

# ARQ and Packet Combining with Post-reception Selection Diversity

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**Abstract**—This paper studies automatic repeat (ARQ) request schemes in the context of selection combining (SC). We consider both pre-reception selection combining (PPR-SC) and post-reception selection combining (POR-SC) schemes with ARQ, in a 3-branch diversity system in non-interleaved Rayleigh fading channel. These POR-SC schemes operate on a block by block basis, without needing channel sensing and branch selection at every bit (or symbol). Simulation results show that the throughputs of ARQ for POR-SC schemes are improved when compared with that for the conventional SC. Significant performance improvement can be achieved by applying packet combining to POR-SC.

**Keywords**—ARQ; selection diversity; SC; SAH; Post-reception; Pre-reception; EARQ; packet combining; Rayleigh fading

## I. INTRODUCTION

Selection combining (SC) is one of the simplest implementations of space diversity reception for combating fading in mobile communication systems. In the classical post-detection selection combining (hereafter ideal selection combining, or ISC) [1], the branch with the best signal to noise ratio (SNR) is chosen to the output. However, the assumption that, selection of a branch occurs at every symbol period, needing a continuous channel monitoring, makes the ISC somewhat impractical [2]. A more practical SC is actually a selection and hold (hereafter, SAH-SC) policy where the selection of a branch is done at a regular time interval [3]. Note that, while ISC operates on bit-by-bit basis, SAH-SC works on block-by-block basis. Clearly, ISC is a limiting case of SAH-SC, as the selection interval becomes zero. The class of selection combining where the diversity branch is chosen before reception (which encompasses both ISC and SAH-SC) is mentioned as pre-reception selection combining or PRR-SC hereafter.

This article concerns reliable data transmission in diversity systems using automatic repeat request (ARQ) schemes. In a wireless channel, often a combination of forward error correction (FEC) coding and interleaving (IL) is used in conjunction to ARQ, in combating fading [4] [5]. However, the combination of IL and FEC provides lower code rate and higher complexity, and also not suited for slotted multiple access schemes. Further, when diversity combining is used for mitigating fading, use of IL and FEC is not only

counterintuitive with increased system complexity, but may also be redundant and counterproductive. In [6], Chuang shows that detection-only coding combined with ARQ and diversity selection without the IL+FEC combination, can achieve a reasonable throughput in a Rayleigh fading environment. However, the diversity selection in [6] operates in the bit level, i.e. in ISC mode. In a selection diversity based ARQ scheme however, a block level diversity for example, the SAH-SC is more important. Also note that, both ISC and SAH-SC needs explicit channel sensing before reception, which increases the complexity of the diversity scheme. However, the issue of channel sensing is not clearly addressed in [6]. Actually, when an ARQ scheme is used, the correctness of the block itself would imply the choice of the diversity branch, and obviate the need of an explicit channel sensing mechanism. Note that, such a concept is also relevant during soft handover in a CDMA, which itself provides an antenna diversity, based upon block reception. In contrast to PRR-SC, we define here a class of diversity reception where the choice of the diversity branch is defined by correct reception of a data block of a packet (equipped with cyclic redundancy check, or CRC), coined as post-reception selection combining (POR-SC). Since all the diversity branches are received in POR-SC, this scheme also provides a scope of further combining if a block is received erroneous at all the diversity branches. While such a combining in a 2-branch diversity system using the EARQ algorithm [7] is presented in [8], the present article concerns with development of a new algorithm for a 3-branch POR-SC scheme, coined as, error locator algorithm (ELA).

Performance of an SC scheme is usually provided in terms of average bit error rate (ABER). However, for ARQ schemes, performance normally is measured in terms of throughput, not as ABER. In this article also, we study the performance of the ARQ scheme with POR-SC and ELA algorithm in 3-branch diversity without the combination of IL and FEC, in terms of efficiency of the ARQ scheme. An analytical method of studying ARQ scheme in a fading channel is highly approximate, depends upon a number of assumptions, and may often be misleading. Hence, we use extensive simulation to study this scheme. Two ARQ schemes based POR-SC are considered in this paper: the first one is ARQ-POR-SC (i.e. NSD in [8]), where if a packet is received correctly at any of the diversity branches, it is accepted. A retransmission is requested otherwise. The second scheme is based on ELA-3

