Midterm Review

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Agenda

1. Cobb-Douglas Example (EV only)[1]
2. 2006 Midterm Question 3 (won't cover in section)
3. 2005 Midterm Question 4
4. 2004 Midterm Question 1
5. 2005 Midterm Question 1 (won't cover in section)

Some Midterm Tips

1. Since this is open book, make a cheat sheet with the important equations and bring a calculator.
2. Try not to get stuck on one question. You need to pace yourself. Try to get to all of the questions.
3. Read the questions carefully and make sure that you're actually answering each of the questions. A lot of people didn't fully answer questions on the problem sets.
4. If you get stuck on the math, describe the process and what the answer might look like. Show Chris what you know. Also, take a step back and really look at the functions/equations that you are being asked to analyze. Some interesting intuition might leap out at you.

[1] We'll cover only those parts relevant to EV

Cobb Douglas Example

See solutions posted on course website.

2006 Midterm Question 3 (20 points)

Based on empirical analysis, you have the following information about a typical poor consumer's demand for heating oil:

Own price elasticity of demand	-2
Wealth elasticity of demand	10
Current price	10
Current consumption	120
Current wealth	12,000

For the coming heating season, the government is expecting an increase in the price of heating oil to 12. It is interested in determining how much to increase public assistance so that the poor people can maintain their current standard of living after the increase in the price of heating oil.

Question 3a) According to economic theory, how should the government determine the required increase in public assistance? Briefly explain your answer. Be sure to relate it to the concepts studied in class.

Question 3b) Using the data provided above, estimate how much public assistance should be increased in order to maintain the poor consumers' standard of living.

[Parts c-e have been added for the purposes of section; they were not on the original exam]

Question 3c) Now assume that the price increase was due to a tax. Calculate the deadweight loss of the tax using the concept from b.

Question 3d) Calculate the Laspeyres Index for the price change. If we used the Laspeyres Index to compensate the consumer, how much would we increase public assistance? Explain the intuition behind the relationship between this value and the value you found in part b).

Question 3e) Estimate the Paasche Index for the price change. If we used the Paasche Index to compensate the consumer, how much would we increase public assistance?

Solutions

Note that the information given to us is in terms of elasticities. Therefore, one step that we'll probably have to consider is turning these elasticities into the derivatives we've seen in section and in our problem sets.

Question 3a) The correct concept is CV, which is equal to the change in wealth necessary for the consumer to achieve the same utility as before the price change after the price change has occurred. Thus, CV is the increase in wealth needed for the consumer, or the amount of wealth that a social planner has to give to the consumer (hence the minus sign for price increases), so that he or she is able to maintain the same utility level. Formally, CV is calculated as follows:

$$CV = e(p^1, u^1) - e(p^1, u^0) = e(p^0, u^0) - e(p^1, u^0) = w - e(p^1, u^0).$$

Question 3b) We use a linear approximation of the Hicksian demand curve.[2] Whenever we approximate a non-linear function, the first-step is to determine the point around which we want to approximate. Where is it going to be for this question?

$$\begin{aligned} \epsilon^p_{oil} &= -2 = \frac{dx}{dp} \cdot \frac{p}{x} = \frac{dx}{dp} \cdot \frac{10}{120} \Rightarrow \frac{dx}{dp} = -24 & (1) \\ \epsilon^w_{oil} &= 10 = \frac{dx}{dw} \cdot \frac{w}{x} = \frac{dx}{dw} \cdot \frac{12,000}{120} \Rightarrow \frac{dx}{dw} = 0.1 & (2) \end{aligned}$$

Now we use the Slutsky Equation and plug the results from equations 1 and 2 to approximate the slope of the Hicksian at the *old* price-wealth vector.

$$\frac{dh}{dp} = \frac{dx}{dp} + \frac{dx}{dw} \cdot x = -24 + 0.1 \cdot 120 = -12 \tag{3}$$

The approximate change in the Hicksian demand is:

$$\begin{aligned} \Delta h &= \frac{dh}{dp} \cdot \Delta p = -12 \cdot 2 = -24 & (4) \\ h(p = 12, \bar{p}^0, u^0) &\approx h(10) + \frac{dh}{dp} \cdot \Delta p = 120 - 24 = 96 & (5) \end{aligned}$$

Therefore,

$$CV = -\frac{1}{2} \cdot 2 \cdot (96 + 120) = -216.$$

Question 3c) To calculate the DWL, we use the following equation, where T is equal to the total tax revenue:

$$DWL = |CV| - T = 216 - 2 \cdot 96 = 24.$$

It may be easier for you to draw a picture.

