Adaptive Sliding-Mode Control of Nonholonomic Wheeled Mobile Robot

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Abstract—**This paper designs an adaptive sliding mode dynamic controller for the trajectory tracking of wheeled mobile robot. First, a kinematic controller is introduced for the wheeled mobile robot. Then, the adaptive sliding mode dynamic controller (ASMDC) is proposed to make the real velocity of the wheeled mobile robot reach the desired velocity command, in presence of system uncertainties and disturbances. The convergence of the complete equations of motion of wheeled mobile robot is proved by the Lyapunov stability theory. Computer simulation results illustrate the effectiveness of the proposed controller.**

Keywords—**Adaptive control, Dynamic control, Kinematic control, Lyapunov method, Sliding mode control, Wheeled mobile robot, Trajectory tracking**.

I. INTRODUCTION

Issues involving the control of nonholonomic WMRs have been considered over recent decades [1-4, 9, 10, 18, 19, 20, 21]. The system design of WMRs can be based on a kinematic or dynamic model. Many studies [1-3] are interested in kinematic tracking problems without taking into account the dynamics of WMRs. Nevertheless, considering only a kinematic model in a real trajectory problem is insufficient to reach a good tracking performance, since the error between the output of the velocity controller and the real velocity of WMR. Thus dynamic model [4] is more frequently adopted in real trajectory tracking of WMR. Several researches have focused on a dynamic model using back-stepping control [5], sliding mode control [6, 7], adaptive control [8, 9, 10], neural control [11, 12] or fuzzy control [13, 14] in order to overcome trajectory tracking problems.

Classical SMC which offer many great proprieties such as ability to deal with uncertainties, small tracking error and fast response, is a powerful control scheme for nonlinear systems. But, the discontinuous nature of control law of classical SMC engenders an undesired chattering phenomenon. Soltine and Li [15] adopted a boundary layer method to eliminate chattering phenomena. Nevertheless, if some uncertainties remain in the system, the bound still requires to be estimated to obtain robustness and convergence of SMC. The problem of perturbations estimation was investigated in [15, 16, 17] for particular class of nonlinear uncertain systems. But, these problems stayed mainly unresolved. Therefore, combined adaptive and sliding mode controllers have been considered in [18, 19, 20, 21, 22] as a method to reduce the chattering phenomena and to enhance trajectory tracking.

In this study, adaptive control and sliding-mode control are combined to treat both uncertainties and disturbances in the whole WMR system. The stability analysis is performed using Lyapunov theory.

This paper is organized as follows: Section 2 presents the kinematic controller design. The complete equations of motion of ASMDC are described in section 3. Computer simulation results of SMDC and ASMDC are given in section 4. Finally, section 5 concludes this paper.

II. KINEMATIC CONTROLLER DESIGN

The nonholonomic WMR shown in Fig 1 is a vehicle with two driving wheels and a front passive wheel. The two wheels have the same radius denoted by r and are separated by $2b$. P_o defines the origin of the robot coordinate system and P_c is the center of mass of the robot which is located at a distance d from P_o.

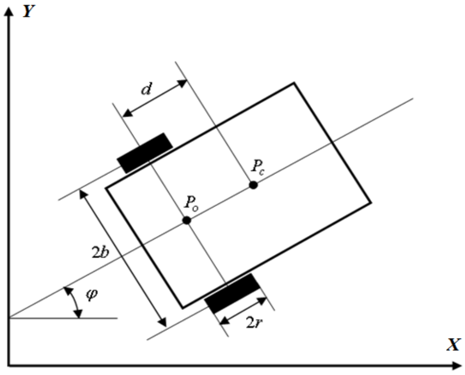

Figure 1. Nonholonomic mobile robot

The kinematic model of the WMR under nonholonomic constraint of pure rolling and non-slipping is defined as follows:

$$\dot{q} = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} = S(q).\upsilon \tag{1}$$

where $S(q) \in \Re^{3\times 2}$ and $\upsilon \in \Re^2$ are the full rank velocity transformation matrix and velocity vector, respectively. $v \in \Re$ denotes the linear velocity of the WMR and $w \in \Re$ denotes the angular velocities of the WMR. The $q(t), \dot{q}(t) \in \Re^3$ are defined as:

$$q = [x_0 \ y_0 \ \varphi]^T, \ \dot{q} = [\dot{x}_0 \dot{y}_0 \dot{\varphi}]^T \tag{2}$$

where (x_0, y_0) and $(\dot{x}_0, \dot{y}_0)$ are the actual position and linear velocity of P_0, φ is the heading angle of the mobile robot, and $\dot{\varphi}$ is the angular velocity of P_0

