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(1) VC Dimension

must lie in [a,b] wher a & b are real numbers.

examples

VC = 1 ✓

a = 1 b = 7 ⇒ [1, 7]

Input	Output
0	(0)
3	(1)

VC = 2

a = 1 b = 7

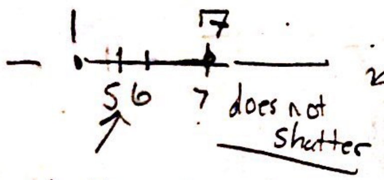
Input	output
0, 8	→ (0, 0)
0, 6	→ (0, 1)
2, 9	→ (1, 0)
2, 4	→ (1, 1)

VC = 3 X

a = 1 b = 7

Input	output
10, 11, 12	(0, 0, 0)
-1, 0, 2	(0, 0, 1)
6, 9, 10	(0, 1, 0)
	(1, 0, 0)
5, 6, 10	(0, 1, 1)
5, ~~6~~ 7	(1, 0, 1) +
5, 6, 8	(1, 1, 0)
2, 3, 4	(1, 1, 1)

VC dimenision = 2



Number Line

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a) $f(x|\theta) = \frac{1}{\theta-1} e^{\frac{-x}{\theta-1}} \quad x>0, \theta>1$

Step 1: Convert to liklihood func..

$$L(\theta) = \prod_{i=1}^{n} \left(\frac{1}{\theta-1}\right) e^{\frac{-x}{\theta-1}} \Rightarrow \sum_{i=1}^{N} \ln\left(\frac{1}{\theta-1}\right) e^{\frac{-x_i}{\theta-1}}$$

$$\ln\left(\frac{1}{\theta-1}\right) + \sum_{i=1}^{N} \frac{-x_i}{\theta-1} \ln(e) \Rightarrow \ln\left(\frac{1}{\theta-1}\right) + \sum_{i=1}^{N} \frac{-x_i}{\theta-1}$$

Step 2: derivative

$$\frac{\partial \ln\left(\frac{1}{\theta-1}\right) + \sum_{i=1}^{N} \frac{-x_i}{\theta-1}}{d\theta}$$

$$= \frac{-1}{(\theta-1)^3} + \sum_{i=1}^{N} \frac{x_i}{(\theta-1)^2}$$

Step 3 set to 0

$$0 = \frac{-1}{(\theta-1)^3} + \sum_{i=1}^{N} \frac{x_i}{(\theta-1)^2} \Rightarrow \frac{1}{(\theta-1)^3} = \sum_{i=1}^{N} \frac{x_i}{(\theta-1)^2}$$

$$\frac{(\theta-1)^2}{(\theta-1)^3} = \sum_{i=1}^{N} x_i \Rightarrow \frac{1}{(\theta-1)} = \sum_{i=1}^{N} x_i$$

$$\Downarrow$$

$$\boxed{\theta-1 = \frac{1}{\sum_{i=1}^{N} x_i}}$$

$$\Downarrow$$

$$\frac{1}{\theta-1} \cdot (\theta-1)^{-1}$$

$$-1(\theta-1)^{-2}$$

$$\boxed{\theta = \frac{1}{\sum_{i=1}^{N} x_i} + 1}$$

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b) $f(x|\theta) = (\theta-1)x^{\theta-2}$, $0 \le x \le 1$, $\theta > 1$

Step 1: likelihood function

$$L(\theta) = \prod_{i=1}^{n} f(x|\theta) = \prod_{i=1}^{n} (\theta-1)x^{\theta-2} = \sum_{i=1}^{n} \ln[(\theta_i-1)x^{\theta_i-2}]$$

$$\sum_{i=1}^{n} \ln(\theta_i-1) + (\theta_i-2)\ln(x)$$

Step 2: differentiate wrt θ — Product Rule

$$\frac{\partial L(\theta)}{\partial \theta} = \frac{\sum_{i=1}^{n} \partial \ln(\theta_i-1) + (\theta_i-2)\ln(x)}{\partial \theta} \Rightarrow \sum_i^n \frac{1}{\theta_i-1} + (1)(\ln(x)) + (\theta_i-2)(1)$$

