Mathematical Modeling of LoRaWAN Performance with Bi-directional Traffic

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***Abstract*—LoRaWAN is gaining momentum in the arena of IoT connectivity technologies thanks to the low cost, ease of deployment, and support for adaptable transmission rates and bidirectional communications. The research community has then been working to develop suitable analytical models that can be instrumental in the study of this technology. In this work we propose a mathematical model that makes it possible to accurately estimate the packet success probability of a LoRaWAN network in presence of bi-directional traffic, i.e., both uplink (UL) and downlink (DL) transmissions, and that accounts for the most critical features of the LoRa chipset and the LoRaWAN standard. The proposed model, furthermore, makes it possible to study the effect of different parameters configurations, thus offering a valid tool to investigate possible improvements to the system configuration. The proposed model is first validated by comparison with some accurate simulation results and, then, its potential is exemplified by analyzing the system settings that yield the best network performance.**

I. Introduction

Low Power Wide Area Networks (LPWANs) are a relatively new family of wireless communication technologies that are being marketed as able to support a number of Internet of Things (IoT) use cases involving very large number of nodes with low traffic demand. The main features of these new wireless solutions, indeed, entail long communication ranges and low energy consumption, which are achieved by compromising on throughput. Among the new LPWAN standards, the LoRaWAN technology is gathering growing interest both from the academic and industry communities [1], also thanks to accessibility to the technical specifications, released by the LoRa Alliance as an open standard. This has spurred a number of performance studies, aimed at identifying the potentials and limitations of the technology.

LoRaWAN performance studies broadly follow three approaches: field trials, simulations and analytical modeling. While empirical studies can reveal performance issues of the system, they are often limited in scope and generality by the typical practical constraints of experimental work, like network size, specificity of the considered scenarios, limited accessibility to the protocol parameters and so on. Simulation frameworks, conversely, can help the study of LoRaWAN networks at the system level, but are often limited by the computational power that is required to simulate large deployments. Mathematical models, finally, are usually based on ideal assumptions, which may affect the accuracy of the results. Nonetheless, mathematical models can be used to quickly evaluate the effect of different parameter configurations, or to study the system in limiting conditions, thus making it possible to better focus simulation and experimental studies on the aspects that are expected to have major impact. Therefore, mathematical modeling is still a fundamental tool for the performance analysis of a system.

In this work, we propose a mathematical model for a LoRaWAN network that, conversely to other models presented in the literature, includes the features of the radio chipset and of the LoRaWAN standard that have a major impact on the system performance, in particular in presence of bi-directional traffic. Furthermore, the model is parameterized to account for various network configurations and assumptions, thus making it possible to study the effect of different configuration and management choices on the performance of the whole LoRaWAN system. The model considers a single LoRaWAN cell, i.e., a unique gateway, but can be further improved by combining the proposed expressions to formulate the network performance when multiple gateways are employed.

The rest of this paper is organized as follows. Section II provides a quick overview of the technology, describing the most interesting characteristics of the LoRa chipset and the LoRaWAN standard. Section III briefly describes some previous mathematical models that have been proposed in the literature. In Section IV we present our model and show some results; finally, Section V draws the conclusion and describes possible future developments.

II. Technology overview

This section presents some key features of the LoRa modulation and chipset, and provides an overview of the LoRaWAN standard, highlighting the features and properties that have a major impact on the performance and, hence, will be considered in our model.

A. The LoRa modulation

LoRa is a proprietary modulation scheme based on Chirp Spread Spectrum (CSS), patented by Semtech. The radio is characterized by a long communication range, ranging from about 2 km in urban areas up to 15 km in line-of-sight rural scenarios. Furthermore, it supports different Spreading Factors (SFs), which make it possible to trade the achievable coverage range for the transmission bitrate. The SF index can vary

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between 7 and 12:[1] the higher the SF, the longer the on-air transmission time of a packet (and, hence, the lower the bitrate), but the lower the sensitivity for correct reception (i.e., the longer the reception range).

It is important to notice that the different SFs are almost orthogonal, so that multiple signals transmitted with different SFs can be simultaneously decoded at the receiver with high success probability. Instead, when multiple packets are simultaneously transmitted with the same SF, they usually collide at the receiver, generating destructive interference. However, the strongest signal may be successfully decoded if its received power is at least CR dB higher than that of the interfering packets (capture effect). CR is the co-channel rejection parameter, which is specified in LoRa chip datasheets [3], [4].

