Section Notes 2

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February 3, 2011

Agenda

1. Monopoly and Price Discrimination
2. Bertrand (Price Competition) vs. Cournot (Quantity Competition)
3. Stackelberg Model (variant of Cournot)
4. Kreps-Sheinkman (1983) (won't cover in section)

1 Price Discrimination and Monopoly

Types of price discrimination[1]

1. First Degree Price Discrimination: Monopolist observes the type of the consumer and is able to calculate each consumer's demand correspondences (or their willingness to pay). The monopolist uses this to charge each individual his or her willingness to pay. AKA perfect price discrimination or individual pricing. Generally, the first-best (FB) outcome can be implemented because with perfect price discrimination there may not be any DWL. Note that the monopolist seller gets all of the surplus. See Figure 3.

2. Second Degree Price Discrimination: Monopolist is unable to observe the types of each consumer, and so proposes a menu of prices and quantities. Consumers self-select themselves into their optimal quantity and price, which can be derived using the Revelation Principle (see MWG 14.C.2 for more information). Screening models. Ex) Intel's 486 DX and SX, and 487 micro processors, where SX is just a damaged version of the DX, and 487 was an upgrade of the SX to the DX.

3. Third Degree Price Discrimination: Monopolist is uable to observe types, but is able to observe something immutable that is related to a consumer's willingness to pay. Monopolist charges based on this characteristic.

[1] This taxonomy isn't really helpful in understanding price discrimination, but almost all of the text books have it.

1.1 Basic Monopolist's Problem

Recall that a producer/seller of goods in a competitive market must solve its profit maximization problem which can be written as follows:

$$\max_q pq - c(q) \Rightarrow p = c'(q^C),$$

or the price (which is exogeneous) is equal to the marginal cost.

The monopolist's problem is slightly different:
[2]

$$\max_q p(q)q - c(q) \Rightarrow p(q^M) + p'(q^M)q^M = c'(q^M) \tag{1}$$

where the LHS of equation 1 is the marginal revenue $\left(\frac{d[p(q)q]}{dq}\right)$ of the monopolist and the RHS is the marginal cost. We see that $p(q^M) > c'(q^M)$ because $p'(\cdot) < 0$, which means that the price under a monopoly exceeds the marginal cost of production. This in turn means that $q^M < q^C$.

1.2 Harvard Football Example

Assume that the good of interest is The Game football tickets, which are being supplied by a monopolistic seller with cost function equal to: $c(Q) = \frac{Q^2}{20}$, where Q is the total number of tickets supplied to the market, including both those supplied to alumni and to students: $Q = q_a + q_s$.

Further assume that there are two types of consumers: alumni and students. The demand function for alumni can be written as: $q_a(p_a) = 100 - p_a$ and $q_s(p_s) = 200 - 10p_s$. The monopolist has information on whether a consumer is a student or an alumni and so is able to engage in Third Degree Price Discrimination.

1.2.1 No Students

If we assume that students are in reading period and unable to attend, the monopolist's problems is:

$$\max_{q_a = Q} \{p_a(Q) \cdot Q - c(Q)\} = \max_Q \left\{(100 - Q) \cdot Q - \frac{Q^2}{20}\right\} \tag{2}$$

the FOCs are (assuming an interior solution):

$$100 - 2Q - \frac{Q}{10} = 0 \Rightarrow Q^* = q_a^* \approx 47.6$$

and the resulting optimal price and profit are: $p_a^* \approx 52.4$ and $\pi_{npd}^* = q_a^* \cdot p_a^* - c(q_a^*) \approx 2381$.

[2]Notice that we use the inverse demand function $p(\cdot)$ in the objective function.

