

Problem 1a:

Model 2:

$$\text{likelihood function} = -\frac{1}{2} (x - m_i)^T S^{-1} (x - m_i)$$

MLE estimates are...

$$m_i = \frac{\sum_t r_i^t x^t}{\sum_t r_i^t}$$
$$S = \frac{\sum_t r_i^t (x^t - m_i)(x^t - m_i)^T}{\sum_t r_i^t}$$

Model 3

$$\text{likelihood function} = -\log(2\pi) - \frac{1}{2} \log(\det(S)) - \frac{1}{2} (x - \mu)^T S^{-1} (x - \mu)$$

$$m_i = \frac{\sum_t r_i^t x^t}{\sum_t r_i^t}$$

$$S = \frac{\sum_t r_i^t (x^t - m_i)(x^t - m_i)^T}{\sum_t r_i^t}$$

To derive diagonal matrix, take avg of diagonal in  $S$ ...  
=  $\alpha$  ... Then

$$S = I \alpha$$