The WMR has the nonholonomic constraint that the driving wheels purely roll and do not slip. This nonholonomic constraint ($m = 1$) is written as the following:

$$\dot{y}_0 \cos\varphi - \dot{x}_0 \sin\varphi = 0 \tag{3}$$

The trajectory tracking problem is formulated by defining a reference mobile robot that gives a trajectory for the actual one to follow:

$$\begin{aligned} \dot{x}_r &= v_r \cos\varphi_r, \\ \dot{y}_r &= v_r \sin\varphi_r, \\ \dot{\varphi}_r &= w_r, \\ q_r &= [x_r \ y_r \ \varphi_r]^T, \ \upsilon_r(t) = [v_r, w_r]^T. \end{aligned} \tag{4}$$

where q_r denotes the reference time varying position and orientation trajectory, and $v_r(t)$ denotes the reference time varying linear velocity and $w_r(t)$ denotes the reference time varying angular velocity.

The posture tracking error between the reference robot and the actual robot can be expressed as:

$$q_e = [e_1 \ e_2 \ e_3]^T = T\tilde{q} \tag{5}$$

where $T = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\tilde{q} = \begin{bmatrix} x_r - x \\ y_r - y \\ \varphi_r - \varphi \end{bmatrix}$.

The derivative of the posture tracking error given in (5) can be written as follows:

$$\dot{q}_e = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -v + we_2 + v_r \cos e_3 \\ -we_1 + v_r \sin e_3 \\ -w + w_r \end{bmatrix} \tag{6}$$

The classic kinematic controller based on Backstepping method is designed to select the smooth velocity input [11]:

$$\upsilon_c = \begin{bmatrix} v_c \\ w_c \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 + k_1 e_1 \\ w_r + k_2 v_r e_2 + k_3 v_r \sin e_3 \end{bmatrix} \tag{7}$$

where k_1, k_2 and k_3 are positive constants.

III. Dynamic Controller Design

In this section we introduce two dynamic controllers, in order to guaranty that the motion of WMR can follow the desired velocity generated by the kinematic controller. First, we define the dynamic model of WMR. Second, we design a sliding-mode dynamic controller (SMDC) for the mobile robot. Finally, the ASMDC is formulated for solving the problem caused by system uncertainties and external disturbances. The stability proof of ASMDC is presented and confirmed by the Lyapunov stability theorem.

A. Dynamic model of WMR

According to the Euler-Lagrangian formulation, the nonholonomic wheeled mobile robot considering model errors and disturbances can be described as:

$$M(q)\ddot{q}+V_m(q,\dot{q})\dot{q}+F(\dot{q})+G(q) = B(q)(\tau+\tau_d)-A^T(q)\lambda \tag{8}$$

where $M(q)\in R^{nxn}$ is a symmetric positive definite inertia matrix of the system, $V_m(q,\dot{q})\in R^{nxn}$, is a centripetal an Coriolis matrix, $G(q)\in R^{nx1}$ is the gravitational vector, $F(\dot{q})\in R^{nx1}$ denotes the surface friction, $\tau_d\in R^{nx1}$ denotes bounded unknown disturbance including unstructured dynamic, $B(q)\in R^{nx(n-m)}$ is the input transformation matrix, $\tau\in R^{(n-m)x1}$ is a control input vector, $A\in R^{nxm}$ is a matrix associated with nonholonomic constraints, $\lambda\in R^{mx1}$ is the vector of Lagrange multipliers, and $\dot{q}$ and $\ddot{q}$ denote velocity and acceleration vectors respectively. Assuming that the mobile robot moves in the horizontal plane, in this case, $G(q)=0$. The parameter matrices and vector in (8) are given by:

$$M(q)=\begin{bmatrix} m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & I\end{bmatrix},\ A^T(q)=\begin{bmatrix}-\sin\theta\\ \cos\theta\\ 0\end{bmatrix},$$

$$G(q)=0,\ B(q)=\frac{1}{r}\begin{bmatrix}\cos\theta & \cos\theta\\ \sin\theta & \sin\theta\\ R & R\end{bmatrix},$$

$$V_m(q,\dot{q})=0,$$

$$\tau=\begin{bmatrix}\tau_r\\ \tau_l\end{bmatrix},\ \lambda=-m(\dot{x}_c\cos\theta+\dot{y}_c\sin\theta)\dot{\theta},$$

where m is the mass of WMR, I is the moment inertia of the WMR about its center. The τ_r and τ_l are torque control inputs generated by the right and left DC motor, respectively.