B. The LoRaWAN standard

The LoRaWAN standard [5], published by the LoRa Alliance, aims at leveraging the features of the LoRa modulation. It defines a star-of-stars network topology, consisting of three kinds of devices:

- The *Network Server (NS)*, a centralized network controller that collects messages, sends replies and controls the network parameters setting;
- The *End Devices (EDs)*, peripheral nodes, typically equipped with different types of sensors, that communicate with the Network Server, through the Gateways.
- The *Gateways (GWs)*, intermediate nodes that relay the traffic between the NS to the EDs. The communication with the EDs occurs through the LoRa wireless standard, while the connection to the NSs can be implemented by using any IP technology.

In general, the standard defines three classes of EDs, denoted as A, B, and C, which correspond to different capabilities of the devices. In this study we focus on Class A EDs, which are by far the most common at the moment. Class A devices access the wireless channel only when they have an UL packet to transmit to the NS and, for the remaining time, they switch off their radio interface to save energy.

In Europe, the system operates in the 868 MHz ISM band, where the standard defines four parallel subchannels, out of which one is reserved for DL transmissions only. When ready to transmit an UL packet, the ED picks at random one of the remaining three subchannels (centered at 868.1 MHz, 868.3 MHz and 868.5 MHz) and transmits its signal, following an ALOHA channel access scheme. The GWs are always listening for valid signals on all the available UL subchannels and forward any received packet to the NS, which is in charge of filtering out the possible replicas. After an UL transmission, the ED can receive DL packets from the GW selected by the NS. To this end, the ED opens two receive windows, respectively 1 and 2 seconds after the completion of the UL transmission. The first window is opened on the same subchannel used for the previous UL transmission, while the second window is always opened on the subchannel reserved for DL transmissions (centered at 869.525 MHz).

[1] Note that, for a SF s, the duration of the chirp symbol is expanded by a factor 2^s (see [2]).

TABLE I
LoRaWAN DEFAULT CHANNELS, DC AND POWER LIMITATIONS.

Frequency (MHz)	Direction	DC	Power limit
868.1	DL, UL	1%	14 dBm
868.3	DL, UL	1%	14 dBm
868.5	DL, UL	1%	14 dBm
869.525	DL	10%	27 dBm

Devices operating in the unlicensed 868 MHz band are subject to Duty Cycle (DC) and transmission power restrictions, as summarized in Table I. Note that the three UL frequencies share the same DC restrictions, since they are located in the same sub-band. Therefore, when a transmission is performed in one of these frequencies, the other two channels must also respect the waiting time imposed by the DC.

C. The SX1301 GW chip

The architecture of the SX1301 chip [4] currently employed in LoRa GWs presents some features that can have a relevant impact on the system performance. The chip contains ten programmable reception paths, which work in parallel and are connected to the same antenna. Eight of these paths can be used to receive packets from the EDs. The center frequency of each reception path can be individually configured, so that it is possible to simultaneously receive multiple packets on the same subchannel. Moreover, each reception path can decode signals modulated with any SF. Therefore, the chip is, in principle, able to demodulate up to 8 packets simultaneously. However, if all the reception paths listening on a given subchannel are engaged, a new packet arriving on the same channel will be lost. Furthermore, the GW can perform a single DL transmission at a time and cannot transmit and receive simultaneously. As a consequence, DL transmissions can severely impact the UL system capacity.

III. STATE OF THE ART LoRaWAN MODELS

Some mathematical models for LoRaWAN have been published in the literature. The author of [6] proposes closed-form expressions for collision and packet loss probabilities, based on the assumption of perfect SF orthogonality, but neglecting the capture effect, so that overlapping transmissions with equal SF always result in a collision. The authors of [7] investigate the performance of LoRaWAN UL transmissions in terms of latency, collision rate and throughput, considering SFs orthogonality and assuming exponential packet inter-arrival times. Again, no capture effect is considered, but DC constraints are introduced. In [8], [9], Bankov et al. provide a first model of a LoRaWAN network considering retransmissions and capture effect. The authors assume perfect orthogonality between SFs, Poisson packet generation at the nodes, and systematic transmission of two Acknowledgments (ACKs), one for each receive window. However, the multiple

TABLE II
CONFIGURABLE PARAMETERS.