(termed as ELA-POR-SC), where all the received copies are checked for the correctness, such as in ARQ-POR-SC. If there is any correct copy, it is accepted. Otherwise, these erroneous copies are combined using the ELA algorithm elaborated afterwards, in an attempt to retrieve the transmitted packet. A retransmission is request if the combined packet is also erroneous. The different between ELA-POR-SC and ARQ-POR-SC is that ELA-POR-SC applies packet combining in ARQ scheme, where ARQ-POR-SC does not. Throughput performances of ARQ schemes with both PPR-SC and POR-SC are studied using computer simulation. The simulation is carried out in multiple uncorrelated no-interleaved Rayleigh fading channels. We assume in the computer simulation that the round-trip delay is negligible, the return channel is error free, error detection is perfect, and hard decision receivers are used. The purpose of this work is to provide simply, feasible and efficient ARQ schemes in diversity systems for reliable data transmission.

This paper is organized as follows. In section 2, the POR-SC schemes in conjunction with ARQ are elaborated briefly. In section 3, throughputs of ARQ based POR-SC schemes in AWGN channel analysis briefly to gain some insight. Throughput performances of ARQ with POR-SC and PPR-SC in Rayleigh fading channel is evaluated using simulation and results are provided in section 4. We assume a BPSK modulation throughout the study. Finally, some conclusions are drawn in section 5.

## II. ARQ SCHEMES BASED SC IN THREE-BRANCH DIVERSITY SYSTEM

We consider a 3-branch diversity system as shown in Fig. 1, where a packet M is transmitted over three diversity branches, and received by three different receivers. Each block is appended with a cyclic redundancy check (CRC) to aid in detection of errors. We however assume a single return channel, the structure of which is not elaborated in this article. Three SC in conjunction with ARQ in the block level are considered here:

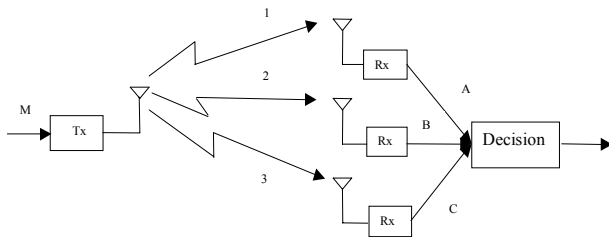


Figure 1. POR-SC in a 3-branch diversity

### A. ARQ based on SAH-SC(ARQ-SAH-SC)

Here, at the beginning of each packet received, the instantaneous SNRs of the received signals at the three diversity branches are compared, and the best one is selected to the output. The received packet is checked for its correctness

using CRC. A retransmission is requested if the received packet is erroneous.

### B. ARQ-POR-SC

The ARQ-POR-SC perhaps the basic form of POR-SC. It monitors the correctness of all the received copies, and then selects a correct copy to the decision output if there is any, otherwise, all the copies are discarded, and a retransmission is requested.

### C. ELA-POR-SC

ELA-POR-SC is an enhance version of ARQ-POR-SC. The scheme operates as following: If any one of the received copies is correct, it is accepted, just in the same way as in the ARQ-POR-SC scheme. However, if all the copies are erroneous, packet combining is used to combine these copies in order to retrieve the correct packet. The 3-copy combining algorithm presented here is however, different than the 2-copy algorithm EARQ [7], and works as follows. Let A, B, and C are the three received copies of the same transmitted packet over the three diversity branches, Let,

$$\begin{aligned} AA &= (A \oplus C) + (A \oplus B) \\ BB &= (A \oplus B) + (B \oplus C) \\ CC &= (A \oplus C) + (B \oplus C) \end{aligned} \quad (1)$$

where “ $\oplus$ ” denotes an “XOR” and “+” denotes an “AND” operation. In AA, BB and CC, every ‘1’ indicates an error in the copy of A, B and C respectively. However, if there is an error in the corresponding position in any of the pairs of A, B and C (termed as a “double error” in [7, 8]), ELA fails to locate it correctly. In this case, ELA can be used to estimate the errors in a received packet. However, if no double error is present in these copies, the correct copy is retrieved as:

$$S = AA \oplus A (= BB \oplus B = CC \oplus C) \quad (2)$$

Note that, the retrieval process fails not only in the presence of a double error, but also in presence of a triple error (however rare be it), which escapes the ELA, but is detected by using the CRC. In both these cases, the erroneous copies are discarded and a retransmission is requested using a NACK sent back to the transmitter. The flow chart of ELA-POR-SC is shown in Fig. 2.