For control purpose, the constraint term $A^T(q)\lambda$ in (8) should be removed by an appropriate transform. Substituting (1) and its differentiation for Eq. (8) and pre-multiplying by $S^T(q)$, we obtain

$$\bar{M}(q)\dot{v}+\bar{V}_m(q,\dot{q})v+\bar{F}(\dot{q})=\bar{B}(q)(\tau+\bar{\tau}_d) \tag{9}$$

where

$$\bar{M}=S^TMS\in R^{2x2},\ \bar{V}_m=S^T(M\dot{S}+V_mS)\in R^{2x2},$$
$$\bar{F}=S^TF\in R^{2x1},\ \bar{\tau}_d=S^T\tau_d,\ \text{and}\ \bar{B}=S^TB.$$

The effect of $\bar{V}_m$ can be removed from (10), in view of the distance between COM and the coordinate center of WMR is zero. The variables in (10) are defined as: $\bar{M}(q)=\begin{bmatrix} m & 0\\ 0 & I\end{bmatrix}$ and $\bar{B}(q)=\frac{1}{r}\begin{bmatrix}1 & 1\\ R & -R\end{bmatrix}$. By considering the surface friction and the disturbance torque as the external disturbances, we can reduce the dynamic equation (9) into the nominal dynamic model:

$$\dot{v}(t)=E.\tau(t), \tag{10}$$

where the system matrix E is:

$$E=\bar{M}^{-1}(q)\bar{B}(q)=\frac{1}{m.r.I}\begin{bmatrix} I & I\\ Rm & -Rm\end{bmatrix}. \tag{11}$$

B. Sliding-mode dynamic control

In this subsection, the SMC method is employed in designing a dynamic tracking controller which allows the actual velocities of WMR coincide with the control velocities generated from the kinematic controller. First, we design SMC for nominal system where disturbances are zero and the system parameters are known. In order to find the torque input and estimate the desired velocities v_c, we propose the auxiliary velocity tracking error and its derivative as:

$$e_c(t)=\begin{bmatrix}e_{c1} & e_{c2}\end{bmatrix}^T=v_c(t)-v(t) \tag{12}$$

$$\dot{e}_c(t)=\dot{v}_c(t)-\dot{v}(t) \tag{13}$$

By selecting the PI_type sliding surface , the sliding surface is defined as:

$$s(t)=\begin{bmatrix}s_1(t)\\ s_2(t)\end{bmatrix}=e_c(t)+\beta\int e_c(\tau)d\tau \tag{14}$$

where β is positive sliding surface integral constant. Yet, if the system is on the sliding surface $s(t)=0$, $e_c(t)=-\beta\int_0^t e_c(\tau)d\tau$ the tracking error $e_c(\infty)\to 0$ since $\beta\succ 0$.

The derivative of the sliding surface $s(t)$ becomes:

$$\dot{s}(t)=\dot{e}_c(t)+\beta e_c(t) \tag{15}$$

From the concept of equivalent control law τ_{eq} is stated by recognizing that $\dot{s}(t)=0$ is a necessary condition for the state trajectory to stay in the sliding surface. Thus substituting (10) for (15), we obtain:

$$\dot{s}(t)=(\dot{v}_c(t)-E\tau)+\beta e_c(t)=0 \tag{16}$$

Therefore the equivalent control law τ_{eq} is:

$$\tau_{eq}(t)=E^{-1}\left[\dot{v}_c(t)+\beta e_c(t)\right] \tag{17}$$

where $E^{-1}=-\dfrac{r}{2R}\begin{bmatrix}-Rm & -I\\ -Rm & I\end{bmatrix}$

Assuming that all parameters are known, the equivalent control law τ_{eq} can keep the system state on the sliding surface. Unfortunately, uncertainties and disturbances always exist in real applications. To maintain the sliding condition, the discontinuous control law τ_{sw} should be introduced. Therefore, the control law is presented as follows:

$$\begin{aligned}\tau(t)&=\tau_{eq}+\tau_{sw}\\ &=E^{-1}\left[\dot{v}_c(t)+\beta e_c(t)+K.sign(s)\right]\end{aligned} \tag{18}$$

where $K=\begin{bmatrix}K_1 & 0\\ 0 & K_2\end{bmatrix}$ and K_i is positive constant, and