Parameter	Value	Description
(a)	`tx priority`	ACK transmission has priority
(a)	`rx priority`	packet reception has priority
(b)	`DC enabled`	DC constraint at the GW
(b)	`DC disabled`	no DC constraint at the GW
(c)	`2acks`	always two ACKs sent
(c)	`1ack`	only one ACK sent

reception path architecture and the DC restrictions at the GWs are neglected.

In this paper, we expand the previous studies by proposing a model that, besides the capture effect (which is accounted for as in [9]), also includes other features, such as the limited number of reception paths at the GW and the DC restrictions in both UL and DL. Moreover, our model makes it possible to consider different configurations of the LoRaWAN network, taking into account factors such as the number of ACKs that the NS sends in reply to ED confirmed data packets, and whether or not the transmission of DL packets by the GW should preempt the reception of any UL packet.

IV. PROPOSED MODEL

In this section we present our mathematical model by first listing the underlying assumptions and then describing the way different features of the system have been accounted for in the model.

A. Assumptions

Our model is based on the following assumptions.

1) Propagation model and scenario: we consider a single LoRaWAN cell, with a unique GW. The network is located in an open-air scenario, where only path loss affects the transmission channel.[2] Furthermore, the channel is assumed to be symmetric. The SFs are assigned to nodes according to their distance to the receiver, with the objective of minimizing the on-air packet transmissions time, while guaranteeing an average received power above the sensitivity threshold.

2) Network traffic: the EDs generate confirmed data packets according to independent Poisson point processes, with aggregate packet generation rate λ. A packet transmitted by an ED is marked as failed when no ACK is received in either the first or the second receive window. Lost packets are not retransmitted.

3) SF orthogonality: for tractability, we assume perfect orthogonality between SFs, i.e., collisions can happen only between packets employing the same SF. This assumption is quite common in the literature and the quasi-orthogonality of the SFs has been confirmed by simulation and experimental studies.

Besides this basic set of assumptions, the model can optionally include these other assumptions (also summarized in Table II).

[2]Note that the chirp modulation alleviates the effect of small fading, which is typically neglected in the literature for this technology.

(a) *Transmission/reception prioritization at the GW:* the LoRaWAN standard gives no indication on whether the transmit operation should have priority over reception at the GW. In [4], however, it is clearly stated that transmission operations imply the interruption of all reception procedures that are in progress. This implies that, if transmission were given the priority, the reception of a number of parallel UL packets would be canceled in order to transmit one single ACK packet, which may be detrimental to the overall network performance in certain cases. The proposed model makes it possible to decide whether or not the transmission of ACKs should be prioritized over the reception of uplink data packets, thus allowing for the analysis of this aspect. In case reception is prioritized, ACKs are only transmitted if the gateway is not listening to any UL packet. Note that the impossibility oh having simultaneous trasmission and reception operations is a chipset's limitation: the model considering GWs supporting concurrent packet transmission and reception can be obtained by taking eqs. (8) and (14) equal to 1.

(b) *GW Duty Cycle (DC):* the common interpretation of the DC restriction mandated by the ETSI regulations for 868 MHz band [10] is that all emitters should respect the DC constraint, irrespective of their roles. While this interpretation is reasonable in scenarios with mostly uplink traffic, it can be very penalizing in presence of confirmed or bidirectional traffic, since the GW will be subject to the same restrictions of any EDs, while also being required to reply to all their confirmed UL packets. In order to assess the actual impact of this restriction, the model makes it possible to decide whether to apply DC requirements at the GW.

(c) *Number of ACKs:* the standard [5] does not specify in which receive window the ACK should be sent. The more conservative interpretation is that the NS sends the ACK twice, one for each receive window opened by the ED after each UL transmission. This approach, however, increases the load of the GW, consumes the DC in two DL subchannels, and increases the dropping rate of UL packets if the transmission of ACKs is prioritized. For this reason, we factor in our model an optional ACK model, where the GW sends only one ACK, preferably in first receive window (RX1) or in second receive window (RX2) when the first option is not available because of DC restrictions or other causes.

