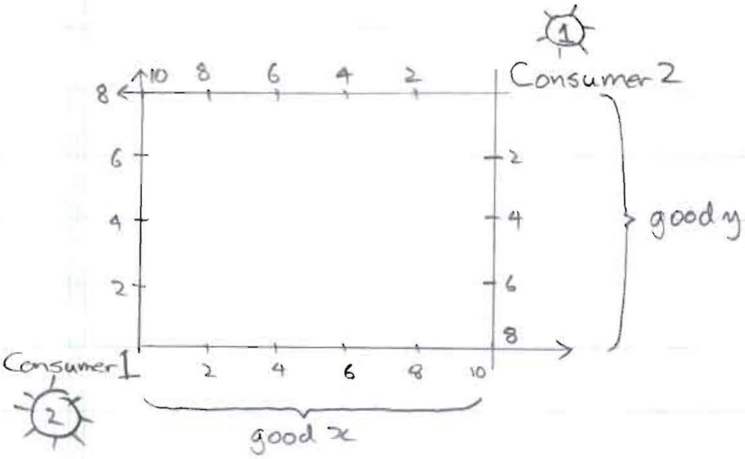
[redacted]

Edgeworth Box



A Pareto Optimal Allocation is an allocation $\langle x_1^*, x_2^*, y_1^*, y_2^*$ such that Consumer 1's utility can't be increased without decreasing Consumer 2's utility and vice versa.

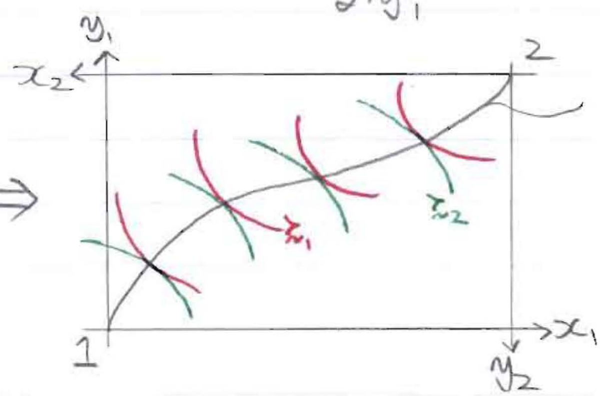
Therefore, an allocation $(\vec{x}^*, \vec{y}^*)$ is Pareto Optimal if $\nexists\ (\vec{x}', \vec{y}')$ s.t. $u_i(x_i', y_i') \geq u_i(x_i^*, y_i^*)\ \forall i$ AND $u_i(x_i', y_i') > u_i(x_i^*\ y_i^*)$ for some i.

Note that $\vec{x}^* = (x_1^*, \ldots, x_i^*, \cdots x_I^*)$, $\vec{y}^* = (y_1^*, \ldots, y_i^*, \cdots y_I^*)$.

The implication is that Pareto Optimal allocations $(\vec{x}^*, \vec{y}^*)$ will be at points in the Edgeworth Box where the indifference curves of Consumer 1 and Consumer 2 are tangent to each other:

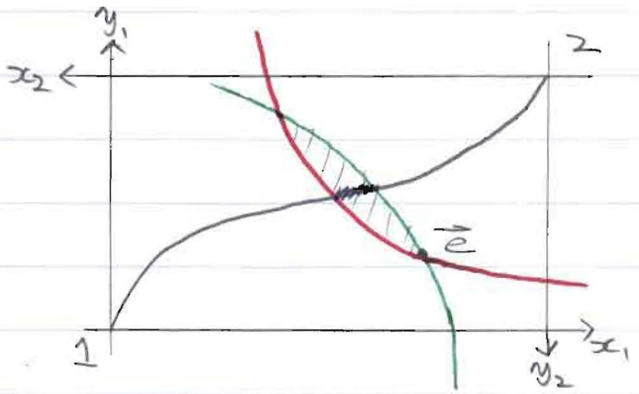
$$\frac{\dfrac{\partial u_1(x_1^*, y_1^*)}{\partial x_1}}{\dfrac{\partial u_1(x_1^*, y_1^*)}{\partial y_1}} = \frac{\dfrac{\partial u_2(x_2^*, y_2^*)}{\partial x_2}}{\dfrac{\partial u_2(x_2^*, y_2^*)}{\partial y_2}}. \iff MRS_{x_1 y_1} = MRS_{x_2 y_2}$$

$\Rightarrow$



Pareto Set $\equiv \{(\vec{x}, \vec{y}) \mid MRS_{x_1 y_1} = MRS_{x_2 y_2}\}$

Now we add in an initial endowment
$\vec{e} = (\bar{e}_x, \bar{e}_y) = (e_{x_1} e_{x_2} e_{y_1} e_{y_2})$



From this Box, do you see that the initial allocation $\vec{e}$ is NOT Pareto Optimal? All allocations in the shaded region are prefer by both Consumer 1 and 2.

