

## Assumptions

### Production function

- ▢ The Solow model focuses on four variables: output ( $Y$ ), capital ( $K$ ), labor ( $L$ ), and “knowledge” or the “effectiveness of labor( $A$ )”
- ▢ The output is production ( $Y$ ) here
- ▢ Our production function  $Y(t) = F(K(t), A(t)L(t))$
- ▢  $A$  and  $L$  enter multiplicatively.  $AL$  is referred to as effective labor.
- ▢ Entering in this fashion ( $AL$ ), is known as labor-augmenting or Harrod-neutral. Others,
  - ▢ entering in the form  $Y = F(AK, L)$  technological progress is capital augmenting;
  - ▢ entering in the form  $Y = AF(K, L)$  technological progress is Hicks-neutral.

Assumptions

constant return to scale

$$F(cK, cAL) = cF(K, AL) \text{ for all } c \geq 0 \quad (1)$$

$f = (k)$

marginal products of capital and labor

$$f(0) = 0, f'(k) > 0, f''(k) < 0$$

Inada conditions 稻田条件

$$\lim_{k \rightarrow 0} f'(k) = \infty$$

$$\lim_{k \rightarrow \infty} f'(k) = 0$$

柯布道格拉斯满足

$$\begin{aligned} f(k) &= F\left(\frac{K}{AL}, 1\right) \\ &= \left(\frac{K}{AL}\right)^\alpha \\ &= k^\alpha \end{aligned}$$

$$\text{自然对数增长: } \frac{d \ln X(t)}{dt} = \frac{L'(t)}{L(t)} = n \quad (2)$$

劳动力和技术增长

$$\frac{A'(t)}{A(t)} = g \quad (3)$$

$$X(t) = X(0) \cdot e^{ct}$$

In a closed economy, output  $Y$  (the produced good is homogeneous and it can change from consumption into investment good at no cost.)

$$Y(t) = I(t) + C(t)$$

simply the 分配过程

$$I(t) = s \cdot Y(t)$$

引入资本存量 $K(t)$  其中 $\delta$ 是折旧率

$$\dot{K}(t) = I(t) - \delta \cdot K(t)$$

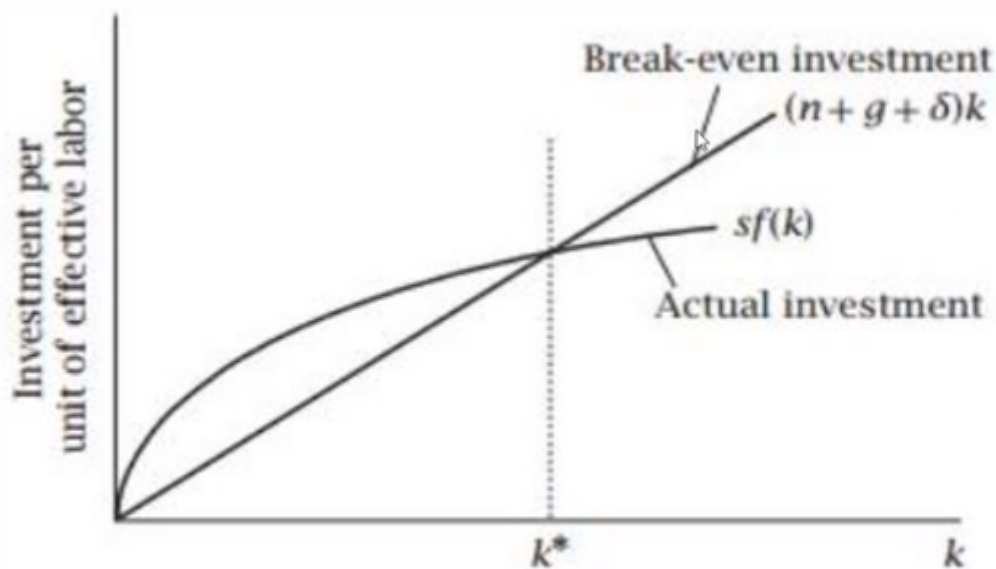
$$\dot{K}(t) = s \cdot Y(t) - \delta \cdot K(t)$$

$$k(t) = \frac{K(t)}{A(t)L(t)}$$

对上式子两侧求导，带入得到

$$\dot{k}(t) = s \cdot f(k(t)) - (n + g + \delta) \cdot k(t)$$

如下



Actual and break-even investment (持平投资)

Break-even investment is to keep  $k$  at existing level

the balanced growth path

$$\dot{K}/K = n + g$$

$$\dot{Y}/Y = n + g$$

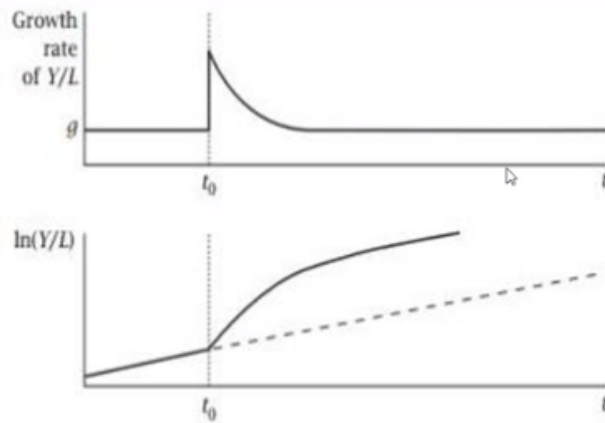
$$(\dot{K}/L)/(K/L) = g$$

$$(\dot{Y}/L)/(Y/L) = g$$

The impact of a change in the saving rate

养老金是一种强制储蓄

当 $s$ 上升时,  $k^*$  会增加



The increase of the growth rate

A higher path parallel

## The balanced growth paths

### The golden-rule (黄金律)

- ▶  $s$  is at the level that causes  $f'(k^*)$  to just equal  $n + g + \delta$  that is, the  $f(k)$  and  $(n + g + \delta) \cdot k$  are parallel at  $k = k^*$ .
- ▶ A marginal change in  $s$  has no effect on consumption in the long run, and consumption is at its maximum possible level among balanced growth paths (Why?).
- ▶ This value of  $k^*$  is known as the golden-rule level of the capital stock.