[2] See Section Note 4.

Question 3d) Assume that the non-oil goods are a composite commodity y with price $q = 1$.[3] Then we can calculate the Laspeyres index as follows:

$$\begin{aligned} PI_{Laspeyres} &= \frac{p'x^0 + q'y^0}{px^0 + qy^0} \\ &= \frac{12 \cdot 120 + 10,800}{10 \cdot 120 + 10,800} = 1.02 \\ &\quad (\because p^0x^0 + q^0y^0 = w \rightarrow 10 \cdot 120 + 1 \cdot y^0 = 12,000 \rightarrow y^0 = 10,800) \end{aligned}$$

$$Compensation = |PI_{Laspeyres} - 1| \cdot w = 240 > |CV|$$

Recall that the Laspeyres Index generally results in over-compensation, since it does not take into account that the consumer facing a price increase on good x can re-optimize.

Question 3e)

$$\begin{aligned} PI_{Paasche} &= \frac{p'x' + q'y'}{px' + qy'} \\ &= \frac{12 \cdot 72 + 11,136}{10 \cdot 72 + 11,136} \approx 1.0121 \\ &\quad (\because p'x' + q'y' = w \rightarrow 12 \cdot 72 + 1 \cdot y' = 12,000 \rightarrow y' = 11,136) \end{aligned}$$

$$Compensation = |PI_{Paasche} - 1| \cdot w \approx 145.20 < |CV|$$

The Paasche Index generally results in under-compensation.

The Fisher Ideal Index takes the geometric mean of the Laspeyres and Paasche Indices[4] to try to balance out the over and under compensation, respectively:

$$PI_{Fisher} = \sqrt{PI_{Laspeyres} \cdot PI_{Paasche}} \approx 1.016$$

$$Compensation = |PI_{Fisher} - 1| \cdot w = 192 < |CV|,$$

but this is not generally the case for all price increases.

[3] If you don't know what a composite commodity is, don't worry. Think about it as a basket of all other goods, aside from heating oil, that a consumer consumes. You'll learn more about this as we do partial equilibrium.

[4] This is the text book definition of the Fischer Ideal Index.

2005 Midterm Question 4 (20 points)

Solutions

The relevant Bernoulli utility function is:

$$u(x) = cx^{\alpha},$$

where $c > 0$ and $\alpha > 0$. We know that agents/consumers are indifferent between the following lotteries over the respective outcomes:

$$\begin{aligned} L_1 &= \left[\frac{1}{10^6}, 1 - \frac{1}{10^6}\right]; \; x_1 = \left[\$10^6, \, 0\right] \\ L_2 &= \left[\frac{1}{2 \cdot 10^6}, \frac{1}{20,000}, 1 - \frac{1}{2 \cdot 10^6} - \frac{1}{20,000}\right]; \; x_2 = \left[\$10^6, \, 1000, \, 0\right] \end{aligned}$$

and

$$L_1 \sim L_2$$

Question a and b) Since agents are indifferent between the two lotteries, it must be the case that $U(L_1) = U(L_2)$. Let's calculate each.

$$U(L_1) = \frac{1}{10^6} \cdot c(10^6)^{\alpha} + \left(1 - \frac{1}{10^6}\right) \cdot c(0)^{\alpha} = \frac{1}{10^6} \cdot c \cdot 10^{6\alpha}$$

$$U(L_2) = \frac{1}{2 \cdot 10^6} \cdot c(10^6)^{\alpha} + \frac{1}{20,000} \cdot c(10^3)^{\alpha} + \left(1 - \frac{1}{2 \cdot 10^6} - \frac{1}{20,000}\right) \cdot c(0)^{\alpha}$$

