Section Notes 3

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Agenda

1. Indirect Utility Function
2. Expenditure Minimization Problem ("EMP")
3. Duality and Slutsky Equation
4. Hicksian vs Walrasian Demand
5. CV and EV
6. Additional Topics: Roy's Identity, the Envelope Theorem

1 Indirect Utility Function

Recall the following problem from last week. The utility function was of the Cobb-Douglas form and resulted in some very nice properties and comparative statics (go back to last week's section notes for review).

Problem 1. How much of good 1 and good 2 should I consume to maximize my utility given wealth w; prices p_1, p_2; and a utility function such that $u(x_1, x_2) = x_1 x_2^2$?

Recall that we used monotonic transformation to change the objective function to: $\ln u(x_1, x_2) = \ln x_1 + 2 \ln x_2$.

The Lagrangian was as follows:

$$\mathcal{L}(x_1, x_2) = \ln x_1 + 2 \ln x_2 - \lambda[p_1 x_1 + p_2 x_2 - w] \tag{1}$$

The solution to the above UMP was as follows:

$$x_1^*(\vec{p}, w) \equiv \frac{1}{3}\left(\frac{w}{p_1}\right) \tag{2}$$

$$x_2^*(\vec{p}, w) \equiv \frac{2}{3}\left(\frac{w}{p_2}\right) \tag{3}$$

$$\lambda^*(\vec{p}, w) \equiv \frac{3}{w} \tag{4}$$

.[1] Then we define the indirect utility function as follows:

$$v(\vec{p}, w) = u(x_1^*(\vec{p}, w), x_2^*(\vec{p}, w))$$

. Understanding the relationship between the utility function and the indirect utility function is very important. Note that $u : X \to \Re^1$, but $v : P \times W \to \Re^1$, where $P \subset \Re_+^L$ and $W \subset \Re_+^1$, which represent sets of price vectors and wealth values, respectively. In short, the utility function "maps" bundles of goods to levels of utility, while the indirect utility function maps paramter values to levels of utility. A utility maximizing economic agent will reach a certain level of utility when faced with a given price vector and wealth value by solving her UMP. For our problem from last week:

$$v(\vec{p}, w) = \ln(x_1^*) + 2\ln(x_2^*) = \ln\left(\frac{w}{3p_1}\right) + 2\ln\left(\frac{2w}{3p_2}\right)$$

. Now let's analyze:

$$\frac{\partial v(\vec{p}, w)}{\partial w} = \left(\frac{3p_1}{w} \times \frac{1}{3p_1}\right) + 2\left(\frac{3p_2}{2w} \times \frac{2}{3p_2}\right) = \frac{1}{w} + \frac{2}{w} = \frac{3}{w} = \lambda^*(\vec{p}, w) \tag{5}$$

. This is the meaning of the Lagrange multiplier, which I alluded to in last section. Furthermore,

$$\frac{\partial v(\vec{p}, w)}{\partial p_1} = \frac{3p_1}{w} \times \left(-\frac{w}{3p_1^2}\right) = -\frac{1}{p_1} < 0. \tag{6}$$

2 EMP[2]

Problem 2. How much of good 1 and good 2 should I consume to minimize my expenditure given wealth w; prices p_1, p_2; utility level $\bar{u}$; and a utility function such that $u(x_1, x_2) = \ln x_1 + 2\ln x_2$?

1. What is the objective? Minimize expenditure, which is equal to $\vec{p} \cdot \vec{h} = p_1h_1 + p_2h_2$. What is the objective function?
2. What are the choice variables (the endogenous variables)? What are the parameters (exogenous / state variables)?
3. What are the constraints? The utility constraint and non-negativity of the endogenous variables.

[1] Notice that I used identities. The equality between the LHS and RHS must hold for all values of p_1, p_2, w.

[2] A good exercise is to set up the UMP and EMP side by side and compare the two optimizations.

(a) The utility constraint (if it can be termed as such), tells us that the economic agent wants to attain *at least* a certain level of utility, which we can write as follows: $u(h_1, h_2) = \ln h_1 + 2\ln h_2 \geq \bar{u}$. Note that like the UMP, this constraint will *bind* (hold with equality).