$\operatorname{sgn}(s)=\left[\operatorname{sgn}(s_1)\ \operatorname{sgn}(s_2)\right]^T$

The dynamic equation (10) in presence of uncertainties and disturbances becomes:

$$\dot{v}(t)=E\tau+\tau_d(t)=\bar{E}\tau(t)+\Delta E.\tau(t)+\tau_d(t) \tag{19}$$

where $\bar{E}$ is denoted as the nominal part of the system matrix E which is introduced by the parameters of WMRs, m, r, I and b. And ΔE denoted the uncertainties of system matrix E . τ_d is the vector of external disturbances. Thus, we can introduce $\delta(t)$ as the upper bound of uncertainties:

$$\delta(t)=\begin{bmatrix}\delta_1(t)\\ \delta_2(t)\end{bmatrix}=\Delta E\tau(t)+\tau_d(t) \tag{20}$$

Then the dynamic equation can be written as

$$\dot{v}(t)=\bar{E}.\tau(t)+\delta(t), \tag{21}$$

Hence, the sliding mode control law (18) can be written as:

$$\begin{aligned}\tau(t)&=\tau_{eq}+\tau_{sw}\\ &=\bar{E}^{-1}\left[\dot{v}_c(t)+\beta e_c(t)+K.sign(s)\right]\end{aligned} \tag{22}$$

where the switching gain K is chosen to compensate for the system uncertainties and disturbances.

In order to reduce the chattering phenomenon, we will use the arctangent function. Thus, replacing the sgn function by the arctangent function $o(s,\varepsilon)$ in (22), implies:

$$\begin{aligned}\tau(t)&=\tau_{eq}+\tau_{sw}\\ &=\bar{E}^{-1}\left[\dot{v}_c(t)+\beta e_c(t)+K.o(s,\varepsilon)\right]\end{aligned} \tag{23}$$

where $o(s,\varepsilon)=\dfrac{2}{\pi}arct(\dfrac{\varepsilon}{s}),$ ε is a small positive constant.

C. Adaptive sliding-mode dynamic controller

Due to the difficulties encountered in measure of parameter variations of the WMR and the exact value of external disturbances in real applications, we propose an adaptive sliding-mode dynamic controller to estimate the upper bound of $|\delta_i(t)|$. We suppose that the optimal bound of δ^* and γ^* exist. The adaptive algorithm can be expressed as follows:

$$\dot{\hat{\gamma}}=\begin{bmatrix}\dot{\hat{\gamma}}_1 & 0\\ 0 & \dot{\hat{\gamma}}_2\end{bmatrix}=\begin{bmatrix}\rho_1 s_1 o(s_1,\varepsilon_1) & 0\\ 0 & \rho_2 s_2 o(s_2,\varepsilon_2)\end{bmatrix}, \tag{24}$$

where $\dot{\hat{\gamma}}$ is the estimated value of d γ^* and $\rho_i(i=1,2)$ is denoted as positive adaptation gain. The ASMDC for the WMR is designed as follows:

$$\begin{aligned}\tau(t)&=\tau_{eq}+\tau_{sw}\\ &=\bar{E}^{-1}\left[\dot{v}_c(t)+\beta e_c(t)+\hat{\gamma}(t).o(s,\varepsilon)\right],\end{aligned} \tag{25}$$

And we define the estimation error as:

$$\tilde{\gamma}=\hat{\gamma}(t)-\gamma^*. \tag{26}$$

The main goal is to choose an adaptive law to update the estimate $\hat{\gamma}(t)$ such that $s(t)$ converge to a zero vector.

