

Design and Analysis of Algorithms Lab

Academic Year: 2020 - 21

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Department of Computer Science and Engineering
Ecole Centrale School of Engineering



- 1 Use Selection Sort and Insertion Sort techniques to sort a set of student records by considering a specified field (Hall Ticket Number, Name, or Team Number).
- 2 Use Selection Sort and Insertion Sort techniques to sort a set of student records by considering all the fields in a specific order (Team Number, Hall Ticket Number, and Name).

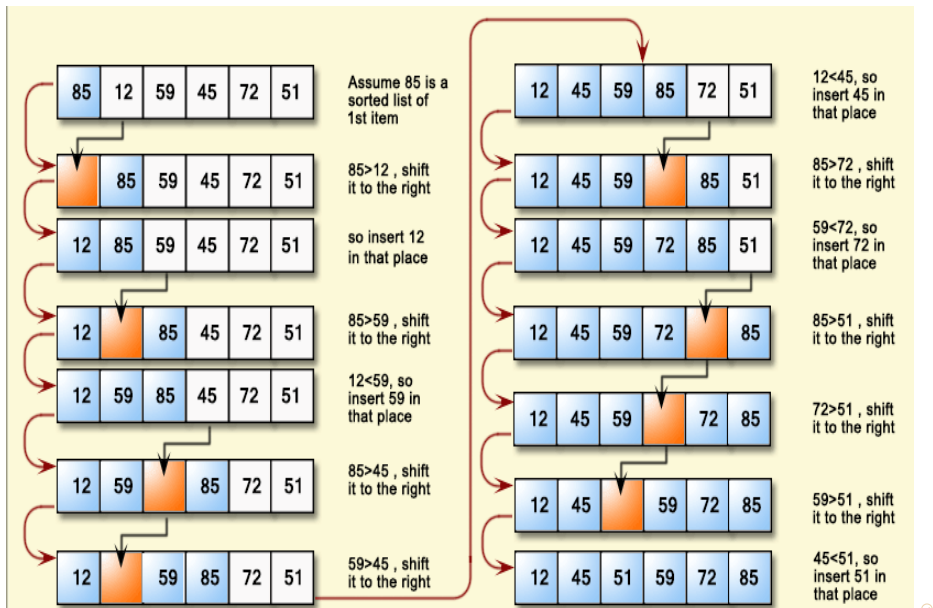
Note:

- ▶ Input should be read from a file **DAA Lab_input1.txt**
- ▶ Output should be written into a file **DAA Lab_output1.txt**

Logic: Selection Sort



Logic: Insertion Sort



DAA Lab 2 Due Date: February 14, 2021

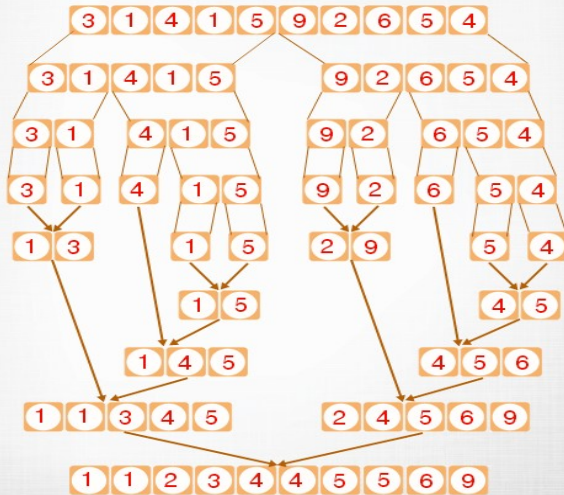
- 1 Use Merge Sort and Quick Sort techniques to sort a set of student records by considering a specified field (Hall Ticket Number, Name, or Team Number).
- 2 Use Merge Sort and Quick Sort techniques to sort a set of student records by considering all the fields in a specific order (Team Number, Hall Ticket Number, and Name).