(b) In the UMP, we could monotonically transform the utility function to ease calculation. Should we do the same here when given a problem? Without getting into the specifics, the answer is no. The reason has to do with the fact that now, the value of the utility function actually has meaning! The level of utility has to be greater than $\bar{u}$. Notice the difference between my section notes and Sam's.

4. How do we set up the problem? Once set up, how do we solve? How do we interpret the answer? Can we use pictures like the ones we saw in Section Notes 1 to help us?

Let's set up the problem and solve it:

$$\min_{h_1 h_2 \geq \geq 0} e = p_1 h_1 + p_2 h_2$$

s.t.

$$\ln h_1 + 2\ln h_2 \geq \bar{u}$$

. The resulting Lagrangian is as follows:[3]

$$\mathcal{L}(h_1, h_2; \bar{u}) = p_1 h_1 + p_2 h_2 - \lambda[\ln h_1 + 2\ln h_2 - \bar{u}] \tag{7}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}(h_1^*, h_2^*, \lambda^*)}{\partial h_1} &= p_1 - \lambda^* \frac{1}{h_1^*} = 0 \Rightarrow \lambda^* = p_1 h_1^* \\
\frac{\partial \mathcal{L}(h_1^*, h_2^*, \lambda^*)}{\partial h_2} &= p_2 - \lambda^* \frac{2}{h_2^*} = 0 \Rightarrow \lambda^* = \frac{p_2 h_2^*}{2} \\
\frac{\partial \mathcal{L}(h_1^*, h_2^*, \lambda^*)}{\partial \lambda} &= -\ln h_1 - 2\ln h_2 + \ln e^{\bar{u}} = 0
\end{aligned}$$

. We have a system of three equations and three unknowns which we can solve for (recall that $\ln 1 = 0$).

$$h_1^*(p_1, p_2, \bar{u}) \equiv \exp\left(\frac{\bar{u}}{3}\right) \times \left(\frac{p_2}{2p_1}\right)^{\frac{2}{3}}$$

$$h_2^*(p_1, p_2, \bar{u}) \equiv \exp\left(\frac{\bar{u}}{3}\right) \times \left(\frac{p_2}{2p_1}\right)^{-\frac{1}{3}}$$

[3] What type of Lagrangian is this? Do you see that we haven't considered the non-negativity constraints in the Lagrangian below?

$$\lambda^*(p_1, p_2, \bar{u}) \equiv \exp\left(\frac{\bar{u}}{3}\right) \times p_1^{\frac{1}{3}} \left(\frac{p_2}{2}\right)^{\frac{2}{3}}$$

. Remember that the solutions are the Hicksian demand functions. With the above Hicksian demand functions, we can solve for the expenditure function which is equal to:

$$\begin{aligned} e(\vec{p}, \bar{u}) &= \vec{p} \cdot \vec{h}^*(\vec{p}, \bar{u}) = \sum_{i=1}^{L} p_i h_i(\vec{p}, \bar{u}) = p_1 h_1^*(\vec{p}, \bar{u}) + p_2 h_2^*(\vec{p}, \bar{u}) \\ &= \exp\left(\frac{\bar{u}}{3}\right) \left[\frac{1}{2^{\frac{2}{3}}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} + \frac{2}{2^{\frac{2}{3}}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}}\right] = \exp\left(\frac{\bar{u}}{3}\right) \times \frac{3}{2^{\frac{2}{3}}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} \end{aligned} \tag{8}$$

. Notice the following three equations (or comparative statics):

$$\frac{\partial e(\vec{p}, \bar{u})}{\partial p_1} = \exp\left(\frac{\bar{u}}{3}\right) \times \frac{1}{3} \times \frac{3}{2^{\frac{2}{3}}} p_1^{-\frac{2}{3}} p_2^{\frac{2}{3}} = \exp\left(\frac{\bar{u}}{3}\right) \times \left(\frac{p_2}{2p_1}\right)^{\frac{2}{3}} = h_1^*(\vec{p}, \bar{u}) \tag{9}$$

