

## Practice Problem 5.3

### Part (a)

At  $p^* = 7$ ,  $Q^* = 28$ . In turn, each firm produces  $q_1^* = q_2^* = 14$ . Profit for each firm at this price/quantity vector is equal to:

$$\pi_i(p_i = 7) = p^* \cdot q_i^* - C(q_i^*) = 7 \cdot 14 - \frac{1}{2} \cdot 14^2 = 0.$$

We need to check that this is an equilibrium. Consider a deviation by firm 1, such that it reduces its price to  $7 - \varepsilon$ . Then firm 1's payoff is:

$$\pi_1(p_1 = 7 - \varepsilon) = 7 \cdot 28 - \frac{1}{2} \cdot 28^2 < 0.$$

The payoff from deviation is less than that from the candidate equilibrium price/quantity vector. The same applies for firm 2 and so this price/quantity vector constitutes an equilibrium.

### Part (b)

At  $p^* = 15$ ,  $Q^* = 20$ . In turn, each firm produces  $q_1^* = q_2^* = 10$ . Profit for each firm at this price/quantity vector is equal to:

$$\pi_i(p_i = 15) = p^* \cdot q_i^* - C(q_i^*) = 15 \cdot 10 - \frac{1}{2} \cdot 10^2 = 100 > 0.$$

We need to check that this is an equilibrium. Consider a deviation by firm 1, such that it reduces its price to  $10 - \varepsilon$ . Then firm 1's payoff is:

$$\pi_1(p_1 = 10 - \varepsilon) = 10 \cdot 20 - \frac{1}{2} \cdot 20^2 = 0.$$

Again, the payoff from deviation is less than that from the candidate equilibrium price/quantity vector. The same applies for firm 2 and so this price/quantity vector constitutes an equilibrium.

### Part (c)

The reason that firms can get positive profits in equilibrium is because cost functions are convex. If firm 1 undercuts firm 2, firm 1's benefit is selling to the entire market. However, this increase in market share has a trade-off: namely that since marginal costs are increasing, the increased total cost may offset the increase in revenue from selling to the entire market.