## III. PERFORMANCE ANALYSIS OF ARQ BASED POR-SC IN AWGN CHANNEL

When BPSK modulation scheme is used, the bit error probability  $p$  in an AWGN channel is given by [9]

$$p_e = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b R_c}{N_0}} \right) \quad (3)$$

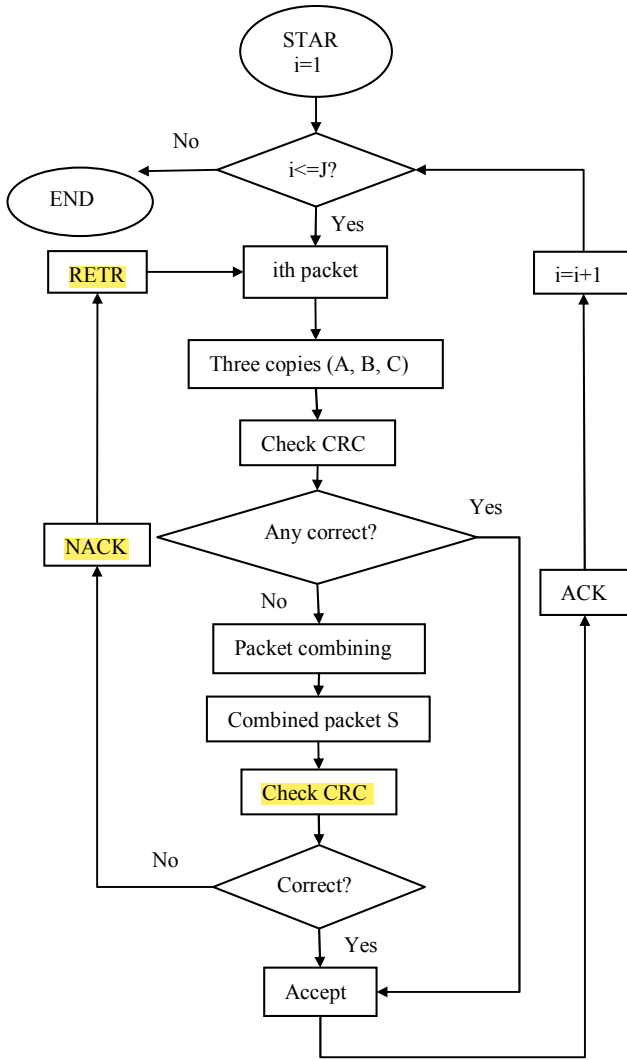


Figure 2. The flow chart of ELA-POR-SC.  $J$  is the total number of transmitted packets.

where  $E_b/N_0$  is the bit energy-to-noise ratio,  $R_c$  is the channel code rate, and  $\text{erfc}$  is the complementary error function. Considering an  $n$ -bit packet, which consisting  $k$  information bits and  $n-k$  CRC bits,  $R_c = k/n$ .

Since no error-correction code is considered, a block is received correctly only if all  $n$  bits in the packet are received without error. Thus, the probability of successfully receiving a packet  $P_{suc}$  in a single branch is given by

$$P_{suc} = (1 - p)^n \quad (4)$$

In ARQ-SC, a packet is successful received when there is at least a correct packet over three diversity branches, and is given by

$$P_1 = 1 - (1 - P_{suc})^3 = 1 - (1 - (1 - p)^n)^3 \quad (5)$$

The throughput for ARQ-SC is

$$\eta_1 = \frac{k}{n} P_1 \quad (6)$$

In ARQ-ELA, when three copies are erroneous, a packet is retrieved correctly when the combined copy does not have a double (including triple) error. The probability of a packet that can be successfully retrieved is given by:

$$\begin{aligned}
 P_2 &= \sum_{k_1=1}^{n-2} \left\{ \binom{n}{k_1} p^{k_1} (1-p)^{n-k_1} (1-p)^{k_1} \times \right. \\
 &\quad \times \sum_{k_2=1}^{n-k_1-1} \left[ \binom{n-k_1-1}{k_2} p^{k_2} (1-p)^{n-k_1-k_2-1} (1-p)^{k_1+k_2} \times \right. \\
 &\quad \times \left. \sum_{k_3=1}^{n-k_1-k_2} \binom{n-k_1-k_2}{k_3} p^{k_3} (1-p)^{n-k_1-k_2-k_3} \right] \left. \right\} \\
 &= \sum_{k_1=1}^{n-2} \sum_{k_2=1}^{n-k_1-1} \sum_{k_3=1}^{n-k_1-k_2} \left[ \binom{n}{k_1} \binom{n-k_1-1}{k_2} \binom{n-k_1-k_2}{k_3} \right] \times \\
 &\quad \times p^{k_1+k_2+k_3} (1-p)^{3n-k_1-k_2-k_3-1} \quad (7)
 \end{aligned}$$