Setting the two equations above equal to each other we see that:

$$\begin{aligned} \frac{1}{10^6} \cdot c \cdot 10^{6\alpha} &= \frac{1}{2 \cdot 10^6} \cdot c \cdot 10^{6\alpha} + \frac{1}{20,000} \cdot c \cdot 10^{3\alpha} \\ \frac{1}{2 \cdot 10^6} \cdot c \cdot 10^{6\alpha} &= \frac{1}{20,000} \cdot c \cdot 10^{3\alpha} \\ 10^{3\alpha} &= \frac{2 \times 10^6}{2 \times 10^4} = 100 \\ \therefore 3\alpha &= 2 \\ \alpha &= \frac{2}{3}. \end{aligned}$$

The important thing to notice is that c is not determined by the indifference between the two lotteries. Recall that positive linear transformations of the expected utility function represent the same preferences over lotteries. Why? Recall our discussion on how the expected utility function was a linear combination of the Bernoulli utility functions.

Question c) Let's solve by using the expected utilities. Define a new lottery $L_3 = [p, 1-p]$ over the outcome $x_3 = [1000, 0]$. Then we can calculate the following:

$$U(L_3) = p \cdot c \cdot 10^{3\alpha} + (1-p) \cdot c \cdot 0^{3\alpha} = 100cp \left(\because \alpha = \frac{2}{3} \right).$$

Since $U(L_3) = U(L_1)$,

$$\begin{aligned} 100cp &= \frac{1}{10^6} \cdot c \cdot 10^{6\alpha} \\ p &= \frac{1}{10^4}. \end{aligned}$$

2004 Midterm Question 1 (30 points)

Consider a two-commodity world and a consumer with the following utility function:

$$u(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}} \right)^2.$$

The prices of goods 1 and 2 are p_1 and p_2, respectively, and the consumer has initial wealth w.

Question 1a) State the consumer's utility maximization problem (3 points).

Question 1b) Will this consumer's Walrasian demand function satisfy Walras' Law? Explain your answer (3 points).

Question 1c) Derive the consumer's Walrasian demand function and indirect utility functions (12 points).

Question 1d) Suppose that the consumer has wealth $w = 260$. Initially, prices are $(p_1, p_2) = (6, 8)$. Suppose prices change to $(p_1', p_2') = (5, 12)$. What is the (exact) EV of this price change (12 points)?

Solutions

Question 1a)

$$\begin{aligned} & \max_{x_1, x_2} \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}} \right)^2 \\ s.t \quad & \vec{p} \cdot \vec{x} \leq w \end{aligned}$$

Question 1b) Yes. The utility function above is strongly monotone (more is better), which implies Walras' Law. More formally, notice that:

$$\frac{\partial u}{\partial x_1} > 0, \frac{\partial u}{\partial x_2} > 0.$$

Question 1c) We need to set up the Lagrangian and solve, but before that is there anything we can do to make the function above more tractable? If you don't do this during the exam, you'd be stuck on this question for quite a while.

$$\mathfrak{L}(x_1, x_2, \lambda) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}} - \lambda\left(p_1 x_1 + p_2 x_2 - w\right) \tag{6}$$

We get the following first order conditions (if we assume an interior solution, which is a pretty good assumption given the fact that the objective function is concave):

$$\begin{aligned}
\frac{\partial \mathfrak{L}(x_1^*, x_2^*, \lambda^*)}{\partial x_1} &= \frac{1}{2\sqrt{x_1^*}} - \lambda^* p_1 = 0 \\
\frac{\partial \mathfrak{L}(x_1^*, x_2^*, \lambda^*)}{\partial x_2} &= \frac{1}{2\sqrt{x_2^*}} - \lambda^* p_2 = 0 \\
\frac{\partial \mathfrak{L}(x_1^*, x_2^*, \lambda^*)}{\partial \lambda} &= p_1 x_1 + p_2 x_2 - w = 0.
\end{aligned}$$

Solving the three equations for the three unknowns, we get the following (use the symmetry):

$$\begin{aligned}
x_1^*(\vec{p}, w) &= \frac{w}{\left(p_1 + \frac{p_1^2}{p_2}\right)} \\
x_2^*(\vec{p}, w) &= \frac{w}{\left(p_2 + \frac{p_2^2}{p_1}\right)}.
\end{aligned}$$

Both of which are homogeneous of degree 0 in the price-wealth vector.

