Monopolistic Screening

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Agenda

1. Insurance Model Set-Up
2. Full Information (First-Degree Price Discrimination)
3. Asymmetric Information (Second-Degree Price Discrimination)
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What's the key difference between screening and signaling?

- in a screening problem, the uninformed party moves first and offers contracts
- in a signaling problem, the informed party moves first

1 Insurance Model Set-Up

1.1 Demand Side

We have a population of *risk averse* expected utility maximizing consumers. These consumers move second and they accept or reject the firm's offer. A representative consumer has wealth equal to W; probability of losing L equal to p_i, where $i \in \{G, B\}$, and such that $p_G < p_B$. "G" is for a good type of consumer with low risk of loss and "B" is a bad type of consumer with a high risk of loss. Assume that the probability of the consumer being a good type is equal to λ, which means that the average risk (or pooled level of risk) is equal to $\bar{p} = \lambda p_G + (1 - \lambda) p_B$. Further assume that the expected utility of a consumer of type i is equal to:

$$U_i = p_i \cdot u(consumption\, in\, s_2) + (1 - p_i) \cdot u(consumption\, in\, s_1),$$

where s_1 and s_2 denotes the level of consumption in State 1 where there is no loss; and level of consumption in State 2 where there is a loss, respectively. Also, $u'(\cdot) > 0$ and $u''(\cdot) < 0$ for simplicity.

What have we just assumed here?

- The level of risk aversion is equal for both good and bad types of consumers.[1]
- We have the same utility function in both the loss state (State 2) and the no loss state (State 1).

1.2 Supply Side

Risk-neutral profit-maximizing monopolistic firm which moves first and offers potentially multiple contracts which can be completely characterized by the following vector: $\vec{\alpha} = [M, R]$,[2] where "M" denotes the premium, and "R" denotes the reimbursement amount.

Taking the demand and supply side together we have the following trivial results:

- A consumer of type $i \in \{G, B\}$ who has purchased insurance will consume $s_2 = W - M - L + R$ in State 2 and with probability $1 - p_i$ she will consume $s_1 = W - M$ in State 1. Without insurance, consumption is equal to $s_{2,no\,insurance} = W - L$ and $s_{1,no\,insurance} = W$, respectively. Note that consumption in each state is completely defined by "M" and "R".
- With probability p_i a firm who has sold insurance will have payoff equal to $M - R$ in State 2 and with probability $1 - p_i$ it will have payoff equal to M in State 1.

1.3 Graphical Representation and Important Assumptions

See Appendix A-1 for a graphical representation. Carefully note the axes of the graph.

Also note that because of the assumptions that we placed on expected utility functions of the two types of consumers, the expected utility functions satisfy the single crossing principle: $\forall \vec{s_i}$

$$\frac{\partial s_{2i}}{\partial s_{1i}} = -\frac{(1 - p_i)u'(s_{1i})}{p_i u'(s_{2i})},$$

which is how we calculated the Marginal Rate of Substitution in Econ 2020a.[3] Therefore, we see that at every point $\vec{s_i}$ in the graph shown in Appendix A-1, the slope of the level set through such point is greater for the good type than the bad type.

Finally, I've drawn in the "suns" for the graph in A-1. Can you figure out why they were placed in their respective positions?

[1] Recall our measures of risk aversion from Econ 2020a, and convince yourself.

[2] Rothschild and Stiglitz (1976) use the notation $\vec{\alpha} = (\alpha_1, \alpha_2)$ and so I'm trying to stick as close to their notation and the notation from class.

[3] If you can't remember, go back to my section notes from Econ 2020a on the Implicit Function Theorem.