Note:

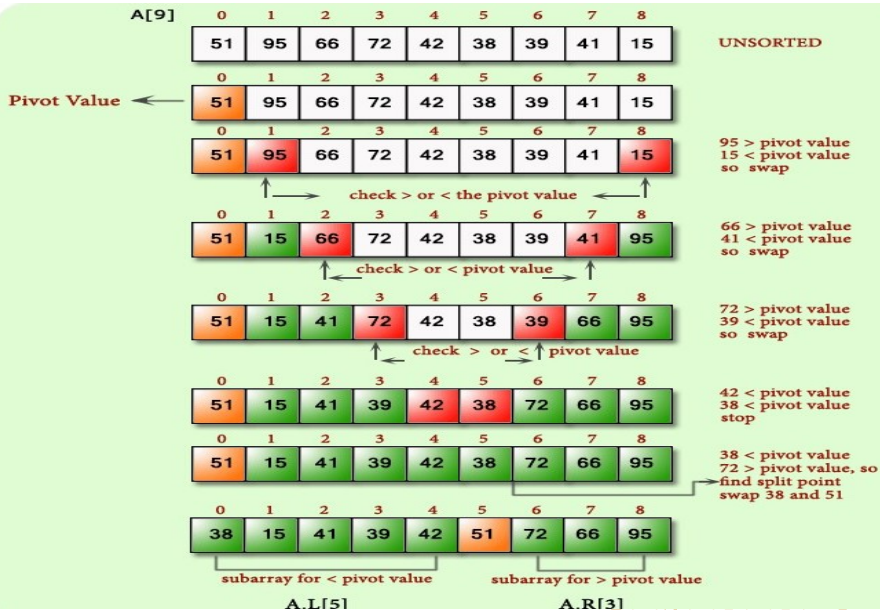
- ▶ Input should be read from a file **DAA Lab_input1.txt**
- ▶ Output should be written into a file **DAA Lab_output1.txt**

Logic: Merge Sort

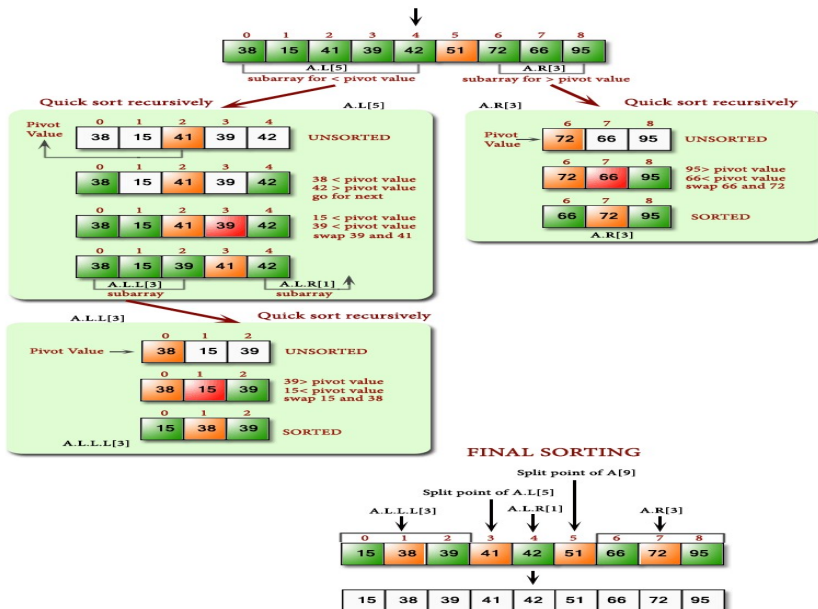
MERGE SORTING ON 3,1,4,1,5,9,2,6,5,4



Logic: Quick Sort



Logic: Quick Sort



- 1 Use Linear Search technique to search a student record by considering a specified field (Hall Ticket Number, Name, or Team Number).
- 2 Use Binary Search technique to search a student record by considering a specified field (Hall Ticket Number, Name, or Team Number).

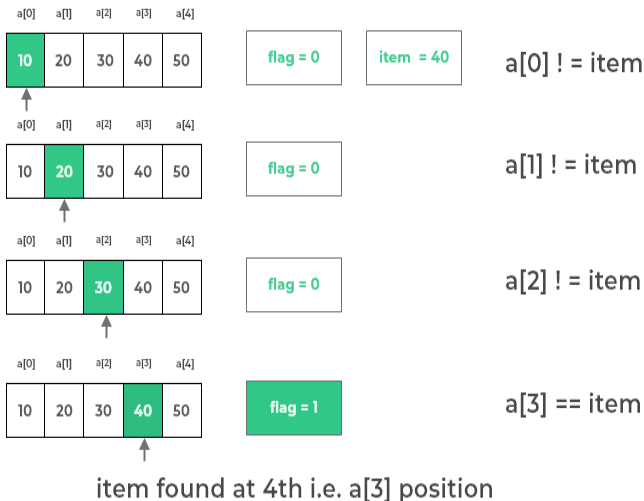
Note:

- ▶ Input should be read from a file **DAALab_input1.txt**
- ▶ Output should be written into a file **DAALab_output1.txt**

Bonus:

- 1 Use Fibonacci Search technique to search a student record by considering a specified field (Hall Ticket Number, Name, or Team Number).

