

Reliability and Energy Consumption of LoRa With Bidirectional Traffic

Siddhartha S. Borkotoky^{1b}, Member, IEEE, Jorge F. Schmidt^{2b}, Udo Schilcher^{2b},
Prameela Battula^{1b}, and Sonu Rath^{1b}

Abstract—We analyze the reliability and energy consumption of uplink message delivery in bidirectional LoRa networks that employ acknowledgements and retransmissions. Our frame-collision analysis with symbol-level timing accuracy shows that the ACKs—introduced to improve reliability—significantly impact the end devices’ power consumption via the collisions they introduce. We demonstrate that a critical design parameter affecting reliability and energy consumption is the maximum number of times an end device retransmits an unacknowledged frame. Our analytical framework can be used to appropriately choose this parameter by taking node density, traffic intensity, target reliability, and the energy budget into account.

Index Terms—LoRaWAN, reliability, energy consumption.

I. INTRODUCTION

WITH its wide coverage and low-power operations, LoRa has emerged as a key player within the IoT ecosystem. LoRa supports bidirectional communications between end devices (EDs) and gateways (GWs). The MAC protocol built on LoRa PHY (LoRaWAN) utilizes this feature through the *confirmed uplink* (CU) mode, in which EDs request acknowledgments (ACKs) from the GW and retransmit unacknowledged frames. While aimed at improving reliability, bidirectional communications have shortcomings [1]–[3]. The retransmissions increase frame collisions. Besides, commercially available GWs are half-duplex and cannot receive uplink frames while transmitting ACKs. Thus, ACKs may themselves prevent the reception of some frames, leading to retransmissions that drain batteries at the energy-constrained EDs. A mechanism to predict network performance is therefore necessary to make design decisions such as whether to use CU in a given network and, if used, how many retransmissions per message to allow to reach a target message-delivery

performance within the EDs’ energy budget. This letter presents an analytical framework for that purpose. Our key contributions include the modeling of frame overlap conditions with symbol-level timing accuracy, the incorporation of experimental findings reported in the literature to accurately model frame-loss events, and the evaluation of the impact of network parameters such as the number of nodes, uplink traffic intensity, and the maximum permitted retransmissions per frame on the delivery and energy performance. The analysis provides insights into the parameters that affect reliability and energy expenditure, and helps in network planning and protocol design.

II. RELATED WORK

Bidirectional LoRa’s performance is analyzed in [4]–[6]. Retransmissions are not considered in [4] and [6], whereas fading is ignored in [5] and [6], although both phenomena critically impact system performance. The model in [4] assumes a frame to be always lost upon collision with another frame. In [5], the total power in all colliding frames combined is used to determine frame-loss occurrences. In [6], any frame that collides with multiple other frames is considered lost. In practice, LoRa frame loss depends on power differences between individual colliding frames, and a frame may survive collision with multiple others regardless of the cumulative interference power if each individual interferer is weaker than the frame in question by a certain margin [7]. Frame loss also depends on which portion of a frame is impacted by interference, and a frame may survive the corruption of the first few preamble symbols [8]. Furthermore, [4]–[6] focus exclusively on frame errors. The energy-consumption aspect—which is critical for battery-powered EDs—is not addressed. Energy consumption is analyzed in [9] under the implicit assumption of bidirectional communications (in the sense that a frame is retransmitted until correct reception), but the analysis therein does not consider frame losses enforced due to ACK transmissions and uses the cumulative interference power to determine frame loss events.

III. SYSTEM MODEL AND ASSUMPTIONS

We consider n EDs E_0, E_1, \dots, E_{n-1} uniformly distributed over a circle of radius \mathcal{R} with a half-duplex GW at the center. The EDs transmit using Class A (pure ALOHA) with power p_t , choosing for each frame one of n_f non-overlapping bands at random. (For details on LoRa, see [10] and [11].) A fraction μ_c of the EDs operate in the CU mode; we refer to such EDs as CU-EDs. Upon receiving a CU-ED’s frame, the GW transmits an ACK after t_δ s. Only one ACK is sent following a received frame. If an ACK is not received following an uplink frame, a CU-ED retransmits the frame after a random backoff

Manuscript received August 14, 2021; accepted September 2, 2021. Date of publication September 15, 2021; date of current version November 11, 2021. This work was supported by the Science and Engineering Research Board (SERB), Government of India, through its Start-up Research Grant (SRG) under SRG/2020/001491, and by the K-project DeSSnet (Dependable, secure and time-aware ED networks), which is funded within the context of COMET—Competence Centers for Excellent Technologies by the Austrian Ministry for Transport, Innovation and Technology (BMVIT), the Federal Ministry for Digital and Economic Affairs (BMDW), and the federal states of Styria and Carinthia; the COMET program is conducted by the Austrian Research Promotion Agency (FFG). The associate editor coordinating the review of this letter and approving it for publication was M. H. Cheung. (Corresponding author: Siddhartha S. Borkotoky.)