$$= \sum_i^n \frac{1}{\theta_i-1} + \ln(x) + (\theta_i-2)$$

Step 3: set to 0

$$\sum_i^n \frac{1}{\theta_i-1} + \ln(x) + (\theta_i-2) = 0$$

$$-\ln(x) = \frac{1}{(\theta_i-1)} + (\theta_i-2) \Longrightarrow -\ln(x) = \frac{1}{(\theta_i-1)} + \frac{(\theta_i-2)(\theta_i-1)}{(\theta_i-1)}$$

$$-\ln(x) = \frac{(\theta_i-2)\cancel{(\theta_i-1)}}{\cancel{(\theta_i-1)}} = -\ln(x) = \theta_i - 2$$

$$\boxed{-\ln(x) + 2 = \sum_i^n \theta_i}$$

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(2)

c) $f(x|\theta) = 1/\theta$ for $0 \le x \le \theta$

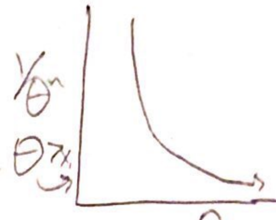
$L(\theta) = \prod_{i}^{n} f(x|\theta) \Rightarrow \prod_{i}^{n} 1/\theta_i \Rightarrow \frac{1}{\theta^n}$ when $\max X_i < \theta$

We know that the $\max(X_i) < \theta$

thus,

likelihood $= 0$ $(0, \max(x_i))$

likelihood ≥ 0 $(\max(x_i), \infty))$



To find θ, find minimum value of $\max(x_i) \Rightarrow \theta(x_n) = \max(x_i)$

and, $\theta(x) = X_{(n)} < \theta$

so, $E_\theta \max(x_i) < \theta$

Expected Value of θ

$X_{(n)} = \max_{1 \le i \le n} X_i \Longrightarrow P(X_n < x)$

each have same distribution so...

$F_{x_n}(x) = x/\theta$ for $0 \le x \le \theta$

$f_{x_n}(x) = \frac{x^n}{\theta^n}$

$\frac{d \frac{x^n}{\theta^n}}{\partial x} = \frac{n x^{n-1}}{\theta^n} \Longrightarrow E_\theta = \int_0^\theta x \frac{n x^{n-1}}{\theta^n} dx = \frac{n}{\theta^n} \Rightarrow$

$\max(x_i) \int_0^\theta x^n dx = \frac{x^{n+1}}{n+1}$

$E_\theta \max(x_i) = \frac{n}{\theta^n} \cdot \frac{x^{n+1}}{n+1} \Big|_0^\theta \Rightarrow \frac{n x^{n+1}}{\theta^n (n+1)} \Big|_0^\theta \; \theta - \frac{n \theta^{n+1}}{\theta^n (n+1)} \Rightarrow \underline{\frac{n}{n+1} \theta}$

$X_{(n)} = \frac{n}{n+1} \theta \Longrightarrow \boxed{\frac{\max(x_i)(n+1)}{n} = \theta}$

Please Note: I used math.arizona.edu/~jwatkins/O-mle.pdf to help me through this problem... I was lost :(

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a) Given

$P(C_1)$ & $P(C_2)$ = Priors

$P_1 = P(x=0|C_1)$
$P_2 = P(x=0|C_2)$ } likelihood

Note:

$P(C_1) + P(C_2) = 1$

$\frac{\text{Prior} \cdot \text{likelihood}}{\text{evidence}}$

evidence = $p(x=0) = p(x=0|C_1)P(C_1) + p(x=0|C_2)P(C_2)$

substitute

$= P_1 P(C_1) + P_2 P(C_2)$ = evidence

$$\boxed{\begin{aligned} P(C_1|x=0) &= \frac{P(C_1)\,P_1}{P_1 P(C_1) + P_2 P(C_2)} \\ P(C_2|x=0) &= \frac{P(C_2)\,P_2}{P_1 P(C_1) + P_2 P(C_2)} \\ P(C_1|x=1) &= \frac{P(C_1)(1-P_1)}{(1-P_1)(P(C_1)) + (1-P_2)(P(C_2))} \\ P(C_2|x=1) &= \frac{P(C_2)(1-P_2)}{(1-P_1)(P(C_1)) + (1-P_2)(P(C_2))} \end{aligned}}$$

