Section Notes 9

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1 Competitive Insurance Model Set-Up

Please read Rothschild & Stiglitz (1976)[1]

1.1 Demand Side

We have a population of *risk averse* expected utility maximizing consumers. These consumers move second and they accept or reject a firm's offer. A representative consumer has wealth equal to W; probability of losing L equal to p_i, where $i \in \{G, B\}$, and such that $p_G < p_B$. "G" is for a good type of consumer with low risk of loss and "B" is a bad type of consumer with a high risk of loss. Assume that the probability of the consumer being a good type is equal to λ, which means that the average risk (or pooled level of risk) is equal to $\bar{p} = \lambda p_G + (1-\lambda)p_B$. Further assume that the expected utility of a consumer of type i is equal to:

$$U_i = p_i \cdot u(consumption\, in\, s_2) + (1 - p_i) \cdot u(consumption\, in\, s_1),$$

[1] Rothschild, Michael and Joseph Stiglitz. 1976. *Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information.* The Quarterly Journal of Economics. 90(4): 629-49.

where s_1 and s_2 denotes the level of consumption in State 1 where there is no loss; and level of consumption in State 2 where there is a loss, respectively. Also, $u'(\cdot) > 0$ and $u''(\cdot) < 0$ for simplicity.

What have we just assumed here?

- The level of risk aversion is equal for both good and bad types of consumers.[2]
- We have the same utility function in both the loss state (State 2) and the no loss state (State 1).

1.2 Supply Side

Multiple *risk neutral* profit-maximizing monopolistic firm which move first and offer potentially multiple contracts which can be completely characterized by the following vector: $\vec{\kappa} = [M, R]$,[3] where "M" denotes the premium, and "R" denotes the reimbursement amount.

Taking the demand and supply side together we have the following trivial results:

- A consumer of type $i \in \{G, B\}$ who has purchased insurance will consume, with probability p_i, $s_2 = W - M - L + R$ in State 2 and with probability $1 - p_i$ she will consume $s_1 = W - M$ in State 1. Without insurance, consumption is equal to $s_{2,no\,insurance} = W - L$ and $s_{1,no\,insurance} = W$, respectively. Note that consumption in each state is completely defined by "M" and "R".
- With probability p_i a firm who has sold insurance will have payoff equal to $M - R$ in State 2 and with probability $1 - p_i$ it will have payoff equal to M in State 1.

The firms' expected profit can be written as follows:

$$\begin{aligned} \mathbb{E}[\pi] &= p_i(M - R) + (1 - p_i)(M) \\ &= p_i(W - L - s_2) + (1 - p_i)(W - s_1) \\ &= (W - p_iL) - (p_i s_2 + (1 - p_i)s_1) \end{aligned} \tag{1}$$

where the first term is equal to the uninsured expected consumption and the second term is equal to the insured expected consumption. This can be rewritten in a more convenient form as follows:

$$s_2 = -\frac{(1 - p_i)}{p_i} s_1 + \frac{1}{p_i} \cdot \{(W - p_iL) - \mathbb{E}[\pi]\} \tag{2}$$

Do you see why the "sun" is in the lower left hand corner when we draw the $[s_1, s_2]$ Euclidean plane?

[2]Recall our measures of risk aversion from Econ 2020a, and convince yourself.

[3]Rothschild and Stiglitz (1976) use the notation $\vec{\alpha} = (\alpha_1, \alpha_2)$.

1.3 Equilibrium Requirements

1.3.1 Consumer's Problem

Given consumption in the two states, consumers maximize among available contracts:

$$\max_{[M,R]\in\Gamma_{insurer}} U_i = p_i \cdot u(W - M - L + R) + (1 - p_i) \cdot u(W - M)$$

where $\Gamma_{insurer}$ denotes the contracts that the insurer offers.

1.3.2 Insurer's Problem

Given that consumer maximize utility, the insurer provides contracts which satisfy the following conditions:

- No contract offered by a firm makes negative profit.
- No contract not offered would make positive profit.

In short, the firm makes zero profits in equilibrium. Recall from Sam's lecture why this was important.

2 Main Results

1. In a separating equilibrium, the bad types (high risk consumers) are fully insured and the good types (low risk consumers) are not fully insured.
2. No pooling equilibrium exists.
3. There may be no equilibrium in pure strategies.

3 Example[4]

The parameter values are as follows:
$w = 1000 \quad L = 900 \quad p_G = 0.1 \quad p_B = 0.6 \quad \lambda = 0.2 \quad u(s) = \sqrt{s}$

- What does the 45 degree line represent? It represents equal consumption in both states of the world: $s_1 = s_2$. If the consumer is able to purchase an insurance contract which puts her on the 45 degree line, then she has full insurance.
- What if the consumer is uninsured? Then we're at point $A = [W,\, W - L] = [1000,\, 100]$

[4]The goal is to derive all values of Rothschild and Stiglitz (1976), Figure III, once we know the parameter values.

- The zero profit lines, $\overline{AB}$ and $\overline{AG}$, denote actuarily fair contracts.[5] In other words, all points on the $\overline{AB}$ and $\overline{AG}$ describe consumption in state 1 and state 2 such that insurance is priced to be actuarily fair. To see this, substitute $s_1 = W - M$ and $s_2 = W - M - L + R$ into equation 2 after setting $\mathbb{E}[\pi] = 0$.

3.1 Separating Equilibrium

3.1.1 Actuarily Fair Line for Bad Types and Full Insurance

Using equation 1 (or equation 2), we can write the actuarily fair line for bad types ($\overline{AB}$) as follows:

$$
\begin{aligned}
(W - p_B L) - (p_B s_2 + (1 - p_B) \cdot s_1) &= 0 \\
\Rightarrow (1000 - (0.6 \cdot 900)) - (0.6 s_2 + (1 - 0.6) \cdot s_1) &= 0 \\
\Rightarrow s_2 = 766.7 - \frac{2}{3} s_1 & \qquad (3)
\end{aligned}
$$

Note that the slope of the actuarily fair line for the bad type is equal to $-\frac{1-p_B}{p_B} = -\frac{0.4}{0.6}$

Now using equation 3, we can solve for the full insurance, zero-profit point for the bad types, or point α_B in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 766.7 - \frac{2}{3} s \Rightarrow s = s_1 = s_2 = 460$$

3.1.2 Actuarily Fair Line for Good Types and Full Insurance

Using equation 1 (or equation 2), we can write the actuarily fair line for good types ($\overline{AG}$) as follows:

$$
\begin{aligned}
(W - p_G L) - (p_G \cdot s_2 + (1 - p_G) \cdot s_1) &= 0 \\
\Rightarrow (1000 - (0.1 \cdot 900)) - (0.1 \cdot s_2 + (1 - 0.1) \cdot s_1) &= 0 \\
\Rightarrow s_2 = 9100 - 9 \cdot s_1 & \qquad (4)
\end{aligned}
$$

Note that the slope of the actuarily fair line for the good type is equal to $-\frac{1-p_G}{p_G} = -\frac{0.9}{0.1}$

Now using equation 4, we can solve for the full insurance, zero-profit point for the good types, or point β in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 9100 - 9s \Rightarrow s = s_1 = s_2 = 910$$

[5] Recall that if an insurance contract is actuarily fair, the premium should equal the expected payout, or $M = p_i R$.