Linear Search



Binary Search

$$M = \frac{(L + R)}{2}$$

or

$$M = L + \frac{(R - L)}{2}$$

Search 15

0	1	2	3	4	5	6	7	8	9
3	5	7	9	12	15	16	18	19	22

L = 0

M = 4

R = 9

0	1	2	3	4	5	6	7	8	9
3	5	7	9	12	15	16	18	19	22

L = 5

M = 7

R = 9

0	1	2	3	4	5	6	7	8	9
3	5	7	9	12	15	16	18	19	22

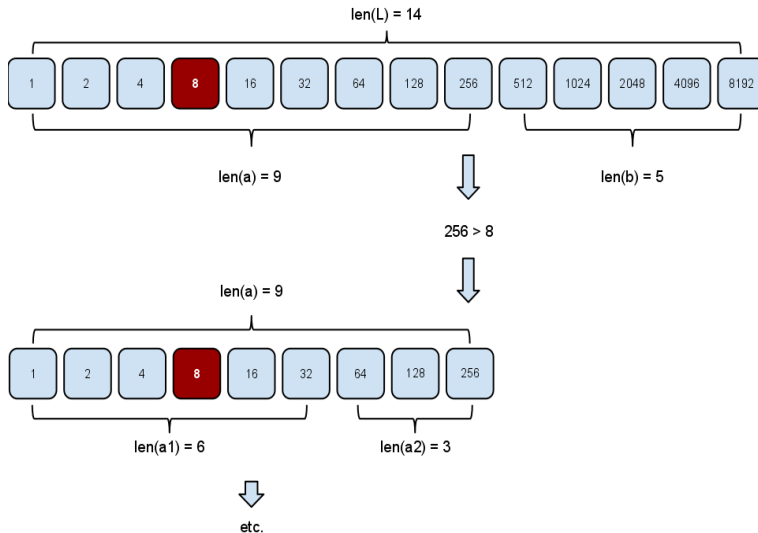
M = 5

L = 5 R = 6

0	1	2	3	4	5	6	7	8	9
3	5	7	9	12	15	16	18	19	22

Found at M = 5

Logic: Fibonacci Search



- 1 Use a Tree Sort technique to sort a set of student records by considering Hall Ticket Number.
- 2 Develop a program to multiply two square-matrices of order 1024×1024 using Block Matrix Multiplications by considering the block sizes: 4, 8, 16, 32, and 64. Use `gettimeofday()` for calculating *runtime* (the average of 5 runs). Draw a plot using *runtime* and block-size.

Note:

- ▶ Input should be read from a file **DAALab_input1.txt**
- ▶ Output should be written into a file **DAALab_output1.txt**

Logic: Tree Sort

Elements of Input Array

14

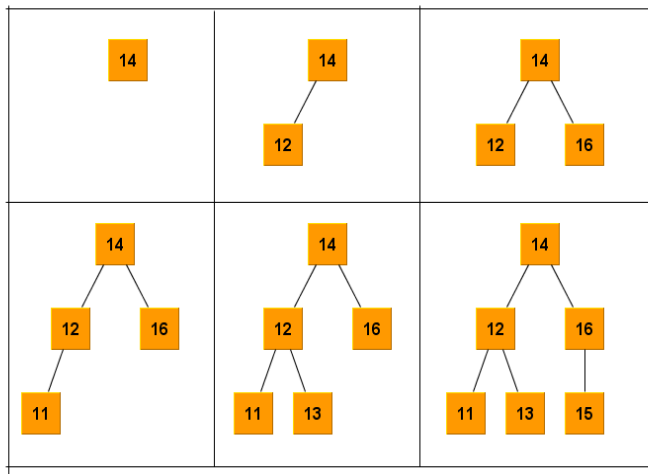
12

16

13

11

15



- 1 Develop a program for the Defective Chessboard problem ($N=1024$, 2048 , and 4096). Use *gettimeofday()* for calculating *runtime* (the average of 5 runs).
- 2 Develop a program to multiply two square-matrices of order 1024×1024 using Strassen's Matrix Multiplication. Use *gettimeofday()* for calculating *runtime* (the average of 5 runs).

Bonus Problem Statements:

- 1 Given an array of n numbers and a positive integer i , write a program to find the i^{th} smallest element that runs in $O(n)$ time.
- 2 Given two sorted arrays, each consisting of n numbers, write a program to find the median of $2n$ elements that runs in $O(\log n)$ time.

