q) = 5q^2$ and $\forall i$ the ownership share $\theta_{ij=1} = \frac{1}{100}$.

2.2.1 Producers

Each producer solves the PMP (in our problem, we only have one firm) to derive the net supply function of good x. In short, we are looking for the optimal value of q^* in terms of the parameter p. Therefore,

$$\max_{q} \Pi(p) = pq - c(q) = pq - 5q^2$$

. The FOCs result in:

$$\frac{\partial \Pi(q^*)}{\partial q} = p - 10q^* = 0$$

[5] and so the supply function can be written as:

$$q^*(p) = \frac{p}{10} \tag{2}$$

, and the optimal profit function is:

$$\Pi^*(p) = \frac{p^2}{10} - \frac{5p^2}{100} = \frac{p^2}{20}$$

2.2.2 Consumers

Each consumer solves his or her own utility maximization problem subject to a wealth constraint:

$$\max_{m_i, x_i} u_i(m_i, x_i) = m_i + \ln x_i \, s.t. \, m_i + px_i \leq \frac{w_m}{100} + \frac{\Pi^*(p)}{100}$$

[5] $\frac{\partial^2 \Pi(q^*)}{\partial q^2} = -10 < 0$, so we're at a maximum.

, where we know that at the optimum, the constraint will bind. This is important since we don't have to resort to the Lagrangian method. We can rewrite the problem as an unconstrained maximization:[6]

$$\max_{x_i} u_i(x_i) = -px_i + \frac{w_m}{100} + \frac{\Pi^*(p)}{100} + \ln x_i$$

. The FOCs result in:

$$\frac{\partial u(x_i^*)}{\partial x_i} = -p + \frac{1}{x_i^*} = 0 \Rightarrow x_i^*(p) = \frac{1}{p}. \tag{3}$$

[7]

2.2.3 Markets Clear at the Equilibrium Price p^*

Since supply must equal demand at the equilibrium price, we have from equations 2 and 3 that the following equation must hold at the equilibrium p^*:

$$q^*(p^*) = \sum_{i=1}^{I} x_i^*(p^*) \Rightarrow \frac{p^*}{10} = \frac{100}{p^*} \Rightarrow p^* = 10\sqrt{10} \approx 31.6 \tag{4}$$

, which means that the equilibrium level of good x that is consumed is equal to:

$$q^*(31.6) = \sum_{i=1}^{I} x_i^*(31.6) = \sqrt{10}$$

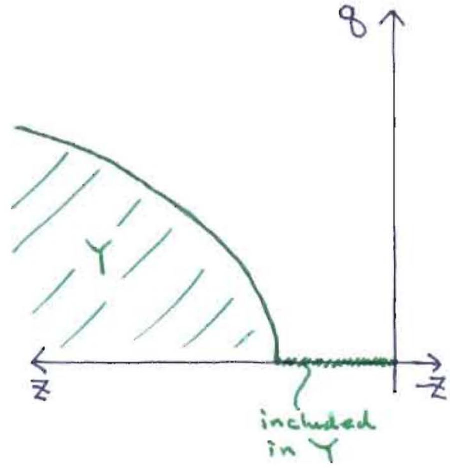
[6]Do we have to worry about corner solutions? Recall that m_i is allowed to be negative (meaning that the consumer can borrow) and that the Inada condition for the non-numeraire good applies.

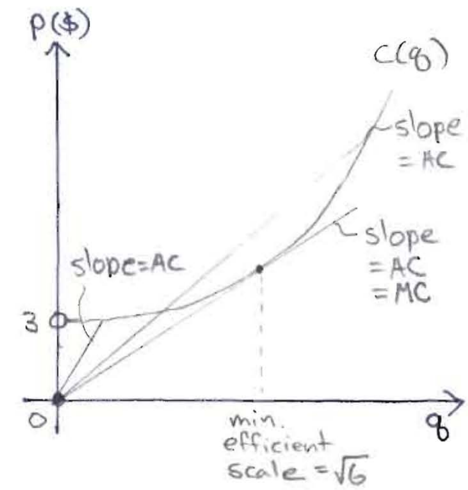
[7]Note how the ownership share does not enter into the FOC. Remember that we're using a quasilinear utility function, so there are no wealth effects.

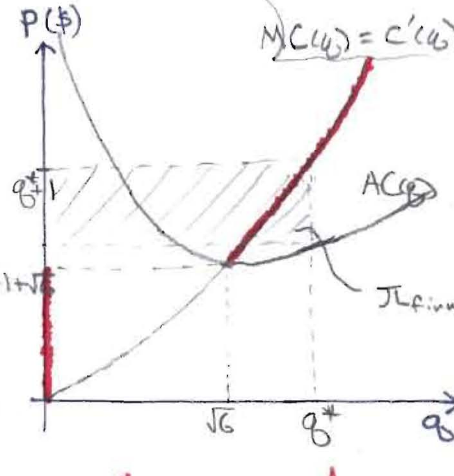
Fixed Costs

1) non-sunk

MWG uses $C'_v(q)$ for marginal variable costs.

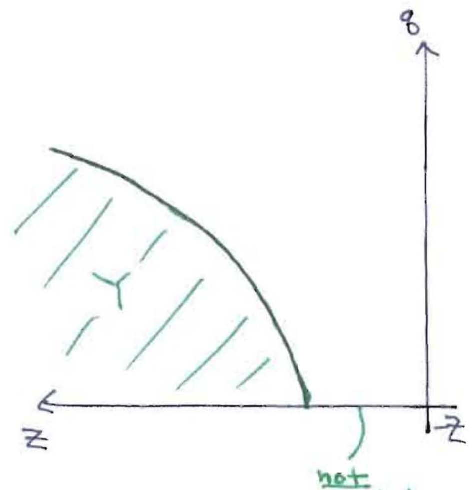


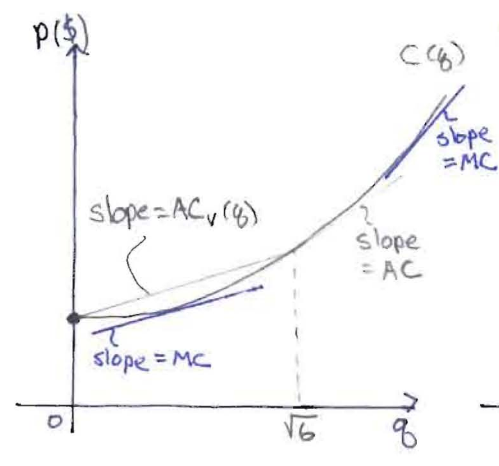


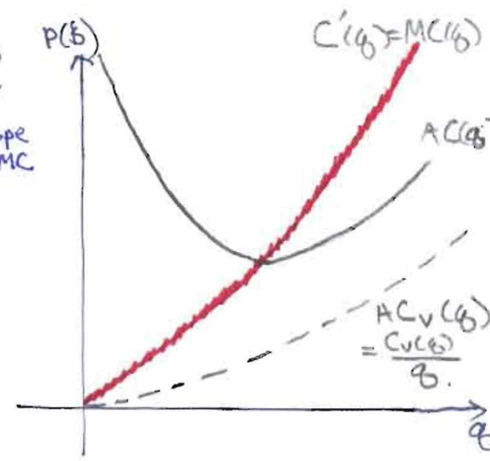


$q^*(p)$ in red.

2) sunk







$q^*(p)$ in red