DUALITY

$$\frac{\partial h_i}{\partial p_j}(\vec{p},\bar{u}) = \frac{\partial x_i}{\partial p_j}(\vec{p},w) - \frac{\partial x_i}{\partial w}(\vec{p},w)\cdot x_j(\vec{p},w)$$

UMP $\longleftrightarrow$ EMP

$$x_1^*(\vec{p},w) \equiv \frac{w}{3p_1} \qquad \vec{h}(\vec{p},\bar{u}) \equiv \vec{x}(\vec{p},e(\vec{p},\bar{u})) \qquad h_1^*(\vec{p},\bar{u}) \equiv e^{\frac{\bar{u}}{3}}\cdot\left(\frac{p_2}{2p_1}\right)^{\frac{2}{3}}$$

$$x_2^*(\vec{p},w) \equiv \frac{2w}{3p_2} \qquad \vec{x}(\vec{p},w) \equiv \vec{h}(\vec{p},v(\vec{p},w)) \qquad h_2^*(\vec{p},\bar{u}) \equiv e^{\frac{\bar{u}}{3}}\cdot\left(\frac{p_2}{2p_1}\right)^{-\frac{1}{3}}$$

$$x_i^*(\vec{p},w) = -\frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial w}}$$

(Roy's)

$$v(\vec{p},w) = u(\vec{x}^*(\vec{p},w))$$

$$e(\vec{p},\bar{u}) = \vec{p}\circ\vec{h}^*(\vec{p},\bar{u})$$

$$(= p_1\cdot h_1^*(\vec{p},\bar{u}) + p_2\cdot h_2^*(\vec{p},\bar{u}))$$

$$\frac{\partial e(\vec{p},\bar{u})}{\partial p_i} = h_i^*(\vec{p},u)$$

$$V(\vec{p},w) \equiv \ln\left(\frac{w}{3p_1}\right) + 2\ln\left(\frac{2w}{3p_2}\right) \longleftrightarrow e(\vec{p},\bar{u}) \equiv e^{\frac{\bar{u}}{3}}\cdot\frac{3}{2^{\frac{2}{3}}}\cdot p_1^{\frac{1}{3}}p_2^{\frac{2}{3}}$$

$$w \equiv e(\vec{p},v(\vec{p},w))$$

AND

$$\bar{u} \equiv V(\vec{p},e(\vec{p},\bar{u}))$$

$$\therefore\ \bar{u} = \ln\left(\frac{e(\vec{p},\bar{u})}{3p_1}\right) + 2\ln\left(\frac{2e(\vec{p},\bar{u})}{3p_2}\right)$$

$$w = e^{\frac{v(\vec{p},w)}{3}}\cdot\frac{3}{2^{\frac{2}{3}}}\cdot p_1^{\frac{1}{3}}\cdot p_2^{\frac{2}{3}}$$