3.1.3 Indifference Curve $\bar{U}_B$ and Solving for α_G

How do we pin down the consumption in each state for α_G? We use the fact that the indifference curve through the point α_B also passes through α_G and the fact that α_G lies on the good types actuarily fair line.

1. Solve for the level of $\bar{U}_B$:

$$\bar{U}_B = (1-p_B)\cdot u(460)+p_B\cdot u(460) = (1-p_B)\sqrt{460}+p_B\sqrt{460} = \sqrt{460} \approx 21.4$$

2. The bad type is indifferent between α_B and α_G. Therefore, if we let $\alpha_G = [s_1, s_2]$, we have:

$$U_B\left(\alpha_G = [s_1, s_2]\right) = p_B\sqrt{s_2} + (1-p_B)\sqrt{s_1} = 21.4. \tag{5}$$

3. Further, the point α_G lies on the good types actuarily fair line which we solved for in equation 4:

$$s_2 = 9100 - 9\cdot s_1 \tag{6}$$

4. Solve the system of equations involving equations 5 and 6, which results (after some very laborious algebra) in:

$$\alpha_G = [s_1, s_2] = [987,\ 217] \tag{7}$$

3.1.4 Separating Equilibrium?

The separating equilibrium results in the bad type consuming 460 in each period and the good type consuming 987 in state 1 and 217 in state 2. We can write this as:460

$$\alpha_B = [460,\ 460]\,;\ \alpha_G = [987,\ 217]\,.$$

Further, since we know the values of W and L, we can solve for $\vec{\kappa}_B = [M_B, R_B]$ and $\vec{\kappa}_G = [M_G, R_G]$. I won't bother going through the algebra here or below.

Is this result consistent with Main Result 1. above?

3.1.5 Certainty Equivalent for Good Types

To put the good type's consumption α_G into perspective, we can also ask: Given the probability of loss for the good type and her consumption α_G in the separating equilibrium, what is the good type's certainty equivalent?[6]

We can answer this question using the same methodology we used to solve for the separating equilibrium.

1. We can solve for $\bar{U}_G$ by solving the following:

$$\bar{U}_G = p_G\cdot u(217)+(1-p_G)\cdot u(987) = 0.1\sqrt{217}+(1-0.1)\sqrt{987} = 29.75 \tag{8}$$

[6] We defined "CE" in Econ 2020a.

2. At the CE, the good type's utility, which I denote as $\bar{U}_G^{CE}$, is as follows:

$$\bar{U}_G^{CE} = p_G \cdot u(s) + (1 - p_G) \cdot u(s) = \sqrt{s} \tag{9}$$

3. Setting equation 8 to equal equation 9, we can solve for the CE consumption for the good type, s:

$$\sqrt{s} = 29.75 \Rightarrow s = 885.$$

3.2 Pooling Equilibrium

At the pooling equilibrium (if it exists), both the good types and the bad types will choose the same contract. Keeping this in mind, we follow the same procedures that we used to find the separating equilibrium above.

Two points worth re-emphasize are the following:

- No contract offered by a firm makes negative profit.
- No contract not offered would make positive profit.

I've written this twice because this tells us that any contract that is offered by the insurer must lie on a zero profit/actuarily fair line.

3.2.1 Actuarily Fair Line and the Pooling Equilibrium

We need to first consider what the probability of state 2 occurring given that both good and bad types will buy the same contract. Denote the pooled probability of state 2:

$$\bar{p} = \lambda p_G + (1 - \lambda) p_B = (0.2 \times 0.1) + (0.8 \times 0.6) = 0.5$$

Using equation 1 (or equation 2), we can write the actuarily fair line for pooled contracts ($\overline{AP}$) as follows:

$$\begin{aligned} (W - \bar{p}L) - (\bar{p} \cdot s_2 + (1 - \bar{p}) \cdot s_1) &= 0 \\ \Rightarrow (1000 - (0.5 \cdot 900)) - (0.5 \cdot s_2 + (1 - 0.5) \cdot s_1) &= 0 \\ \Rightarrow s_2 = 1100 - s_1 & \end{aligned} \tag{10}$$

Note that the slope of the actuarily fair line for the pooled type is equal to $-\frac{1-\bar{p}}{\bar{p}} = -\frac{0.5}{0.5}$

Now using equation 11, we can solve for the full insurance, zero-profit point for the pooled contract, or point P in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 1100 - s \Rightarrow s = s_1 = s_2 = 750$$

Again, we can use $P = \vec{s}_p = [750,\, 750]$ to solve for the premium and the redistribution (since we know W and L). This in turn fully identifies the pooling insurance contract $\vec{\kappa}_p = [M_p,\, R_p]$.

3.2.2 Non-Existence of Pooling Equilibrium

I use a graphical argument to show that $P = \vec{s}_p = [750,\, 750]$ is not a pooling equilibrium despite satisfying the conditions above. Consider the point ε, which we can further write as $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ in Figure III. If an insurer were to offer this consumption bundle, the good type consumer would prefer $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ to $\vec{s}_p = [750,\, 750]$ because of the single crossing principle. Furthermore, $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ is below the pooled actuarily fair/zero profit line $\overline{AP}$, which means that the insurer can make positive profits if they were to offer this contract. Because of what I (re)emphasized at the beginning of this section on pooling equilibrium, the immediate result is that there does not exist a pooling equilibrium. This is Main Result 2.

Also note that because $\vec{s}_p = [750,\, 750]$ lies below the indifference curve $\bar{U}_G$, which passes through the separating equilibrium α_G, the good types do not prefer the pooling consumption bundle to the separating equilibrium.

