Section Notes 1

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Agenda

1. Introduction to Game Theory (basic definitions, assumptions, and representations)
2. Dominance and Iterated Dominance
3. Mixed Strategies

Textbooks

Of course, you can rely on MWG for the readings and you can also try the bible of game theory: Fudenberg and Tirole. But these two books are probably better for more advanced classes in Game Theory and as references in the future (especially FT). Two great sources for this class and a firm grounding in Game Theory are:

1. Robert Gibbons, 1992, *Game Theory for Applied Economists*, Princeton University Press.
2. Martin J. Osborne, 2004, *An Introduction to Game Theory*, Oxford University Press.

I especially recommend Gibbons (1992).

1 Introduction to Game Theory

What is a game? You might see the following types of notation when dealing with games:

$$\begin{aligned} \Gamma_n &= [I, \{S_i\}, \{u_i(\cdot)\}] \\ \Gamma_e &= [\mathcal{I}, I, \mathfrak{A}, p(\cdot), \alpha(\cdot), \mathcal{H}, \{\Delta(S_i)\}, \{u_i(\cdot)\}] \end{aligned}$$

. All the above is saying is that, at its core, a game has a set of players, for each player a set of actions, a strategy space for each player, and payoff functions (or outcomes; or preferences over the set of action profiles). Don't worry about these types of representations.

1.1 Common Definitions (less formal than MWG or FT)

Here are a few terms that you'll come across in the next few weeks:

- Players: This is easy. Only one thing to add: you might see the expression "in a finite game" in various propositions or definitions within game theory, in which case, $I < \infty$.

- Actions: When it is a player's turn to move, what will she do? Will she choose UP or DOWN; COOPERATE or DEFECT; LEFT, RIGHT, or CENTER; q or q', where $q, q' \in \mathbb{R}_+$. The actions available to a player can be finite or infinite.

- Information Sets: Concept used to represent a game where we assume that the players DO NOT have *perfect information*.[1] In a sequential (or dynamic) game, which is represented in an *extensive form*,[2] an information set is represented by drawing a circle around the relevant nodes.

- Strategy: "A strategy is a *complete contingent plan*, or *decision rule*, that specifies how the player will act in *every possible distinguishable circumstance* in which she might be called upon to move. ... Thus, a player's strategy amounts to a specification of how she plans to move at each one of her information sets, should it be reached during play of the game."[3] You should always try to keep the concepts of "strategy" and "action" separate and know the difference between the two. In a static, simultaneous move game, action and strategy may be the same. However, in more complicated (and so interesting) games, this will not be the case.

- Outcomes/utilities/payoffs: This should be self explanatory. Note that unlike in Econ 2020a, the arguments of player i's utility function is not only her own strategy, but the strategy of the other players: $u_i(s_i, \vec{s}_{-i})$.

- Histories: Don't worry about this now, but the history of player i, denoted H_i, is the set of sequences after which player i is called upon to move or select an action, $a \in \mathcal{A}(h)$, or her action set. We'll discuss histories when we get into dynamic, extensive form games.

- Pure Strategies: Although we tend to think in terms of pure strategies and much of applied game theory focuses on pure strategies, it may be

[1] See below for a formal definition of "perfect information."

[2] I won't cover "extensive form" vs "normal (reduced) form" games. Chris did a good job in class and you'll get practice in this week's problem set.

[3] MWG p.228, emphasis in original.

helpful to think of pure strategies as mixed strategies with a degenerate probability distribution. In short, a pure strategy is a mixed strategy that "assigns probability 1 to the action a_i [which is] equivalent to her simply choosing the action a_i...."[4]

- Mixed Strategies: This is covered in more detail below.
- Static vs. Dynamic / Extensive-Form vs Normal-Form / Simultaneous vs Sequential: These should be pretty clear.

Example. Now let's look at the following examples:

[4] Osborne (2004, p.108).

1.2 Common Assumptions

Note that all games will not have all of these assumptions. The point is that in more advanced games, we're going to try to relax some of these assumptions.

- Complete Information: Each of the players' payoff functions, which are dependent on the strategies of each player, is *common knowledge* to all of the other players.
- Perfect Information: "A game is one of *perfect information* if each information set contains a single decision node. Otherwise, it is a game of *imperfect information*."[5] If we assume perfect information, then each player knows exactly where they are on the game-tree.
- Common Knowledge: "[A] basic postulate of game theory that all players know the structure of the game, know that their rivals know it, know that their rivals know that they know it, and so on."[6] One aspect of "common knowledge" that we assume in most of our games is that each of the players is rational, knows that the other players know she is rational, knows that she knows it, and so on.

