Section Notes 8

Wonbin Kang and Sam Richardson

March 31, 2011

Agenda

1. Information economics overview
2. General model of adverse selection
3. Adverse selection in insurance
4. Adverse selection in auctions[1]

1 Information economics overview

Information economics combines the game theory concepts we learned in Econ 2020a; and the price theory and general equilibrium theory we learned in the Econ 2020b.

1.1 Fundamental Welfare Theorems

Recall that the two theorems are:

Theorem 1.1. *First Fundamental Welfare Theorem: If a competitive equilibrium (Walrasian equilibrium) exists, then the equilibrium 1) is equal to a Pareto equilibrium; 2) is Pareto optimal; or 3) leads to a Pareto efficient allocation of resources..*

Note that bullets 1), 2), and 3) are equivalent statements.

Theorem 1.2. *Second Fundamental Welfare Theorem: Any Pareto optimal equilibrium (or, any Pareto efficient allocation) can be sustained by a competitive equilibrium.*

Recall the assumptions needed for the Fundamental Welfare Theorems:[2]

1. Complete markets: no unpriced goods

[1] We won't cover in section.

[2] Note that the Second Fundamental Welfare Theorem requires one more condition: convexity of preferences.

2. No market power: all economic agents are price-takers (competitive markets for all goods).[3]
3. Complete information for all agents. This assumption fails if we have uncertainty in the payoffs or when there is asymmetric or imperfect information. Information economics uses the tools of game theory to identify equilibrium when the complete information assumption fails.

1.2 Timeline of Information Economics Models

The models we will look at in the second half of this semester have some variation of the following timeline:[4]

- $t = 0$: Nature determines the agents type (in many texts, denoted as θ)
- [$t = \frac{1}{2}$: signaling and screening models allow actions here]
- $t = 1$: Principal designs / offers contract
- $t = 2$: Agents accept or reject contract offer
- $t = 3$: Nature determines outcomes and payoffs are realized

2 General Model of Adverse Selection

Consider the following set up:

- seller's value is $V \sim F(v)$, which means that seller is of type equal to $v \in [v_L, v_H]$;
- buyer's value is equal to $h(v)$, where $h(\cdot)$ is an increasing function;
- informational assumption: seller knows v and buyer knows the distribution;
- assume that the seller has all of the bargaining power and the buyer makes offers to trade the good at some price p.

Definition 2.1. At any Rational Expectation Equilibrium of the model above, the equilibrium price p^* is equal to the expected value to the buyer conditional on trade. Therefore, *ex ante*, buyers will receive zero surplus in any REE, and *any equilibrium price p^* that satisfies the buyer's zero surplus condition is a REE.*

We further classify the REEs as follows:

[3] At the end of Econ 2020a, we discussed failures of this assumption: namely, monopolies and externalities.

[4] Some people find it helpful to draw the timeline of a game/model as in Appendix A-1.

Definition 2.2. A REE is "unstable" if a small deviation in the price offered by one buyer would increase the payoffs for both that buyer and the seller. A REE is "stable" if no small deviation in price by one buyer would increase the payoffs for both that buyer and the seller.[5]

To find the set of Rational Expectation outcomes,[6] the first question is to check and see if there exist boundary equilibrium outcomes: the equilibrium price is equal to the lower bound of seller's values or the equilibrium price is equal to at least the upper bound of seller's values.

- If $p^* = v_L$, the probability the good is sold is equal to zero. Further, this REE is stable if and only if $h(v_L) < v_L$. If $h(v_L) > v_L$, then the buyer and the seller is better off by a slightly higher bid by the buyer.

- If $p^* \geq v_H$, the probability the good is sold is equal to one, and so the following holds: $(\mathbb{E}[h(v)|V \leq p^*] = \mathbb{E}[h(v)])$. Note that if $\mathbb{E}[h(v)] \leq v_H$, then the buyer receives a negative payoff. Therefore, it must be the case that $\mathbb{E}[h(v)] > v_H$ with the corresponding REE price $p^* = \mathbb{E}[h(v)]$. In short, although all sellers would be willing to sell at $p^* = v_H$, competition among the buyers results in an equilibrium price greater than v_H.

Next we need to ask: given that a trade occurs at some equilibrium price, what is the expected value of the trade to the buyer? In short, we need to know:[7]

$$\mathbb{E}[h(v)|V \leq p^*]$$

and whether $\mathbb{E}[h(v)|V \leq p^*] \lesseqgtr p^*$. If $\mathbb{E}[h(v)|V \leq p^*] > p^*$, then an offer of p^* would be profitable to the buyer. If $\mathbb{E}[h(v)|V \leq p^*] < p^*$, then an offer of p^* would be unprofitable to the buyer. Note that at the REE, the zero surplus condition must hold for the buyer.

