# Section Notes 7

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# Agenda

1. Externalities
2. Entry Deterrence

## 1 Bilateral Externalities

### 1.1 Definition

Externalities exist when the actions of one economic agent *directly* affect the utility of other agents. However, if the effect of one agent's action is *indirectly* through prices, then it isn't an externality.[1] Don't get too hung up on the definition. It isn't always clear whether some action is an externality or not.

Formally, an externality occurs when we can write the utility function of an agent $i$ as follows:

$$u_i(\vec{x}, \vec{y}) = f(x_i, y_i) + g(x_j, y_j) \tag{1}$$

where $i \neq j$. Recall that in all of our previous models the utility of agent $i$ was usually determined by his or her consumption bundle. Note that the utility function above is like the utility functions that we saw in game theory: my utility is determined by the strategies of my opponents.

### 1.2 Example

Assume that the good/activity of interest is playing the bag pipes in-doors. Let $h_1 \geq 0$ denote agent 1's time spent playing the bag-pipes, then we can write agent 1's utility function as follows:

$$u_1(m_1, h_1) = m_1 + \phi_1(h_1) \tag{2}$$

[1] Viner (1931) coined the expression "pecuniary externality."

where $\phi_1'(\cdot) > 0$ and $\phi_1''(\cdot) < 0$. If we also assume that agent 1's budget constraint is: $m_1 + ph_1 \leq w_1$, we can plug the binding constraint into the objective function to get to the following optimization problem for agent 1:

$$\max_{h_1} [w_1 - ph_1 + \phi_1(h_1)] \tag{3}$$

Problem 3 has the following first order condition:

$$\phi_1'(h_1^*) = p = 0 \tag{4}$$

[2] and assume that $p = 0$. For those interested, why we assume that $p = 0$ can be found below in subsection 1.5.

Agent 2 is agent 1's neighbor and her utility function[3] is:

$$u_2(m_2, h_1) = m_2 + \phi_2(h_1)$$

where $\phi_2'(\cdot) < 0$ and $\phi_2''(\cdot) < 0$, which means that playing the bag pipes is a negative externality.

If we let agent 2 decide the level of $h_1$, we would have $h_1^* = 0$. Can you see why?

The socially optimal level of playing bag pipes indoors, which we denote as $h_1^{FB}$ will solve:

$$\max_{h_1} \{u_1(m_1, h_1) + u_2(m_2, h_1)\} \Leftrightarrow \max_{h_1} \{\phi_1(h_1) + \phi_2(h_1)\} \tag{5}$$

The first order conditions for the optimization problem 5 is:

$$\phi_1'(h_1^{FB}) + \phi_2'(h_1^{FB}) = 0. \tag{6}$$

Now compare FOCs 4 and 6:

$$\phi_1'(h_1^{FB}) = -\phi_2'(h_1^{FB}) > 0 = \phi_1'(h_1^*) \Leftrightarrow \phi_1'(h_1^{FB}) > \phi_1'(h_1^*)$$

therefore, by concavity of $\phi$, we have:

$$h_1^{FB} < h_1^*. \tag{7}$$

Inequality 7 tells us that when we're dealing with a negative externality, we have too much of the externality than is socially optimal. See Figure 1 at the end of the notes.

Try this for when we have a positive externality (public good).

[2] Assuming that there exists an interior solution. For this part of the class, we normally assume that an interior solution exists and don't deal with the K-T conditions.

[3] Notice that agent 2 doesn't get to choose $h_1$, but it enters her utility function.

### 1.3 Pareto Optimality of $h_1^*$

Is the level of bag pipe playing defined by 4 above Pareto optimal? It would seem as though that you can't improve agent 2's utility without decreasing agent 1's utility (decreasing $h_1$would benefit agent 2 but would hurt agent 1, and vice-versa).

However, $h_1^*$is not Pareto optimal. See Figure 2 at the end of the notes. By decreasing $h_1^*$by $\Delta h > 0$, the increase in utility for agent 2 is relatively larger than the decrease in utility for agent 1. Therefore, agent 2 can compensate agent 1, using transfers of the numeraire good, to make agent 1 as well off as when he was playing bag pipes to his heart's content, or at the optimal level, $h_1^*$.

