

# Selection Diversity Based ARQ Schemes and Their Enhancements Using Packet Combining in a Slow Rayleigh Fading Channel

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## ABSTRACT

Antenna diversity is a widely studied strategy to combat fading processes in wireless systems. While packet combining may further enhance receiver diversity, this issue has not received much attention. In this article, we study ARQ schemes with or without receiver diversity. An important outcome of this work is that significant performance gain can be achieved with packet combining along with selection diversity.

## I. INTRODUCTION

Selection combining (SC) is considered one of the simplest implementations of space diversity reception. In the post detection SC scheme, one of  $M$  receivers (Figure 1) receiving identical transmission from a single transmitter, is chosen to the output. The SC scheme is considered feasible only for TDMA (time division multiple access) systems, but not for continuous transmission systems [1]. However, its performance is evaluated under the assumption of continuous branch selection. When the branches are uncorrelated and identical, the mean SNR of the combined signal is given by [1,2]

$$\langle \gamma_s \rangle = \Gamma \sum_{k=1}^M \frac{1}{k}, \quad (1)$$

where  $\Gamma$  is the average SNR at an individual branch, and  $M$  is the number of branches. Average bit error rate (BER) is also often used performance measure for SC. For BPSK modulation and coherent demodulation in flat Rayleigh fading channel, the average BER is given by [3, p. 723]

$$P_2 = \left( \frac{1-\mu}{2} \right)^M \sum_{k=0}^{M-1} \binom{M-1+k}{k} \left( \frac{1+\mu}{2} \right)^k, \quad (2)$$

where

$$\mu = \sqrt{\frac{\Gamma}{1+\Gamma}}. \quad (3)$$

For both real-time (speech, video) and non-real-time (data) services, block or packet error rate (BLER or PER)<sup>1</sup> is the most relevant measure of system performance. However, neither average SNR nor average BER provides a simple estimation of BLER,

<sup>1</sup>Block and packet are often used interchangeably. Here, by packet we mean a block of  $n$  bits consisting of  $k$  information bits and  $n-k$  frame check sequence (FCS) bits for error detection, and used in the context of data transfer using an automatic repeat request (ARQ) scheme. We consider here that the FCS provides with perfect error detection.

except for a random error channel or if a combination of ideal interleaving (IL) and forward error correction (FEC) coding is used to randomise the errors. However, use of the combination of IL and FEC defeats the very purpose of using SC as a means of combating fading in the first place. For reliable data transfer, ARQ schemes are generally used. It is also evident that efficiency of an ARQ scheme in a fading channel without the combination of IL and FEC is in general higher than that with the combination of IL and FEC [4], and increases delay. This should also be true in SC environment. While other measures like average fade duration (AFD) and level crossing rate (LCR) are also studied in context of SC recently [5], they are also insufficient to provide an estimate of BLER in an SC environment. Interestingly, since a TDMA environment is a basic necessity for an SC scheme, physical estimation of PER is probably more straight forward than average SNR (or signal power) or average BER. However, due to the present inadequacy of analytical methods, simulation is probably the only way to judge the PER level performance of an SC scheme.

In this article, we consider the performance of an ARQ scheme based on SC in a slow Rayleigh fading channel. An interesting issue in this context is, how the SC is implemented. We provide a simple selection strategy which works on a block by block basis, and does not require an SNR (or signal power) measurement. This scheme is conceptually similar to the stop-and-wait (SAW) scheme studied in [6]. Another issue considered here is post-SC combining. While in classical SC, only one branch with the highest SNR (or signal power) is chosen, there are recent reports on further combining of several outputs with maximal ratio combining (MRC) [7] or equal gain combining. Note that these schemes use soft decision outputs. However, hard decision outputs can also be used in a post-SC combining. In [8], Isioka et al. studied a simple packet combining scheme using weighted majority logic decision. In this article, we consider the EARQ type of packet combining as described in [9].

Rest of the article is organised as follows. After this introduction, we describe the underlying ARQ scheme, and the ARQ scheme in an SC environment, with the selection principle in Section II. We further elaborate the packet combining scheme in the context of ARQ based on selection diversity. A brief analysis of these schemes in AWGN channel is provided in Section III to gain some insight. Next follows a performance evaluation of these schemes in Rayleigh fading channel. Owing to analytical complexities, we resort to simulation here. Finally, the main points are summarised in Section V.