(d) *SF employed in RX2:* The ACKs returned in RX1 are transmitted with the SF used for the corresponding UL packet, while those transmitted in RX2 make always use of SF 12. While this choice increases the robustness of the DL transmission, it can also needlessly increase the DC consumption at the gateway, occupying the channel for a longer time, and may yield increased interference to neighboring GWs. In order to assess the performance impact of this setting, our model specifies the on-air time of the ACKs sent in the RX2 as a configurable parameter (T_{RX2}^{ack}), thus enabling the study of different SF selection criteria for the transmission of the ACKs in RX2.

B. Model description

We call T_i^{data} and T_i^{ack} the time-on-air of a data packet and an ACK message, respectively, sent with SF i, while T_{RX2}^{ack} is the time duration of an ACK packet in RX2.

We assume F channels are used for both UL and DL, while a single channel with less strict DC restrictions is reserved to DL transmissions in RX2 (reflecting the configuration of Table I). Denoting by p_i the probability that a mote employs SF i, the average packet arrival rate on a given channel for SF i is equal to

$$r_i = \frac{\lambda p_i}{F}. \tag{1}$$

1) Capture effect: We assume that, in case of collision between packets with equal SF, one packet is captured when its received power exceeds the power of the other colliding nodes by at least CR dB, for the entire frame reception period. Since the received power is only affected by path loss, the capture probability only depends on the distances of the colliding EDs from the GWs. Given that a collision between packets using the same SF i occurs, we define the following conditional probabilities:

- $\mathbb{S}_i^{GW}$: probability that an UL packet transmitted with SF i is captured at the GW;
- $\mathbb{S}_i^{mote}$: probability that a DL packet transmitted with SF i is captured by the intended ED;
- $\mathbb{F}_i^{both}$: the probability that two (or more) packets simultaneously transmitted on the same channel and with the same SF i are all lost.

The probabilities $\mathbb{S}_i^{GW}$ and $\mathbb{S}_i^{mote}$ are computed as in [9] considering only collisions between two packets. If more packets collide, the success probabilities for both are assumed equal to zero. Moreover, since in case of collision between packets with equal SF only one can survive, we have

$$\mathbb{F}_i^{both} = 1 - \mathbb{S}_i^{*}\,, \tag{2}$$

where $\mathbb{S}_i^{*}$ is either $\mathbb{S}_i^{GW}$ or $\mathbb{S}_i^{mote}$, depending on whether the collision occurs at the GW or ED, respectively.

2) ACK generation and transmission processes: We define P_i^{data} and P_i^{ack} the probabilities that a data packet and the corresponding ACK are correctly received, given that both are transmitted with SF i. Now, the generation of ACKs are subject to the successful reception of UL packets by the GW, in any frequency channel. Therefore, the rate of UL packets that are successfully received is given by

$$\sum_{f=1}^{F} r_i\, P_i^{data} = \lambda\, p_i\, P_i^{data}\,. \tag{3}$$

Note that this packet arrival process is still Poisson, being the result of a stochastic filtering of the original Poisson packet generation process.

Since we consider only confirmed traffic, each successfully received packet triggers an ACK, which will be transmitted in RX1 or RX2, or both, depending on the GW conditions and on the considered assumptions.

Now, let $\lambda_{RX1}^{(i)}$ and $\lambda_{RX2}^{(i)}$ denote the average rate at which ACKs are transmitted in RX1 and RX2, respectively, in response to UL packets sent with SF i. We have:

$$\lambda_{RX1}^{(i)} = \lambda p_i P_i^{data} Q_{RX1}\,; \tag{4a}$$

$$\lambda_{RX2}^{(i)} = \lambda p_i P_i^{data} Q_{RX2}\,; \tag{4b}$$

where Q_{RX1} and Q_{RX2} are the probability that the GW does transmit in RX1 and RX2, respectively. The actual transmission of an ACK in a given window occurs only if the GW is not already transmitting and the DC does not prevent transmissions. In the case of `1ack`, the ACK transmission is scheduled first in RX1 and, alternatively, in RX2. If the transmission is prevented in both windows, the ACK is dropped.

