Problem 1 part 1

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$$\underset{w}{\text{minimize}} \; \|Xw-y\|^2$$

X: $n \times m$
w: $m \times 1$
y: $n \times 1$

Start

$\|Xw-y\|^2$

$\downarrow$ (inner dot product)

$\langle Xw-y, Xw-y \rangle$

$\downarrow$ ($\langle A,B \rangle = A^T B$)

$(Xw-y)^T (Xw-y)$

$\downarrow$ distribute transpose

$((Xw)^T - y^T)(Xw-y)$

$\downarrow$ FOIL

$(Xw)^T Xw - (Xw)^T y - y^T (Xw) - y^T y$

$\downarrow$ $(A,B)^T = B^T A$

$w^T X^T Xw - 2w^T X^T y - y^T y$

$\downarrow$ now, derivative wrt to w

$\frac{\partial}{\partial w} w^T X^T Xw - 2w^T X^T y - y^T y$

$\downarrow$ partial derivative

$2X^T Xw - 2X^T y$

$\downarrow$ now, set to 0

$0 = 2X^T Xw - 2X^T y$

$0 = 2X^T Xw - 2X^T y$

$\downarrow$ add $2X^T y$

$2X^T y = 2X^T Xw$

$\downarrow$ cancel the 2's

$X^T y = X^T Xw$

$\downarrow$ inverse

$(X^T X)^{-1} X^T y = (X^T X) w (X^T X)^{-1}$

$\downarrow$ cancel

$\boxed{(X^T X)^{-1} X^T y = w}$ END

verify dimensions.

$(m \times n)(n \times m)$
$(m \times m)(m \times n)$
$(m \times n)(n \times 1) = m \times 1 = m \times 1$ ✓

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Problem 1 Part 2

minimize $||Xw-y||^2 + \lambda||w||^2$

Start

$||Xw-y||^2 + \lambda||w||^2$

↓ inner dot product

$\underbrace{w^T X^T X w - 2w^T X^T y - y^T y}_{\text{from previous problem...}} + \lambda w^T w$

↓ $\frac{\partial}{\partial w}$

$2X^T X w - 2X^T y + 2\lambda w$

↓ set to 0

$0 = 2X^T X w - 2X^T y + 2\lambda w$

↓ add $2X^T y$ to both sides

$2X^T y = 2X^T X w + 2\lambda w$

↓ divide by 2

$X^T y = X^T X w + \lambda w$

↓ take out w

$X^T y = w(X^T X + \lambda)$

$X^T y = w(X^T X + \lambda)$

↓ inverse

$(X^T X + \lambda)^{-1}(X^T y) = w$

↓ Cancel

$\boxed{(X^T X + \lambda I)^{-1}(X^T y) = w}$

add I matrix to add to matrix and take Inverse

Problem 2

Part 1

$Pr(H) = p$

$Pr(T) = 1-p$

Sequence = HHTTH

Each event is independent

$p \cdot p \cdot (1-p) \cdot (1-p) \cdot p$

↓ simpify

$\boxed{p^3 \cdot (1-p)^2}$

natural Logarithm

use $p^3 \cdot (1-p)^2$ for ln properties

$\ln(p^3 \cdot (1-p)^2)$

$= \ln(p^3) + \ln((1-p)^2)$

$= \boxed{3\ln(p) + 2\ln(1-p)}$

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Problem 2

part 2

a) Coin A = $\frac{1}{2}$

Coin B = $\frac{2}{3}$

$P(A) = P(B) = \frac{1}{2}$

Data = D = HHTTH

$P(D) = p^3 \cdot (1-p)^2$ (from previous problem)

so,

$P(A \cap D) = P(A) \cdot P(D) = \frac{1}{2}\left(\frac{1}{2}^3 * (1-\frac{1}{2})^2\right)$

$= \boxed{0.015625}$

b) $P(B \cap D) = P(B) \cdot P(D) = \frac{1}{2}\left(\frac{2}{3}^3 * (1-\frac{2}{3})^2\right)$

$\approx \boxed{0.016461}$

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Problem 2

Part 3

$3\ln(p) + 2\ln(1-p)$ ← from previous problem

$\frac{d}{dp} 3\ln(p) + 2\ln(1-p)$

$3\left(\frac{1}{p}\right) + 2\frac{1}{1-p}(-1)$

$= \frac{3}{p} - \frac{2}{1-p}$

$= \frac{3(1-p)}{p-p^2} - \frac{2p}{p-p^2}$

$= \frac{3-5p}{p-p^2}$

Find roots

$p-p^2 = 0$

$p = 0$ and 1

set $\frac{3-5p}{p-p^2}$ to 0

$\frac{3-5p}{p-p^2} = 0$

$3-5p = 0$

$3 = 5p$

$\boxed{\frac{3}{5} = p}$

$\boxed{p = \Pr(H) = \frac{3}{5}}$ in order to maximize the probability of observing H, H, T, T, H.

$(p)^3 \cdot (1-p)^2$

$= \left(\frac{3}{5}\right)^3 * \left(1-\frac{3}{5}\right)^2$

$= 0.216 * 0.16$

$= 0.03456$

$\Pr(H, H, T, T, H)$ given coin bias $p = \frac{3}{5}$ is ...

$\boxed{0.03456}$

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