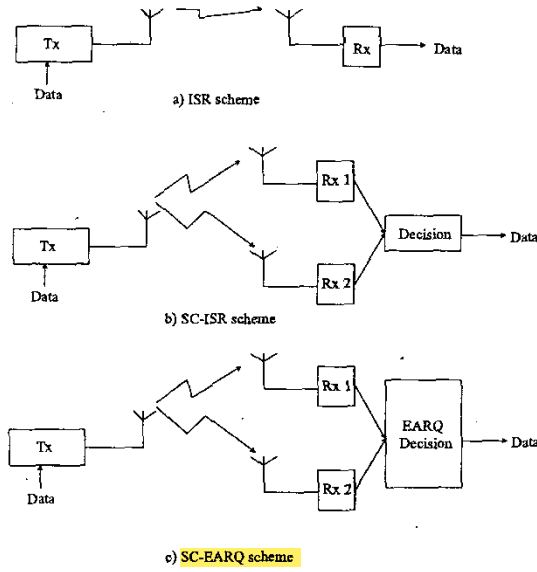


Fig. 1. Schematic diagrams of the three ARQ schemes.

## II. THE DIVERSITY ARQ SCHEMES

In this section, we outline SC based ARQ schemes studied in this article. Figure 1 illustrates the operation of each scheme.

### A. Basic ARQ scheme

The ideal selective-repeat (ISR) ARQ scheme, assuming negligible round-trip delay and an error-free feedback channel, is considered in this study. This scheme forms the basis of the SC based ARQ, and further packet combining using an EARQ type of packet combining.

### B. ARQ based on selection diversity (SC-ISR)

In this article, we consider post-selection combining, i.e.,  $M$  receivers complete with individual antennas receive identical transmission from a transmitter. In the present SC-ISR scheme, each received copy is checked for its correctness, using the usual cyclic redundancy check (CRC) bits. A correct packet is chosen to the output. Note that receiver selection here is done after reception of a block, in contrast to the classical SC scheme, where the receiver with the highest SNR (or power level) is chosen before reception of a block. In study, we consider a two antenna diversity, and the channels are independent Rayleigh slow fading channels.

### C. The EARQ scheme with selection diversity (SC-EARQ)

The EARQ scheme was proposed as a simple improvement to the basic ARQ scheme in a time diversity system. In this paper, we use this packet combining scheme to enhance performance of the SC-ISR scheme elaborated above. The scheme operates as follows (Figure 1c). If at least one of the received copies over a diversity branch is correct, it is accepted. However, if both copies are received erroneous, the bit-wise modulo-2 sum of the copies is computed to locate the errors in a combined copy. The decision process involves a brute force bit-by-bit inversion and CRC check

in an attempt to retrieve the correct packet. The retrieval process fails if there is at least one bit position where both the copies have an error (termed as a 'double' error in [9]), or the number of errors in the combined copy exceeds a predefined number  $N_{\max}$  (to keep the computational complexity within reasonable limits). The EARQ scheme discussed here is a truncated version of the scheme proposed in [9] in the sense that, if the packet combining process fails for a pair of erroneous copies, then they are discarded and a retransmission is asked by the decision process.

## III. PERFORMANCE ANALYSIS IN AWGN CHANNEL

If the BPSK modulation scheme is used, and the channel code rate is  $R_c$ , the bit error probability in an AWGN channel with the SNR equal to  $E_b/N_0$  is given by [10]

$$p = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b R_c}{N_0}} \right) \quad (4)$$

When  $n$ -bit packets are used, consisting of  $k$  information bits and  $n - k$  CRC bits, then  $R_c = k/n$ , and the throughput of the basic SR scheme is given by [11]

$$\eta_{\text{SR}} = R_c (1 - p)^n = \frac{k}{n} (1 - p)^n. \quad (5)$$

In the SC-ISR scheme, the probability of successfully receiving a packet is that of a successful reception over either of the diversity branches, and is given by

$$P_1 = 1 - (1 - (1 - p)^n)^2. \quad (6)$$

And the throughput is

$$\eta_{\text{SC}} = \frac{k}{n} P_1. \quad (7)$$

In the SC-EARQ scheme, when both copies are erroneous, a packet is retrieved correctly if the combined copy does not have a double error, and if the total number of errors in the combined copy is at most  $N_{\max}$ . The probability that both these conditions are satisfied is

$$\begin{aligned} P_2 &= \sum_{k_1=1}^{N_{\max}-1} \left\{ \binom{n}{k_1} p^{k_1} (1-p)^{n-k_1} \times \right. \\ &\quad \left. \times (1-p)^{k_1} \sum_{k_2=1}^{N_{\max}-k_1} \left[ \binom{n-k_1}{k_2} p^{k_2} (1-p)^{n-k_1-k_2} \right] \right\} \\ &= \sum_{k_1=1}^{N_{\max}-1} \sum_{k_2=1}^{N_{\max}-k_1} \binom{n}{k_1} \binom{n-k_1}{k_2} p^{k_1+k_2} (1-p)^{2n-k_1-k_2}, \end{aligned} \quad (8)$$

and the throughput of the EARQ scheme is given by

$$\eta_{\text{EARQ}} = \frac{k}{n} (P_1 + P_2). \quad (9)$$

Figure 2 provides a plotting of these schemes with  $k = 84$  and  $n = 100$ . It can be seen that, while SC-ISR provides considerable improvement over ISR, a packet combining scheme like EARQ can enhance the performance of the SC-ISR scheme still further, with very reasonable additional computational complexity.