3.3 Separating and Pooling Equilibrium (with high λ)

3.3.1 Actuarily Fair Line and the Pooling Equilibrium with high λ

Assuming that there are a lot of good types and $\lambda = 0.95$, we can solve for the pooled probability of state 2 occurring:

$$\bar{p}' = \lambda p_G + (1-\lambda)p_B = (0.95 \times 0.1) + (0.05 \times 0.6) = \frac{1}{8}$$

Using equation 1 (or equation 2), we can write the actuarily fair line for pooled contracts ($\overline{AP'}$) as follows:

$$\begin{aligned}
(W - \bar{p}'L) - (\bar{p}'s_2 + (1-\bar{p}') \cdot s_1) &= 0 \\
\Rightarrow \left(1000 - \left(\frac{1}{8} \cdot 900\right)\right) - \left(\frac{1}{8} \cdot s_2 + (1-0.5) \cdot s_1\right) &= 0 \\
\Rightarrow s_2 = 7100 - 7s_1 & \qquad (11)
\end{aligned}$$

Note that the slope of the actuarily fair line for the pooled type is equal to $-\frac{1-\bar{p}'}{\bar{p}'} = -\frac{\frac{7}{8}}{\frac{1}{8}}$

Now using equation 11, we can solve for the full insurance, zero-profit point for the pooled contract, or point P' in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 7100 - 8s \Rightarrow s = s_1 = s_2 = 887.5$$

Again, we can use $P' = \vec{s}_{p'} = [887.5,\, 887.5]$ to solve for the premium and the redistribution (since we know W and L). This in turn fully identifies the pooling insurance contract $\vec{\kappa}_{p'} = [M_p,\, R_p]$, where $\lambda = 0.95$.

3.3.2 Non-Existence of Equilibrium

Since we know by subsection 3.2.2 that pooling equilibria do not exist in this model, we might be worried about the existence of the separating equilibrium. The consumption bundle $P' = \vec{s}_{p'} = [887.5, 887.5]$ is preferred by the good type to α_G, so if we had contracts α_G and α_B, a firm could make positive profits by offering a contract like γ. The result is that there is no equilibrium, which is equivalent to Main Result 3.

4 Monopolistic Insurance Model Set-Up

4.1 Demand Side

We have a population of *risk averse* expected utility maximizing consumers. These consumers move second and they accept or reject the firm's offer. A representative consumer has wealth equal to W; probability of losing L equal to p_i, where $i \in \{G, B\}$, and such that $p_G < p_B$. "G" is for a good type of consumer with low risk of loss and "B" is a bad type of consumer with a high risk of loss. Assume that the probability of the consumer being a good type is equal to λ, which means that the average risk (or pooled level of risk) is equal to $\bar{p} = \lambda p_G + (1-\lambda)p_B$. Further assume that the expected utility of a consumer of type i is equal to:

$$U_i = p_i \cdot u(consumption\, in\, s_2) + (1 - p_i) \cdot u(consumption\, in\, s_1),$$

where s_1 and s_2 denotes the level of consumption in State 1 where there is no loss; and level of consumption in State 2 where there is a loss, respectively. Also, $u'(\cdot) > 0$ and $u''(\cdot) < 0$ for simplicity.

What have we just assumed here?

- The level of risk aversion is equal for both good and bad types of consumers.[7]
- We have the same utility function in both the loss state (State 2) and the no loss state (State 1).

4.2 Supply Side

Risk-neutral profit-maximizing monopolistic firm which moves first and offers potentially multiple contracts which can be completely characterized by the following vector: $\vec{\alpha} = [M, R]$,[8] where "M" denotes the premium, and "R" denotes the reimbursement amount.

Taking the demand and supply side together we have the following trivial results:

[7] Recall our measures of risk aversion from Econ 2020a, and convince yourself.

[8] Rothschild and Stiglitz (1976) use the notation $\vec{\alpha} = (\alpha_1, \alpha_2)$ and so I'm trying to stick as close to their notation and the notation from class.

- A consumer of type $i \in \{G, B\}$ who has purchased insurance will consume $s_2 = W - M - L + R$ in State 2 and with probability $1 - p_i$ she will consume $s_1 = W - M$ in State 1. Without insurance, consumption is equal to $s_{2,no\,insurance} = W - L$ and $s_{1,no\,insurance} = W$, respectively. Note that consumption in each state is completely defined by "M" and "R".

- With probability p_i a firm who has sold insurance will have payoff equal to $M - R$ in State 2 and with probability $1 - p_i$ it will have payoff equal to M in State 1.

4.3 Graphical Representation and Important Assumptions

See Appendix A-1 for a graphical representation. Carefully note the axes of the graph.

Also note that because of the assumptions that we placed on expected utility functions of the two types of consumers, the expected utility functions satisfy the single crossing principle: $\forall \vec{s_i}$

$$\frac{\partial s_{2i}}{\partial s_{1i}} = -\frac{(1 - p_i)u'(s_{1i})}{p_i u'(s_{2i})},$$

which is how we calculated the Marginal Rate of Substitution in Econ 2020a.[9] Therefore, we see that at every point $\vec{s_i}$ in the graph shown in Appendix A-1, the slope of the level set through such point is greater for the good type than the bad type.

Finally, I've drawn in the "suns" for the graph in A-1. Can you figure out why they were placed in their respective positions?

1. For the firm,

$$\begin{aligned} \mathbb{E}[\pi] &= p_i(M - R) + (1 - p_i)(M) \\ &= p_i(W - L - s_2) + (1 - p_i)(W - s_1) \\ &= (W - p_i L) - (p_i s_2 + (1 - p_i)s_1) \end{aligned}$$

   where the first term is equal to the uninsured expected consumption and the second term is equal to the insured expected consumption. We can reorganize this as:

$$s_2 = -\frac{(1 - p_i)}{p_i} s_1 + \frac{1}{p_i} \cdot \{(W - p_i L) - \mathbb{E}[\pi]\} \tag{12}$$

   which shows us that the iso profit lines are linear in the $[s_1, s_2]$ Euclidean space and profit is increasing towards the lower left in Appendix A-1.

[9]If you can't remember, go back to my section notes from Econ 2020a on the Implicit Function Theorem.

2. Can you figure out why the consumer wants to have equal consumption in both states of the world? What assumption is key here? Even if you can't show it mathematically, this should be intuitively clear to you. (HINT: Set up the full information expected profit maximization problem subject to the consumer's IR; write out the Lagrangian; and solve assuming interior solutions; or see Section 5 below)

4.4 Equilibrium Requirements

Note that the equilibrium concept we want to apply is SPNE (or trivially PBE). If we backward induct, we can figure out how consumers maximize their expected utility by choosing the intended contract. Recall the two conditions from lecture:

4.4.1 Participation Constraint/Individual Rationality Constraint

This means that the consumer will receive more than his or her reservation value (payoff if she does nothing, or in this case, not buy insurance).