We hence have

$$Q_{RX1} = P_{RX1} \tag{5a}$$

$$Q_{RX2} = \begin{cases} P_{RX2}, & \text{if } \texttt{2acks}, \\ P_{RX2}(1-P_{RX1}) & \text{if } \texttt{1ack}. \end{cases} \tag{5b}$$

where P_{RX1} and P_{RX2} are the probabilities that the GW *is allowed* to transmit in RX1 and RX2, respectively. These probabilities, in turn, depend on the considered assumptions, and can be expressed as

$$P_{RX1,2} = \alpha \cdot \beta \cdot \delta, \tag{6}$$

where α accounts for the assumption (b), i.e., the DC constraint at the GW, β depends on the assumption (a) regarding the prioritization of DL transmissions over UL receptions, and δ captures the probability that the GW is not already transmitting. For analytical tractability, we approximate all transmission processes as Poisson processes.[3] To compute α, we recall that the frequencies used in RX1 are subject to a DC of 1%. Therefore, an ACK can be transmitted in RX1 only if no other ACKs were transmitted in the previous interval of length $100\,T_i^{ack}$. Similar considerations apply to transmissions in RX2, which is however subject to a DC of 10%. Therefore, we have

$$\alpha = \begin{cases} 1 & \text{if } \texttt{DC disabled}, \\ e^{-\sum_i^{SF}(99T_i^{ack})\lambda_{RX1}^{(i)}} & \text{if } \texttt{DC enabled}, P_{RX1}, \\ e^{-\sum_i^{SF}(9T_{RX2}^{ack})\lambda_{RX2}^{(i)}} & \text{if } \texttt{DC enabled}, P_{RX2}. \end{cases} \tag{7}$$

Note that the considered intervals have a duration that is T_i^{ack} shorter than necessary according to the DC constraint, since the events corresponding to the missing interval are already taken into account by the $delta$ factor.

The optional prioritization of ACK transmission over reception is accounted for in β, which is defined as

[3] We remark that this assumption does not hold in general, but it simplifies the analysis, at the cost of some loss of accuracy in the model.

$$\beta = \begin{cases} 1 & \text{if } \texttt{tx priority}, \\ P_{0,paths} & \text{if } \texttt{rx priority}. \end{cases} \tag{8}$$

where $P_{0,path}$ is the probability that there receiver is idle, and will be computed later on (see Eq. (17)).

Finally, the probability that the GW is not transmitting can be expressed as follows:

$$\delta = e^{-\sum_i^{SF} T_i^{ack} \lambda_{RX1}^{(i)}} \cdot e^{-\sum_i^{SF} T_{RX2}^{ack} \lambda_{RX2}^{(i)}}. \tag{9}$$

Note that P_{RX1} and P_{RX2} do not necessarily sum to 1, as it can happen that, in high traffic conditions, neither of the receive windows can be used, e.g., because of DC limitations.

We need now to compute the probability that the ACK is correctly received by the intended ED. If the ACK is transmitted in RX1, with SF i, it is successfully received when there are no other simultaneous transmissions with the same SF, or if there is exactly one overlapping transmission with equal SF i, whose interference is at least CR lower than the received signal power of the ACK. Hence, we have

$$\begin{aligned} P_i^{ack1} &= e^{-(T_i^{data}+T_i^{ack})r_i} \\ &+ (T_i^{data}+T_i^{ack}) r_i e^{-(T_i^{data}+T_i^{ack})r_i} \mathbb{S}_i^{mote}. \end{aligned} \tag{10}$$

Note that the vulnerability interval $(T_i^{data}+T_i^{ack})$ accounts both for the possible overlapping of the ACK transmission with ongoing data transmissions and with new transmissions from other EDs starting while the ACK is being sent. Since RX2 is always opened in the subchannel reserved for DL transmissions, there is no risk of collisions and, hence, under the considered assumptions, ACKs sent in RX2 are always successful, i.e., $P^{ack2} = 1$.

Therefore, the overall probability that an ACK for an UL packet sent with SF i is successfully received by an ED is

$$P_i^{ack} = Q_{RX1} \cdot P_i^{ack1} + Q_{RX2} \cdot P_i^{ack2}. \tag{11}$$

3) Successful UL transmission probability: Finally, we consider the probability that a data frame sent with SF i is successfully received by the GW, and define it as:

$$P_i^{data} = P_i^{data\ rx} \cdot P_{rxPath} \cdot \delta \cdot \gamma. \tag{12}$$

The $P_i^{data\ rx}$ factor accounts for the probability of surviving to possible collisions in the channel, and is computed as follows:

$$P_i^{data\ rx} = e^{-(2T_i^{data})\, r_i} + \left(2T_i^{data}\right) r_i\, e^{-(2T_i^{data})\, r_i}\, \mathbb{S}_i^{GW}. \tag{13}$$

As in previous equations, δ describes the probability that the GW is not in transmission state. Moreover, we introduce the factor γ to account for the transmission/reception priority, see (a). More specifically, γ is one if reception if prioritized over transmission and, otherwise, it is the probability that the packet reception is interrupted by an ACK transmission. We hence have:

TABLE III

POSSIBLE ALLOCATION OF GW RECEPTION PATHS.