$$\frac{\partial e(\vec{p}, \bar{u})}{\partial p_2} = \exp\left(\frac{\bar{u}}{3}\right) \times \frac{2}{3} \times \frac{3}{2^{\frac{2}{3}}} p_1^{\frac{1}{3}} p_2^{-\frac{1}{3}} = \exp\left(\frac{\bar{u}}{3}\right) \times \left(\frac{p_2}{2p_1}\right)^{-\frac{1}{3}} = h_2^*(\vec{p}, \bar{u}) \tag{10}$$

$$\frac{\partial e(\vec{p}, \bar{u})}{\partial \bar{u}} = \exp\left(\frac{\bar{u}}{3}\right) \frac{1}{3} \frac{3}{2^{\frac{2}{3}}} p_1^{\frac{1}{3}} p_2^{\frac{2}{3}} = \exp\left(\frac{\bar{u}}{3}\right) p_1^{\frac{1}{3}} \left(\frac{p_2}{2}\right)^{\frac{2}{3}} = \lambda^*(p_1, p_2, \bar{u}) \tag{11}$$

. Equations 9 and 10 are very important and you should remember this (no need to know the general proof). Equation 11 is a really good place to see if you've understood the envelope theorem since this is a direct consequence. Note that $e(\vec{p}, \bar{u}) = \lambda^*(\vec{p}, \bar{u})$ (compare equations 8 and 11). This is just a coincidence and will normally not be the case.

During lecture there seemed to be some confusion on the concavity of the expenditure function. A full proof would require understanding of Substitution matrix, semi-definiteness of quadratic matrices, and the relationship between second order conditions and quadratic matrices. But notice what happens if we solve for $\frac{\partial^2 e(\vec{p}, \bar{u})}{\partial p_1^2} = \frac{\partial h_1^*(\vec{p}, \bar{u})}{\partial p_1}$. Do you see that the second derivative of the expenditure function is negative in our example?

3 Duality and Slutsky

A nice representation of the duality of the UMP and the EMP can be found in Figure 3.G.3. However, the figure has a typo: the bottom left arrow between the boxes labeled "$e(p, u)$" and "$v(p, w)$" should be: "$u = v(p, e(p, u))$" on top and "$e = e(p, v(p, w))$" on the bottom. Now let's look at the duality of the UMP and the EMP using the Cobb-Douglas utility function that we saw above. Use our examples above to draw Figure 3.G.3.

(See "Duality" at the end of the Section Notes)

Now let's look at the Slutsky Equation. You don't have to know the derivation of the equation itself, however, you should be comfortable with interpreting it. The general form of the Slutsky Equation where there are a total of L goods is as follows:

$$\frac{\partial h_i(\vec{p},\bar{u})}{\partial p_j} = \frac{\partial x_i(\vec{p},w)}{\partial p_j} + \frac{\partial x_i(\vec{p},w)}{\partial w} \times x_j(\vec{p},w)$$

$$\Rightarrow \frac{\partial x_i(\vec{p},w)}{\partial p_j} = \frac{\partial h_i(\vec{p},\bar{u})}{\partial p_j} - \frac{\partial x_i(\vec{p},w)}{\partial w} \times x_j(\vec{p},w) \tag{12}$$

. We interpret the Slutsky equation as follows:

1. The first term in the RHS of equation 12, $\frac{\partial h_i(\vec{p},\bar{u})}{\partial p_j}$, is the *substitution effect.*[4]
   (a) If $\frac{\partial h_i(\vec{p},\bar{u})}{\partial p_j} > 0$, then goods i and j are substitutes.
   (b) If $\frac{\partial h_i(\vec{p},\bar{u})}{\partial p_j} < 0$, the goods i and j are complements.
   (c) However, if $\frac{\partial x_i(\vec{p},w)}{\partial p_j} \gtrless 0$, then goods i and j are either *gross* substitutes or *gross* complements, respectively.
2. The second term in the RHS of equation 12, $\frac{\partial x_i(\vec{p},w)}{\partial w} \times x_j(\vec{p},w)$, is the *wealth effect.*[5]

The important thing to note is that we are able to decompose the change in the Walrasian demand function (which we are able to observe) to substitution and wealth effects. We'd love to be able to use the Hicksian demand functions whenever possible, but the problem is that we can't observe Hicksian demand functions (how are we going to observe $\bar{u}$ for each individual agent?).