Theorem 1. *Both the posture tracking error q_e and the velocity tracking error e_c of the complete equations of motion of WMR with uncertainties and disturbances (21) will asymptotically converge to zero vectors, if the kinematic controller (7) and the ASMDC (25) are applied.*

Proof. Let the Lyapunov function candidate be defined as:

$$L=L_1+L_2 \tag{27}$$

where

$$L_1 = \frac{1}{2}(e_1{}^2 + e_2{}^2) + \frac{1-\cos e_3}{k_2} \tag{28}$$

$$L_2 = \frac{1}{2}s^T(t)s(t) + \frac{1}{2}((\frac{1}{\rho_1})\tilde{\gamma}_1{}^2 + (\frac{1}{\rho_2})\tilde{\gamma}_2{}^2) \tag{29}$$

Substituting (6) and (7) for the time derivative of V_1 in (28), we obtain :

$$\dot{L}_1 = -k_1 e_1{}^2 - \frac{k_3 v_r \sin^2 e_3}{k_2} \le 0. \tag{30}$$

Therefore, if the reference velocity $v_r \ge 0$ then $\dot{L}_1 \le 0$. Differentiating (29), we obtain

$$\begin{aligned} \dot{L}_2 &= s^T(t)\dot{s} + \frac{1}{\rho_1}\tilde{\gamma}_1\dot{\tilde{\gamma}}_1 + \frac{1}{\rho_2}\tilde{\gamma}_2\dot{\tilde{\gamma}}_2 \\ &= s^T(t)\left[\dot{v}_c(t) - \dot{v}(t) + \beta e_c(t)\right] \\ &\quad + \frac{1}{\rho_1}\tilde{\gamma}_1\dot{\tilde{\gamma}}_1 + \frac{1}{\rho_2}\tilde{\gamma}_2\dot{\tilde{\gamma}}_2 \end{aligned} \tag{31}$$

$\dot{\tilde{\gamma}}$ is equal to $\dot{\hat{\gamma}}$ because γ^* is constant. Substituting (21) and (25) for (31), we obtain:

$$\begin{aligned} \dot{L}_2 &= s^T(t)\left[-\hat{\gamma}o(s,\varepsilon) - \delta(t)\right] + \frac{1}{\rho_1}\tilde{\gamma}_1\dot{\hat{\gamma}}_1 + \frac{1}{\rho_2}\tilde{\gamma}_2\dot{\hat{\gamma}}_2 \\ &= s^T(t)\left[-(\gamma^* + \tilde{\gamma}(t))o(s,\varepsilon) - \delta(t)\right] \\ &\quad + \frac{1}{\rho_1}\tilde{\gamma}_1\dot{\hat{\gamma}}_1 + \frac{1}{\rho_2}\tilde{\gamma}_2\dot{\hat{\gamma}}_2 \\ &= s^T(t)\left[-\gamma^* o(s,\varepsilon) - \delta(t)\right] \\ &\quad + \sum_{i=1}^{2}\tilde{\gamma}_i(t)\left[\frac{1}{\rho_i}\dot{\hat{\gamma}}_i - s_i o(s,\varepsilon)\right] \end{aligned} \tag{32}$$

From the adaptive law (26), one can get:

$$\frac{1}{\rho_i}\dot{\hat{\gamma}}_i - s_i o(s,\varepsilon) = 0, \forall t \ge 0 \tag{33}$$

Substituting (33) for (32)

$$\begin{aligned} \dot{L}_2 &= s^T(t)\left[-\gamma^* o(s,\varepsilon) - \delta(t)\right] \\ &= -\left[\gamma_1{}^* s_1 o(s_1,\varepsilon) + \gamma_2{}^* s_2 o(s_2,\varepsilon) + \delta_1(t)s_1 + \delta_2\right. \\ &\le \sum_{i=1}^{2}\left|\delta_i(t)\right| s_i o(s_i,\varepsilon) - \gamma_i{}^* s_i o(s_i,\varepsilon) \\ &= -\sum_{i=1}^{2} s_i o(s_i,\varepsilon)(\gamma_i{}^* - \left|\delta_i(t)\right|) \le 0 \end{aligned} \tag{34}$$

Then introducing the following term:

$$\kappa(t) = \sum_{i=1}^{2} s_i o(s_i,\varepsilon)(\gamma_i{}^* - \left|\delta_i(t)\right|) = -\dot{L}_2 \tag{35}$$

Because L_2 is non increasing and bounded, we can find:

$$\int_0^t \kappa(\tau)d\tau \le L_2(s(0),\tilde{\gamma}(0)) - L_2(s(t),\tilde{\gamma}(t)) \tag{36}$$

Also, as κ is bounded, it can be shown that $\lim_{t\to\infty}\kappa(t) = 0$ by Barbalat's lemma. We have also $\lim_{t\to\infty} s(t) = 0$. From (30) and (34) we can conclude that $\dot{L}$ is negative semi-definite. That is the posture tracking q_e and sliding surface s approach zeros vectors. It is noted in (14) that once $s = 0$, then $e_c = -\beta\int e_c(\tau)d\tau$ and it is clear that $e_c(\infty) \to 0$. This completes the proof of theorem.