Now we're asked to solve for the indirect utility function, which is an identity which takes prices and wealth as its arguments:

$$v(\vec{p}, w) = \left(\sqrt{x_1^*} + \sqrt{x_2^*}\right)^2 = \left(\sqrt{\frac{w}{\left(p_1 + \frac{p_1^2}{p_2}\right)}} + \sqrt{\frac{w}{\left(p_2 + \frac{p_2^2}{p_1}\right)}}\right)^2 = \frac{w(p_1 + p_2)}{p_1 p_2}$$

Question 1d) Note that there are many ways to calculate the EV. However, why do you think that the problems above asked you to solve for the indirect utility function and why do you think it asks for the "exact" EV? We don't want to use approximation using the Slutsky equation here. What we want to use is:

$$\begin{aligned}
v(\vec{p}^{\,0}, w + EV) &= v(\vec{p}^{\,1}, w) \\
\frac{(260 + EV)(6 + 8)}{6 \cdot 8} &= \frac{260 \cdot (5 + 12)}{5 \cdot 12} \\
\therefore EV &\approx -7.4286
\end{aligned}$$

2005 Midterm Question 1 (20 points)

A consumer has utility function $u(x_1, x_2) = v(x_1)+v(x_2)$,where $v(\cdot)$ is a strictly increasing, strictly concave function with $v'(0) = \infty$. The positive prices of the two goods are p_1, p_2, respectively, and the consumer has positive initial wealth equal to w.

Question 1a) Write down the consumer's utility maximization problem.

Question 1b) Let x_1^* and x_2^* denote the consumer's optimal choices of x_1 and x_2. Derive the conditions that characterize x_1^* and x_2^*.

Question 1c) Under what conditions on the exogenous variables will x_1^* be strictly greater than x_2^*?

Question 1d) Derive an expression relating $\frac{dx_1^*}{dw}$ and $\frac{dx_2^*}{dw}$ to the shape of the function $v(\cdot)$.

Solutions

Question 1a)

$$\begin{array}{lc} & \max_{x_1,x_2} v(x_1) + v(x_2) \\ s.t & \vec{p}\cdot\vec{x} \leq w \end{array}$$

Question 1b) We need to set up a Lagrangian. However, is there anything we can do to simplify the maximization? No.

$$\mathfrak{L}(x_1, x_2, \lambda) = v(x_1) + v(x_2) - \lambda\left(p_1x_1 + p_2x_2 - w\right)$$

We get the following first order conditions (Note that we know that there will be an interior solution here. Why? It is because of (one of) the Inada condition above,$v'(0) = \infty$):

$$\begin{aligned} \frac{\partial\mathfrak{L}(x_1^*, x_2^*, \lambda^*)}{\partial x_1} &= v'(x_1^*) - \lambda^* p_1 = 0 \\ \frac{\partial\mathfrak{L}(x_1^*, x_2^*, \lambda^*)}{\partial x_2} &= v'(x_2^*) - \lambda^* p_2 = 0 \\ \frac{\partial\mathfrak{L}(x_1^*, x_2^*, \lambda^*)}{\partial \lambda} &= p_1x_1 + p_2x_2 - w = 0 \end{aligned}$$

Solving for these equations to the extent possible, we have the following equations:

$$\frac{v'(x_1^*)}{v'(x_2^*)} = \frac{p_1}{p_2} \tag{7}$$

$$p_1 x_1^* + p_2 x_2^* = w.$$

Question 1c) Since $v(\cdot)$ is stictly concave, $x_1^* > x_2^*$ iff $v'(x_1^*) < v'(x_2^*)$, which occurs iff $\frac{p_1}{p_2} < 1$ or $p_1 < p_2$.

Question 1d) I included this because it uses the Implicit Function Theorem.

Using equation 7, which is a first order condition, we can define the following identity:

$$v'(x_1^*) \equiv \frac{p_1}{p_2} v'(x_2^*),$$

differentiating both sides of the identity with regards to w we get:

$$v''(x_1^*) \cdot \frac{dx_1^*}{dw} \equiv \frac{p_1}{p_2} v''(x_2^*) \cdot \frac{dx_2^*}{dw}.$$

Cobb Douglas EV, CV Example
Solutions

Suppose that the utility function is $u(x, y) = xy$ with initial wealth $W^0 = 60$ and initial prices $p^0 = 2$, $q^0 = 2$ at time 0.