Frequency (MHz)	Allocated reception paths
868.1	3
868.3	3
868.5	2

$$\gamma = \begin{cases} 1 & \text{if } \texttt{rx priority}, \\ e^{-\sum_i^{SF} T_i^{data} (\lambda_{RX1}^{(i)}+\lambda_{RX2}^{(i)})} & \text{if } \texttt{tx priority}. \end{cases} \tag{14}$$

The P_{rxPath} term in Eq. (12) accounts for the probability of finding at least one path available for reception. We use the channel allocation described in Table III.[4] Since channel selection is random and uniformly distributed, before transmission the ED chooses a channel associated to 3 reception paths with probability 2/3 and a channel associated with 2 reception paths with probability 1/3. If we denote by P_{rxPath}^a and P_{rxPath}^b the probability of finding one available reception path in a channel with three and two allocated reception paths, respectively, we get:

$$P_{rxPath} = \frac{2}{3} \cdot P_{rxPath}^a + \frac{1}{3} \cdot P_{rxPath}^b, \tag{15}$$

with

$$P_{rxPath}^a = P_{0,path}^a + P_{1,path}^a + P_{2,paths}^a \tag{16a}$$

$$P_{rxPath}^b = P_{0,path}^a + P_{1,path}^b, \tag{16b}$$

and

$$P_{0,path} = (P_{0,path}^a)^2 \cdot (P_{0,path}^b), \tag{17}$$

where the probability $P_{n,path}$ is the probability that a packet arriving at the GW finds n paths busy. Note that the expression of $P_{0,path}$ is due to the fact that a packet arriving at the GW finds all the reception paths available if no packets are being received in any of the three channels. Derivation of these expressions is provided in the Appendix.

Finally, the total success probability is given by the sum of the success probabilities for each SF:

$$P_{succ} = \sum_i^{SF} p_i \cdot P_i^{data} \cdot P_i^{ack}. \tag{18}$$

C. Model validation

To verify the accuracy of the model, we compared it with realistic LoRaWAN simulations obtained by using ns-3 [11], [12]. The simulator considers the GW to have eight reception paths, configured as in Table III and performs only one ACK transmission for each received confirmed packet,

[4] If a different channel allocation is preferred, the formulas to compute P_{rxPath} are obtained analogously, evaluating for each channel the probability that at least one reception path remains free, considering all possible SFs.

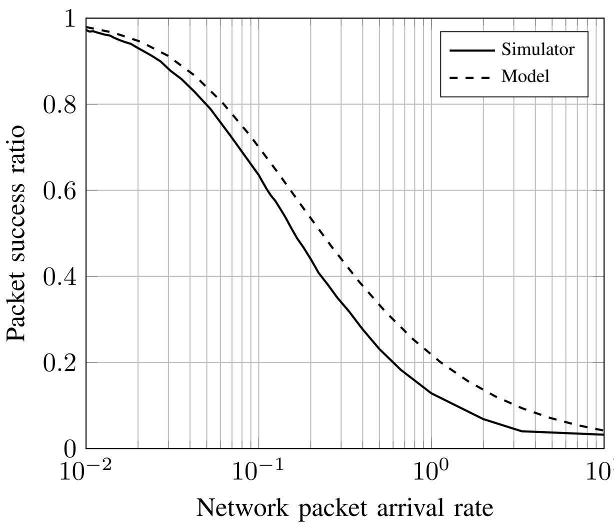

Fig. 1. Comparison of model with ns-3 LoRaWAN simulator.