- $(IR_G) : U_G(\vec{s}_G) \geq U_G(no\,insurance) \Rightarrow p_G \cdot u(s_{2G}) + (1 - p_G) \cdot u(s_{1G}) \geq p_G \cdot u(W - L) + (1 - p_G) \cdot u(W)$
- $(IR_B) : U_B(\vec{s}_B) \geq U_B(no\,insurance) \Rightarrow p_B \cdot u(s_{2B}) + (1 - p_B) \cdot u(s_{1B}) \geq p_B \cdot u(W - L) + (1 - p_B) \cdot u(W)$

4.4.2 Incentive Compatibility Constraints

This means that each type of consumer will prefer the contract intended for her type than any other contract intended for another type of consumer.

- $(IC_G) : U_G(\vec{s}_G) \geq U_G(\vec{s}_B) \Rightarrow p_G \cdot u(s_{2G}) + (1 - p_G) \cdot u(s_{1G}) \geq p_G \cdot u(s_{2B}) + (1 - p_G) \cdot u(s_{1B})$
- $(IC_B) : U_B(\vec{s}_B) \geq U_B(\vec{s}_G) \Rightarrow p_B \cdot u(s_{2B}) + (1 - p_B) \cdot u(s_{1B}) \geq p_B \cdot u(s_{2G}) + (1 - p_B) \cdot u(s_{1G})$

Give these four constraints, the insurer will choose its optimal policy. Try to understand why the IC and IR constraints bind with equality in some instances and remain slack in other instances.

5 Full Information (First-Degree Price Discrimination)

Since the firm is able to observe whether the consumer is a good type or a bad type, the insurer is able to provide insurance such that it will maximize its profit conditioning on the probabilities of loss. The result is that the entire surplus generated by this trade will go to the insurer. Note that the IC constraints are not necessary when we have full information. The insurer solves the following problem for each i:

$$\max_{s_{1i}, s_{2i}} \{W - p_i L - [p_i s_{2i} + (1-p_i) s_{1i}]\}$$

subject to the Individual Rationality constraint of each i:

$$(IR_i):\ p_i \cdot u(s_{2i}) + (1-p_i) \cdot u(s_{1i}) \geq p_i \cdot u(W-L) + (1-p_i) \cdot u(W)$$

which will bind with equality or else the insurer can always reduce its payout (R) in State 2 (which makes the insurer better off) without violating the IR constraint.

Where have we seen a similar optimization problem? Think back to the UMPs, CMPs, and PMPs from Econ 2020a. The solution to the optimization problem should be fairly easy once you realize the following:

1. Do you see that you have a linear objective function in the choice variables? If you don't see this, check out equation 12 above.

2. The IR constraint can be written as:

   $$p_i \cdot u(s_{2i}) + (1-p_i) \cdot u(s_{1i}) \geq \underline{\mathrm{u}}$$

   where $\underline{\mathrm{u}}$ is a constant since W, L, and p_i are exogenous parameters to the model.

3. In short, we have an optimization problem where the optimum will be at the point where the objective function will be tangent to the convex constraint set.[10] There are several ways to solve this problem, but the easiest seems to be that at the optimum $[s^*_{1i}, s^*_{2i}]$ the following must hold:

$$\begin{aligned} -\frac{(1-p_i)}{p_i} &= \frac{\partial s_{2i}}{\partial s_{1i}} \Rightarrow -\frac{(1-p_i)}{p_i} = -\frac{(1-p_i)u'(s_{1i})}{p_i u'(s_{2i})} \Rightarrow u'(s_{1i}) = u'(s_{2i}) \\ \therefore &\quad s_{1i} = s_{2i} \end{aligned} \tag{13}$$

The important thing to understand here is that we are at a Pareto Optimum (or the "First Best") when we have full information and First Degree Price Discrimination. Further, because the IR constraint binds at the optimum contract, the insurer gets all of the surplus and the consumer gets her reservation utility.

[10] Convexity of the constraint set is implied by the assumptions we placed on the Bernoulli utility functions.

6 Asymmetric Information (Second-Degree Price Discrimination)

6.1 Optimization Problem of the Insurer

If the insurer is unable to distinguish between the different types of consumers, then its problem can be written as follows:

$$\max_{s_{1G},s_{2G},s_{1B},s_{2B}} \lambda \left\{W - p_G L - [p_G s_{2G} + (1-p_G)s_{1G}]\right\} + (1-\lambda)\left\{W - p_B L - [p_B s_{2B} + (1-p_B)s_{1B}]\right\}$$

subject to the IR constraints for both consumer types and the IC constraints for both consumer types:

- $(IR_G) : p_G \cdot u(s_{2G}) + (1-p_G)\cdot u(s_{1G}) \geq p_G \cdot u(W-L) + (1-p_G)\cdot u(W) = \bar{u}_0^G$
- $(IR_B) : p_B \cdot u(s_{2B}) + (1-p_B)\cdot u(s_{1B}) \geq p_B \cdot u(W-L) + (1-p_B)\cdot u(W) = \bar{u}_0^B$
- $(IC_G) : p_G \cdot u(s_{2G}) + (1-p_G)\cdot u(s_{1G}) \geq p_G \cdot u(s_{2B}) + (1-p_G)\cdot u(s_{1B})$
- $(IC_B) : p_B \cdot u(s_{2B}) + (1-p_B)\cdot u(s_{1B}) \geq p_B \cdot u(s_{2G}) + (1-p_B)\cdot u(s_{1G})$

6.2 Results: Binding Constraints

Now let's look at some qualitative results of the optimization problem facing the insurer. Which of the constraints above bind with equality?

- Consider the following:

$$\begin{aligned} p_B \cdot u(s_{2B}) + (1-p_B)\cdot u(s_{1B}) &\geq p_B \cdot u(s_{2G}) + (1-p_B)\cdot u(s_{1G}) \\ &> p_G \cdot u(s_{2G}) + (1-p_G)\cdot u(s_{1G}) \\ &\geq p_G \cdot u(W-L) + (1-p_G)\cdot u(W) \\ &> p_B \cdot u(W-L) + (1-p_B)\cdot u(W) \end{aligned}$$

  The first weak inequality comes from IC_B; the second strict inequality comes from the single crossing property;[11] the third weak inequality comes from the IR_G; and the fourth inequality is because $p_B > p_G$. Therefore, we have that:

$$p_B \cdot u(s_{2B}) + (1-p_B)\cdot u(s_{1B}) > p_B \cdot u(W-L) + (1-p_B)\cdot u(W)$$

  which means that the IR_B is slack and does not bind. This is interpreted as *the high risk consumers (or bad types) receive a rent.*[12]

[11] Understanding this can be a little bit tricky, but try totally differentiating the expected utility functions $U_G(no\,insurance)$ and $U_B(no\,insurance)$. You'll notice that $dU_B > dU_G$.

[12] See Appendix A-2.