(a) Compute EV <u>without using the Hicksian demand function</u> for a price increase on good 1 at time 1 to $p^1 = 3$. Assume that price q and wealth w do not change from time 0 to time 1 so that $W^1 = 60$ and $q^1 = 2$.

Under initial conditions,

$$x^0(p^0, q^0, W) = W / 2p^0 = 15;$$
$$y^0(p^0, q^0, W) = W / 2q^0 = 15.$$
$$u^0 = v(p^0, q^0, W) = x^0y^0 = 225.$$

Under new conditions,

$$x^1(p^1, q^1, W) = W / 2p^1 = 10$$
$$y^1 (p^1, q^1, W) = W / 2q^1 = 15.$$
$$u^1 = v(p^1, q^1, W) = x^1y^1 = 150.$$

When the utility function is this straightforward, it is frequently possible to compute CV and EV directly from the definition of the utility function without having to resort to integrating the Hicksian demand function.

The definition of EV is $EV = u(p^0, u^1) – e(p^0, u^0)$.

By definition, $e(p^0, u^0) = W^0 = 60$.

So we only need to compute $u(p^0, u^1)$ to solve for EV. $e(p^0, u^1)$ is defined as minimal expenditure (i.e. wealth) necessary to achieve utility u^1 at prices p^0. Returning to the definition of Cobb-Douglas demand functions, we know that

$$x(p, q, W) = W / 2p;$$
$$y(p, q, W) = W / 2q.$$

At initial prices, $p = 2$, $q = 2$ and so the optimal bundle for wealth W yields utility $W^2 / 16$. From above we know that $u^1 = 150$.

Therefore $e(p^0, u^1)$ is given by wealth W such that $W^2 / 16 = 150$, or

$$W = \sqrt{2400} = 48.99.$$

Therefore, $EV = u(p^0, u^1) – e(p^0, u^0) = 48.99 – 60 = -11.01$.

(b) Compute CV <u>without using the Hicksian demand function</u> for the same price increase studied in part (a).

The definition of CV is $CV = u(p^1, u^1) – e(p^1, u^0)$.

By definition, $e(p^1, u^1) = W^1 = 60$.

So we only need to compute $u(p^1, u^0)$ to solve for EV. $e(p^1, u^0)$ is defined as minimal expenditure (i.e. wealth) necessary to achieve utility u^0 at prices p^1.

At new prices, p = 3, q = 2 and so the optimal bundle given wealth W is x = W / 6 and y = W / 4, corresponding to utility $W^2 / 24$.

From above we know that $u^0 = 225$.

Therefore $e(p^1, u^0)$ is given by wealth W such that $W^2 / 24 = 225$, or

$$W = \sqrt{5400} = 73.48.$$

Therefore, $CV = u(p^0, u^1) – e(p^0, u^0) = 60 – 73.48 = -13.48$.

Note that EV and CV have the appropriate qualitative properties for a price increase: (1) both EV and CV are negative; (2) EV is greater than CV in magnitude since (Cobb-Douglas utility implies both goods are normal) because u^0 is greater than u^1.

(c) Now use Hicksian demand to compute EV.

For EV we want to find the Hicksian demand corresponding to utility u^1.

In essence, we need to solve EMP for optimal bundles given $u^1 = 150$, q = 2 and with varying values of p.

With wealth W, and prices (p, 2), the optimal bundles in CP are x = W / 2p and y = W / 4, producing utility $W^2 / 8p$.

EMP min px + 2y s.t $xy \geq 150$

yields optimized solution = W that solves $W^2 / 8p = 150$.

i.e. W = SQRT (1200 p).

This produces optimal bundles hx = W / 2p = SQRT(300 / p),
hy = W / 4 = SQRT(75 p).

Note also that these Hicksian demands produce the desired utility since hx * hy = SQRT(300 * 75) = 150.
The relevant Hicksian integral is then

$$\begin{aligned}
\textbf{EV} \quad &= -\int_2^3 (\sqrt{300/p})\,dp \\
&= \sqrt{300}\ \mathbf{2p^{1/2}} \big|_{p=2}^{p=3} \\
&= \sqrt{1200} * \sqrt{3} - \sqrt{1200} * \sqrt{2} \\
&= \mathbf{-\ 11.01.}
\end{aligned}$$

(d) Now use Hicksian demand to compute CV.