Further, let's assume that the utility function of the agent takes the quasi-linear form. What would the Slutsky equation for a non-numeraire good look like? It would be:

$$\frac{\partial x_i(\vec{p},w)}{\partial p_j} = \frac{\partial h_i(\vec{p},\bar{u})}{\partial p_j}$$

, because $\frac{\partial x_i(\vec{p},w)}{\partial w} = 0$ if good i is a non-numeraire good. Therefore, the slopes of the Walrasian and the Hicksian demand functions would be the same.

[4]Distinguish this from substitute goods and complementary goods, which are given a more precise definition below.

[5]Recall that one of the utility functions that we discussed in a previous section had the property that $\frac{\partial x_i(\vec{p},w)}{\partial w} = 0$ for some of the goods in the set X. Do you remember which function it was and which goods would not have a wealth effect? Answer is quasilinear utility functions and all of the non-numeraire goods.

4 Hicksian vs. Walrasian Demand

Recall from our discussion on duality, that the following must hold: $\vec{h}^*(\vec{p}, \bar{u}) = \vec{x}^*(\vec{p}, e(\vec{p}, \bar{u}))$ and $\vec{x}^*(\vec{p}, w) = \vec{h}^*(\vec{p}, v(\vec{p}, w)$. Therefore, solving the UMP and the EMP will give you the same bundle of goods. However, then the question becomes, why do we need two representations of the same solution? The short answer is that the two functions take different arguments (wealth in the Walrasian, and fixed utility in the Hicksian) and so will behave differently. Let's look at the relationship between the two and why the Hicksian is sometimes referred to as the compensated demand function.

(See "Duality" at the end of the Section Notes)

The Slutsky equation also provides us with insight into the relationship between the two demand functions. Recall that $\frac{\partial x_i(\vec{p},w)}{\partial p_j} = \frac{\partial h_i(\vec{p},\bar{u})}{\partial p_j} - \frac{\partial x_i(\vec{p},w)}{\partial w} \times x_j(\vec{p}, w)$, which implies that $\frac{\partial x_i(\vec{p},w)}{\partial p_i} = \frac{\partial h_i(\vec{p},\bar{u})}{\partial p_i} - \frac{\partial x_i(\vec{p},w)}{\partial w} \times x_i(\vec{p}, w)$. Note also that the Hicksian is always nonincreasing in p_i. Therefore, when we draw the Hicksian and Walrasian demand functions together, we see the following:

1. If the good i is normal, then $\frac{\partial x_i(\vec{p},w)}{\partial w} > 0$, which means that $\frac{\partial x_i(\vec{p},w)}{\partial p_i} < \frac{\partial h_i(\vec{p},\bar{u})}{\partial p_i} < 0$, which means that the slope of the Hicksian will be steeper than that of the Walrasian.

2. If the good i is inferior, then $\frac{\partial x_i(\vec{p},w)}{\partial w} < 0$, which means that $\frac{\partial x_i(\vec{p},w)}{\partial p_i} > \frac{\partial h_i(\vec{p},\bar{u})}{\partial p_i}$, which means that the slope of the Hicksian will be flatter than that of the Walrasian.[6]

5 CV and EV

Compensating Variation	Equivalent Variation
$e(\vec{p}^0, u^0) - e(\vec{p}^1, u^0) = w - e(\vec{p}^1, u^0)$	$e(\vec{p}^0, u^1) - e(\vec{p}^1, u^1) = e(\vec{p}^0, u^1) - w$
$\int_{p_1^1}^{p_1^0} h_1(p_1, \vec{p}_{-1}, u^0) dp_1$	$\int_{p_1^1}^{p_1^0} h_1(p_1, \vec{p}_{-1}, u^1) dp_1$
$v(\vec{p}^0, w) = v(\vec{p}^1, w - CV)$	$v(\vec{p}^0, w + EV) = v(\vec{p}^1, w)$

The final calculation give us the intuition of what CV and EV are.

CV measures the net revenue of the social planner who must compensate the consumer for the price change after it occurs, bringing her back to her original utility level.

EV measures the dollar amount that the consumer would be indifferent from accepting in lieu of the price change.