IV. SIMULATION RESULTS

To demonstrate the performance and robustness of the proposed controller, in the presence of uncertainties and disturbances, extensive simulations for wheeled mobile robot shown in Fig. 1 are presented in this study.

Actual parameters of wheeled mobile robot are:

$b = 0.15\ m,\ r = 0.03\ m,\ I = 3.75\ kg\ m^2,\ m = 4\ kg.$

The reference trajectory as a function of time is selected as: $x_r(t) = 0.2t + 0.3,\ y_r(t) = 0.5 + 0.25\sin(0.2\pi t).$

It consists of a sinusoidal path for the wheeled mobile robot.

The variation of mass and inertia of WMR is described as follows: $\begin{cases} m: 4 \to 6kg \\ I: 3.75 \to 5kg\ m^2 \end{cases}$

The external disturbance imposed on the wheels of mobile robot is set at t>10s as: $\tau_d = \left[2\sin(3t)\ \ 2\cos(3t)\right]^T$

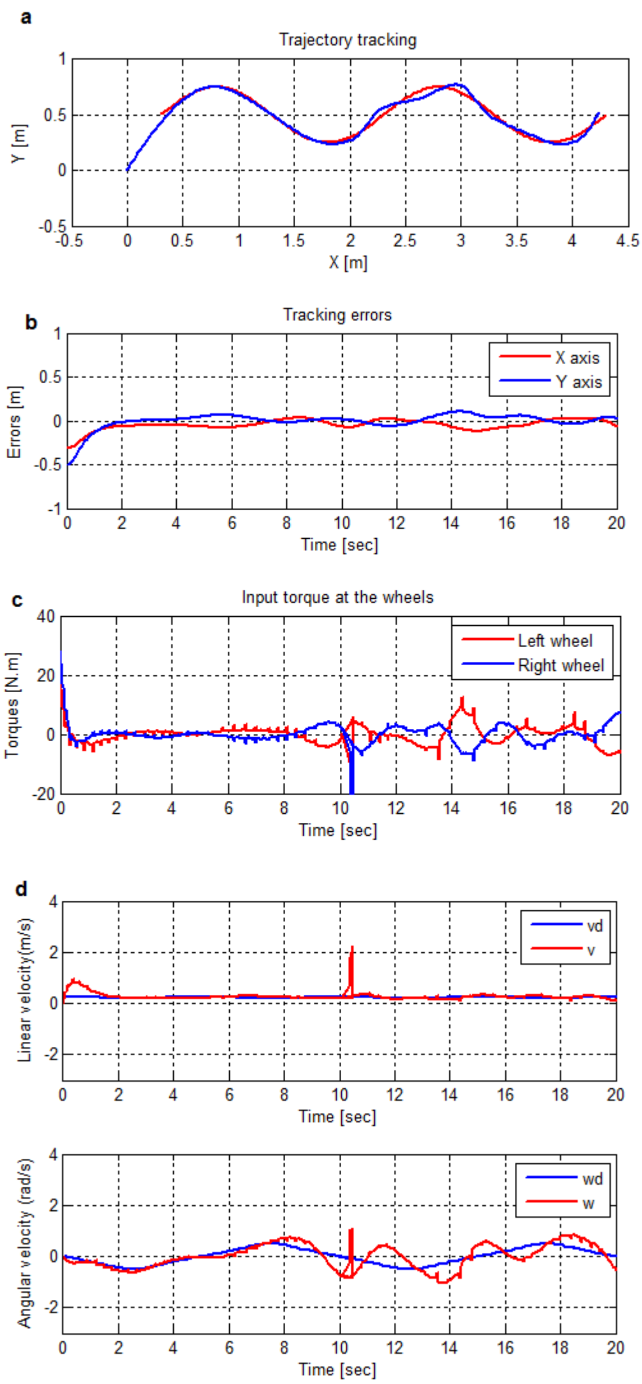

Figure. 2. Tracking performance with SMDC under uncertainties and disturbances: (a) The trajectory tracking. (b) The tracking errors. (c) Input torques at the wheels. (d) Linear and angular velocities.