For CV we want to find the Hicksian demand corresponding to utility u^0.

In essence, we need to solve EMP for optimal bundles given $u^1 = 225$, q = 2 and with varying values of p.

This is exactly the same as in (c), but with change in target utility from 150 to 225.

EMP min px + 2y s.t $xy \geq 225$

yields optimized solution = W that solves $W^2 / 8p = 225$.
i.e. W = SQRT (1800 p).

This produces optimal bundles hx = W / 2p = SQRT(450 / p),
hy = W / 4 = SQRT(1800 p / 16).

Note also that these Hicksian demands produce the desired utility since hx * hy = SQRT(450 * 1800 / 4) = 225.

The relevant Hicksian integral is then

$$
\begin{aligned}
\textbf{CV} &= -\int_2^3 (\sqrt{450/p})\,dp \\
&= \sqrt{450}\ \mathbf{2p^{1/2}}\ \Big|_{p=2}^{p=3} \\
&= \sqrt{1800} * \sqrt{3}\ \mathbf{-}\ \sqrt{1800} * \sqrt{2} \\
&= \mathbf{-\ 13.48.}
\end{aligned}
$$

(e) Now compute the Area Variation associated with the change in price.

AV is simply the integral of Walrasian demand as a function of the price change. Walrasian demand for good x is W / 2p, so this is straightforward:

$$
\begin{aligned}
\textbf{AV} &= -\int_2^3 W/2p\,dp \\
&= \mathbf{-(W/2)\ ln\ p}\ \Big|_{p=2}^{p=3} \\
&= \mathbf{-[30\ ln\ 3 - 30\ ln\ 2]} \\
&= \mathbf{-\ 12.16.}
\end{aligned}
$$

As expected, AV falls between EV and CV.

(f) Now use a linear approximation to the Hicksian to produce a linear approximation to EV and CV – appropriate for a case where we can estimate the Walrasian demand function, but cannot estimate the Hicksian demand function.

If we know the Walrasian demand function x(p, q, W) = W / 2p, we can

(1) find the intersection of Walrasian and Hicksian at old and new prices.
OLD Price: p = 2, x = 15.
NEW price: p = 3, x = 10.

(2) use the Slutsky equation to identify $\partial h_x / \partial p$ at p = 2 and p = 3.

$$dx / dp = dh / dp - x\, dx / dw$$

OR

$$dh / dp = dx / dp + x\, dx / dw$$

We know $dx / dp = -W / 2p^2$; $dx / dw = 1 / 2p$.

So

$$\begin{aligned} dh / dp &= -W / 2p^2 + x / 2p \\ &= -W / 2p^2 + W / 4p^2 \quad \text{since } x = W / 2p. \\ &= -W / 4p^2 \end{aligned}$$

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APPROXIMATING CV

At W = 60, p = 2 (initial conditions, corresponding to u^0),

$$dh / dp = -60 / 16 = -15 / 4.$$

Approximate Hicksian $h_x(p, u^0)$ with linear function through (x = 15, p = 2), slope -15/4.

$$h_x(p) = 22.5 - 15p / 4.$$

Approximated CV

$$\begin{aligned} &= -\int_{p_0}^{p_1} h_x(p, u^0)\, dp \\ &= \int_{2}^{3_1} (22.5 - 15p/4)\, dp \\ &= 22.5\,p - 15p^2/8 \quad \Big|_{p=2}^{p=3} \\ &= -13.125 \end{aligned}$$

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APPROXIMATING EV

At W = 60, p = 3 (new conditions, corresponding to u^1),

$$dh / dp = -60 / 36 = -15 / 9.$$

Approximate Hicksian $h_x(p, u^0)$ with linear function through (x = 10, p = 3), slope -15/9.

$$h_x(p) = 15 - 15p / 9.$$

Approximated EV $= -\int_{p_0}^{p_1} h_x(p, u^0)\, dp$

$= \int_{2}^{3_1} (15 - 15p/9)\, dp$

= 15 p – 15p^2/18 $\Big|_{p=2}^{p=3}$

= -10.83.

= -13.125

Note that it seems clear that the linear approximations are more accurate representations of actual EV and CV than is AV.