1.2.2 Third Degree Price Discrimination

Now the monopolist solves the following problem:

$$\max_{q_a, q_s} \{p_a(q_a) \cdot q_a + p_s(q_s) \cdot q_s - c(q_a + q_s)\} \quad \Rightarrow$$

$$\max_{q_a, q_s} \left\{ (100 - q_a) \cdot q_a + \left(20 - \frac{q_s}{10}\right) \cdot q_s - \frac{(q_a + q_s)^2}{20} \right\}$$

which has the following FOCs:

$$100 - 2q_a = \frac{(q_a + q_s)}{10} \quad (3)$$

$$20 - \frac{q_s}{5} = \frac{(q_a + q_s)}{10} \quad (4)$$

Solving the system of linear equations 3 and 4 results in $q_a^* \approx 45.2$, $p_a^* \approx 54.8$, $q_s^* \approx 51.6$, $p_s^* \approx 14.84$, and $\pi_{pd}^* = q_a^* \cdot p_a^* + q_s^* \cdot p_s^* - c(q_a^* + q_s^*) \approx 2774 > \pi_{npd}^*$.

How much is produced (sold) when only alumni are sold to? How much is produced (sold) when the monopolist seller is able to price discriminate? Intuition?

1.3 Monopolistic Seller and Two-Type Buyer[3]

As noted above, second degree price discrimination involves the monopolistic seller offering a menu of price and quantity contracts. The following is a simple abstract model of second degree price discrimination. See also the Miller Notes on Monopoly.

Consider a setting where there is a monopolistic seller and a buyer who is of two types (a high value type or a low value type and we use the subscript $i \in \{L, H\}$ to denote each type). The probability of a low-type is equal to β. Assume that the price charged by the monopolist is equal to T and the quantity produced/sold is equal to q. The monopolist has a linear cost function equal to $c \cdot q$. The buyer who is of type i has separable utility of the following form: $\theta_i u(q) - T$, where θ_i is a parameter denoting the relative utility the buyer gets from consuming the monopolist's good. Assume further that the reservation utility of the buyer is normalized to equal zero (0) and that $u(0) = 0$.

1.3.1 First Degree Price Discrimination (First-Best)

Assume that the seller is able to perfectly observe the type of the buyer. The result is that the seller doesn't need to offer a menu of price and quantity contracts, but can simply make a take-it-or-leave-it one price and quantity contract offer to the buyer. What is the monopolist's constrained optimization problem?

[3]Note that some of this will be revisited in the context of adverse selection.

$$\max_{T_i,q_i} [T_i - cq_i] \; s.t. \, \theta_i u(q_i) - T_i \geq 0$$

which becomes the following unconstrained optimization problem:

$$\max_{T_i,q_i} [\theta_i u(q_i) - cq_i]$$

The FOCs result in the optimal quantity q^{FB} such that:

$$\begin{aligned} \theta_H u'\left(q_H^{FB}\right) &= c \\ \theta_L u'\left(q_L^{FB}\right) &= c \end{aligned}$$

In conclusion, the monopolist seller who know the buyer's type i will make a take-it-or-leave-it offer of:

$$\left[T_i = \theta_i u\left(q_i^{FB}\right), \, q_i^{FB}\right]$$

1.3.2 Second Degree Price Discrimination (Second-Best)[4]

For second degree price discrimination, we assume that the seller is unable to perfectly observe the type of the buyer. However, she does know the ex ante distribution of types, $\Pr(\theta_i = \theta_L) = \beta$. Assuming that monopolist is able to offer a menu of price/quantity contracts and that the buyers will self-select into the optimal contract, we can write the seller's constrained maximization problem as follows:

$$\max_{[T_H,T_L,q_H,q_L]} [\beta (T_L - cq_L) + (1-\beta)(T_H - cq_H)] \; s.t.$$

$$\begin{aligned} \theta_H u(q_H) - T_H &\geq \theta_H u(q_L) - T_L \;\; (IC_H) \\ \theta_L u(q_L) - T_L &\geq \theta_L u(q_H) - T_H \;\; (IC_L) \\ \theta_H u(q_H) - T_H &\geq 0 \;\; (IR_H) \\ \theta_L u(q_L) - T_L &\geq 0 \;\; (IR_L) \end{aligned}$$

Proposition 1. *Of the four constraints, the IC_H and the IR_L are the only two constraints that will bind. The remaining two constraints will remain slack at the optimum.*

I'll skip the proof of the proposition. But a good rule of thumb to have is that the the incentive compatibility constraint of the type who wants to act like the other type will bind while his individual rationality constraint is slack.