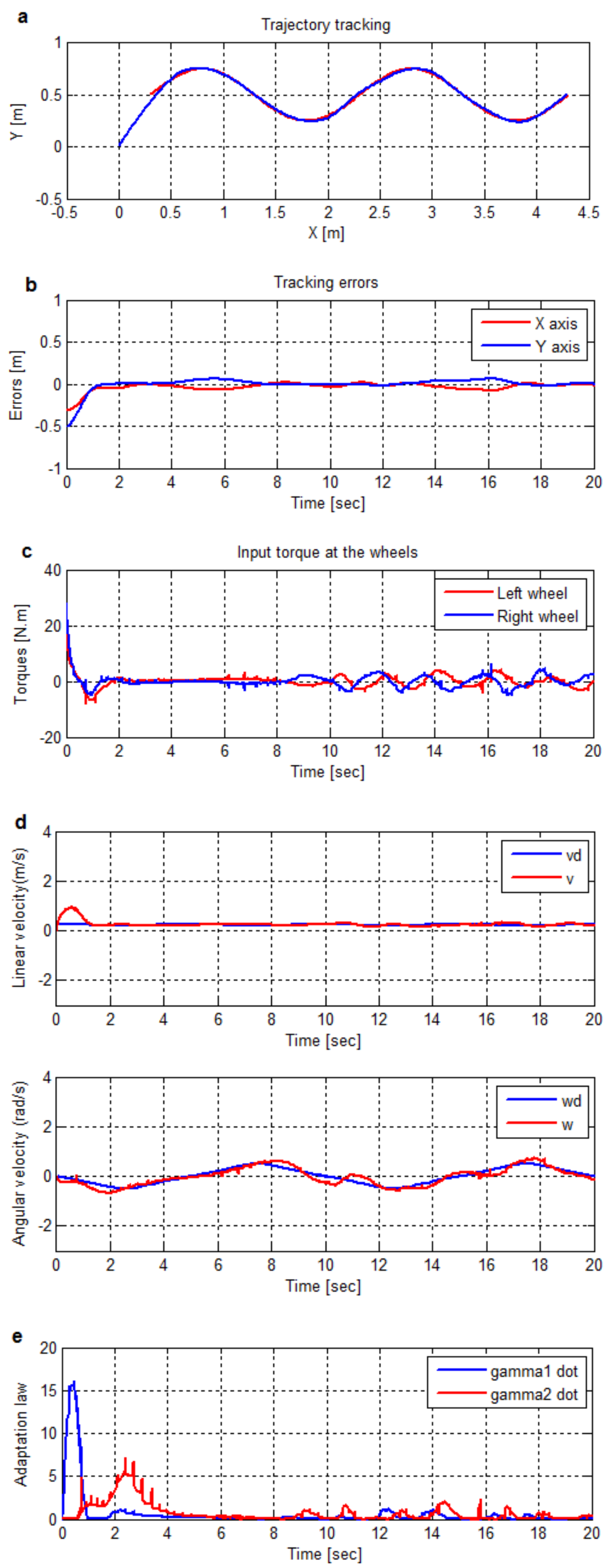

Figure. 3. Tracking performance with ASMDC under uncertainties and disturbances: (a) The trajectory tracking. (b)

The tracking errors. (c) Input torques at the wheels. (d) Linear and angular velocities. (e) The variation in adaptive law.

Figures 2 and 3 reveal that the effect of parameters variations are almost zeros for SMDC and ASMDC. These two controllers have excellent ability to remove the parametric uncertainties (mass and inertia of WMR). However, there is obviously a difference when external disturbances occurred. After 10s, the tracking ability of SMDC is highly degraded, but the tracking efficacy of ASMDC is still maintained. In addition the control torques shown in Figures 3.c is very smooth, if we compare it with the control torques obtained by SMDC (Figure 2.c). Figure 3.e illustrates the adaptive law (24) and they tend toward some finite values.

V. CONCLUSION

In this paper, sliding mode and adaptive sliding mode dynamic controllers have been presented to control a noholonomic mobile robot in presence of disturbances and uncertainties. The Lyapunov stability theory has been utilized to verify the convergence of the trajectory-tracking errors. Simulation results have demonstrated that the ASMDC scheme gives the best performance in comparison with SMDC as system uncertainties and external disturbances appeared.