Siddhartha S. Borkotoky, Prameela Battula, and Sonu Rath are with the Indian Institute of Technology Bhubaneswar, Khordha 752050, India (e-mail: borkotoky@iitbbs.ac.in; battulaprameela4284@gmail.com; sr42@iitbbs.ac.in).

Jorge F. Schmidt is with the Institute of Networked and Embedded Systems, University of Klagenfurt, 9020 Klagenfurt, Austria, and also with Lakeside Labs GmbH, 9020 Klagenfurt, Austria (e-mail: jorge.schmidt@aau.at).

Udo Schilcher is with the Institute of Networked and Embedded Systems, University of Klagenfurt, 9020 Klagenfurt, Austria (e-mail: udo.schilcher@aau.at).

Digital Object Identifier 10.1109/LCOMM.2021.31113134

1558-2558 © 2021 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.

See <https://www.ieee.org/publications/rights/index.html> for more information.

time. A frame is retransmitted until it is acknowledged or R_m retransmissions are made. We call the first transmission of a frame a *fresh frame*.

We denote the receiver sensitivity by ζ , and model the network links as independent and identically distributed (i.i.d.) block-fading channels with pathloss exponent α . The received power at the GW from an ED located at a distance d is $R = \gamma p_t A d^{-\alpha}$, where A is the power-fading coefficient that remains constant throughout a frame and is independent from one frame to the next, and γ is a hardware constant. We illustrate our analytical approach for Rayleigh fading, but it is trivial to incorporate other fading and shadowing models.

A frame contains a preamble for acquisition and synchronization, followed by the payload. The preamble comprises 8 symbols. The frame duration depends on the spreading factor (SF), which is an integer between 7 and 12; larger SFs provide stronger noise immunity at the cost of longer symbol durations. All EDs are assumed to use the same SF. When signals from such EDs collide, the strongest signal is correctly demodulated if its power exceeds that of each of the other signals by a SF-dependent threshold ξ [8]. For successful acquisition of a frame by the receiver, at least 5 of the preamble symbols must be detected [8]. Note that our analysis can be easily extended to networks where multiple SFs are used. For example, the method in [12] for modeling inter-SF collisions can be directly incorporated into our framework.

The interval between two consecutive fresh frames from an ED is modeled as exponentially distributed with mean λ^{-1} . We assume that λ^{-1} is much larger than typical frame lengths. This is consistent with typical LoRa use cases where EDs transmit small amounts of data relatively infrequently.

IV. ANALYTICAL EVALUATION

Our reliability measure is the *message-failure probability* (MFP), which is the probability that a CU-ED's message remains undelivered to the GW after reaching the maximum retransmission attempts. Our energy-expenditure measure is the *expected transmission count* (ETC), which is the average number of times a CU-ED transmits the same frame. We wish to determine the MFP and ETC for a CU-ED at distance d_0 from the GW. Let $P_F(d_0)$ be the probability that the GW fails to receive the CU-ED's transmission. It follows that

$$\text{MFP}(d_0) = (P_F(d_0))^{R_m+1}, \quad (1)$$

and

$$\begin{aligned} \text{ETC}(d_0) = (1 - P_F(d_0)) \sum_{i=1}^{R_m} i (P_F(d_0))^{i-1} \\ + (R_m + 1) (P_F(d_0))^{R_m}. \end{aligned} \quad (2)$$

In the analysis that follows, l_s and l_f denote the duration of a modulation symbol and an uplink frame, respectively, and l_a is the duration of an ACK frame.

A. Analysis of Frame Starting Times

Let $P_{\text{st}}(x, y, d)$ be the probability that a frame from an ED at distance d from the GW begins within the interval $[x, y]$. Let $P_{\text{st}}^{(0)}(x, y)$ and $P_{\text{st}}^{(i)}(x, y, d)$, $i \geq 1$, denote the same probability but for a fresh frame and the i th retransmission of

a frame, respectively. Note that the start of a fresh frame is independent of d , but retransmission probabilities depend on d .