Thus, the throughput of ELA-POR-SC is given by:

$$\eta_2 = \frac{k}{n} (P_1 + P_2) \quad (8)$$

#### IV. SIMULATION RESULTS

##### A. Computer simulation

Throughput performances of ARQ schemes with SAH-SC (ARQ-SAH-SC) and POR-SC (ARQ-POR-SC and ELA-POR-SC) in a 3-branch diversity system are studied using simulation. The 3-branch uncorrelated identical Rayleigh fading channels are generated using method of exact Doppler spread (MEDS) presented in [10]. The maximum Doppler frequency is determined by the carrier frequency 2GHz and the mobile velocity  $v$ . The received signal is contaminated with additive white Gaussian noise with two-sided power spectral density  $N_0/2$  BPSK modulation with coherent demodulation is assumed. The number of CRC parity bits is 16. The raw bit rate is  $R$  is 9.6kbps. At least 1 million packets are transmitted in each simulation point. The 'hold' time is equal to the duration of a packet.

##### B. Simulation results

Throughput performance comparisons are made between ARQ-SAH-SC, ARQ-POR-SC and ELA-POR-SC. Fig. 3 shows the throughput performances of three block level SCs in conjunction with ARQ as functions of the average SNR, when mobile velocity  $v=1\text{m/s}$  and packet size  $b=100$  bits. As can be seen, the performance of both ARQ-POR-SC and ELA-POR-

SC are improved compared to ARQ-SAH-SC, when the average SNR is lower than 15dB. The difference in throughput between the ARQ-POR-SC and ARQ-SAH-SC is small. This is since, in flat fading environment, the channel changes relatively slow within packet duration of 100 bits, whether the selection is done either before or after a packet received, does not make big difference. However, when the average SNR is lower than 10 dB, ELA-POR-SC provides significant improvement over the other two. This is expected since the throughput of ELA-POR-SC improves by retrieving a correct packet from the erroneous received packets.

Fig. 4 shows the throughput as a function of the average SNR, for three schemes, when mobile velocity  $v=1m/s$  and packet size  $b=400$  bits. Here the difference between ARQ-SAH-SC and ARQ-POR-SC is more significant as compared to Fig. 3. This is due to the fact that, the channel changes significantly within 400 bits, and the selection of a branch only based on the instantaneous channel condition in the first symbol (bit) arrival is rather inaccurate in ARQ-SAH-SC, when compared to ARQ-POR-SC and ELA-POR-SC, which provides further combining.

Fig. 5 shows the throughput as a function of the average SNR, for the three schemes, when mobile velocity  $v=20m/s$  and packet size  $b=100$  bits. It can be seen that, the performance of both ELA-POR-SC and ARQ-POR-SC are improved significantly compared to ARQ-SAH-SC, even when the average SNR is very low. A diversity gain of more than 10dB is seen at the 70% throughput level for ELA-POR-SC over ARQ-SAH-SC, and more than 6dB is seen at the same throughput level for ARQ-POR-SC and ARQ-SAH-SC.

The above results also corroborate with the fact that, throughput of pure ARQ schemes in Rayleigh fading channel depends on the packet size. To further illustrate this, the effects of the packet sizes in ELA-POR-SC are shown in Fig. 6. It shows that the throughput for packet size of 100 bits is higher than that of 400 bits when average SNR is lower than 8dB, but the results changes vice versa when average SNR is higher than 8dB. Therefore, when SNR is low, it is better to transmit packets with small packet sizes, otherwise, bigger packet sizes is a good choice, if SNR is high.

## V. CONCLUSION

In this paper, we have studied the performance of ARQ in conjunction with three-block level SC schemes. The ARQ schemes are simulated in a multiple uncorrelated fading channel as given in [10]. Performance of the ARQ schemes with POR-SC is compared with ARQ with SAH-SC, for 3-branch diversity cases. Simulation results show that ARQ combined with POR-SC is provide performance improvement over ARQ with SAH-SC, and this advantage increases as fading become faster. Among the three schemes, ELA-POR-SC provides the best throughput performance, even at poor channel condition. Another advantage of ARQ with POR-SC schemes is the removal of the need of explicit channel sensing, making the scheme simply and easy to implement.