- Let's look at the IR_G. If the Individual Rationality constraint for the good types (or low risk consumers) is slack, this means that both Individual Rationality constraints (for the Good and the Bad type) are slack. The insurer can reduce payments to both good and bad types in the state s_2 without violating either of their Individual Rationality constraints—*i.e.* there is a profitable deviation for the insurer. Therefore, it must be the case that IR_G binds with equality:

$$p_G \cdot u(s_{2G}) + (1 - p_G) \cdot u(s_{1G}) = p_G \cdot u(W - L) + (1 - p_G) \cdot u(W)$$

which is interpreted as the *low risk, good types of consumers do not receive a rent above their Individual Rationality constraint.*[13]

- It must also be the case that IC_B binds with equality because if it was slack, then the insurer could always increase its profit by increasing the premium to bad type consumers. Furthermore, IC_G must be slack. If the IC_G was binding with equality, then the insurer could again increase its profits by increasing the premium charged to the bad types. The interpretation of this is that *the high risk, bad type consumers are indifferent between the contracts offered by the insurer.*[14]

6.3 Results: Pooling and Efficiency

In the model above, pooling is never optimal because the separate contracts always result in higher profits for the insurer. This should be intuitive. If not, see Appendix A-5, and keep in mind that the insurer's profits are higher the closer its iso profit line is to the origin.

Also, note that the bad types will get full insurance because:

1. The iso profit line is tangent to the level curve of the bad type consumer's expected utility function. The principle is the same as the one we used to solve the insurer's optimization problem in 13 above.[15]

2. Furthermore, because $\vec{s}_B$ which maximizes the insurer's profit lies below the indifference curve through the no insurance allocation (A) for the good type consumer, we don't have to worry about this contract attracting the low risk, good type consumers.

Therefore, we have the following two results:

1. *High risk, bad type consumers get the efficient quantity of insurance*—i.e. *their contract puts them on the 45 degree line.*

2. *Low risk, good type consumers get less than the efficient quantity of insurance because of the single crossing property and because IC_B binds with equality.*

[13]See Appendix A-3.
[14]See Appendix A-4.
[15]See Appendix A-6 for a graphical representation.

When will firm only sell to one type?

1. When there are lots of bad types and few good types—*i.e.* when $\lambda \to 0$.

2. Each price increase for the bad types results in less profits from the good types because you have to decrease the price to good types to make sure that the IC_B continues to bind.

3. By offering separate contracts, the insurer can create more profit by offering the bad types a contract with a higher premium (which reduces s_{1B} and s_{2B}, hence the arrow denoted ρ in Appendix A-5). However, because IC_B has to bind with equality, this means that the contract for the low risk types (good types) will decrease payoff in s_{2G} but must increase s_{1G} (which is denoted by the arrow ρ' in A-5), which means that the contract offered to the good types decreases premiums, but also decreases payoffs in the bad state.

4. If there are very few good types, the insurer may want to simply maximize profits from the bad types and offer no contract to the good types. The result would be that IR_B binds with equality and a negative quantity to the good types such that $s_1 > W$.

7 Summary of Qualitative Results

1. High risk, bad type consumers get the efficient quantity of insurance—i.e. their contract puts them on the 45 degree line.

2. Low risk, good type consumers get less than the efficient quantity of insurance because of the single crossing property and because IC_B binds with equality.

3. The high risk consumers (or bad types) receive a rent.

4. Low risk, good types of consumers do not receive a rent above their Individual Rationality constraint.

5. the high risk, bad type consumers are indifferent between the contracts offered by the insurer.

Section Notes 10

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Agenda

1. Competitive Screening (Insurance Model Set Up)
2. Main Results
3. Example

1 Competitive Insurance Model Set-Up

Please read Rothschild & Stiglitz (1976)[1]

1.1 Demand Side

We have a population of *risk averse* expected utility maximizing consumers. These consumers move second and they accept or reject a firm's offer. A representative consumer has wealth equal to W; probability of losing L equal to p_i, where $i \in \{G, B\}$, and such that $p_G < p_B$. "G" is for a good type of consumer with low risk of loss and "B" is a bad type of consumer with a high risk of loss. Assume that the probability of the consumer being a good type is equal to λ, which means that the average risk (or pooled level of risk) is equal to $\bar{p} = \lambda p_G + (1-\lambda)p_B$. Further assume that the expected utility of a consumer of type i is equal to:

$$U_i = p_i \cdot u(consumption\, in\, s_2) + (1-p_i) \cdot u(consumption\, in\, s_1),$$

where s_1 and s_2 denotes the level of consumption in State 1 where there is no loss; and level of consumption in State 2 where there is a loss, respectively. Also, $u'(\cdot) > 0$ and $u''(\cdot) < 0$ for simplicity.

What have we just assumed here?

[1] Rothschild, Michael and Joseph Stiglitz. 1976. *Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information.* The Quarterly Journal of Economics. 90(4): 629-49.

- The level of risk aversion is equal for both good and bad types of consumers.[2]
- We have the same utility function in both the loss state (State 2) and the no loss state (State 1).

1.2 Supply Side

Multiple *risk neutral* profit-maximizing monopolistic firm which move first and offer potentially multiple contracts which can be completely characterized by the following vector: $\vec{\kappa} = [M, R]$,[3] where "M" denotes the premium, and "R" denotes the reimbursement amount.

Taking the demand and supply side together we have the following trivial results:

- A consumer of type $i \in \{G, B\}$ who has purchased insurance will consume, with probability p_i, $s_2 = W - M - L + R$ in State 2 and with probability $1 - p_i$ she will consume $s_1 = W - M$ in State 1. Without insurance, consumption is equal to $s_{2,no\,insurance} = W - L$ and $s_{1,no\,insurance} = W$, respectively. Note that consumption in each state is completely defined by "M" and "R".
- With probability p_i a firm who has sold insurance will have payoff equal to $M - R$ in State 2 and with probability $1 - p_i$ it will have payoff equal to M in State 1.

The firms' expected profit can be written as follows:

$$\begin{aligned} \mathbb{E}[\pi] &= p_i(M - R) + (1 - p_i)(M) \\ &= p_i(W - L - s_2) + (1 - p_i)(W - s_1) \\ &= (W - p_i L) - (p_i s_2 + (1 - p_i)s_1) \end{aligned} \tag{1}$$

where the first term is equal to the uninsured expected consumption and the second term is equal to the insured expected consumption. This can be rewritten in a more convenient form as follows:

$$s_2 = -\frac{(1 - p_i)}{p_i} s_1 + \frac{1}{p_i} \cdot \{(W - p_i L) - \mathbb{E}[\pi]\} \tag{2}$$

Do you see why the "sun" is in the lower left hand corner when we draw the $[s_1, s_2]$ Euclidean plane?