Logic: Defective Chessboard

A chessboard that has one unavailable square. We have to cover the remaining squares using triominos.

(Triomino is an **L** shaped object and it is formed with three squares.)



2x2



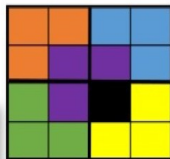
2x2



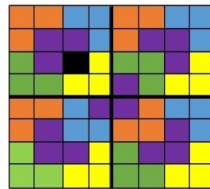
2x2



2x2



4x4

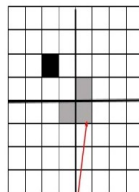
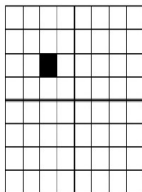
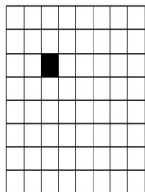


8x8

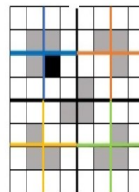
Black color square is the defective one.

Number of triomino's required for an $n \times n$ defective chess board: $\frac{n^2-1}{3}$.

8 X 8 Defective Chessboard



Creation of defective box



DIVISION OF
PROBLEM INTO SUB
PROBLEM

- 1 Divide the chessboard into 4 equal parts.
- 2 Identify the part which has the defective square and put a triomino that cover all the remaining three parts.
- 3 Now assume that all 4 parts are defective chessboards.
- 4 Repeat the steps 1 to 3 until all the squares are covered with triominoes.

Defective Chessboard: Analysis

$$\begin{aligned}T(n) &= 4 \cdot T\left(\frac{n}{2}\right) + \mathcal{O}(1) \\&= 4 \cdot T\left(\frac{n}{2}\right) + \text{constant} \\&= 4 \cdot T\left(\frac{n}{2}\right) + \text{constant} \\&= \Theta(n^2)\end{aligned}$$

Reasoning:

From case 1 of Master Theorem, where $a=4$, $b=2$, and $f(n) = \mathcal{O}(1)$

$$n^{\log_b a} = n^{\log_2 4}$$

$$f(n) = n^{\log_2 4 - \epsilon}, \text{ where } \epsilon = 2$$

So, $f(n)$ is polynomially less than $n^{\log_2 4} = n^2$.