[4]It should be noted that the contract menu below is not the only viable contract. Contracts with linear prices and/or a simple two-part tariff are all possible contracts that the monopolistic seller can use. However, the contract which maximizes the seller's profits is as discussed below.

On the other hand, the individual rationality constraint of the type that does not have an incentive to act like the other type will bind while his incentive compatibility constraint will be slack.

The result of Proposition 1 above is that the constrained optimization problem above can be rewritten as:

$$\max_{[T_H, T_L, q_H, q_L]} \left[\beta\left(T_L - cq_L\right) + (1-\beta)\left(T_H - cq_H\right)\right] \ s.t.$$

$$\begin{aligned} \theta_H u(q_H) - T_H &= \theta_H u(q_L) - T_L \\ \theta_L u(q_L) - T_L &= 0 \end{aligned}$$

which in turn can be written as the following unconstrained optimization problem:

$$\max_{[T_H, T_L, q_H, q_L]} \left[\beta\left(\theta_L u(q_L) - cq_L\right) + (1-\beta)\left(\theta_H u(q_H) - cq_H - \left(\theta_H - \theta_L\right) u(q_L)\right)\right]$$

The FOCs are:

$$\begin{aligned} \theta_H u'\left(q_H^{SB}\right) &= c \\ \theta_L u'\left(q_L^{SB}\right) &= \frac{c}{1 - \left(\frac{1-\beta}{\beta} \cdot \frac{\theta_H - \theta_L}{\theta_L}\right)} > c \end{aligned}$$

which implies that $q_H^{FB} = q_H^{SB} > q_L^{SB}$ and that $q_L^{FB} > q_L^{SB}$.

In conclusion, with optimal second degree price discrimination:

1. High types consume the same as in the first best optimal quantity, but the low type's consumption is distorted to less than the optimal level of consumption.
2. The low type's consumer surplus is equal to zero (which must be the case, since his IR constraint binds with equality), while the high type receives an informational rent.

2 Bertrand vs. Cournot Duopoly

Proposition 2. *In the Bertrand Duopoly model, there is a unique Nash Equilibrium (p_1^*, p_2^*), where the two identical firms set their prices equal to the marginal cost c.*

- Both firms earn zero profits.
- Since price is equal to marginal cost, we get the perfectly competitive outcome with only two firms.

- Because of Bertrand competition, each firm faces infinitely elastic demand correspondences given the price of the other firm.
- Therefore, monopoly is the only form of price distortion that can arise.

Proposition 3. *In the Cournot Duopoly model where per unit cost $c > 0$ for both firms and the inverse demand function $p(q)$ is such that $p'(q) < 0$ and $p(0) > c$; any Nash Equilibrium (q_1^*, q_2^*) will result in market prices greater than c and smaller than the monopoly price.*

- Firms earn positive profits and we are not in a perfectly competitive equilibrium.
- Oligopolies arise under the Cournot model.

Let's look at an example with linear inverse demand functions and constant marginal costs. Assume that both firms have marginal costs equal to c, and the inverse demand function is linear: $P(Q) = \alpha - Q$, where $Q = q_1 + q_2$. Further assume that $\alpha > c$. The profit function for each firm can be written as:

$$\begin{aligned} \pi_1(q_1; q_2) &= q_1 P(q_1 + q_2) - cq_1 = (\alpha - c - q_2)q_1 - q_1^2 \\ \pi_2(q_2; q_1) &= q_2 P(q_1 + q_2) - cq_2 = (\alpha - c - q_1)q_2 - q_2^2 \end{aligned}$$

We can derive the best response functions for each firm by using the FOCs of each firms' profit maximization problem. Therefore, the best response function for firm i is:[5]

$$q_i = b_i(q_j) = \frac{\alpha - c - q_j}{2} \tag{5}$$

By the definition of Nash Equilibrium, we know that the following two equations must hold at (q_1^*, q_2^*):

$$\begin{aligned} q_1^* &= \frac{\alpha - c - q_2^*}{2} \\ q_2^* &= \frac{\alpha - c - q_1^*}{2} \end{aligned}$$

Solving the two equations above defines the Nash Equilibrium:

$$(q_1^*, q_2^*) = \left(\frac{\alpha - c}{3}, \frac{\alpha - c}{3} \right) \tag{6}$$

which results in total output $Q_{Cournot} = \frac{2}{3} \cdot (\alpha - c)$ and equilibrium price $P_{Cournot} = P\left(\frac{2}{3} \cdot (\alpha - c)\right) = \frac{1}{3} \cdot (\alpha + 2c)$.