REFERENCES

[1] S. Blazic, A novel trajectory-tracking control law for wheeled mobile robots, Robotics and Autonomous Systems 59 (2011) 1001–1007.

[2] G. Yanfeng, Z. Hua,Y. Yanhui,a, Back-Stepping and Neural Network Control of a Mobile Robot for Curved Weld Seam Tracking, Procedia Engineering 15 (2011) 38 – 44

[3] W. E. Dixon, Z. P. Jiang, D. M. Dawson, Global exponential setpoint control of wheeled mobile robots: a Lyapunov approach, Automatica 36 (2000) 1741-1746.

[4] R. Ferro and FL. Lewis, Control of a nonholonomic mobile robot: backstepping kinematics into dynamics, Journal Robot System (1997) 1393-9

[5] B. Zhou, J. Han, X. Dai, Backstepping Based Global Exponential Stabilization of a Tracked Mobile Robot with Slipping Perturbation, Journal of Bionic Engineering 8 (2011) 69–76.

[6] J. Yang, R. Ma, Y. Zhang, C. Zhao, Sliding Mode Control for Trajectory Tracking of Intelligent Vehicle, International Conference on Medical Physics and Biomedical Engineering, Physics Procedia 33 (2012) 1160 – 1167.

[7] L. Gracia, F. Garelli, A. Sala, Integrated sliding-mode algorithms in robot tracking applications, Robotics and Computer-Integrated Manufacturing 29 (2013) 53–62.

[8] F N. Martins , W C. Celeste , R. Carelli, M r Sarcinelli-Filho, T F. Bastos-Filho, An adaptive dynamic controller for autonomous mobile robot trajectory tracking, Control Engineering Practice 16 (2008) 1354– 1363.

[9] F. Pourboghrat , M P. Karlsson, Adaptive control of dynamic mobile robots with nonholonomic constraints, Computers and Electrical Engineering 28 (2002) 241–253.

[10] F. Gao, F. Yuan, H. Yao, Robust adaptive control for nonholonomic systems with nonlinear parameterization, Nonlinear Analysis: Real World Applications 11 (2010) 3242_3250.

[11] R. Fierro, F. L. Lewis, Control of a Nonholonomic Mobile Robot Using Neural Networks, IEEE transactions on neural networks, vol. 9, no. 4, july 1998.

[12] J. Ye, Tracking control for nonholonomic mobile robots: Integrating the analog neural network into the backstepping technique, Neurocomputing 71 (2008) 3373–3378.

[13] S K. Pradhan, D R. Parhi, A K. Panda, Fuzzy logic techniques for navigation of several mobile robots, Applied Soft Computing 9 (2009) 290–304.

[14] Y. Cai , Q. Zhan, X. Xi, Path tracking control of a spherical mobile robot, Mechanism and Machine Theory 51 (2012) 58–73.

[15] J.J. E Slotine, W. Li, Applied Nonlinear Control, Prentice-Hall Inc., 1991.

[16] H. Elmali, N. Olgac, Theory and implementation of sliding mode control with perturbation estimation, IEEE Int. Conf. Robotics Autom. 3 (1992) 2114–2119.

[17] N. Kim, C.W. Lee, P.H. Chang, Sliding mode control with perturbation estimation: application to motion control of parallel manipulators, Control Eng. Pract. 6 (11) (1998) 1321–1330.

[18] C.-Y. Chen, T.-H S. Li, Y.-C. Yeh, EP-based kinematic control and adaptive fuzzy sliding-mode dynamic control for wheeled mobile robots.

[19] T. Sun, H. Pei, Y. Pan, H. Zhou, C. Zhang, Neural network-based sliding mode adaptive control for robot manipulators, Neurocomputing 74 (2011) 2377-2384.

[20] M. Zeinali, L. Notash, Adaptive sliding control with uncertainty estimator for robot manipulators, Mechanism and Machine Theory 45 (2010) 80-90.

[21] C.-Y.Chen, T.-H S. Li, Y.-C. Yeh, C.-C. Chang, Design and implementation of an adaptive sliding-mode dynamic controller for wheeled mobile robots, Mechatronics 19 (2009) 156-166.

[22] N. Chen, F. Song, G. li, X. Sun, C. Ai, An adaptive sliding mode backstepping control for mobile manipulator with nonholonomic constraints, Commun Nonlinear Sci Numer Simulat (2013).