[2]Recall our measures of risk aversion from Econ 2020a, and convince yourself.
[3]Rothschild and Stiglitz (1976) use the notation $\vec{\alpha} = (\alpha_1, \alpha_2)$.

1.3 Equilibrium Requirements

1.3.1 Consumer's Problem

Given consumption in the two states, consumers maximize among available contracts:

$$\max_{[M,R]\in\Gamma_{insurer}} U_i = p_i \cdot u(W - M - L + R) + (1 - p_i) \cdot u(W - M)$$

where $\Gamma_{insurer}$ denotes the contracts that the insurer offers.

1.3.2 Insurer's Problem

Given that consumer maximize utility, the insurer provides contracts which satisfy the following conditions:

- No contract offered by a firm makes negative profit.
- No contract not offered would make positive profit.

In short, the firm makes zero profits in equilibrium. Recall from Julian's lecture why this was important.

2 Main Results

1. In a separating equilibrium, the bad types (high risk consumers) are fully insured and the good types (low risk consumers) are not fully insured.
2. No pooling equilibrium exists.
3. There may be no equilibrium in pure strategies.

3 Example[4]

The parameter values are as follows:
$w = 1000 \quad L = 900 \quad p_G = 0.1 \quad p_B = 0.6 \quad \lambda = 0.2 \quad u(s) = \sqrt{s}$

- What does the 45 degree line represent? It represents equal consumption in both states of the world: $s_1 = s_2$. If the consumer is able to purchase an insurance contract which puts her on the 45 degree line, then she has full insurance.
- What if the consumer is uninsured? Then we're at point $A = [W, W - L] = [1000, 100]$

[4] The goal is to derive all values of Rothschild and Stiglitz (1976), Figure III, once we know the parameter values.

- The zero profit lines, $\overline{AB}$ and $\overline{AG}$, denote actuarily fair contracts.[5] In other words, all points on the $\overline{AB}$ and $\overline{AG}$ describe consumption in state 1 and state 2 such that insurance is priced to be actuarily fair. To see this, substitute $s_1 = W - M$ and $s_2 = W - M - L + R$ into equation 2 after setting $\mathbb{E}[\pi]$ =0.

3.1 Separating Equilibrium

3.1.1 Actuarily Fair Line for Bad Types and Full Insurance

Using equation 1 (or equation 2), we can write the actuarily fair line for bad types ($\overline{AB}$) as follows:

$$\begin{aligned}(W - p_B L) - (p_B s_2 + (1 - p_B) \cdot s_1) &= 0 \\ \Rightarrow (1000 - (0.6 \cdot 900)) - (0.6 s_2 + (1 - 0.6) \cdot s_1) &= 0 \\ \Rightarrow s_2 = 766.7 - \frac{2}{3} s_1 & \qquad (3)\end{aligned}$$

Note that the slope of the actuarily fair line for the bad type is equal to $-\frac{1-p_B}{p_B} = -\frac{0.4}{0.6}$

Now using equation 4, we can solve for the full insurance, zero-profit point for the bad types, or point α_B in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 766.7 - \frac{2}{3} s \Rightarrow s = s_1 = s_2 = 460$$

3.1.2 Actuarily Fair Line for Good Types and Full Insurance

Using equation 1 (or equation 2), we can write the actuarily fair line for good types ($\overline{AG}$) as follows:

$$\begin{aligned}(W - p_G L) - (p_G \cdot s_2 + (1 - p_G) \cdot s_1) &= 0 \\ \Rightarrow (1000 - (0.1 \cdot 900)) - (0.1 \cdot s_2 + (1 - 0.1) \cdot s_1) &= 0 \\ \Rightarrow s_2 = 9100 - 9 \cdot s_1 & \qquad (4)\end{aligned}$$

Note that the slope of the actuarily fair line for the good type is equal to $-\frac{1-p_G}{p_G} = -\frac{0.9}{0.1}$

Now using equation 4, we can solve for the full insurance, zero-profit point for the good types, or point β in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 9100 - 9s \Rightarrow s = s_1 = s_2 = 910$$

[5]Recall that if an insurance contract is actuarily fair, the premium should equal to the expected payout, or $M = p_i R$.

3.1.3 Indifference Curve $\bar{U}_B$ and Solving for α_G

How do we pin down the consumption in each state for α_G? We use the fact that the indifference curve through the point α_B also passes through α_G and the fact that α_G lies on the good types actuarily fair line.

1. Solve for the level of $\bar{U}_B$:

$$\bar{U}_B = (1-p_B)\cdot u(460)+p_B\cdot u(460) = (1-p_B)\sqrt{460}+p_B\sqrt{460} = \sqrt{460} \approx 21.4$$

2. The bad type is indifferent between α_B and α_G. Therefore, if we let $\alpha_G = [s_1, s_2]$, we have:

$$U_B\left(\alpha_G = [s_1, s_2]\right) = p_B\sqrt{s_2} + (1-p_B)\sqrt{s_1} = 21.4. \tag{5}$$

3. Further, the point α_G lies on the good types actuarily fair line which we solved for in equation 4:

$$s_2 = 9100 - 9 \cdot s_1 \tag{6}$$

4. Solve the system of equations involving equations 5 and 6, which results (after some very laborious algebra) in:

$$\alpha_G = [s_1, s_2] = [987,\ 217] \tag{7}$$

3.1.4 Separating Equilibrium?

The separating equilibrium results in the bad type consuming 460 in each period and the good type consuming 987 in state 1 and 217 in state 2. We can write this as:460

$$\alpha_B = [460,\ 460]\,;\ \alpha_G = [987,\ 217]\,.$$

Further, since we know the values of W and L, we can solve for $\vec{\kappa}_B = [M_B, R_B]$ and $\vec{\kappa}_G = [M_G, R_G]$. I won't bother going through the algebra here or below.

Is this result consistent with Main Result 1. above?

3.1.5 Certainty Equivalent for Good Types

To put the good type's consumption α_G into perspective, we can also ask: Give the probability of loss for the good type and her consumption α_G in the separating equilibrium, what is the good type's certainty equivalent?[6]

We can answer this question using the same methodology we used to solve for the separating equilibrium.

1. We can solve for $\bar{U}_G$ by solving the following:

$$\bar{U}_G = p_G\cdot u(217)+(1-p_G)\cdot u(987) = 0.1\sqrt{217}+(1-0.1)\sqrt{987} = 29.75 \tag{8}$$

[6] We defined "CE" in Econ 2020a.