Note that if there is a price increase, this is going to be "bad" for consumer welfare and so CV and EV will be negative.

[6]These are tricky concepts to get down because the steepness of the curves will depend on the *absolute* value of the slopes and the axes are flipped when we work with demand functions.

Example 3. From 2003 midterm, question 2

The government is deciding between a 10% tax on apples or bananas. The government has the following information available to it

	Price	Quantity	$\frac{\partial x_i}{\partial p_i}$	$\frac{\partial x_i}{\partial w}$
Apples (a)	1	50	-100	0
Bananas (b)	2	30	-50	0.5

.

Notice that a 10% tax would result in the following price vectors (depending on whether apples or bananas are taxed): $\vec{p}^a = \begin{bmatrix} p_a^a \\ p_b^a \end{bmatrix} = \begin{bmatrix} 1.1 \\ 2 \end{bmatrix}$ and $\vec{p}^b = \begin{bmatrix} p_a^b \\ p_b^b \end{bmatrix} = \begin{bmatrix} 1 \\ 2.2 \end{bmatrix}$. Let's use the notation $\vec{p}^0 = \begin{bmatrix} p_a^0 \\ p_b^0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ to denote the pre-tax price.

Problem. Is EV or CV is more appropriate in comparing the welfare changes of this tax?

The answer is EV because we need to consider the difference of the EVs and CVs:

$$EV\left(\vec{p}^0, \vec{p}^a, w\right) - EV\left(\vec{p}^0, \vec{p}^b, w\right) = e\left(\vec{p}^0, u^a\right) - e\left(\vec{p}^0, u^b\right) \tag{13}$$

, and

$$CV\left(\vec{p}^0, \vec{p}^a, w\right) - CV\left(\vec{p}^0, \vec{p}^b, w\right) = e\left(\vec{p}^b, u^a\right) - e\left(\vec{p}^a, u^b\right) \tag{14}$$

. However, in equation 14, the base price for the calculation of welfare are in different prices (the two new prices).

CV is good when considering a single policy, and if we are the government that actually has to compensate people

Problem. Which good, apples or bananas, should we tax?

Let's first consider the apple tax, which would change the price vector from $\vec{p}^0$ to $\vec{p}^a$.

How many apples will be demanded?

$$x_a(\vec{p}^a, w) = x_a(\vec{p}^0, w) + \frac{\partial x_a(\vec{p}^0, w)}{\partial p_a} \times \Delta p_a = 50 + (-100) \times (0.1) = 40 \tag{15}$$

What is the tax revenue going to be?

$$t \times x_a(\vec{p}^a, w) = 0.1 \times 40 = 4$$

What is the EV for a tax on apples? Recall that in order to calculate the EV, we need to figure out what the Hicksian demand curve at $\vec{p}^a$ looks like. For purposes of this problem, we can assume that the demand functions will be linear (another interpretation is that we are approximating a non-linear demand function by its derivative). Because there is no wealth effect for apples, we find that:

$$\frac{\partial h_a(\vec{p}^a, \bar{u})}{\partial p_a} = \frac{\partial x_a(\vec{p}^a, w)}{\partial p_a} + \frac{\partial x_a(\vec{p}^a, w)}{\partial w} \times x_a(\vec{p}^a, w) = -100$$

. Which allows us to calculate the EV as follows:

$$EV = \int_{p_a^a=1.1}^{p_a^0=1} h_a(p_a, p_b^0 = p_b^a, u^a)dp_a = -(0.1) \times \frac{40+50}{2} = -4.5 = CV = \Delta CS \tag{16}$$

Now let's consider the tax on bananas. The price vector changes from $\vec{p}^0$ to $\vec{p}^b$. We go through the exact same steps as before, but bear in mind that we now have a wealth effect.