Lemma 1: The probability that a fresh frame transmission starts within the interval $[x, y]$, where $0 \leq x < y < \infty$ and $y - x \ll \lambda^{-1}$, is approximated by

$$P_{\text{st}}^{(0)}(x, y) \approx \lambda(y - x). \quad (3)$$

Proof: Since the inter-arrival times for fresh frames from any given ED are exponential with rate λ , the number of fresh frames generated within the interval $[x, y]$ is approximately Poisson with rate $\lambda(y - x)$. Thus the probability that at least one frame starts during $[x, y]$ is $1 - e^{-\lambda(y-x)}$. But since $[x, y]$ is very small relative to the mean inter-arrival time, we can assume that the probability of multiple fresh frame starting within $[x, y]$ is negligible. Consequently, we have

$$P_{\text{st}}^{(0)}(x, y) \approx 1 - e^{-\lambda(y-x)} \approx \lambda(y - x), \quad (4)$$

where we utilize $e^{-\lambda(y-x)} \approx 1 - \lambda(y - x)$ for $\lambda(y - x) \ll 1$. ■

Lemma 2: The probability that the i th retransmission of a frame from a CU-ED at distance d from the GW starts within the interval $[x, y]$ is

$$P_{\text{st}}^{(i)}(x, y, d) \approx \lambda(y - x) (P_F(d))^i. \quad (5)$$

Proof: Upon non-arrival of an ACK, the CU-ED waits for a random backoff time and retransmits the frame. Consider the i th retransmission of the frame begins at time T_i . If the corresponding fresh frame started at time T_0 , then $T_i = T_0 + il_f + \sum_{j=1}^i B_j$, where B_j is the backoff time preceding the j th retransmission. Noting that the i th retransmission can occur only if an ACK did not arrive for any of the past i transmissions, we have

$$\begin{aligned} P_{\text{st}}^{(i)}(x, y, d) &= P(x \leq T_i \leq y) (P_F(d))^i \\ &= P\left(x \leq T_0 + il_f + \sum_{j=1}^i B_j \leq y\right) (P_F(d))^i \\ &= P\left(x - il_f - \sum_{j=1}^i B_j \leq T_0 \leq y - il_f - \sum_{j=1}^i B_j\right) (P_F(d))^i. \end{aligned} \quad (6)$$

Use of (3) yields $P_{\text{st}}^{(i)}(x, y, d) \approx \lambda(y - x) (P_F(d))^i$. ■

Lemma 3: If $0 < P_F(d) < 1$, the probability that a transmission from an arbitrary ED at distance d from the GW starts within the interval $[x, y]$ is

$$P_{\text{st}}(x, y, d) \approx \lambda(y - x) \left(1 + \mu_c \left(\frac{1 - (P_F(d))^{R_m+1}}{1 - P_F(d)} - 1\right)\right). \quad (7)$$

Proof: The probability that a transmission from an arbitrary ED (which may be a CU-ED with probability μ_c) at distance d from the GW starts within $[x, y]$ is

$$\begin{aligned} P_{\text{st}}(x, y, d) &\approx P_{\text{st}}^{(0)}(x, y) + \mu_c \sum_{i=1}^{R_m} P_{\text{st}}^{(i)}(x, y, d) \\ &\approx \lambda(y - x) \left(1 + \mu_c \sum_{i=1}^{R_m} (P_F(d))^i\right). \end{aligned} \quad (8)$$

Applying the expression for the sum of a geometric series, we arrive at (7). ■

Substituting $\mu_c = 1$ in (7), we obtain the start time probability for a CU-ED at distance d to be

$$\tilde{P}_{\text{st}}(x, y, d) \approx \lambda(y - x) \frac{1 - (P_F(d))^{R_m+1}}{1 - P_F(d)}. \quad (9)$$

B. Failure Probability

To derive $P_F(d_0)$, consider the transmission of a frame F from the CU-ED in question. For this transmission to be successful, F must satisfy two conditions: (A1) survive fading and interference, and (A2) not arrive during an ACK transmission. We denote by $S_{\text{FI}}(d_0)$ and S_{A} the probabilities of events (A1) and (A2), respectively. Thus,

$$P_F(d_0) = 1 - S_{\text{FI}}(d_0)S_{\text{A}}. \quad (10)$$

Derivation of $S_{\text{FI}}(d)$: The received power in F is $R = \gamma p_t A d^{-\alpha}$. Conditioning on the fading power,

$$S_{\text{FI}|A=a}(d_0) = \theta_a(d_0)S_{\text{I}|A=a}(d_0), \quad (11)$$

where $\theta_a(d_0) = 1$ if $p_t a d_0^{-\alpha} \geq \zeta$, and $\theta_a(d_0) = 0$ otherwise. $S_{\text{I}|A=a}(d_0)$ is the conditional probability that frame F with $A = a$ survives interference from other EDs.