2. At the CE, the good type's utility, which I denote as $\bar{U}_G^{CE}$, is as follows:

$$\bar{U}_G^{CE} = p_G \cdot u(s) + (1 - p_G) \cdot u(s) = \sqrt{s} \tag{9}$$

3. Setting equation 8 to equal equation 9, we can solve for the CE consumption for the good type, s:

$$\sqrt{s} = 29.75 \Rightarrow s = 885.$$

3.2 Pooling Equilibrium

At the pooling equilibrium (if it exists), both the good types and the bad types will choose the same contract. Keeping this in mind, we follow the same procedures that we used to find the separating equilibrium above.

Two points worth re-emphasize are the following:

- No contract offered by a firm makes negative profit.
- No contract not offered would make positive profit.

I've written this twice because this tells us that any contract that is offered by the insurer must lie on a zero profit/actuarily fair line.

3.2.1 Actuarily Fair Line and the Pooling Equilibrium

We need to first consider what the probability of state 2 occurring given that both good and bad types will buy the same contract. Denote the pooled probability of state 2:

$$\bar{p} = \lambda p_G + (1 - \lambda) p_B = (0.2 \times 0.1) + (0.8 \times 0.6) = 0.5$$

Using equation 1 (or equation 2), we can write the actuarily fair line for pooled contracts ($\overline{AP}$) as follows:

$$\begin{aligned} (W - \bar{p}L) - (\bar{p} \cdot s_2 + (1 - \bar{p}) \cdot s_1) &= 0 \\ \Rightarrow (1000 - (0.5 \cdot 900)) - (0.5 \cdot s_2 + (1 - 0.5) \cdot s_1) &= 0 \\ \Rightarrow s_2 = 1100 - s_1 & \end{aligned} \tag{10}$$

Note that the slope of the actuarily fair line for the bad type is equal to $-\frac{1-\bar{p}}{\bar{p}} = -\frac{0.5}{0.5}$

Now using equation 11, we can solve for the full insurance, zero-profit point for the pooled contract, or point P in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 1100 - s \Rightarrow s = s_1 = s_2 = 750$$

Again, we can use $P = \vec{s}_p = [750, 750]$ to solve for the premium and the redistribution (since we know W and L). This in turn fully identifies the pooling insurance contract $\vec{\kappa}_p = [M_p, R_p]$.

3.2.2 Non-Existence of Pooling Equilibrium

I use a graphical argument to show that $P = \vec{s}_p = [750,\ 750]$ is not a pooling equilibrium despite satisfying the conditions above. Consider the point ε, which we can further write as $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ in Figure III. If an insurer were to offer this consumption bundle, the good type consumer would prefer $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ to $\vec{s}_p = [750,\ 750]$ because of the single crossing principle. Furthermore, $\varepsilon = [s_{1\varepsilon}, s_{2\varepsilon}]$ is below the pooled actuarily fair/zero profit line $\overline{AP}$, which means that the insurer can make positive profits if they were to offer this contract. Because of what I (re)emphasized at the beginning of this section on pooling equilibrium, the immediate result is that there does not exist a pooling equilibrium. This is Main Result 2.

Also note that because $\vec{s}_p = [750,\ 750]$ lies below the indifference curve $\bar{U}_G$, which passes through the separating equilibrium α_G, the good types do not prefer the pooling consumption bundle to the separating equilibrium.

3.3 Pooling Equilibrium (with high λ)

3.3.1 Actuarily Fair Line and the Pooling Equilibrium with high λ

Assuming that there are a lot of good types and $\lambda = 0.95$, we can solve for the pooled probability of state 2 occurring:

$$\bar{p}' = \lambda p_G + (1-\lambda)p_B = (0.95 \times 0.1) + (0.05 \times 0.6) = \frac{1}{8}$$

Using equation 1 (or equation 2), we can write the actuarily fair line for pooled contracts ($\overline{AP'}$) as follows:

$$\begin{aligned}
(W - \bar{p}'L) - (\bar{p}'s_2 + (1-\bar{p}')\cdot s_1) &= 0 \\
\Rightarrow \left(1000 - \left(\frac{1}{8}\cdot 900\right)\right) - \left(\frac{1}{8}\cdot s_2 + (1-0.5)\cdot s_1\right) &= 0 \\
\Rightarrow s_2 = 7100 - 7s_1 &
\end{aligned} \tag{11}$$

Note that the slope of the actuarily fair line for the bad type is equal to $-\frac{1-\bar{p}'}{\bar{p}'} = -\frac{\frac{7}{8}}{\frac{1}{8}}$

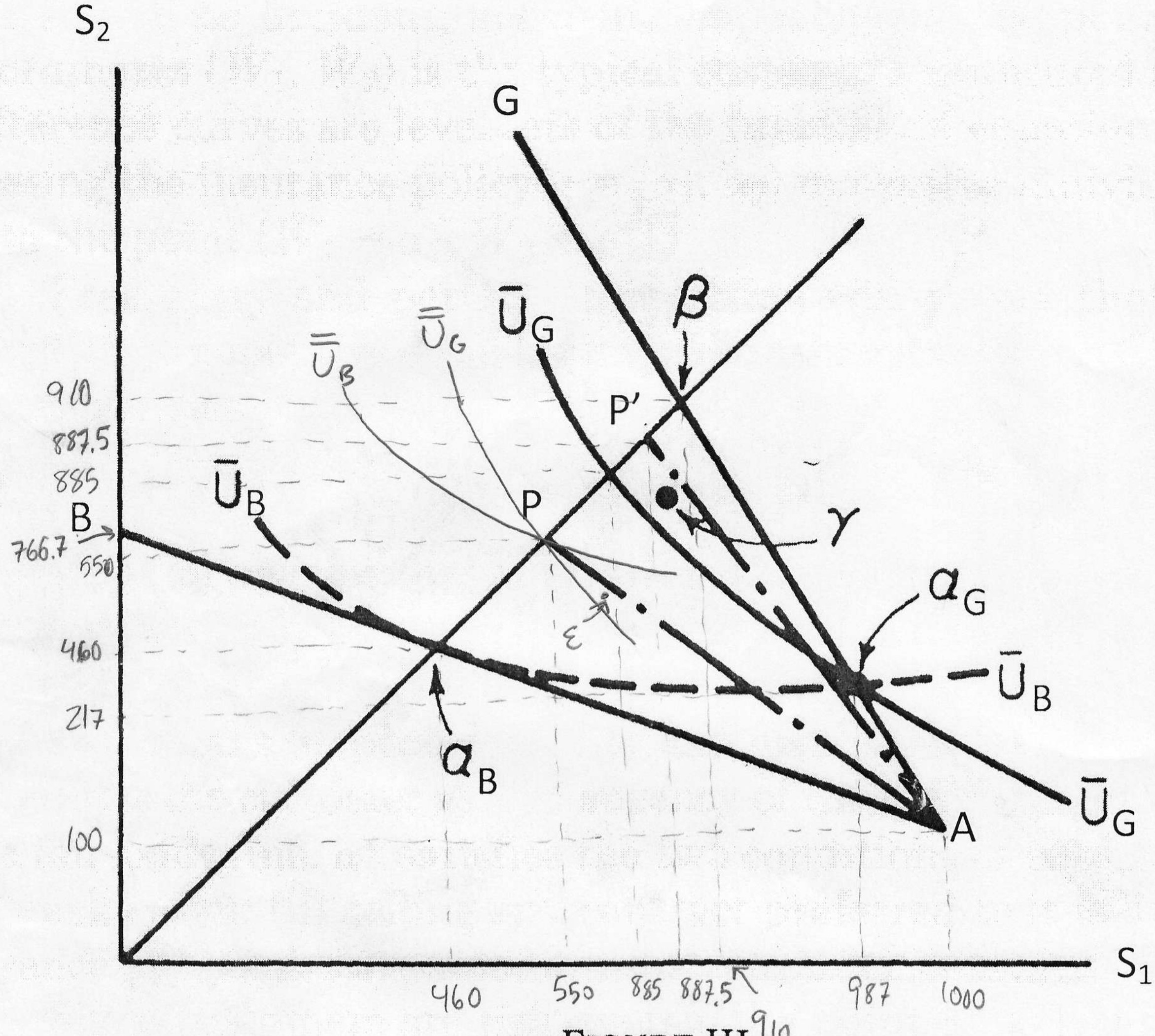
Now using equation 11, we can solve for the full insurance, zero-profit point for the pooled contract, or point P' in Figure III. We solve by setting $s_1 = s_2 = s$, and we find that:

