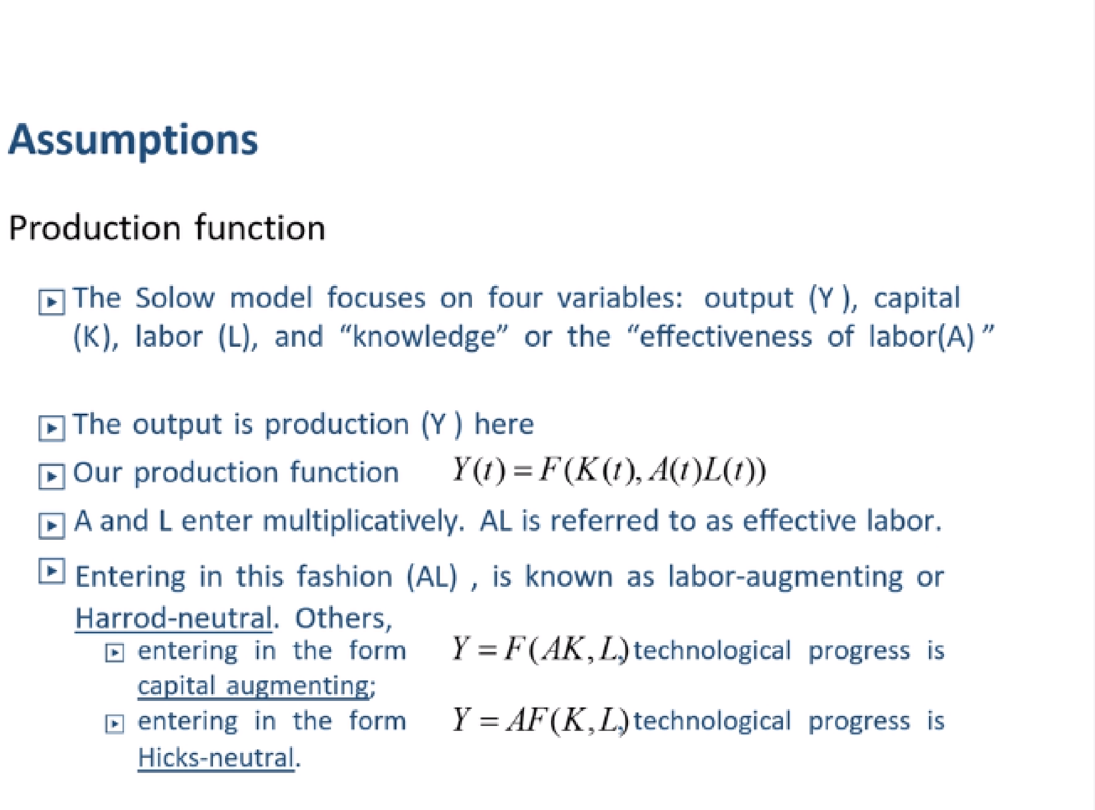
The Solow model



Assumptions

constant return to scale

$$F(cK, cAL) = cF(K, AL) for_a ll_c >= 0 \tag{1}$$

f = (k)

marginal products of capital and labor

$f(0) = 0, f'(k) > 0, f''(k) < 0$

Inada conditions 稻田条件

$\lim_{k\to 0} f'(k) = \infty$
$\lim_{k\to \infty} f'(k) = 0$

柯布道格拉斯满足

$$\begin{aligned} f(k) &= F\left(\frac{K}{AL}, 1\right) \\ &= \left(\frac{K}{AL}\right)^{\alpha} \\ &= k^{\alpha} \end{aligned}$$

$$\text{自然对数增长：} \frac{dlnX(t)}{dt} = \frac{L'(t)}{L(t)} = n \tag{2}$$

劳动力和技术增长

$$\frac{A'(t)}{A(t)} = g \tag{3}$$

$X(t) = X(0) \cdot e^{ct}$

In a closed economy, output Y (the produced good is homogeneous and it can change from consumption into investment good at no cost.)