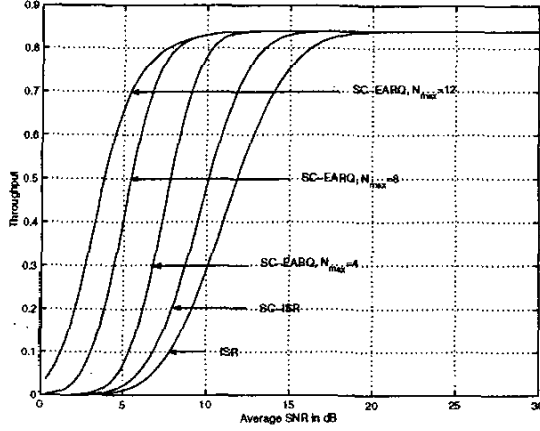


Fig. 2. Throughput curves of the ARQ schemes in AWGN channel, with  $k = 84$  and  $n = 100$ .

#### IV. SIMULATIONS IN RAYLEIGH FADING CHANNEL

##### A. Channel model

The channel model used in the simulations is a flat Rayleigh fading channel. Such a fading process is well approximated by a sum of sinusoids, a method developed by Jakes [1]. The sinusoids have random frequency inside the Doppler spectrum and random phase. The total power of the sinusoids is normalised to unity. The width of the Doppler spectrum is determined by the carrier frequency 2 GHz and the user speed  $v$ . The modulation scheme is BPSK, and a coherent demodulation scheme is assumed.

##### B. Simulation results

In all the simulation results that follow, the number of the CRC parity bits is 16, and the (raw) bit rate is 9.6 kbps.

Throughput is the basic performance measure used in this study. The throughputs of the three ARQ schemes as functions of the average SNR, when  $v = 1$  m/s,  $n = 100$  bits, and the parameter  $N_{\max}$  for EARQ is set to 10, are provided in Figure 3. Figures 4 and 5 are the same, except that in these cases  $v = 10$  m/s and  $v = 20$  m/s, respectively. It can be seen that the effects of SC become more important as the speed increases, and EARQ can further boost the performance significantly.

The simulated throughput performance of the SC-EARQ scheme with three different  $N_{\max}$  values, 4, 8, 12, for 100-bit packets and  $v = 1$  m/s is shown in Figure 6. The setting  $N_{\max} = 4$ , which guarantees a very low computational complexity, provides with almost 50% throughput for SNR values down to 8 dB. However, a higher value of  $N_{\max}$  is recommended for still lower SNR environments.

It is well known that the throughput of simple ARQ schemes in Rayleigh fading channel depends upon the packet size. The packet size is also expected to affect the performance of other schemes mentioned in this article. Here we consider only the effects of the packet size on the SC-EARQ scheme. In Figure 7, throughput of this scheme for packet sizes of 100, 200 and 400 bits is provided, when  $N_{\max} = 10$  and  $v = 1$  m/s. For low SNR values, the smallest packet size is clearly the best choice, but at high SNR

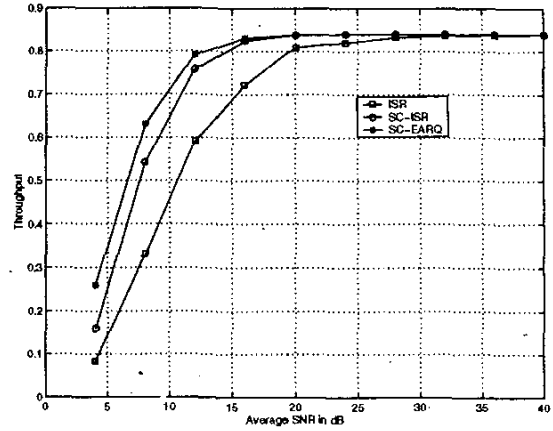


Fig. 3. Simulated throughputs of the three ARQ schemes when the packet size is 100 bits,  $v = 1$  m/s, and  $N_{\max} = 10$ .