Three remarks:

[5] Note that the profit functions for both firms are symmetric. This means that the FOCs will be symmetric leading to symmetric best response functions.

1. $P_{Cournot} = \frac{1}{3} \cdot (\alpha + 2c) > c$ ($\because \alpha > c$).
2. $Q_{Cournot} = \frac{2}{3} \cdot (\alpha - c) > Q_{Monopoly} = \frac{1}{2} \cdot (\alpha - c)$.
3. Each firm earns profits equal to: $\frac{1}{9} \cdot (\alpha - c)^2$[6]

3 Stackelberg Model

One of the assumptions underlying the Cournot model above was that each firm didn't know how much the other firm was producing. However, what happens if firm 2 is now able to observe how much firm 1 produces and produces accordingly?[7] Further, since firm 1 knows that firm 2 is observing its output, will firm 1 change its output from the Cournot equilibrium identified above?

To repeat the set up from Section 1: Assume that both firms have marginal costs equal to c, and the inverse demand function is linear: $P(Q) = \alpha - Q$, where $Q = q_1 + q_2$. Further assume that $\alpha > c$ and that firm 1 produces first and then firm 2 produces.

In equilibrium, let's assume that firm 1 produces q_1^*. The firm 2's profit maximization problem is equal to:

$$\max_{q_2} P(q_1^* + q_2)q_2 - cq_2 = \max_{q_2}(\alpha - c - q_1^*)q_2 - q_2^2$$

the FOC for the above optimization problem results in:

$$q_2^* = b_2(q_1^*) = \frac{\alpha - c - q_1^*}{2} \tag{7}$$

Because firm 1 knows that firm 2 will best respond to any output q_1 that firm 1 produces, firm 1's profit maximization problem is:

$$\max_{q_1} P\left(q_1 + \frac{\alpha - c - q_1^*}{2}\right) q_1 - cq_1 \Rightarrow \max_{q_1} \frac{1}{2} \cdot q_1 \cdot (\alpha - c - q_1) \tag{8}$$

The FOC identifies the optimal level of production for firm 1 as:

$$q_1^* = \frac{\alpha - c}{2} \tag{9}$$

and equation 7 identifies the optimal production of firm 2 as:

$$q_2^* = \frac{\alpha - c}{4}. \tag{10}$$

Two remarks in relation to the Cournot model above:

[6]The monopolist's profit maximization problem is:

$$\max_{Q} P(Q) \cdot Q - cQ \Rightarrow \max_{Q}(\alpha - c)Q - Q^2$$

FOC identifies the monopolist's optimal production level: $Q_{Monopoly} = \frac{\alpha - c}{2}$

[7]In game theory, this is sequential action, and the relevant equilibrium concept is subgame perfect equilibrium.

1. $Q_{Stackelberg} = \frac{3}{4} \cdot (\alpha - c) > Q_{Cournot} = \frac{2}{3} \cdot (\alpha - c) > Q_{Monopoly} = \frac{1}{2} \cdot (\alpha - c)$, which means that the equilibrium price in the Stackelberg model is less than that of the Cournot model.

2. Comparing the profits of each firm to that of the Cournot model:

$$\begin{aligned}
\pi_1^{Stackelberg} &= \frac{1}{8} \cdot (\alpha - c)^2 > \frac{1}{9} \cdot (\alpha - c)^2 = \pi_1^{Cournot} \\
\pi_2^{Stackelberg} &= \frac{1}{16} \cdot (\alpha - c)^2 < \frac{1}{9} \cdot (\alpha - c)^2 = \pi_2^{Cournot}.
\end{aligned}$$

4 Kreps-Scheinkman (1983)

In the Cournot model, we assume that firms compete on the basis of quantity produced. We can interpret this as a firm choosing its production capacity. If that is the case, then what is the equilibrium when firms first choose their capacity levels and then engage in Bertrand price competition? This is the question addressed by David M. Kreps and Jose A. Scheinkman, 1983, *Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes*, Bell Journal of Economics, 14(2): 326-337. The result of the paper is clearly stated in the title of the article.

