An Adaptive ARQ Scheme with Packet Combining for Time Varying Channels

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Abstract—In this letter, dependence of the efficiency of hybrid type-II ARQ schemes on packet size in the context of a simple packet combining scheme is discussed. A simple algorithm of adopting the optimum packet size according to channel bit error rate (BER) is presented. Also, a very simple method of estimating the channel BER is provided.

Index Terms—EARQ, SRJ, XOR, adaptive schemes, bit inversion, hybrid type-II ARQ schemes.

I. INTRODUCTION

FOR reliable data transmission over a noisy channel, automatic repeat request (ARQ) schemes are used. In these schemes, the transmitter appends a frame check sequence (FCS) to the data block to aid the receiver to detect errors. The FCS is thus the minimum coding requirement for reliable exchange of information. Of the three basic ARQ schemes, stop and wait (SW), go back N (GBN) and selective reject (SRJ) [1], SRJ is the most efficient. Throughput $T(p)$ and transmission efficiency η of SRJ scheme in binary symmetric channel (BSC) are given by the well-known equations

$$T(p) = 1 - B(p) = (1 - p_e)^n \qquad (1)$$

$$\eta = \frac{n - h}{n} \cdot T(p) \qquad (2)$$

where $B(p)$ is packet error rate (PER), p_e is bit error rate (BER), n is packet size in bits, and h is the number of FCS and other overhead bits. The optimum packet size for maximizing η at a given BER has been studied extensively in the past (e.g., [2]–[5]) and, for BSC channel, can be easily derived from (2).

Simple ARQ schemes (even with the optimum packet size) may however be wasteful in high-BER and time varying channels as encountered in wireless and mobile systems. In such a channel, hybrid type-II ARQ schemes (see [6], [7] for references) are often used, where data blocks are carefully encoded before transmission so that successive incorrectly received copies of the same packet can be combined to form rate 1/2, or in the general case rate $1/m$, invertible codewords to retrieve the correct packet. Efficiency of type-II schemes at a given BER also depends on packet size. In time varying channels, efficiency of type-II schemes can thus be improved by dynamically adopting the optimum packet size at a given BER. However, such dependence has not been rigorously studied before. Further, the efficiency gain by type-II schemes requires increased processing complexity and corresponding energy consumption, at both the transmitter and the receiver. In mobile terminals, processor power is usually limited, and energy is drawn from a battery. Thus, an optimum scheme for reliable data transfer in mobile environment should not only maximize transmission efficiency, but also minimize the number of transmissions,[1] coding complexity at the transmitter and decoding complexity at the receiver.

A simple idea of packet combining is presented in [8],[2] where erroneous copies are XOR-ed to locate the errors in a combined copy. The correct packet can be retrieved by an exhaustive search method. In [9], an SRJ scheme based on this idea is proposed (the Extended ARQ or EARQ hereafter), and its performance in BSC channel is studied. Since EARQ uses an SRJ transmitter to simply append FCS to the data block, the coding complexity is minimal. Further, since the receiver uses simple hard decision detector,[3] it is compatible to existing hard decision receivers, and the detection complexity is lower than with those using soft detection. Thus, the overall coding-decoding complexity of the EARQ scheme is well contained within the receiver in XOR-ing the erroneous packets and the brute search process. Since the efficiency of the EARQ scheme is also comparable to other (and more complex) schemes, these factors are extremely attractive for energy constrained environments [10], [11].

In this letter, the dependence of efficiency of hybrid type-II schemes on packet size at a given BER is studied. In this context, we take the example of the EARQ scheme, and an upper bound of hybrid type-II schemes, as defined in [9]. We also propose a scheme for throughput optimization in time varying channels, by dynamically adjusting the packet size. We assume that the round trip delay is negligible, so that the underlying ARQ scheme is essentially SRJ, and that the feedback is always received error free. Implications of the

[1] Note that an optimum packet size also minimizes the number of transmissions.

[2] Actually, [8] presents an idea for improving a type-I scheme with type-II functionality.

[3] Use of soft decision is not dealt with in this article.

TABLE I
CROSSOVER VALUES OF BER AND OPTIMUM PACKET SIZES FOR THE THREE ADAPTIVE ARQ SCHEMES

$p_{cr,SRJ}$	10^{-6}	$3.2 \cdot 10^{-6}$	$3.2 \cdot 10^{-5}$	$3.2 \cdot 10^{-4}$	$3.0 \cdot 10^{-3}$
$p_{cr,ideal}$	10^{-6}	$3.2 \cdot 10^{-6}$	$3.2 \cdot 10^{-5}$	$3.1 \cdot 10^{-4}$	$1.6 \cdot 10^{-3}$
$p_{cr,EARQ}$	10^{-6}	$3.2 \cdot 10^{-6}$	$3.2 \cdot 10^{-5}$	$3.1 \cdot 10^{-4}$	$1.6 \cdot 10^{-3}$
n_{SRJ}	4008	1275	400	135	50
n_{ideal}	4026	1289	425	154	110
n_{EARQ}	4026	1289	425	150	110

varying packet size on the feedback scheme are not considered here.