1. For the firm,

$$\begin{aligned}\mathbb{E}[\pi] &= p_i(M-R)+(1-p_i)(M)\\ &= p_i(W-L-s_2)+(1-p_i)(W-s_1)\\ &= (W-p_iL)-(p_is_2+(1-p_i)s_1)\end{aligned}$$

   where the first term is equal to the uninsured expected consumption and the second term is equal to the insured expected consumption. We can reorganize this as:

$$s_2 = -\frac{(1-p_i)}{p_i}s_1 + \frac{1}{p_i}\cdot\{(W-p_iL)-\mathbb{E}[\pi]\} \tag{1}$$

   which shows us that the iso profit lines are linear in the $[s_1, s_2]$ Euclidean space and profit is increasing towards the sun in Appendix A-1.

2. Can you figure out why the consumer wants to have equal consumption in both states of the world? What assumption is key here? Even if you can't show it mathematically, this should be intuitively clear to you. (HINT: Set up the full information expected profit maximization problem subject to the consumer's IR; write out the Lagrangian; and solve assuming interior solutions; or see Section 2 below)

1.4 Equilibrium Requirements

Note that the equilibrium concept we want to apply is SPNE (or trivially PBE). If we backward induct, we can figure out how consumers maximize their expected utility by choosing the intended contract. Recall the two conditions from lecture:

1.4.1 Participation Constraint/Individual Rationality Constraint

This means that the consumer will receive more than his or her reservation value (payoff if she does nothing, or in this case, not buy insurance).

- $(IR_G): U_G(\vec{s}_G) \geq U_G(no\,insurance) \Rightarrow p_G\cdot u(s_{2G})+(1-p_G)\cdot u(s_{1G}) \geq p_G\cdot u(W-L)+(1-p_G)\cdot u(W)$
- $(IR_B): U_B(\vec{s}_B) \geq U_B(no\,insurance) \Rightarrow p_B\cdot u(s_{2B})+(1-p_B)\cdot u(s_{1B}) \geq p_B\cdot u(W-L)+(1-p_B)\cdot u(W)$

1.4.2 Incentive Compatibility Constraints

This means that each type of consumer will prefer the contract intended for her type than any other contract intended for another type of consumer.

- $(IC_G): U_G(\vec{s}_G) \geq U_G(\vec{s}_B) \Rightarrow p_G\cdot u(s_{2G})+(1-p_G)\cdot u(s_{1G}) \geq p_G\cdot u(s_{2B})+(1-p_G)\cdot u(s_{1B})$

- $(IC_B) : U_B(\vec{s}_B) \geq U_B(\vec{s}_G) \Rightarrow p_B \cdot u(s_{2B}) + (1-p_B) \cdot u(s_{1B}) \geq p_B \cdot u(s_{2G}) + (1-p_B) \cdot u(s_{1G})$

Give these four constraints, the insurer will choose its optimal policy. Try to understand why the IC and IR constraints bind with equality in some instances and remain slack in other instances.

2 Full Information (First-Degree Price Discrimination)

Since the firm is able to observe whether the consumer is a good type or a bad type, the insurer is able to provide insurance such that it will maximize its profit conditioning on the probabilities of loss. The result is that the entire surplus generated by this trade will go to the insurer. Note that the IC constraints are not necessary when we have full information. The insurer solves the following problem for each i:

$$\max_{s_{1i}, s_{2i}} \{W - p_i L - [p_i s_{2i} + (1-p_i) s_{1i}]\}$$

subject to the Individual Rationality constraint of each i:

$$(IR_i) : \; p_i \cdot u(s_{2i}) + (1-p_i) \cdot u(s_{1i}) \geq p_i \cdot u(W-L) + (1-p_i) \cdot u(W)$$

which will bind with equality or else the insurer can always reduce its payout (R) in State 2 (which makes the insurer better off) without violating the IR constraint.

Where have we seen a similar optimization problem? Think back to the UMPs, CMPs, and PMPs from Econ 2020a. The solution to the optimization problem should be fairly easy once you realize the following:

1. Do you see that you have a linear objective function in the choice variables? If you don't see this, check out equation 1 above.