Let's examine this result using MWG 12.C.11(a).[8]

Consider a capacity constrained duopoly pricing game. Firm j's capacity is q_j for $j = 1, 2$, and it has a constant cost per unit of output of $c \geq 0$ up to this capacity limit. Assume that the market demand function $x(p)$ is continous and strictly decreasing at all p such that $x(p) > 0$ and that there exists a price $\tilde{p}$ such that $x(\tilde{p}) = q_1 + q_2$. Suppose also that $x(p)$ is concave and let $p(\cdot) = x^{-1}(\cdot)$ denote the inverse demand function.

The rationing scheme if as follows:

If $p_j > p_i$:

$$\begin{aligned}
x_i(p_1, p_2) &= \min\{q_i,\ x(p_i)\} && (11) \\
x_j(p_1, p_2) &= \min\{q_j,\ \max\{x(p_j) - q_i,\ 0\}\} && (12)
\end{aligned}$$

, and if $p_j = p_i = p$:

$$x_i(p_1, p_2) = \min\left\{q_i, \max\left\{\frac{x(p)}{2},\ x(p) - q_j\right\}\right\}, \forall i \in \{1, 2\}.$$

Further assume that the higher value consumers (those consumers with a higher willingness to pay) are served first.

Suppose that $q_1 < b_1(q_2)$ and $q_2 < b_2(q_1)$, where $b_i(q_j)$ is the best response function for a firm i given production level q_j by firm j. Show that $p_1^* = p_2^* = p(q_1 + q_2)$ is a Nash Equilibrium.

[8] I won't cover 12.C.11(b), since you need to know some game theory concepts.

Assume that firm 2 charges $p_2^* = p(q_1+q_2)$, the price at which both firms are producing at their respective capacities. If firm 1 charges $p_1 \leq p(q_1+q_2)$, firm 1 will sell its full capacity level q_1, resulting in profits equal to $(p_1 - c)q_1$, which is less than the profits that firm 1 would get if it charged $p_1^* = p_2^* = p(q_1+q_2)$, $(p_1^* - c)q_1$.[9] Therefore, there is no profitable deviation for firm 1 to charge below p_2^*.

Is there a profitable deviation for firm 1 to charge above $p_2^* = p(q_1+q_2)$? Note that from the rationing scheme set out above in equations 11 and 12, the residual demand for firm 1 can be rewritten as follows:

$$x_1(p_1, p_2) = \min\{q_1,\, x(p_1) - q_2\}$$

. Note that $x(p_1) - q_2$ is the solution to firm 1's profit maximization problem:

$$\max_{x_1} \{p(x_1+q_2) - c\}\, x_1 \, s.t.\, x_1 \leq q_1$$

. Solving the maximization problem results in a best response function: $x_1^* = b_1(q_2)$, which by assumption is greater than firm 1's capacity: $b_1(q_2) > q_1$, meaning that firm 1 can't produce $b_1(q_2)$. Therefore, we have a corner solution to the problem, where $x_1^* = q_1$. Therefore, $p_1 = p(q_1+q_2) = p_2^*$ and we have contradiction.

Therefore, it is optimal for firm 1 to charge $p(q_1+q_2)$ given that firm 2 does the same.

By symmetry, the same argument applies to firm 2 and so we are at an equilibrium.

Since we know the result of the Bertrand price competition given capacity constraints, firms will now maximize profits by solving for the optimal capacities (q_1^*, q_2^*), given prices will equal $p(q_1+q_2)$. Sound familiar? It should, because this is the set up of the Cournot model.

[9] Notice how important the capacity constraints are. Unlike in our previous Bertrand example, firm 1 does *not* produce goods to satisfy the entire demand.