$$\therefore T(n) = \Theta(n^2)$$

Logic: Strassen's Matrix Multiplication

$$M_1 = (A_{11} + A_{22}) \cdot (B_{11} + B_{22})$$

$$M_2 = (A_{21} + A_{22}) \cdot B_{11}$$

$$M_3 = A_{11} \cdot (B_{12} - B_{22})$$

$$M_4 = A_{22} \cdot (B_{21} - B_{11})$$

$$M_5 = (A_{11} + A_{12}) \cdot B_{22}$$

$$M_6 = (A_{21} - A_{11}) \cdot (B_{11} + B_{12})$$

$$M_7 = (A_{12} - A_{22}) \cdot (B_{21} + B_{22})$$

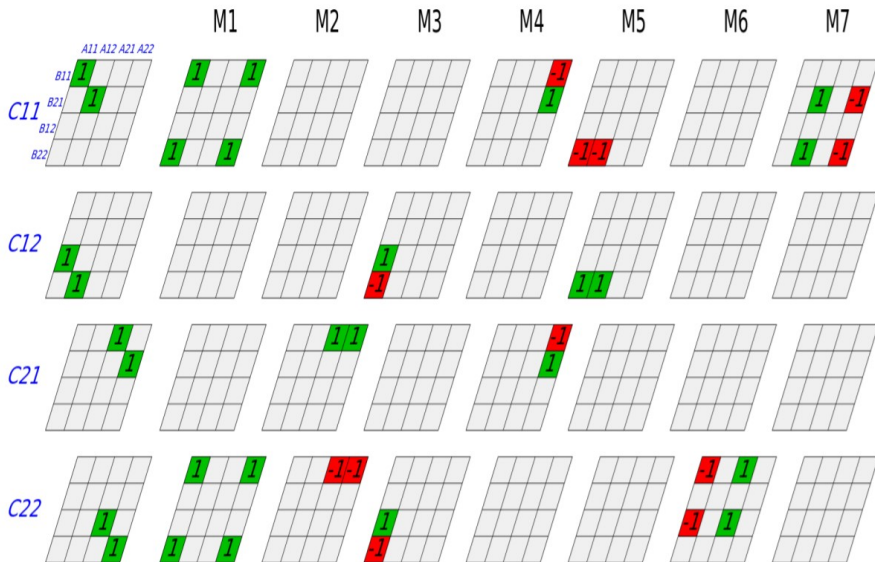
$$C_{11} = M_1 + M_4 - M_5 + M_7$$

$$C_{12} = M_3 + M_5$$

$$C_{21} = M_2 + M_4$$

$$C_{22} = M_1 - M_2 + M_3 + M_6$$

Strassen's Matrix Multiplication



Strassen's Matrix Multiplication: Analysis

$$\begin{aligned}T(n) &= 7 \cdot T\left(\frac{n}{2}\right) + 18 \cdot \mathcal{O}\left(\frac{n^2}{4}\right) \\&= 7 \cdot T\left(\frac{n}{2}\right) + \mathcal{O}(n^2) \\&= 7 \cdot T\left(\frac{n}{2}\right) + c \cdot n^2 \\&= \Theta(n^{2.81})\end{aligned}$$

Reasoning:

From case 1 of Master Theorem, where $a=7$, $b=2$, and $f(n) = \mathcal{O}(n^2)$
 $n^{\log_b a} = n^{\log_2 7}$

$f(n) = n^{\log_2 7 - \epsilon}$, where $\epsilon = 0.81$

So, $f(n)$ is polynomially less than $n^{\log_2 7} = n^{2.81}$.

$\therefore T(n) = \Theta(n^{2.81})$

- ❶ **Kanpsack Problem:** We are given with n objects and a knapsack with capacity M . Let $w_1, w_2, w_3, \dots, w_n$ and p_1, p_2, \dots, p_n be the weights and profits of n objects, respectively. If we place a fraction x_i , ($0 \leq x_i \leq 1$) of object i into the Knapsack, then we get a profit $p_i \cdot x_i$ and kanpsack capacity is reduced by $M - w_i \cdot x_i$. Write a program to find a solution vector $(x_1, x_2, x_3, \dots, x_n)$ in such a way that we have to get the maximum profit.
- ❷ **Job Sequencing with Deadlines:** We are given with a machine and a set of n jobs. Each job i has an integer deadline (d_i) and a profit (p_i). Execution time of any job is one unit. If a job i is executed within its deadline, then we get profit p_i . Write a program to find a solution vector $(x_1, x_2, x_3, \dots, x_n)$ in such a way that we have to get the maximum profit.

An Example of Knapsack Problem

Objects	1	2	3	4	5	6	7
Profit	10	5	15	7	6	18	3
Weight	2	3	5	7	1	4	1

M 15

An Example of Job Sequencing with Deadlines Problem

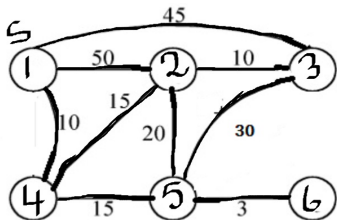
Job	Deadline	Profit
1	2	40
2	4	15
3	3	60
4	2	20
5	3	10
6	1	45
7	1	55

- 1 **Single Source Shortest Path (SSSP):** Given a connected weighted graph (weights represent the distances between two vertices), write a program to find a shortest path from a given source vertex 's' to every other vertex.

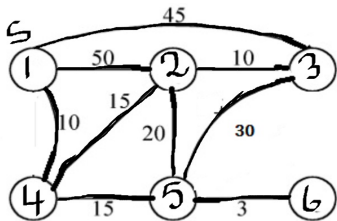
Using the SSSP program find a shortest path between every pair of vertices.

- 2 **Huffman Coding:** Write a program to compress and decompress a file using a Huffman Coding. The uncompressed text file and the original text file should be the same.
(**Size of original file should be ≥ 1 MB**).

Single Source Shortest Path Problem

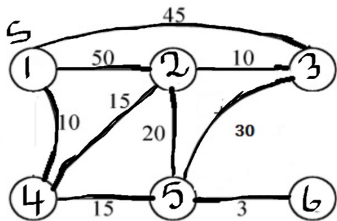


Single Source Shortest Path Problem



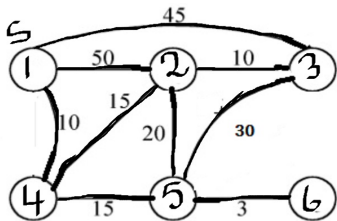
	Adjacency Matrix Representation					
	1	2	3	4	5	6
1	0	50	45	10	---	---
2	50	0	10	15	20	---
3	45	10	0	---	30	---
4	10	15	---	0	15	---
5	---	20	30	15	0	3
6	---	---	---	---	3	0