$$s = 7100 - 8s \Rightarrow s = s_1 = s_2 = 887.5$$

Again, we can use $P' = \vec{s}_{p'} = [887.5,\ 887.5]$ to solve for the premium and the redistribution (since we know W and L). This in turn fully identifies the pooling insurance contract $\vec{\kappa}_{p'} = [M_p,\ R_p]$, where $\lambda = 0.95$.

3.3.2 Non-Existence of Equilibrium

The consumption bundle $P' = \vec{s}_{p'} = [887.5, 887.5]$ is preferred by the good type to α_G, so if we had contracts α_G and α_B, a firm could make positive profits by offering a contract like γ. The result is that there is no equilibrium, which is equivalent to Main Result 3.

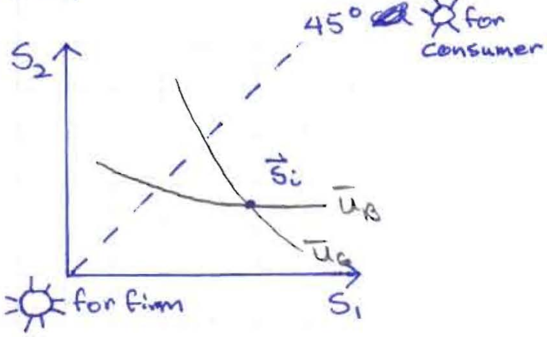


S_2
G
β
$\bar{U}_G$
$\bar{\bar{U}}_G$
$\bar{\bar{U}}_B$
910
887.5
885
P'
$\bar{U}_B$
P
γ
B
766.7
550
a_G
ε
460
$\bar{U}_B$
217
a_B
$\bar{U}_G$
100
A
S_1
460
550
885
887.5
987
1000
910

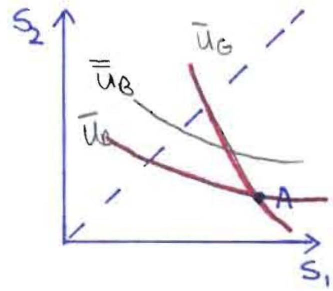
FIGURE III

Monopolistic Screening Appendix

A-1



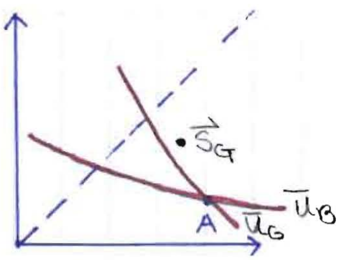
A-2



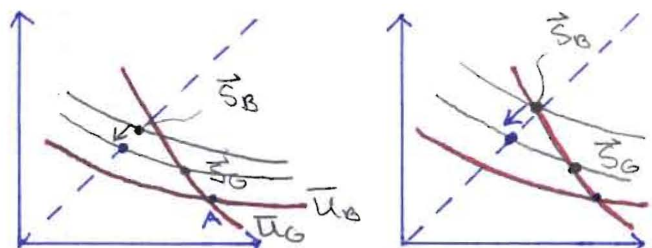
"A" is no insurance

$\bar{\bar{u}}_B$ is indiff. curve of $U_B(\vec{S}_B)$

A-3



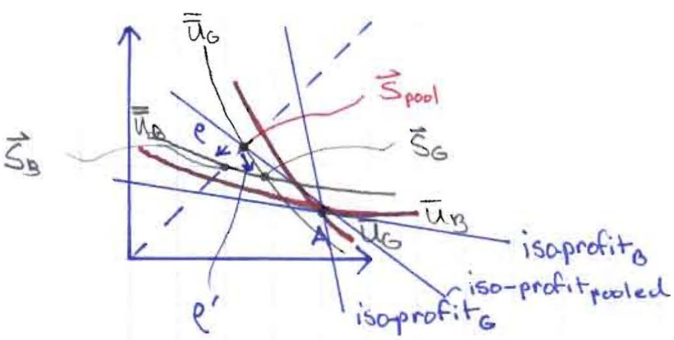
A-4



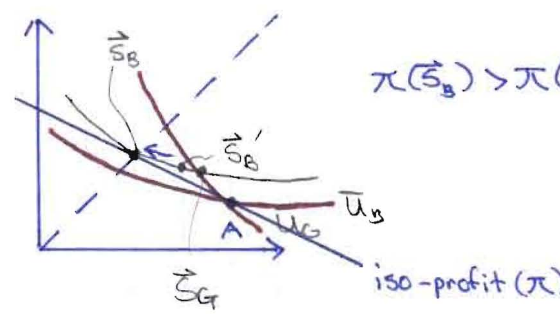
$(IC)_B$ binds

$(IC)_G$ slack.

A-5



A-6



$\pi(\vec{S}_B) > \pi(\vec{S}_B{}')$

$\bar{U}_G$ denotes indiff. curve of U_G(no insurance)
$\bar{U}_B$ " " " " U_B(no insurance) expected utility

For A-5, moving from $\vec{S}_{pool}$ to $\vec{S}_B$ and $\vec{S}_G$ improves profits for the insurer.