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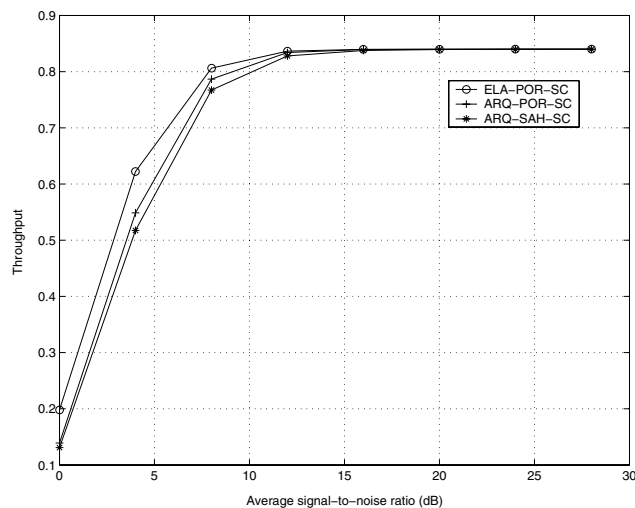


Figure 3. Simulated throughputs of the three SC schemes when the packet size is 100 bits, and  $v=1m/s$ .

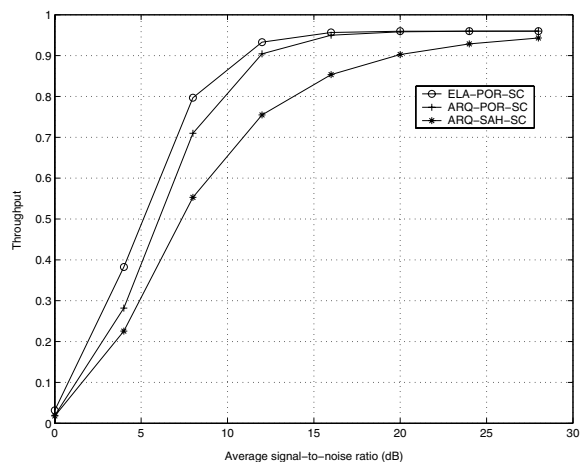


Figure 4. Simulated throughput of the three SC schemes when the packet size is 400 bits, and  $v=1m/s$ .

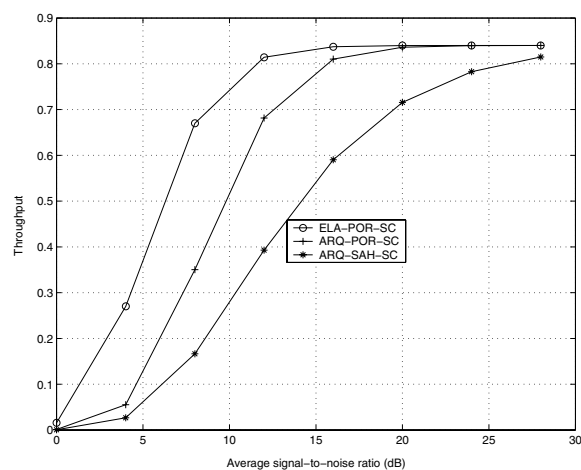


Figure 5. Simulated throughput of the three SC schemes when the packet size is 100 bits, and  $v=20 m/s$ .

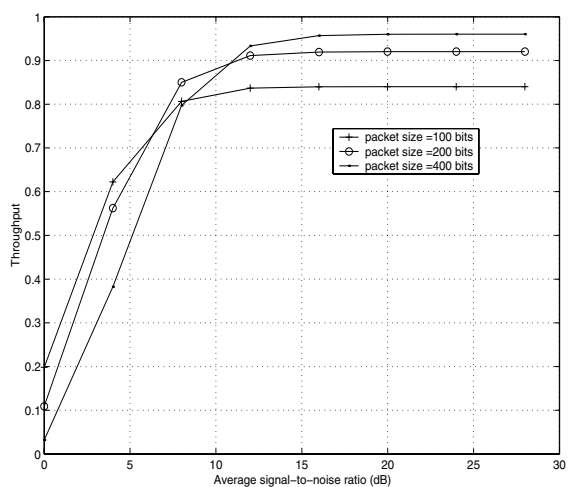


Figure 6. Comparison of the throughput of ELA-POR-SC for three different packet sizes, when  $v=1 m/s$ .