$Y(t) = I(t) + C(t)$

simply the 分配过程

$I(t) = s \cdot Y(t)$

引入资本存量K(t) 其中δ是折旧率

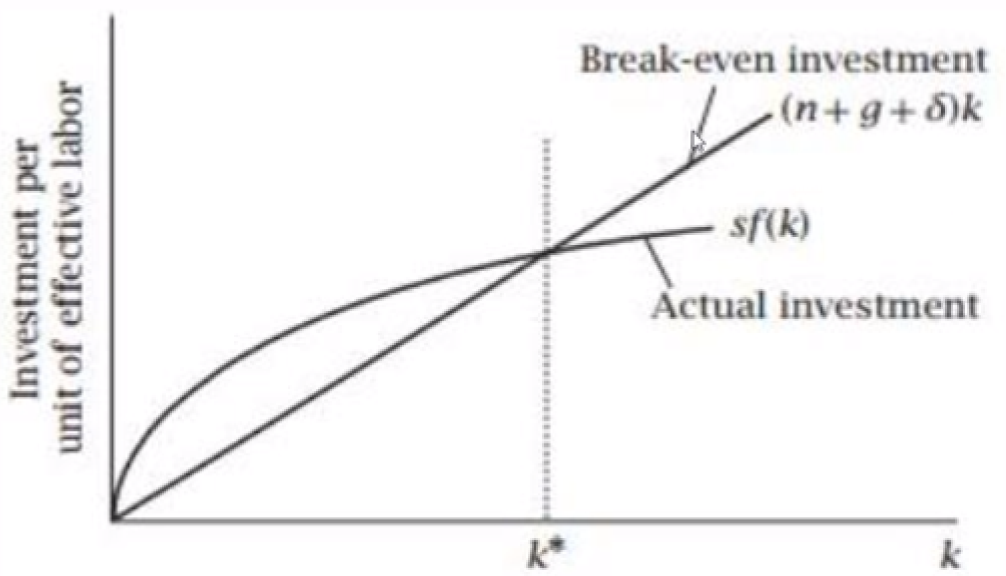
$\dot{K}(t) = I(t) - \delta \cdot K(t)$
$\dot{K}(t) = s \cdot Y(t) - \delta \cdot K(t)$

$k(t) = \frac{K(t)}{A(t)L(t)}$

对上式子两侧求导，带入得到

$\dot{k}(t) = s \cdot f(k(t)) - (n + g + \delta) \cdot k(t)$

如下



Actual and break-even investment （持平投资）
Break-even investment is to keep k at existing level

the balanced growth path

$\dot{K}/K = n + g$

$\dot{Y}/Y = n + g$

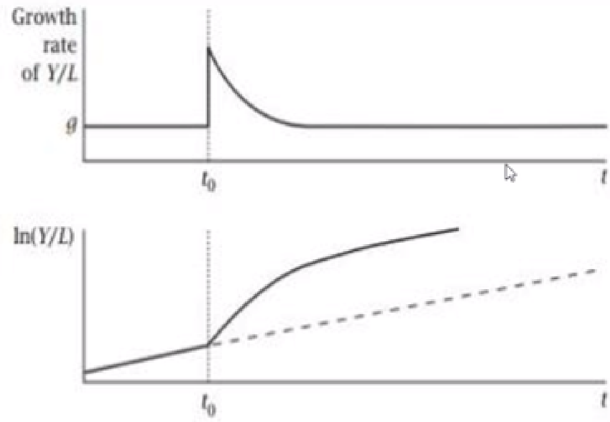
$(K\dot{/}L)/(K/L) = g$

$(Y\dot{/}L)/(Y/L) = g$

The impact of a change in the saving rate

养老金是一种强制储蓄

当s上升时，k star 会增加



The increase of the growth rate

A higher path parallel

The balanced growth paths

The golden-rule （黄金律）

- s is at the level that causes $f'(k^*)$ to just equal $n+g+\delta$ that is, the f(k) and $(n+g+\delta)\cdot k$ are parallel at $k=k^*$.
- A marginal change in s has no effect on consumption in the long run, and consumption is at its maximum possible level among balanced growth paths (Why?).
- This value of k^* is known as the golden-rule level of the capital stock.