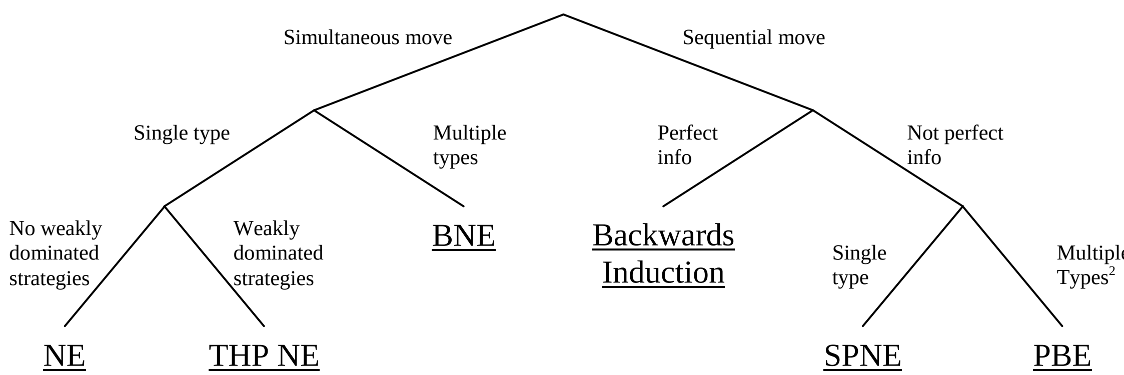
Systematic approach to solving games

1) Write down the game (when feasible). Fill in payoffs for each cell of the matrix or terminal node.
   a) *Normal form* if the game is simultaneous move without multiple types.
   b) *Bayesian Normal Form* if the game is simultaneous move with multiple types.
   c) *Extensive Form* if the game is sequential.

2) What is each player's strategy set?
   a) First, determine what each player's information sets are.
   b) A player's strategy must specify what action to take in each of the player's information sets.

3) When possible, eliminate *strictly* dominated *strategies*.[1]
   a) Do not eliminate *weakly* dominated strategies.
   b) Do not eliminate strictly dominated *actions* (yet).

4) Determine what equilibrium concept to use:



5) Solve. Remember that an equilibrium specifies the players' strategies (their actions at *all* their information sets, not just the information sets that are reached in equilibrium). If asked for the equilibrium *outcome*, you can just specify what actions get played in equilibrium, and the resulting payoffs.

   a) Nash equilibrium:
      i) Find each player's best-response function, which gives that player's optimal strategy, given the strategies played by the other players. (In the normal form, this entails simply underlining best responses.)
      ii) Solve best-response functions simultaneously to get the set of pure-strategy Nash equilibria. (In the normal form, this is just finding cells where all payoffs are underlined.)
      iii) If there is more than one pure-strategy Nash equilibrium, look for equilibria in mixed strategies: if a player is mixing, she must be indifferent between all pure strategies that she plays with positive probability.

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[1] This could theoretically cause problems if you're solving PBE without refinements. I wouldn't worry about it, though.
[2] You may want to use PBE in cases where there are not multiple types. PBE more generally allows us to deal with beliefs.

b) Trembling-hand perfect Nash equilibrium:
   i) Solve for the set of Nash equilibria, as above.
   ii) Eliminate weakly dominated strategies from consideration either as pure strategies or as parts of mixed strategies (but do not iteratively eliminate weakly dominated strategies).[3]

c) Bayes-Nash equilibrium:
   i) Solve exactly as you would solve for Nash equilibria, but expected payoffs will be a function of the probabilities over types.

d) Backwards induction Nash equilibrium:
   i) Find and mark best responses at the terminal decision nodes (the decision nodes closest to the terminal nodes). When there are ties, consider all tied options.
   ii) "Trim the tree": Given the best responses at the terminal decision nodes, find best responses at the decision nodes just before the terminal decision nodes.
   iii) Iterate until you have found best responses at all nodes.

e) Subgame perfect Nash equilibrium:
   i) Identify all subgames. A subgame starts at a singleton node, contains all successor nodes, and does not split information sets.
   ii) Find the set of Nash equilibria in the terminal subgames (the subgames closest to the terminal nodes). When there are multiple Nash equilibria, consider all of them.
   iii) Using the results from (ii), identify the payoffs from reaching each terminal subgame. Using these payoffs, find the set of Nash Equilibria in the subgames one step before the terminal subgames.
   iv) Iterate using this type of backward induction by subgame until you have solved for the whole game.

f) Perfect Bayesian Equilibrium (consists of both strategies and beliefs):
   i) Eliminate strictly dominated actions in any information set.
   ii) Propose equilibrium strategy for one player (usually the sender, who has multiple types).
   iii) Compute other players' beliefs based on the proposed play.
   iv) Calculate the best responses of other players, conditional on their beliefs.
   v) Check if any type of the first player has an incentive to deviate. (When necessary, find the set of off-path beliefs consistent with this play.)
   vi) Repeat until you have checked each possible pure strategy for the first player (not involving strictly dominated actions).
   vii) Apply Cho-Kreps Intuitive Criterion, if asked to do so: if a message is equilibrium dominated for a type, then assign probability zero to the message coming from that type.

[3] This isn't 100% correct, but should suffice for our purposes.