Given the analysis above, let's consider the following cases referring to Figures 13-2 to 13-7 in Chapter 13 of the Avery Notes.

- Consider the case where $h(v_L) < v_L$, or the buyer's value of the good is less than that of the seller's at low values of v. See Figures 13-2 to 13-4. The efficient outcome would have that trade occurs only if the buyer values the good more than the seller or $h(v) > v$.

- Consider the case where $h(v_L) > v_L$. See Figures 13-5 to 13-7.

- Compare the outcomes against the first best outcome where there is no information asymmetry and so the buyer perfectly knows the seller's type.

[5] Do you see why, given a REE, we only need to consider slight deviations to higher prices offered by the buyer to determine whether we are at a stable or unstable equilibrium?

[6] Note that I'm talking in terms of outcomes and not equilibria. The latter would require that I write down the equilibrium strategies of the buyer and the seller.

[7] Note that at an interior REE, where some types of sellers will trade and others will not, the zero surplus condition from the definition above requires that:

$$\mathbb{E}[h(v)|V \leq p^*] = p^*.$$

3 Adverse Selection in Insurance

3.1 Model Set-Up[8]

Risk averse consumer with wealth W and utility function equal to $u(W) = \sqrt{W}$. Two states of the world where in one state the consumer loses all of her wealth and in the second state there are no damages. Risk of losing everything is equal to $p \sim U[0,1]$. The timing of the model is as follows:

- $t = 0$: Nature determines type of consumer, p. The consumer knows her type. If p is high, then the consumer is high risk.
- $t = 1$: Monopolist insurer determines the contract to be offered: insurer offers full insurance[9] at price α_1.[10] Note that the monopolist doesn't know the consumer's type. We assume that the monopolist insurer is profit maximizing and risk neutral.
- $t = 2$: Consumer decides whether to buy or not to buy.
- $t = 3$: Nature determines outcomes and payoffs are realized.

Based on the model above and the assumption of full insurance, consumer with insurance will always receive final wealth equal to $W - \alpha_1$ in both states of the world. If there are no damages, final payoff is $W - \alpha_1$; and if there are damages, final payoff is $W - d - \alpha_1 + \alpha_2 = W - \alpha_1$ ($\because \alpha_2 = d$). A consumer without insurance will receive W in one state and 0 in the other. The expected utility of an uninsured consumer is equal to:

$$U_{uninsured}(W, \alpha_1) = (1-p)\sqrt{W} + p\sqrt{0}.$$

3.2 Equilibrium

What is the equilibrium price and who buys insurance given the fact that the monopolist insurer does not know the consumer's type (which for purposes of this model is equal to p)? What would happen if there is full information? We'll go over the first question, but you should try the second question. The equilibrium for the second question is the "first-best" outcome and $\alpha_1 = p \cdot W$.

1. If insurer sets price equal to α_1, then the consumer will buy insurance iff:

[8] Some people like to draw game trees, but I don't find them very helpful.

[9] "Full insurance" means that in the case of damages d the consumer receives d.

[10] In Rothschild and Stigler (1976)'s model (a very important model we will see later in the semester), the consumer is able to purchase partial insurance which is denoted as a vector $\vec{\alpha} = [\alpha_1, \alpha_2]$ where α_1 is the premium and α_2 is the insurance coverage—*i.e.* the amount of wealth the insurer pays the consumer in case of damages. Where appropriate, I'll use this notation below.

$$
\begin{aligned}
U_{insured}(W, \alpha_1) &\geq U_{uninsured}(W, \alpha_1) \\
\sqrt{W - \alpha_1} &\geq (1-p)\sqrt{W} \\
\sqrt{\frac{W-\alpha_1}{W}} &\geq 1-p \\
p &\geq 1 - \sqrt{\frac{W-\alpha_1}{W}}
\end{aligned}
$$

Therefore, if $\alpha_1 = 0$, all consumers buy insurance but insurers get negative profits, which means that there is a profitable deviation for the insurer. At $\alpha_1 = W$, no one buys insurance.