2. The IR constraint can be written as:

$$p_i \cdot u(s_{2i}) + (1-p_i) \cdot u(s_{1i}) \geq \underline{\mathrm{u}}$$

   where $\underline{\mathrm{u}}$ is a constant since W, L, and p_i are exogenous parameters to the model.

3. In short, we have an optimization problem where the optimum will be at the point where the objective function will be tangent to the convex constraint set.[4] There are several ways to solve this problem, but the easiest

[4]Convexity of the constraint set is implied by the assumptions we placed on the Bernoulli utility functions.

seems to be that at the optimum $[s_{1i}^*, s_{2i}^*]$ the following must hold:

$$\begin{aligned} -\frac{(1-p_i)}{p_i} &= \frac{\partial s_{2i}}{\partial s_{1i}} \Rightarrow -\frac{(1-p_i)}{p_i} = -\frac{(1-p_i)u'(s_{1i})}{p_i u'(s_{2i})} \Rightarrow u'(s_{1i}) = u'(s_{2i}) \\ \therefore \quad & s_{1i} = s_{2i} \end{aligned} \tag{2}$$

The important thing to understand here is that we are at a Pareto Optimum (or the "First Best") when we have full information and First Degree Price Discrimination. Further, because the IR constraint binds at the optimum contract, the insurer gets all of the surplus and the consumer gets her reservation utility.

3 Asymmetric Information (Second-Degree Price Discrimination)

3.1 Optimization Problem of the Insurer

If the insurer is unable to distinguish between the different types of consumers, then its problem can be written as follows:

$$\max_{s_{1G}, s_{2G}, s_{1B}, s_{2B}} \lambda \{W - p_G L - [p_G s_{2G} + (1-p_G)s_{1G}]\} + (1-\lambda)\{W - p_B L - [p_B s_{2B} + (1-p_B)s_{1B}]\}$$

subject to the IR constraints for both consumer types and the IC constraints for both consumer types:

- $(IR_G) : p_G \cdot u(s_{2G}) + (1-p_G) \cdot u(s_{1G}) \geq p_G \cdot u(W-L) + (1-p_G) \cdot u(W) = \bar{u}_0^G$
- $(IR_B) : p_B \cdot u(s_{2B}) + (1-p_B) \cdot u(s_{1B}) \geq p_B \cdot u(W-L) + (1-p_B) \cdot u(W) = \bar{u}_0^B$
- $(IC_G) : p_G \cdot u(s_{2G}) + (1-p_G) \cdot u(s_{1G}) \geq p_G \cdot u(s_{2B}) + (1-p_G) \cdot u(s_{1B})$
- $(IC_B) : p_B \cdot u(s_{2B}) + (1-p_B) \cdot u(s_{1B}) \geq p_B \cdot u(s_{2G}) + (1-p_B) \cdot u(s_{1G})$

3.2 Results: Binding Constraints

Now let's look at some qualitative results of the optimization problem facing the insurer. Which of the constraints above bind with equality?

- Consider the following:

$$\begin{aligned} p_B \cdot u(s_{2B}) + (1-p_B) \cdot u(s_{1B}) &\geq p_B \cdot u(s_{2G}) + (1-p_B) \cdot u(s_{1G}) \\ &> p_G \cdot u(s_{2G}) + (1-p_G) \cdot u(s_{1G}) \\ &\geq p_G \cdot u(W-L) + (1-p_G) \cdot u(W) \\ &> p_B \cdot u(W-L) + (1-p_B) \cdot u(W) \end{aligned}$$

The first weak inequality comes from IC_B; the second strict inequality comes from the single crossing property;[5] the third weak inequality comes from the IR_G; and the fourth inequality is because $p_B > p_G$. Therefore, we have that:

$$p_B \cdot u(s_{2B}) + (1 - p_B) \cdot u(s_{1B}) > p_B \cdot u(W - L) + (1 - p_B) \cdot u(W)$$

which means that the IR_B is slack and does not bind. This is interpreted as *the high risk consumers (or bad types) receive a rent.*[6]