Note:

$P(x=1|C_1) = 1 - p(x=0|C_1)$
$= 1 - P_1$

$p(x=1|C_2) = 1 - p(x=0|C_2)$

use in equation $= 1 - P_2$

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b) $P_{ij} = (x_j = 0 | C_i)$ $\quad i = 1, 2$
$j = 1, 2, \ldots, D$

$$\boxed{P(C_i | x_j = 0) = \frac{P(C_i) P_{ij}}{\sum_{k=1}^{2} P(C_k) \cdot P_{kj}} \qquad P(C_i | x_j = 1) = \frac{P(C_i)(1 - P_{ij})}{\sum_{k=1}^{2} P(C_k) \cdot (1 - P_{kj})}}$$

c) $P_{11} = 0.6$ 8 calculations for each prior, $P(C_i) = 0.2, 0.6, 0.8$ (calculations)

$P_{12} = 0.1$

$P_{21} = 0.6$

$P_{22} = 0.9$

$P(C_1) = 0.2 \quad P(C_2) = 0.8$

$j = 1$

$$P(C_1 | x_1 = 0) = \frac{P(C_1) \cdot P_{11}}{\sum_{k=1}^{2} P(C_k) \cdot P_{kj}} = \frac{0.2 \cdot 0.6}{(0.2)(0.6) + (0.8)(0.6)} = \frac{0.12}{0.6} = \underline{0.2}$$

$$P(C_1 | x_1 = 1) = \frac{P(C_1) \cdot 1 - P_{11}}{\sum_{k=1}^{2} P(C_k) \cdot (1 - P_{k1})} = \frac{(0.2) \cdot (1 - 0.6)}{(0.2)(1 - 0.6) + (0.8)(1 - 0.6)} = \frac{0.08}{0.4} = \underline{0.2}$$

$$P(C_2 | x_1 = 0) = \frac{P(C_2) \cdot P_{21}}{\sum_{k=1}^{2} P(C_k) \cdot P_{k1}} = \frac{(0.8) \cdot (0.6)}{(0.2)(0.6) + (0.8)(0.6)} = \frac{.48}{.6} = \underline{0.8}$$

$$P(C_2 | x_1 = 0) = \frac{P(C_2) \cdot 1 - P_{21}}{\sum_{k=1}^{2} P(C_k) \cdot P_{k1}} = \frac{(0.8)(1 - 0.6)}{(0.2)(1 - 0.6) + (0.8)(1 - 0.6)} = \frac{.32}{0.4} = \underline{0.8}$$

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3c) Continued.

$j=2 \qquad P(C_1)=0.2 \quad P(C_2)=0.8$

$$P(C_1|x_2=0)=\frac{P(C_1)\cdot P_{12}}{\sum_{K=1}^{2} P(C_K)\cdot P_{K2}}=\frac{(0.2)\cdot(0.1)}{(0.2)(0.1)+(0.8)(0.9)}=\frac{.02}{.74}=0.\overline{027}$$

$$P(C_1|x_2=1)=\frac{P(C_1)\cdot(1-P_{12})}{\sum_{K=1}^{2} P(C_K)\cdot(1-P_{K2})}=\frac{(0.2)(1-0.1)}{(0.2)(1-0.1)+(0.8)(1-0.9)}=\frac{.18}{.26}\approx 0.6923$$

$$P(C_2|x_2=0)=\frac{P(C_2)\cdot P_{22}}{\sum_{K=1}^{2} P(C_K)\cdot P_{K2}}=\frac{(0.8)(0.9)}{(0.2)(0.1)+(0.8)(0.9)}=\frac{.72}{.74}=0.\overline{9729}$$

$$P(C_2|x_2=1)=\frac{P(C_2)\cdot(1-P_{22})}{\sum_{K=1}^{2} P(C_K)\cdot(1-P_{K2})}=\frac{(0.8)(1-0.9)}{(0.2)(1-0.1)\cdot(0.8)(1-0.9)}=\frac{.08}{.26}\approx .30769$$