How many bananas will be demanded?

$$x_b(\vec{p}^b, w) = x_b(\vec{p}^0, w) + \frac{\partial x_b(\vec{p}^0, w)}{\partial p_b} \times \Delta p_b = 50 + (-50) \times (0.2) = 20 \tag{17}$$

What is the tax revenue going to be?

$$t \times x_b(\vec{p}^b, w) = 0.2 \times 20 = 4$$

What is the EV for a tax on bananas? As before, we begin by asking what the Hicksian demand curve at $\vec{p}^b$ looks like. There is a wealth effect for bananas, and so we find that:

$$\frac{\partial h_b(\vec{p}^b, \bar{u})}{\partial p_b} = \frac{\partial x_b(\vec{p}^b, w)}{\partial p_b} + \frac{\partial x_b(\vec{p}^b, w)}{\partial w} \times x_b(\vec{p}^b, w) = -50 + 0.5(20) = -40$$

. Which allows us to calculate the EV as follows:

$$EV = \int_{p_b^b=2.2}^{p_b^0=2} h_b(p_a^0 = p_a^b, p_b, u^b)dp_b = -(0.2) \times \frac{20+28}{2} = -4.8 \tag{18}$$

6 Envelope Theorem[7]

In the context of the indirect utility function we looked at in Section 1, the envelope theorem tells us the following:

$$\frac{\partial v}{\partial p_1} = \frac{\partial u(x_1^*(\vec{p}, w), x_2^*(\vec{p}, w))}{\partial p_1} = \frac{\partial \mathcal{L}(x_1^*, x_2^*, \lambda^*)}{\partial p_1}$$
$$= -\lambda^*(\vec{p}, w) \times x_1^*(\vec{p}, w) = -\frac{3}{w} \times \frac{1}{3}\left(\frac{w}{p_1}\right) = -\frac{1}{3p_1}$$

. The second equation above is where the envelope theorem applies. When we're at the optimum, we can ignore how small changes in the parameters affect our optimal consumption bundle. The general form of the theorem with a general constraint is as follows:

[7] Sam recommends that you look at Silberberg Ch. 7. I personally haven't read it. MWG has a chapter on the Envelope Theorem and Simon and Blume (1994) also has a comprehensive treatment of the Envelope Theorem.

Theorem 4. *Let $f, h : \Re^n \times \Re^1 \rightarrow \Re^1$ and let $\vec{x}^*(a) = [x_1^*(a), ..., x_L^*(a)]$ be the solution to the problem optimizing function f over the convex set $C_a \equiv \{\vec{x} \in \Re^n | h(\vec{x}; a) = 0\}$ for a fixed parameter a. Then,*

$$\frac{df(\vec{x}^*(a); a)}{da} = \frac{\partial \mathcal{L}(\vec{x}^*(a), \lambda^*(a); a)}{\partial a}.$$

7 Roy's Identity

Roy's Identity states that if you have continuous utility functions, locally non-satiated preferences, and convex preferences, the following identity will hold:

$$x_i^*(\vec{p}, w) \equiv -\frac{\frac{\partial v}{\partial p_i}}{\frac{\partial v}{\partial w}}$$

. MWG (p.74) provides three proofs. Try the envelope theorem version, now that you know what the envelope theorem is. In our example, we have the numerator and denominator values for Roy's Identity for good 1. Plug in equations 5 and 6, and you see that:

$$-\frac{\frac{\partial v}{\partial p_i}}{\frac{\partial v}{\partial w}} = -\frac{-\frac{1}{p_1}}{\frac{3}{w}} = \frac{w}{3p_1}$$

. Compare this to the demand for good 1 we calculated by solving the UMP, identity 2. This means that the indirect utility function contains the same information as the Walrasian demand function we calculated by optimizing our utility function over our budget set.

DUALITY

$$\frac{\partial h_i}{\partial p_j}(\vec{p},\bar{u}) = \frac{\partial x_i}{\partial p_j}(\vec{p},w) - \frac{\partial x_i}{\partial w}(\vec{p},w)\cdot x_j(\vec{p},w)$$