- Let's look at the IR_G. If the Individual Rationality constraint for the good types (or low risk consumers) is slack, this means that both Individual Rationality constraints (for the Good and the Bad type) are slack. The insurer can reduce payments to both good and bad types in the state s_2 without violating either of their Individual Rationality constraints—*i.e.* there is a profitable deviation for the insurer. Therefore, it must be the case that IR_G binds with equality:

$$p_G \cdot u(s_{2G}) + (1 - p_G) \cdot u(s_{1G}) = p_G \cdot u(W - L) + (1 - p_G) \cdot u(W)$$

  which is interpreted as the *low risk, good types of consumers do not receive a rent above their Individual Rationality constraint.*[7]

- It must also be the case that IC_B binds with equality because if it was slack, then the insurer could always increase its profit by increasing the premium to bad type consumers. Furthermore, IC_G must be slack. If the IC_G was binding with equality, then the insurer could again increase its profits by increasing the premium charged to the bad types. The interpretation of this is that *the high risk, bad type consumers are indifferent between the contracts offered by the insurer.*[8]

3.3 Results: Pooling and Efficiency

In the model above, pooling is never optimal because the separate contracts always result in higher profits for the insurer. This should be intuitive. If not, see Appendix A-5, and keep in mind that the insurer's profits are higher the closer its iso profit line is to the origin.

Also, note that the bad types will get full insurance because:

1. The iso profit line is tangent to the level curve of the bad type consumer's expected utility function. The principle is the same as the one we used to solve the insurer's optimization problem in 2 above.[9]

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[5] Understanding this can be a little bit tricky, but try totally differentiating the expected utility functions $U_G(no\, insurance)$ and $U_B(no\, insurance)$. You'll notice that $dU_B > dU_G$.

[6] See Appendix A-2.

[7] See Appendix A-3.

[8] See Appendix A-4.

[9] See Appendix A-6 for a graphical representation.

2. Furthermore, because $\vec{s}_B$ which maximizes the insurer's profit lies below the indifference curve through the no insurance allocation (A) for the good type consumer, we don't have to worry about this contract attracting the low risk, good type consumers.

Therefore, we have the following two results:

1. *High risk, bad type consumers get the efficient quantity of insurance*—i.e. *their contract puts them on the 45 degree line.*

2. *Low risk, good type consumers get less than the efficient quantity of insurance because of the single crossing property and because IC_B binds with equality.*

When will firm only sell to one type?

1. When there are lots of bad types and few good types—*i.e.* when $\lambda \to 0$.

2. Each price increase for the bad types results in less profits from the good types because you have to decrease the price to good types to make sure that the IC_B continues to bind.

3. By offering separate contracts, the insurer can create more profit by offering the bad types a contract with a higher premium (which reduces s_{1B} and s_{2B}, hence the arrow denoted ρ in Appendix A-5). However, because IC_B has to bind with equality, this means that the contract for the low risk types (good types) will decrease payoff in s_{2G} but must increase s_{1G} (which is denoted by the arrow ρ' in A-5), which means that the contract offered to the good types decreases premiums, but also decreases payoffs in the bad state.

4. If there are very few good types, the insurer may want to simply maximize profits from the bad types and offer no contract to the good types. The result would be that IR_B binds with equality and a negative quantity to the good types such that $s_1 > W$.

4 Summary of Qualitative Results

1. High risk, bad type consumers get the efficient quantity of insurance—i.e. their contract puts them on the 45 degree line.

2. Low risk, good type consumers get less than the efficient quantity of insurance because of the single crossing property and because IC_B binds with equality.

3. The high risk consumers (or bad types) receive a rent.

4. Low risk, good types of consumers do not receive a rent above their Individual Rationality constraint.

5. the high risk, bad type consumers are indifferent between the contracts offered by the insurer.