$j=1 \qquad P(C_1)=0.6 \quad P(C_2)=0.4$

Note, I will stop writing equation before, as they are all listed previously...

$$P(C_1|x_1=0)=\frac{(0.6)(0.6)}{(0.6)(0.6)+(0.4)(0.6)}=\frac{.36}{.6}=0.6$$

$$P(C_1|x_1=1)=\frac{(0.6)(1-0.6)}{(0.6)(1-0.6)+(0.4)(1-0.6)}=\frac{.24}{.4}=0.6$$

$$P(C_2|x_1=0)=\frac{(0.4)(0.6)}{(0.6)(0.6)+(0.4)(0.6)}=\frac{.24}{.6}=0.4$$

$$P(C_2|x_1=1)=\frac{(0.4)(1-0.6)}{(0.6)(1-0.6)+(0.4)(1-0.6)}=\frac{.16}{.4}=0.4$$

3 c) Continued

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$j=2 \quad P(C_1) = 0.6 \quad P(C_2) = 0.4$

$$P(C_1|x_2=0) = \frac{(0.6)(0.1)}{(0.6)(0.1) + (0.4)(0.9)} = \frac{0.06}{.42} \approx \underline{0.1429}$$

$$P(C_1|x_2=1) = \frac{(0.6)(1-0.1)}{(0.6)(1-0.1) + (0.4)(1-0.9)} = \frac{.54}{.58} \approx \underline{0.9310}$$

$$P(C_2|x_2=0) = \frac{(0.4)(0.9)}{(0.6)(0.1) + (0.4)(0.9)} = \frac{.36}{.42} \approx \underline{0.85714}$$

$$P(C_2|x_2=1) = \frac{(0.4)(1-0.9)}{(0.6)(1-0.1) + (0.4)(1-0.9)} = \frac{.04}{.58} \approx \underline{0.06897}$$

$j=1 \quad P(C_1) = 0.8 \quad P(C_2) = 0.2$

$$P(C_1|x_1=0) = \frac{(0.8)(0.6)}{(0.2)(0.6) + (0.8)(0.6)} = \frac{.48}{.6} = \underline{0.8}$$

$$P(C_1|x_1=1) = \frac{(0.8)(1-0.6)}{(0.2)(1-0.6) + 0.8(1-0.6)} = \frac{.32}{0.4} = \underline{0.8}$$

$$P(C_2|x_1=0) \quad \frac{(0.2)(0.6)}{(0.2)(0.6) + (0.8)(0.6)} = \frac{0.12}{0.6} = \underline{0.2}$$

$$P(C_2|x_1=1) \quad \frac{(0.2)(1-0.6)}{(0.2)(1-0.6) + (0.8)(1-0.6)} = \frac{0.08}{0.4} = \underline{0.2}$$

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3c) Continued

j=2 $\quad P(C_1) = 0.8 \quad P(C_2) = 0.2$

$$P(C_1 | x_2 = 0) = \frac{(0.8)(0.1)}{(0.8)(0.1) + (0.2)(0.9)} = \frac{.08}{.26} \approx 0.3077$$

$$P(C_1 | x_2 = 1) = \frac{(0.8)(1-0.1)}{(0.8)(1-0.1) + (0.2)(1-0.9)} = \frac{.72}{.74} \approx 0.\overline{972}$$

$$P(C_2 | x_2 = 0) = \frac{(0.2)(0.9)}{(0.8)(0.1) + (0.2)(0.9)} = \frac{.18}{.26} \approx 0.6923$$

$$P(C_2 | x_2 = 1) = \frac{(0.2)(1-0.9)}{(0.8)(1-0.1) + (0.2)(1-0.9)} = \frac{.02}{.74} \approx 0.\overline{027}$$