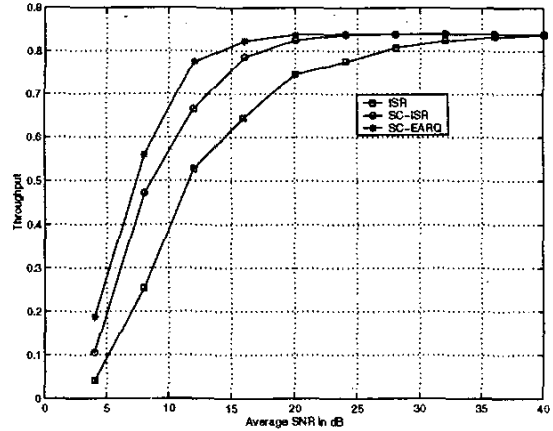


Fig. 4. Simulated throughputs of the three ARQ schemes when the packet size is 100 bits,  $v = 10$  m/s, and  $N_{\max} = 10$ .

levels, the low code rate of the small packets becomes costly, and the bigger packet sizes clearly yield higher throughputs.

#### V. CONCLUSIONS

In this paper, we have studied the performances of three different SR ARQ based error control schemes. Of these, one was the basic ISR scheme, one (SC-ISR) employed selection diversity, and finally the SC-EARQ scheme used packet combining together with receiver diversity. The throughput performance of all these schemes was analysed in the AWGN channel, and simulations based on the Jakes model were used to study their performance in Rayleigh fading channels.

The SC-ISR scheme provides some performance improvement over the basic ISR scheme in fading channel environment. The advantages become more prominent at higher user velocities.

Another major contribution of this paper is to introduce the EARQ scheme in the context of receiver diversity schemes. This scheme achieves considerably high throughput efficiency even at poor SNR conditions, where the throughputs of the other schemes

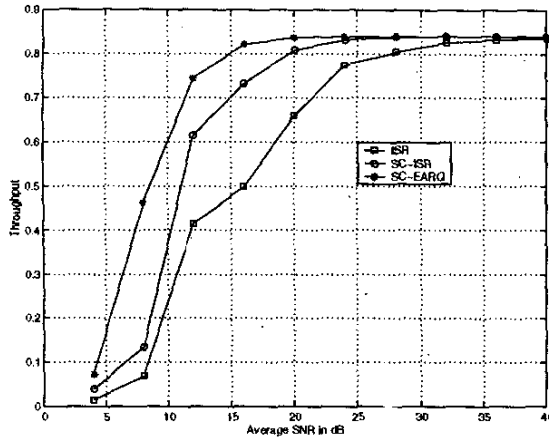


Fig. 5. Simulated throughputs of the three ARQ schemes when the packet size is 100 bits,  $v = 20$  m/s, and  $N_{\max} = 10$ .

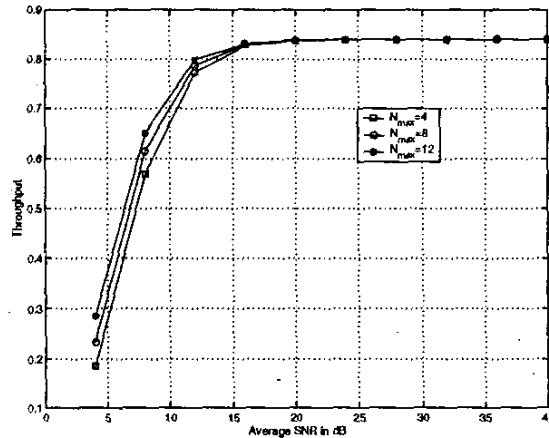


Fig. 6. Simulated throughput of the SC-EARQ scheme for three different values of  $N_{\max}$ , when the packet size is 100 bits and  $v = 1$  m/s.

discussed here fall down rapidly. With judicious choice of the values of the packet size and the parameter  $N_{\max}$ , the SC-EARQ scheme should be a good candidate to be used in diversity combining schemes.

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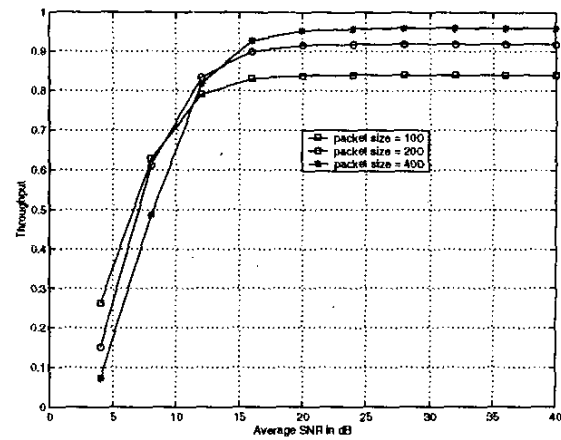


Fig. 7. Simulated throughput of the SC-EARQ scheme for three different packet sizes, when  $N_{\max} = 10$  and  $v = 1$  m/s.

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