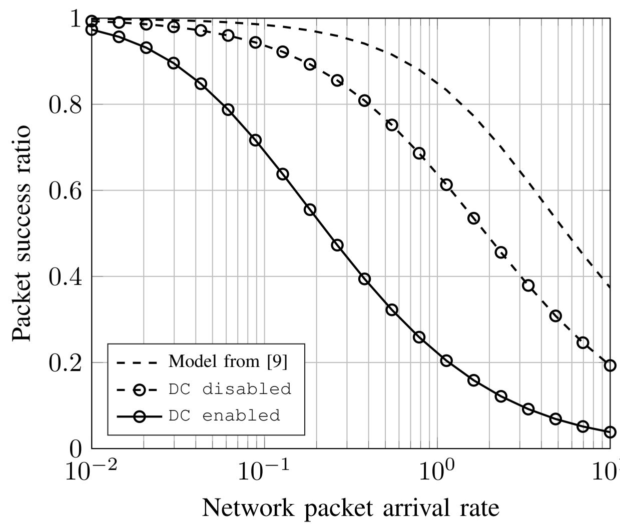

Fig. 2. Comparison of analytical model with different assumptions.

giving priority to ACK transmission over packet reception. We employed the Okumura-Hata path loss model and data packets with payload of 51 bytes, while ACKs carried no payload. The same conditions were considered in the model, for a fair comparison. The plot in Figure 1 compares the packet success rate given by the model (dashed line) and obtained by simulation (solid line), when varying the packet arrival rate λ. We can observe that the theoretical curve follows rather closely the empirical results, though it is slightly optimistic. However, the approximation is very good in the most interesting operating region, i.e., when the packet success ratio is higher than 60%.

D. Parametric study

Among the considered parameters, the DC at the GW has a major impact on network performance. This is apparent in Figure 2, which reports the performance estimation provided by the proposed model with and without DC restrictions at the GW, together with the result provided by the model in [9] (which neglects the DC limitations at the GW and other system features).[5] We can observe that the DC constraint can have a dramatic impact on the packet success rate, in particular for high traffic scenarios. The reason is that each ACK transmission forces the GW to remain silent for certain time, which can additionally be rather long if the ACK is sent with a high SF.

The dashed and solid lines in Figure 3 show the theoretical packet success rate when varying the parameter configuration, but always under the `DC enabled` assumption and by considering usage of SF 12 in the second window, as recommended by default in the standard. Several non-trivial conclusions can be drawn from this plot. First of all, we can observe that systematically transmitting the ACKs in both receive windows has a negative impact on the network performance: each ACK transmission influences the rate at which the following ACKs can be sent, and does not bring significant benefits to the network for the considered traffic rates. Other results, not reported here for space constraints, show that this holds true (to a smaller extent) also when the DC is not enabled. The reason is that, in order to transmit two ACKs, the GW needs to stay longer in transmission mode, thus losing other potentially incoming packets. This effect is especially relevant when transmission is given priority over reception. This reasoning also extends to the case of only 1 ACK being sent for each received UL packet, which is the best solution when reception is prioritized. The dotted line in Figure 3 shows the effect of employing for both ACK transmissions the same SF that was used in the UL: thanks to the fact that in this case the second channel is used for transmission of shorter packets, DC limitations become more lenient, and let the channel be available to transmit ACKs with an higher probability, thus covering more devices and consistently improving the network performance.

Finally, we observe that correct parameter tuning in a LoRaWAN network can have a considerable impact on performance: if we set 0.9 as the minimum acceptable packet success ratio of the network, the best configuration is able to withstand three times the maximum traffic the worst configuration can handle (e.g., if we assume EDs generating one packet each hour, with the best configuration the network is able to support 216 EDs, against the 72 EDs supported with the worst one). Similarly, it can be observed that for a fixed amount of traffic using the best configuration can yield a 10% increase in successful packet ratio.

V. Conclusions

The main contribution of this work consists in a new mathematical model for LoRaWAN networks. The analytical model includes SF orthogonality and the capture effect, which characterize LoRa modulation, and it takes into account realistic features of LoRaWAN chips and regulatory constraints,

[5]For a fair comparison, we implemented in our model the same assumption considered in [9], i.e., 2 ACKs and reception priority.

