Workshop 1: Research Seminar - Wave Function and Reality

Quantum mechanics is one of the most successful theories in the history of physics. However, one of its central quantities, the wave function, has sparked active and highly controversial discussions over the past century. Is the wave function merely a mathematical construct, or does it have a direct connection to physical reality? This question is deeply intertwined with philosophical considerations about the nature of our universe. Do physical objects have intrinsic properties independent of measurement (objectivism), or do properties only emerge during the measurement process, dependent on an observer (relativism)?

Please download and read the following paper (referred to as "the PBR paper"):
M. Pusey, J. Barrett, & T. Rudolph, *On the reality of the quantum state*, Nat. Phys., **8**, 475 (2012)

Work in teams of 3-5 students!

Learning Objectives:

- Understanding the bra-ket formalism
- Familiarity with the postulates of quantum mechanics
- Distinguishing between pure/mixed and separable/entangled states

Task 1: Definitions and Research Question

a) What is objective reality?

b) Explain the meaning and difference between ψ-ontic and ψ-epistemic models.

c) What research question do Pusey, Barrett, and Rudolph address in their paper?

Task 2: Pure and Mixed States

In the following exercises, we use two orthonormal states: $|0\rangle$ and $|1\rangle$, as well as $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$.

a) Compute the overlap (inner product): $\langle 0 \mid +\rangle$.

b) If a wave function is prepared in the state $|+\rangle$, what is the probability of measuring an observable corresponding to $|0\rangle$? If a wave function is prepared in the state $|-\rangle$, what is the probability of measuring an observable corresponding to $|1\rangle$?

c) What is the definition of a pure and a mixed state?

d) Write down the density matrix of a pure state with a 50% probability of measuring $|0\rangle$. Then, write the density matrix of a mixed state with a 50% probability of measuring $|0\rangle$.

Task 3: Separable and Entangled States

Consider a laboratory that prepares a quantum state $|\psi\rangle$, which can be either $|0\rangle$ or $|+\rangle$. Assume a second, independent laboratory prepares another quantum state $|\phi\rangle$, which can also be either $|0\rangle$ or $|+\rangle$.

a) What are the four possible states $|\psi\rangle \otimes |\phi\rangle$? Are these states separable or entangled?

b) Write down a separable state and a maximally entangled state using $|+\rangle$ and $|-\rangle$. Which state from equation (1) of the PBR paper is related to your solution?

c) Compute the overlap of $|0\rangle \otimes |+\rangle$ with the four states $|\xi_i\rangle$ ($i = 1, \ldots, 4$) from equation (1) of the PBR paper. Are the states $|\xi_i\rangle$ separable or entangled?

Task 4: The "No-Go Theorem" by Pusey, Barrett, and Rudolph
We focus on the "simple" example on page 2 of the PBR paper. The quantum states $|0\rangle$ and $|+\rangle$ overlap as demonstrated in Task 2(a). Assume that their classical probability distributions, compatible with the classical state λ, also overlap (see Fig. 1(b) in the PBR paper). The overlap region is called Δ, and the probability of preparing either quantum state resulting in $\lambda \in \Delta$ is denoted by q.

a) What is the probability of preparing a quantum state $|0\rangle \otimes |+\rangle$ resulting in $\lambda_1, \lambda_2 \in \Delta$?

b) A measurement device is capable of measuring the states $|\xi_i\rangle$ given in equation (1) of the PBR paper (see Fig. 2 in the paper). Is every measurement outcome possible for an incoming state $|+\rangle \otimes |+\rangle$?

c) How do Pusey, Barrett, and Rudolph arrive at a contradiction in their argument?

d) According to PBR, is quantum mechanics more likely to be ψ-ontic or ψ-epistemic?

e) Do Pusey, Barrett, and Rudolph suggest that the wave function is a real, physical object? How does their result relate to prominent interpretations of quantum mechanics, such as hidden variable theories or the many-worlds interpretation?