II. APPROACH

Let us consider an ideal type-II ARQ scheme, where if the first transmission is received incorrectly, then the second transmission is assumed either to be correct, or to retrieve the packet correctly when combined with the first copy. Throughput[4] of such a scheme is simply given by

$$T_U(p) = \frac{1}{1 + B(p)}. \tag{3}$$

In the EARQ scheme (a detailed discussion on EARQ appears in [9]), erroneous copies of the same packet are XOR-ed to locate the erred bit positions, and a brute search method is employed to retrieve the correct packet. The operation fails if the erroneous copies have at least one erred bit position in common (or a "double error"), and a further retransmission is asked. The probability of a successful retrieval at the Lth attempt is

$$P(L) = \left[1 - \sum_{r=1}^{L-1} P(r) \right] \cdot [T(p) + (1 - T(p)) \cdot (1 - \alpha_d(L))] \tag{4}$$

and corresponding throughput is given by

$$T_e(p) = \frac{1}{\langle L \rangle} \tag{5}$$

where $\alpha_d(L)$ is the conditional probability that, provided the Lth copy is erred, it has a double error with all previous $L-1$ copies, and $\langle L \rangle$ is the expectation value of L. Estimation of $\alpha_d(L)$ is given in [9] and is not repeated here. The transmission efficiencies of the ideal scheme and EARQ can be had by multiplying the throughput with packet payload $(n - h)/n$. Assuming a 16-bit FCS overhead, efficiencies of the three schemes for BER[5] of $10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$, and $2 \cdot 10^{-2}$, for various packet fsizes up to 1000 bits, are provided in Fig. 1. Important observations from Fig. 1 are as follows.

- Below BER of 10^{-4}, type-II schemes are not needed.
- For BER $\geq 10^{-4}$, packet size greater than 1000 bits is impractical.
- At BER of 10^{-3}, EARQ performs equivalently to the ideal scheme. However, at the maximum efficiency point

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[4] Note that, at high BER, efficiency of this scheme approaches to 50% at infinite packet size, not achievable by any practical type-II ARQ schemes.

[5] We assume that conditions where BER $\gg 10^{-2}$ usually occur for very short periods, and no sustained operation is possible under such situations. In these cases, packets are simply rejected in the usual SRJ fashion.
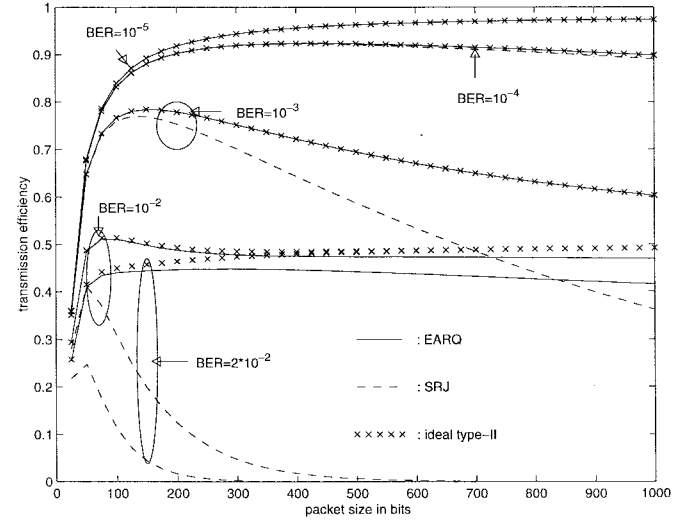

Fig. 1. Efficiencies of SRJ, ideal type-II and EARQ schemes as functions of packet size, at different BER values $\left(10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, \text{ and } 2 \cdot 10^{-2} \right)$.

near 150-bit packet size, type-II schemes are only marginally better than SRJ.
- At BER of 10^{-2}, the efficiency of SRJ is 41% at 49-bit packet size. The maximum achievable efficiency with packet combining is about 51% near 85-bit packet size, where EARQ also performs near-ideally.
- At BER of $2 \cdot 10^{-2}$, efficiency of SRJ is 27% at 37-bit packet size. Achievable efficiency with packet combining stays below 50% (at finite packet sizes). The efficiency of EARQ is about 45% near 300-bit packet size.
- Type-II schemes are much less sensitive to variation in packet size when compared to SRJ.