In short, efficiency is determined by the quantity of $h_1$ and not by how the numeraire good is distributed. This leads us to the Coase Theorem, which in a nut shell states:

**Theorem.** *If property rights are clearly defined, transactions costs are equal to zero, contracts are costlessly and perfectly enforced, parties have complete and perfect information, then the initial allocation of property rights will not matter because bargaining amongst the parties will lead to the Pareto optimal (or efficient) outcome.*

### 1.4 Solutions to Externalities

Possible solutions include:

1. Quotas
2. Pigouvian taxation / subsidies (which forces the party with the property right to internalize the externality)[4]
3. Coasian Bargaining (or as Aghion would say: "TIOLI"[5])
4. Market-making
5. Stigma/social recognition[6]

### 1.5 Short Aside

Assume that the numeraire good is good 1. Define the indirect utility function of agent $i$ who has quasi linear preferences as follows:

[4]In the example above, we could force agent 1 to solve the following optimization problem, which leads to the optimal taxation that results in the Pareto optimal level of bag pipe playing:

$$\max_{h_1} \{\phi_1(h_1) - t \cdot h_1\} \Rightarrow \phi_1'(h_1^{**}) = t \Rightarrow t^* = -\phi_2'(h_1^{FB}) > 0$$

[5]"Take-it-or-leave-it" offers by the person *with the property right/ex ante bargaining power.*

[6]Isn't this just another form of internalization?

$$\begin{aligned} v_i(\vec{p}, w_i, h) &= \max_{x_i \geq \geq 0} u_i(\vec{x}_i, h) = \max_{x_i \geq \geq 0} g_i(\vec{x}_{-1i}, h) + x_{1i} \\ s.t. &\quad \vec{p} \cdot \vec{x}_i \leq w_i \end{aligned}$$

which has the following solutions (or demand correspondences):

$$\vec{x}^*_{-1i}(\vec{p}, h)$$

and

$$x^*_{1i}(\vec{p}, h) = w_1 - \vec{p} \cdot \vec{x}^*_{-1i}(\vec{p}, h)$$

Therefore,

$$v_i(\vec{p}, w_i, h) = g_i(\vec{x}_{-1i}(\vec{p}, h), h) + w_i - \vec{p} \cdot \vec{x}^*_{-1i}(\vec{p}, h) = w_i + \phi_i(\vec{p}, h) = w_i + \phi_i(h) \tag{8}$$

and we have the desired form, where the final equality comes from the assumption that prices are fixed.

## 2 Entry and Deterrence: MWG 12.BB.2

Recall the stages of the game from Julian's lecture:

1. Incumbent $I$ chooses the capacity level of its plant, denoted $k_I$. Capacity costs are denoted $r$ per unit produced.

2. Potential entrant, $E$, decides whether to enter the market at fixed cost $F$.

3. If entrant enters the market, the two firms choose their quantities and the resulting price is a function of total quantity the two firms produce. For the entrant, marginal cost is equal to $r + w$, where $r$ is the capacity costs and $w$ the wages. For the incumbent, marginal costs are equal to $w$ up to the capacity $k_I$ and infinite above $k_I$.

**Problem.** Show that if the incumbent in the entry deterrence model discussed in Appendix B of MWG is indifferent between deterring entry and accommodating it, social welfare is strictly greater if he chooses deterrence. Discuss generally why we might not be too surprised if entry deterrence could in some cases raise social welfare.

See Appendix A-3 which shows MWG Figure 12.BB.8. In this figure, the Incumbent prefers to deter entry.

To answer MWG 12.BB.2, note that from Appendix A-3, we know that if the Incumbent accommodates the Entrant, the Incumbent will produce less than if it deters entry (which is also very intuitive). Therefore, for the Incumbent to be indifferent between deterring and accommodating, it must be the case that the equilibrium price given deterrence is *less than* that given accommodation. Furthermore, if we have accommodation, the fixed costs of production $r$ is duplicated by both the Incumbent and the Entrant, which is costly for society.

As discussed in the Cournot model, each firm does not consider the negative externality it is causing the other firm. Likewise, the Entrant does not consider the externality it is causing on the Incumbent by entering the market.

Based on the above, there may be instances where deterring entry can in fact increase social welfare.

**Problem.** MWG 12.BB.1

Try this problem. The graphical representation is in Appendix A-4.