UMP   EMP

$$x_1^*(\vec{p},w) \equiv \frac{w}{3p_1}$$

$$x_2^*(\vec{p},w) \equiv \frac{2w}{3p_2}$$

$$\vec{h}(\vec{p},\bar{u}) \equiv \vec{x}(\vec{p},e(\vec{p},\bar{u}))$$

$$\vec{x}(\vec{p},w) \equiv \vec{h}(\vec{p},v(\vec{p},w))$$

$$h_1^*(\vec{p},\bar{u}) \equiv e^{\frac{\bar{u}}{3}}\cdot\left(\frac{p_2}{2p_1}\right)^{\frac{2}{3}}$$

$$h_2^*(\vec{p},\bar{u}) \equiv e^{\frac{\bar{u}}{3}}\cdot\left(\frac{p_2}{2p_1}\right)^{-\frac{1}{3}}$$

$$x_i^*(\vec{p},w) = -\frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial w}}$$ (Roy's)

$$v(\vec{p},w) = u(\vec{x}^*(\vec{p},w))$$

$$e(\vec{p},\bar{u}) = \vec{p}\circ\vec{h}^*(\vec{p},\bar{u})$$
$$(= p_1\cdot h_1^*(\vec{p},\bar{u}) + p_2\cdot h_2^*(\vec{p},\bar{u}))$$

$$\frac{\partial e(\vec{p},\bar{u})}{\partial p_i} = h_i^*(\vec{p},u)$$

$$V(\vec{p},w) \equiv \ln\left(\frac{w}{3p_1}\right) + 2\ln\left(\frac{2w}{3p_2}\right) \longleftrightarrow e(\vec{p},\bar{u}) \equiv e^{\frac{\bar{u}}{3}}\cdot\frac{3}{2^{\frac{2}{3}}}\cdot p_1^{\frac{1}{3}}p_2^{\frac{2}{3}}$$

$$w \equiv e(\vec{p},v(\vec{p},w))$$

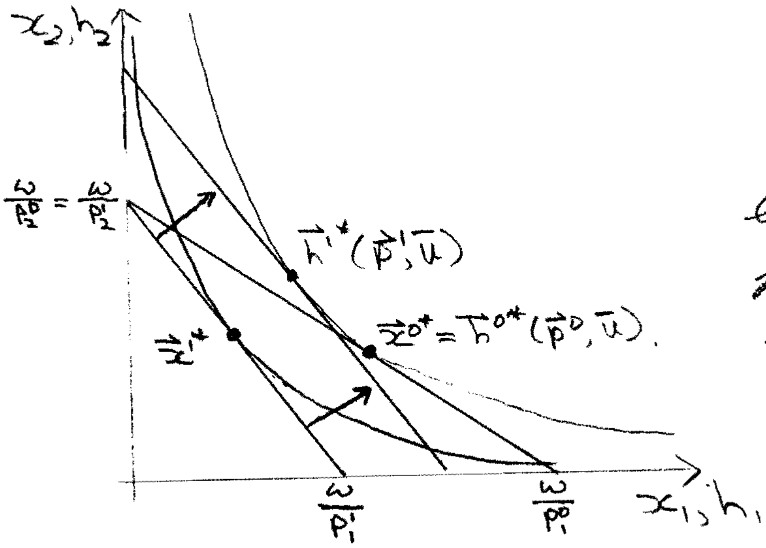
AND

$$\bar{u} \equiv v(\vec{p},e(\vec{p},\bar{u}))$$

$$\therefore\ \bar{u} = \ln\left(\frac{e(\vec{p},\bar{u})}{3p_1}\right) + 2\ln\left(\frac{2e(\vec{p},\bar{u})}{3p_2}\right)$$

$$w = e^{\frac{v(\vec{p},w)}{3}}\cdot\frac{3}{2^{\frac{2}{3}}}\cdot p_1^{\frac{1}{3}}\cdot p_2^{\frac{2}{3}}$$

Hicksian vs Walrasian



Begin with price vector $\vec{p}^0 = \begin{bmatrix} p_1^0 \\ p_2^0 \end{bmatrix}$. The picture to the left depicts an increase in the price of good 1. Let the new price vector be $\vec{p}^1 = \begin{bmatrix} p_1^1 \\ p_2^1 \end{bmatrix}$.

$\Rightarrow \vec{x}^{0*}(\vec{p}^0, \omega) = \vec{h}^{0*}(\vec{p}^0, \bar{u})$, <u>but</u> $\vec{x}^{1*}(\vec{p}^1, \omega) \neq \vec{h}^{1*}(\vec{p}^1, \bar{u})$.

2003 Midterm.