Further, the set of allocations marked in blue is the contract curve: the set of allocations $(\vec{x}^{**}, \vec{y}^{**})$ s.t. they are Pareto Optimal AND preferred by Consumer 1 and 2 to the endowment:

$$C_{contract} \equiv \{(\vec{x}, \vec{y}) \mid MRS_{x_1 y_1} = MRS_{x_2 y_2} \text{ and } U_i(x_i y_i) \geq U_i(e_{x_i} e_{y_i}), \forall i\}$$

What about price ($\vec{p} = (p_x, p_y)$)? Prices play the role of clearing the markets. Note that every $\vec{p}$ does NOT clear markets.

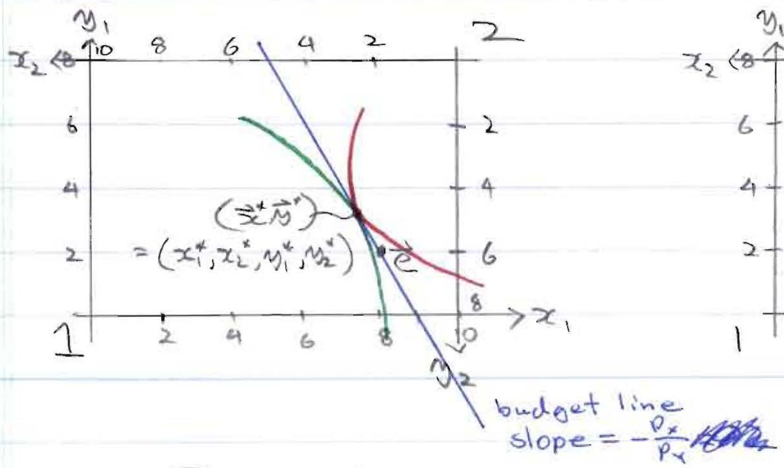


Figure 1

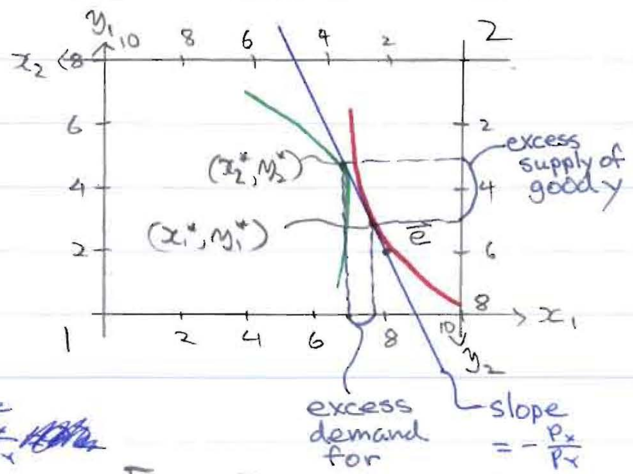
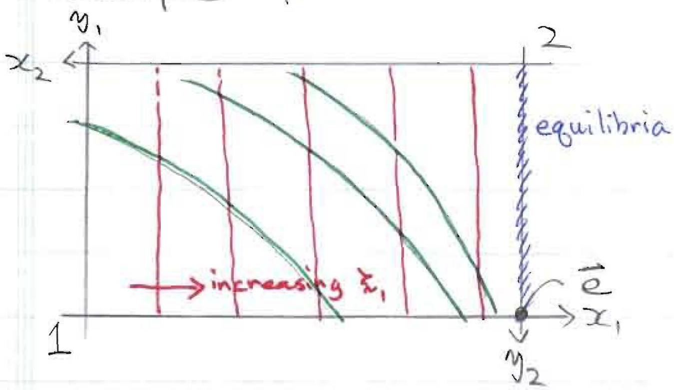


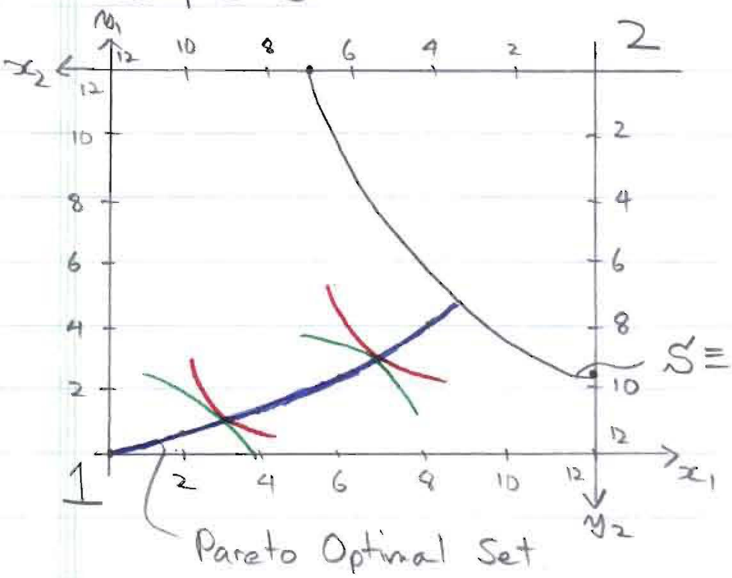
Figure 2

Example 4



Note that the preferences for Consumer 1 are not strongly monotonic.

Example 5



Recall that Pareto Optimal Set was defined by $y_1 = \frac{4x_1}{16-x_1}$

$S \equiv \{(x_1, y_1) \mid x_1^2 y_1 = 300\}$