2. Given a price equal to α_1, the probability that a randomly drawn consumer will buy insurance can be solved for as follows:

$$
\Pr\left(p \geq 1 - \sqrt{\frac{W-\alpha_1}{W}}\right) = 1 - \left(1 - \sqrt{\frac{W-\alpha_1}{W}}\right) = \sqrt{\frac{W-\alpha_1}{W}}
$$

3. Given the probability that a consumer will buy insurance, what is the expected profit for the insurer? To solve for the expected profit, we need to calculated the expected risk of a consumer who has bought insurance:

$$
\begin{aligned}
\mathbb{E}\left[p | bought\, insurance\right] &= \mathbb{E}\left[p \,|\, p \geq 1 - \sqrt{\frac{W-\alpha_1}{W}}\right] \\
&= \frac{1}{2} \cdot \left[1 + 1 - \sqrt{\frac{W-\alpha_1}{W}}\right] \\
&= 1 - \frac{1}{2} \cdot \sqrt{\frac{W-\alpha_1}{W}}
\end{aligned}
$$

Given the consumer's risk as perceived by insurer, the insurer will maximize its profits. To put this model in the context of the signaling models we discussed before: from the perspective of the insurer, the consumer (Sender) sends a signal (buying insurance) and based on this signal the insurer (Receiver) updates its beliefs. Therefore, the expected profit of the insurer on the sale of this insurance policy is equal to:

$$
\mathbb{E}\left(\pi\right) = \alpha_1 - W \cdot \mathbb{E}\left[p | bought\, insurance\right] = \alpha_1 - W\left(1 - \frac{1}{2} \cdot \sqrt{\frac{W-\alpha_1}{W}}\right)
$$

4. Therefore, in equilibrium it must be the case that:

$$
\begin{aligned}
\mathbb{E}(\pi) &= \alpha_1 - W\left(1 - \frac{1}{2}\cdot\sqrt{\frac{W-\alpha_1}{W}}\right) \geq 0 \\
&\Rightarrow \alpha_1 \geq \frac{3w}{4}
\end{aligned}
$$

If we have competitive markets, $\alpha_1 = \frac{3w}{4}$, and only consumers with risk $p \geq \frac{1}{2}$ will buy insurance.

4 Adverse Selection in Auctions

4.1 Short Overview

Winner's Curse typically refers to auctions in which the value of the item is uncertain to the buyers:

- oil exploration rights;
- spectrum auctions;
- free agents in sports

If you take an Industrial Organization class, you'll think a lot about this: in equilibrium, bidders take winner's curse into account when submitting bids.

In FCC spectrum auctions in California, PacBell hired Paul Milgrom to give seminars on winner's curse to all their competitors. The result was that PacBell won the auction at a really good price.

4.2 Model Set-Up

Assume that players are bidding in a second price auction. Assume that player i's estimated value is equal to v_i, and $i \in \{1, 2\}$; and that each players' value of the good/service being auctioned has a distribution equal to $F(v_i)$ and the probability density function of v_i is as provided below. Recall that in a second price auction, it is a weakly dominant strategy for a player's bid to equal her value (which in this model is a random variable).

4.3 Model Equilibrium

1. What is the expectation of player i's valuation?

$$\mathbb{E}(v_i) = \int_{-\infty}^{\infty} v_i f(v_i) dv_i$$

Let's assume that the probability distribution function of v_i is as follows:[11]

$$f(v_i) = \begin{cases} \frac{v_i}{25}, & 0 \leq v_i < 5 \\ \frac{2}{5} - \frac{v_i}{25}, & 5 \leq v_i \leq 10 \\ 0, & otherwise \end{cases}$$

Given the pdf of v_i, we can now calculate the expected value of player i's valuation:

$$\begin{aligned} \mathbb{E}(v_i) &= \int_0^5 \left(v_i \cdot \frac{v_i}{25}\right) dv_i + \int_5^{10} \left\{ v_i \cdot \left(\frac{2}{5} - \frac{v_i}{25}\right)\right\} dv_i \\ &= \frac{v_i^3}{75}|_0^5 + \left(\frac{1}{5}v_i^2 - \frac{v_i^3}{75}\right)|_5^{10} = 5 \end{aligned}$$

2. What is player i's probability of winning given the value of v_i?

$$\Pr(player\, i\, wins|v_i) = \Pr(v_2 < v_1|v_1) = F(v_1),$$

which we can solve for by integrating the pdf we solved for above:[12]

$$F(v_i) = \begin{cases} \frac{v_i^2}{50} & ,0 \leq v_i < 5 \\ \frac{2}{5}v_i - \frac{v_i^2}{50} - 1 & ,5 \leq v_i \leq 10 \end{cases}$$

[11] The graphical representation of the PDF can be found in Appendix A-2.

[12] Note that -1 in the cdf is a "normalizing variable" which is required so that the probability measure properties are met.