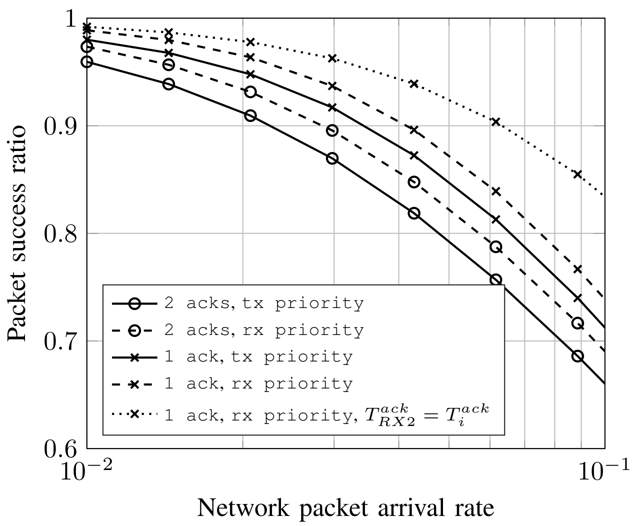

Fig. 3. Comparison of analytical model with different assumptions and `DC enabled`.

such as the DC limitations and the finite number of reception paths at the GW. Moreover, the model is structured in a way that makes it easy to modify a series of parameters, such as the number of ACKs, the SF employed for ACKs in RX2, the presence of DC and the prioritization of ACK transmission over reception. This makes the model suitable to study different network scenarios, investigating possible enhancements or performance limits with minimum effort. After a presentation of the mathematical formalization, we validated the model with simulation results obtained from a full-fledged ns-3 simulator, and showed some examples of how the model is able to predict the performance of various parameter configurations, also identifying the network settings that optimize the packet delivery ratio as the one where a single acknowledgment is sent and reception is given the priority.

As future work, we plan to further improve this model in order to close the gap with the simulation outcomes and to consider the presence of multiple gateways, and to use it to analyze possible enhancements to the system. Finally, it would be interesting to have the possibility of comparing the model outcomes with the results of a real deployment.

Appendix

In the following, we state equations for P_{0path}^a and P_{1path}^a; we call ϑ the probability of having at least one packet that arrived at the GW but could not lock:

$$\vartheta^a = (1 - e^{-\sum_i^{SF} T_i^{data} r_i})(1 - P_{rxPath}^a). \tag{19}$$

Therefore, we obtain:

$$\begin{aligned} P_{0,path}^a &= \Pr[\text{no packet arrivals}] \\ &\quad + \Pr[\text{arrivals that could not lock}] \\ &= e^{-\sum_i^{SF} T_i^{data} r_i} + \vartheta^a. \end{aligned} \tag{20}$$

$$\begin{aligned} P_{1,path}^a &= \Pr[\text{exactly 1 packet arrival that could lock}] \\ &\quad + \Pr[\text{1 packet arrival could lock and others could not}] \\ &= \Big(\sum_i^{SF} T_i^{data} r_i e^{-T_i^{data} r_i} (P_{rxPath}^a) \cdot e^{-\sum_{j \neq i}^{SF} T_j^{data} r_j} \Big) \\ &\quad + \Big(\sum_i^{SF} T_i^{data} r_i e^{-T_i^{data} r_i} (P_{rxPath}^a) \cdot \vartheta^a \Big). \end{aligned} \tag{21}$$

When two packets arrive at the GW, they can either have the same SF, or have different SFs. In this last case, the coefficient 1/2 in $P_{2,paths}$ avoids considering twice the same couple of SFs twice, as we are not interested in the order (e.g., $i = 7$, $j = 8$ and $i = 8$, $j = 7$ must be considered only once):

$$\begin{aligned} P_{2,paths}^a &= \Pr[\text{exacly 2 packets arrivals that could lock}] \\ &\quad + \Pr[\text{2 arrivals could lock and others could not}] \\ &= \Big(\sum_i^{SF} \frac{(T_i^{data} r_i)^2}{2} e^{-T data_i r_i} (P_{rxPath}^a)^2 \Big) \\ &\quad \Big(e^{-\sum_{j \neq i}^{SF} T_j^{data} r_j} + \vartheta^a \Big) \\ &\quad + \frac{1}{2} (P_{rxPath}^a)^2 \Big(\sum_i^{SF} T_i^{data} r_i e^{-T_i^{data} r_i} \\ &\quad \sum_{j \neq i}^{SF} T_j^{data} r_j e^{-T_j^{data} r_j} \Big) \cdot \Big(e^{-\sum_{t \neq i,j}^{SF} T_t^{data} r_t} + \vartheta^a \Big). \end{aligned} \tag{22}$$

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