Single Source Shortest Path Problem



	1	2	3	4	5	6
1	-1	-1	-1	-1	-1	-1
2	-1	1	1	1	-1	-1
3	-1	4	1	1	4	-1
4	-1	4	2	1	4	-1
5	-1	4	2	1	4	5
6	-1	4	2	1	4	5

Single Source Shortest Path Problem



	1	2	3	4	5	6
1	-1	-1	-1	-1	-1	-1
2	-1	1	1	1	-1	-1
3	-1	4	1	1	4	-1
4	-1	4	1	1	4	--
5	-1	4	2	1	4	--
6	-1	4	2	1	4	5

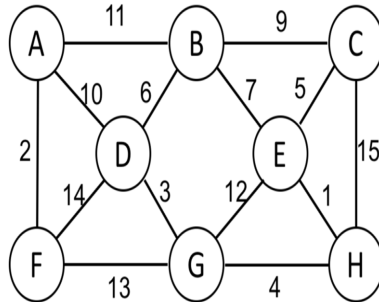
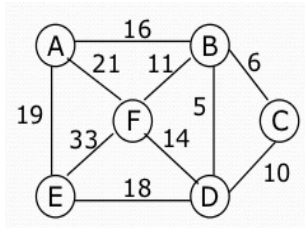
Single Source Shortest Path Problem: Algorithm

Algorithm 1: SSP($n, v_1, \text{Cost}[\text{ }[\text{ }], \text{Dist}[\text{ }], \text{Path}[\text{ }]$)

Result: Shortest Distances from source vertex to all other vertices

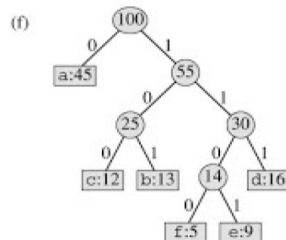
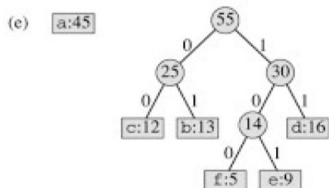
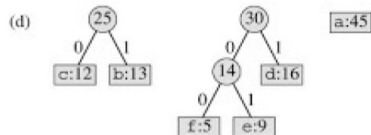
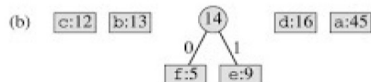
```
1 for  $i = 1$  to  $i \leq n$  do
2    $S[i] = 0; \text{Dist}[i] = \text{Cost}[v_1][i]; \text{Path}[i] = -1;$ 
3  $S[v_1] = 1;$ 
4 for  $i = 2$  to  $i \leq n$  do
5    $u = \text{ChooseMinimumDistanceVertex}(\text{from } V - S);$ 
6    $S[u] = 1;$ 
7   for each  $w$  adjacent to  $u$  and  $S[w] = 0$  do
8     if  $\text{Dist}[w] > \text{Dist}[u] + \text{cost}[u][w]$  then
9        $\text{Dist}[w] = \text{Dist}[u] + \text{cost}[u][w]$ 
10       $\text{Path}[w] = u$ 
```


Find Shortest Paths from vertex A to all other vertices



Huffman Codes

(a) f:5 e:9 c:12 b:13 d:16 a:45



DAA Lab Submission Guide Lines

- ▶ Mail-ID: cs203.daa.mec@gmail.com (Doubt Clarification).
- ▶ Submission Link will be shared.
- ▶ Late Submission (≤ 3 -Days):50% weightage will be given.
- ▶ Write a readme file to understand your solutions.
- ▶ Submit source files only (C or JAVA).

Lab Weightage - 30%.

Lab Instructor: Sri. Brahmaiah G

Reference Books:

- 1 Introduction to Algorithms, 3rd edition, T.H.Cormen, C.E.Leiserson, R.L.Rivest and C.Stein.
- 2 Fundamentals of Computer Algorithms, Ellis Horowitz, Satraj Sahni and Rajasekaran.
- 3 Algorithms, 4th edition, Robert Sedgewick.
- 4 Design and Analysis of Computer Algorithms, Aho, Ullman, and Hopcroft.

Web Resources:

- 1 Algorithms by Robert Sedgewik
- 2 Algorithms by Abdul Bari
- 3 MIT - Open Courseware Videos on Algorithms
- 4 Data Structures and Algorithms