It is thus evident that, unless the channel BER remains steadily higher than $2 \cdot 10^{-2}$, complex coding-decoding schemes (e.g., [6], [12]–[14], etc.) help little in improving efficiency, but they may be constantly demanding on both computational and energy resources. Further, a simple adaptive scheme which chooses the optimum packet size at a given BER would maximize the efficiency and minimize the number of transmissions. This however requires estimation of the channel BER, which is described below.

III. MEASUREMENT OF CHANNEL BER

Estimation of channel BER for employing an adaptive scheme has been studied before [5], [15]. However, observations in [5], that estimation of channel BER needs a lot of overhead and may conflict with the error detection scheme, are not true. Neither is a complicated sequential test as proposed in
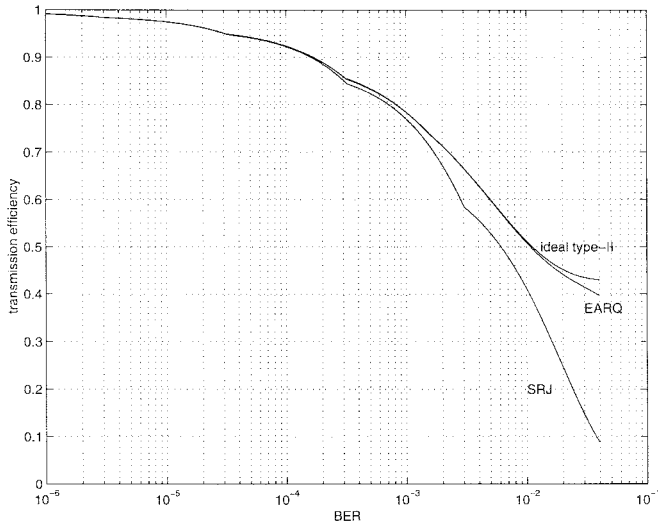
Fig. 2. Optimum efficiencies of adaptive SRJ, ideal type-II and EARQ schemes as functions of BER.

[15] needed. The EARQ scheme by default provides a simple method for measuring instantaneous BER, as follows. The average number of errors in an erred copy is given by

$$\chi = \frac{np_e}{1 - (1 - p_e)^n}. \tag{6}$$

Thus, while the assumption of one erroneous bit per block in [5] is valid at very low BER, it is highly inaccurate for high BER $(\approx 10^{-2})$ and moderate packet sizes $(>100$ bits), where χ tends to np_e. XOR-ing two consecutive erroneous copies both locates the errors and provides an accurate estimate of number of errors in a combined copy (i.e., in $2n$ bits). A medium term[6] average value of BER can further be had by averaging over a number of successive transmissions. Note that the BER measurement with EARQ is so simple that it can also be used with schemes proposed in [5] and [15] or any other adaptive scheme requiring instantaneous BER measurement.

Table I provides the optimum block sizes for SRJ, the ideal type-II and EARQ schemes at different BER's, and BER crossover values. Since efficiency of type-II schemes is much less sensitive to packet size variation than that of SRJ, only a fixed packet size[7] of 110 bits is used in the high-BER region $(>2 \cdot 10^{-3})$. Fig. 2 provides the optimum efficiency curves of the three schemes. It can be seen that, at relatively low BER $(<10^{-3})$, optimized SRJ performs very close to type-II schemes. However, above BER of 10^{-3}, type-II schemes improve the efficiency substantially. EARQ performs almost ideally up to BER of 10^{-2}, and maintains a sufficiently high efficiency when BER is as high as $4 \cdot 10^{-2}$. Computational complexity after this may be prohibitively high, forcing to

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[6]In a time varying channel, averaging the BER over long time is not practical.

[7]Though this packet size is not optimum for either EARQ or the ideal scheme, it represents fairly well the characteristics of these two schemes at BER above 10^{-2}. Further, the relatively small packet size also keeps the computational complexity of the EARQ scheme within reasonable limits.

repeated retransmissions. However, in a time varying channel, if such a highly noisy situation is only short-lived, then packets transmitted in these intervals can be discarded without much loss of efficiency. Though multiple packet sizes are adopted here, only two packet sizes, a smaller size of 110 bits above BER of 10^{-3}, and a bigger packet size of about 750 bits below it, would provide nearly the same performance, but would simplify the scheme considerably.

IV. CONCLUDING REMARKS

The main intention of this letter is to demonstrate that complex coding-decoding schemes are rarely necessary for efficient and reliable transfer of data in a time varying channel unless the channel BER is consistently high $(\gg 2 \cdot 10^{-2})$. Till a BER of 10^{-3}, an SRJ scheme of optimal packet size provides with very satisfactory performance. Efficiency can further be enhanced using the packet combining of the EARQ scheme, and substantial performance can still be achieved when the channel BER goes as high as $4 \cdot 10^{-2}$. Additional advantage of EARQ is that it automatically provides an accurate estimation of the channel BER. Computational complexity of EARQ at the optimum packet size is also very reasonable.

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