2 Dominance and Iterated Dominance

2.1 Definitions

- Player i's strategy s_i is *strictly dominant* iff $u_i(s_i, \vec{s}_{-i}) > u_i(\hat{s}_i, \vec{s}_{-i})$, $\forall \hat{s}_i \in S_i$, $\hat{s}_i \neq s_i$ and $\forall \vec{s}_{-i} \in \times_{-i} S_k$.
- Player i's strategy s_i is *weakly dominant* iff $u_i(s_i, \vec{s}_{-i}) \geq u_i(\hat{s}_i, \vec{s}_{-i})$, $\forall \hat{s}_i \in S_i$, $\hat{s}_i \neq s_i$ and $\forall \vec{s}_{-i} \in \times_{-i} S_k$; and $u_i(s_i, \vec{s}_{-i}) > u_i(\hat{s}_i, \vec{s}_{-i})$, $\forall \hat{s}_i \in S_i$, $\hat{s}_i \neq s_i$ and for some $\vec{s}_{-i} \in \times_{-i} S_k$
- Player i's strategy s_i is *strictly dominated* iff $\exists \hat{s}_i \in S_i$, $\hat{s}_i \neq s_i$ s. t. $u_i(s_i, \vec{s}_{-i}) < u_i(\hat{s}_i, \vec{s}_{-i})$, $\forall \vec{s}_{-i} \in \times_{-i} S_k$.
- Player i's strategy s_i is *weakly dominated* iff $\exists \hat{s}_i \in S_i$, $\hat{s}_i \neq s_i$ s. t. $u_i(s_i, \vec{s}_{-i}) \leq u_i(\hat{s}_i, \vec{s}_{-i})$, $\forall \vec{s}_{-i} \in \times_{-i} S_k$; and $u_i(s_i, \vec{s}_{-i}) < u_i(\hat{s}_i, \vec{s}_{-i})$, for some $\vec{s}_{-i} \in \times_{-i} S_k$.

Note that I used $\vec{s}_{-i} \in \times_{-i} S_k$ to represent the *pure* strategies of the "other" players. However, keep in mind that the definitions above apply to *mixed* strategies ($\vec{\sigma}_{-i} \in \times_{-i} \Sigma_k$) as well, or combinations of both.[7] Also note that although I used Bernoulli utility functions in the definitions, we could switch these out with expected utility functions, $U(\cdot)$.

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[5] MWG Definition 7.C.1 (emphasis in original)
[6] MWG p.226
[7] More on this below when we consider mixed strategies.

2.2 Examples

	L	M	R
U	1,0	1,2	0,1
D	0,3	0,1	2,0

	L	M	R
U	5,3	2,2	2,-2
C	0,0	3,5	0,-2
D	0,0	0,2	3,1

Recall in class Chris noted that the order of eliminating strictly dominated strategies does not matter. However, the same does not apply when we're dealing with *weakly* dominated strategies. Check Problem 2.2 in the Practice Problems to see why.

3 Mixed Strategies

Definition. In the normal form game (simultaneous move), suppose that $S_i \equiv \{s_{i1}, ..., s_{ik}, ..., s_{iK}\}$, then a mixed strategy for player i is a probability distribution over the possible strategies, $\vec{p}_i = [p_{i1}, ..., p_{ik}, ..., p_{iK}]$, where $p_{ik} \in [0,1]$ and $\sum_{k=1}^{K} p_{ik} = 1$. Consider the following games and check to see which pure strategies are strictly dominated.

	L	R
U	-1,3	5,-1
C	5,-1	-1,3
D	0,3	0,2

	L	M	R
U	3,1	1,0	4,0
C	1,3	2,4	1,2
D	0,1	1,1	2,2

In the first example the mixed strategy of the form $\vec{p}^* = [p, 1-p, 0]$, where $p \in \left(\frac{1}{6}, \frac{5}{6}\right)$, strictly dominates strategy D; and in the second example, a mixed strategy of the form $\vec{q}^* = [q, 1-q, 0]$, where $q \in \left(\frac{1}{3}, 1\right)$, strictly dominates strategy D. Therefore, in both examples, strategy D is strictly dominated.