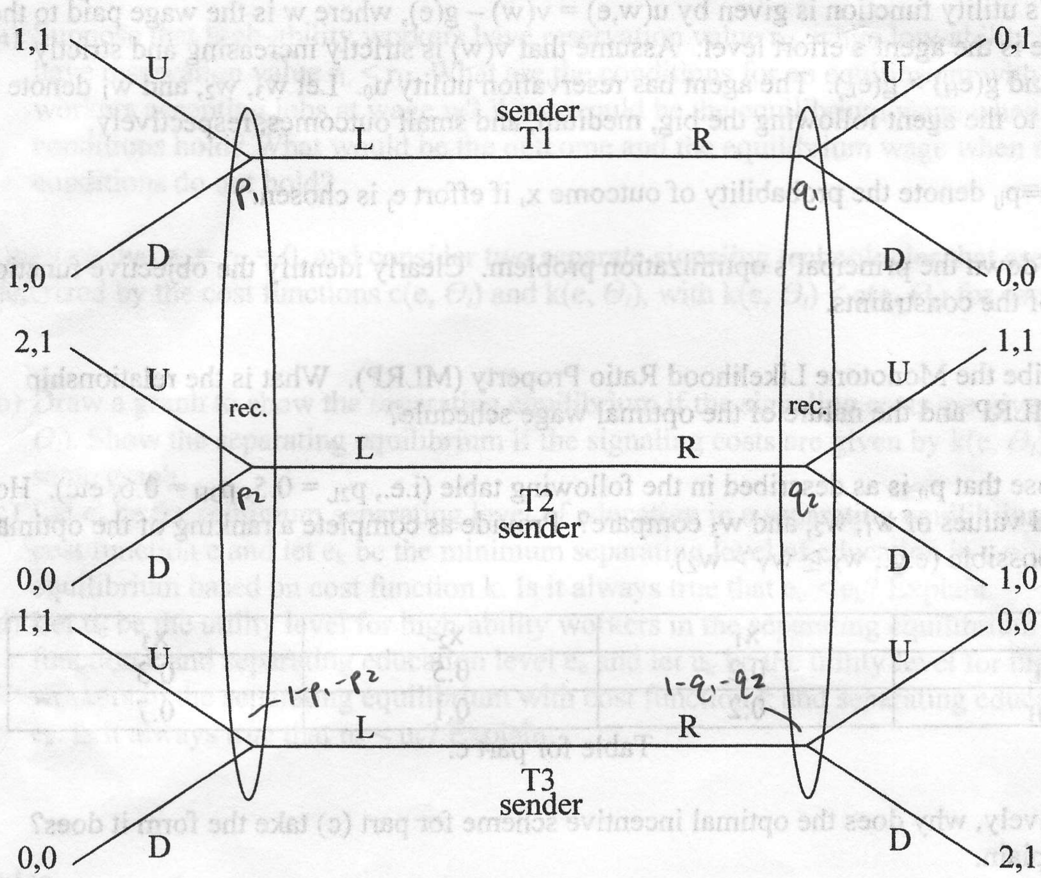
3. Consider the following signal game, in which the sender is type T1, T2, T3, each with probability 1/3 . Find a Perfect Bayesian Equilibrium in which all three sender types play L.



What if you were asked to find the set of all such PBE's in pure strategies?

Is there such a PBE where at least one player mixes?

5. Consider the standard principal-agent problem with moral hazard (hidden effort) that we studied in class. There are three outcome levels, small (x_1), medium (x_2), and big (x_3), and two effort levels, low (e_L) and high (e_H). Assume that output satisfies $x_3 > x_2 > x_1$ and that the principal **minimizes the expected cost of inducing the agent to choose high effort.**

The agent's utility function is given by $u(w,e) = v(w) - g(e)$, where w is the wage paid to the agent and e is the agent's effort level. Assume that v(w) is strictly increasing and strictly concave, and $g(e_H) > g(e_L)$. The agent has reservation utility u_0. Let w_3, w_2, and w_1 denote the wage paid to the agent following the big, medium, and small outcomes, respectively.

Let $p(x_i|e_j) \equiv p_{ij}$ denote the probability of outcome x_i if effort e_j is chosen.

5a) Write down the principal's optimization problem. Clearly identify the objective function and each of the constraints.

5b) Describe the Monotone Likelihood Ratio Property (MLRP). What is the relationship between MLRP and the nature of the optimal wage schedule?

5c) Suppose that p_{ij} is as described in the following table (i.e., $p_{2L} = 0.5$, $p_{3H} = 0.6$, etc). How do the optimal values of w_1, w_2, and w_3 compare? Provide as complete a ranking of the optimal wages as possible (e.g., $w_X \geq w_Y > w_Z$).

	x_1	x_2	x_3
e_L	0.2	0.5	0.3
e_H	0.2	0.1	0.7

Table for part c.

5d) Intuitively, why does the optimal incentive scheme for part (c) take the form it does? Briefly explain.

5e) Suppose that p_{ij} is as described in the following table. Suppose that $v(w) \rightarrow -\infty$ as $w \rightarrow -\infty$ and that the principal can impose arbitrarily large penalties on the agent (i.e., w can be negative). Argue that there is an incentive scheme with expected cost $v^{-1}(u_0 + g(e_H))$ that induces the agent to choose high effort. Explain your answer.

	x_1	x_2	x_3
e_L	0.34	0.33	0.33
e_H	0	0.4	0.6

Table for part e.

5f) Suppose that the principal cannot impose fines on the agent, i.e., that the wages must be non-negative. Does this change the answer to part e). If so, briefly explain why and how. If not, briefly explain why not.

Labor Market with Multiple Types

Consider a model of a competitive labor market with two types of workers: high-ability workers with productivity Θ_H and low-ability workers with productivity $\Theta_L < \Theta_H$. The proportion of high ability workers is λ.

(a) Suppose that high-ability workers have reservation value r_H, while low-ability workers have reservation value $r_L < r_H$. What are the conditions for an equilibrium with all workers accepting jobs at wage w? What would be the equilibrium wage when these conditions hold? What would be the outcome and the equilibrium wage when these conditions do not hold?

Now assume that $r_L = r_H = 0$, and consider two separate signaling technologies that are characterized by the cost functions $c(e, \Theta_i)$ and $k(e, \Theta_i)$, with $k(e, \Theta_i) < c(e, \Theta_i)$ for each pair (e, Θ_i).

(b) Draw a graph to show the separating equilibrium if the signaling costs are given by $c(e, \Theta_i)$. Show the separating equilibrium if the signaling costs are given by $k(e, \Theta_i)$ on the same graph.

(c) Let e_c be the minimum separating level of education in a separating equilibrium based on cost function c and let e_k be the minimum separating level of education in a separating equilibrium based on cost function k. Is it always true that $e_c < e_k$? Explain.

(d) Let u_c be the utility level for high-ability workers in the separating equilibrium with cost function c and separating education level e_c and let u_k be the utility level for high-ability workers in the separating equilibrium with cost function k and separating education level e_k. Is it always true that $u_c < u_k$? Explain.

Agendas

Assume that there are three voters with Euclidian (spatial) preferences over $\mathbb{R}^2$ with ideal points at (-1,0), (0,1), and (1,0). If voters are voting sincerely, construct an agenda to get from (0,0) to (2,2).