To derive $S_{\text{I}|A=a}(d_0)$, we note that F is lost due to interference from an arbitrary ED located at distance D' from the GW if the latter transmits a frame \tilde{F} over the same channel as F and if the following conditions are satisfied: (B1) \tilde{F} has received power \tilde{R} at the GW such that $10 \log_{10}(R/\tilde{R}) < \xi$, and (B2) \tilde{F} overlaps with F . Conditioned on $D' = d'$, and letting $\xi_0 = 10^{0.1\xi}$, the probability that (B1) is satisfied is

$$\begin{aligned} P_w(d_0, d', a) &= P(R/\tilde{R} < \xi_0 | A = a, D' = d') \\ &= P(p_t a d_0^{-\alpha} / p_t A' d'^{-\alpha} < \xi_0) \\ &= P(A' > \xi_0^{-1} a (d_0/d')^{-\alpha}) \\ &= e^{-\xi_0^{-1} a (d_0/d')^{-\alpha}}. \end{aligned} \quad (12)$$

Regarding condition (B2), note that \tilde{F} will not cause the loss of F if no more than the first three preamble symbols of F overlap with \tilde{F} . If F and \tilde{F} occupy the intervals $[T + l_f]$ and $[\tilde{T} + l_f]$, respectively, then the overlap condition necessary to cause a failure is $\tilde{T} \leq T + l_f$ and $T + 3l_s \leq \tilde{T} + l_f$, or equivalently

$$T + 3l_s - l_f \leq \tilde{T} \leq T + l_f. \quad (13)$$

Let $\beta_1 = T + 3l_s - l_f$ and $\beta_2 = T + l_f$. The conditional probability that F survives interference from *one* arbitrary ED is

$$\begin{aligned} S_{\text{I}|A=a}^{(1)}(d_0) &= \mathbb{E}_{D'} \left[1 - n_f^{-1} P_w(d_0, D', a) P_{\text{st}}(\beta_1, \beta_2, D') \right] \\ &= 1 - \mathbb{E}_{D'} \left[n_f^{-1} e^{-\xi_0^{-1} a (d_0/D')^{-\alpha}} P_{\text{st}}(\beta_1, \beta_2, D') \right], \end{aligned} \quad (14)$$

where

$$\mathbb{E}_{D'}[g(D')] = \frac{2}{R^2} \int_0^R v g(v) dv. \quad (15)$$

The parameter $P_{\text{st}}(\beta_1, \beta_2 | v)$, given by (7), depends on the interfering ED's frame-success probability, which complicates the computation. For a simpler solution, we use the approximation $P_{\text{st}}(\beta_1, \beta_2 | v) \approx \lambda(\beta_2 - \beta_1)(1 + \mu_c \bar{R}) = \lambda(2l_f - 3l_s)(1 + \mu_c \bar{R})$, which is the start-time probability if a CU-ED always makes \bar{R} retransmissions. The computation of \bar{R} will be described shortly. We now have

$$S_{\text{I}|A=a}^{(1)}(d_0) \approx 1 - \mathcal{K}(\bar{R}) \mathbb{E}_{D'} \left[e^{-\xi_0^{-1} a (d_0/D')^{-\alpha}} \right], \quad (16)$$

where $\mathcal{K}(j) = n_f^{-1} \lambda(2l_f - 3l_s)(1 + \mu_c j)$. Since there are $n - 1$ EDs other than the sender of F , the probability that F survives interference is

$$S_{\text{I}|A=a}(d_0) \approx \left(S_{\text{I}|A=a}^{(1)}(d_0) \right)^{n-1}. \quad (17)$$

We substitute (16) into (17), use the result to find (11), and finally remove the conditioning on A to obtain

$$S_{\text{FI}}(d_0) = \mathbb{E}_A \left[\theta_A(d_0) \left(1 - \mathcal{K}(\bar{R}) \mathbb{E}_{D'} \left[e^{-\xi_0^{-1} A (d_0/D')^{-\alpha}} \right] \right)^{n-1} \right], \quad (18)$$

where

$$\mathbb{E}_A[g(A)] = \int_0^\infty g(a) e^{-a} da. \quad (19)$$

The parameter \bar{R} is given by

$$\bar{R} = \min\{1/\bar{S}, R_m\}, \quad (20)$$

where \bar{S} is the probability of frame loss (averaged over all EDs) if the network has no ACKs or retransmissions. It is obtained by evaluating (18) with d_0 replaced by D , $\mathcal{K}(\bar{R})$ replaced by $\mathcal{K}(0) = n_f^{-1} \lambda(2l_f - 3l_s)$, and then taking the expectation w.r.t. D .

Derivation of S_{A} : Recall that F occupies the time interval $[T, T + l_f]$. Consider an ACK frame F_a occupying $[T_a, T_a + l_a]$, sent in response to a frame F_i from ED E_i located at a distance D from the GW. If $T_a \leq T$ and $T_a + l_a \geq T + 3l_s$ then F reaches the GW during F_a 's transmission, has at least the first three preamble symbols go undetected, and is lost. The timing conditions combine to

$$T - l_a + 3l_s \leq T_a \leq T. \quad (21)$$

Since only the successful reception of a confirmed uplink frame triggers an ACK, which is sent t_δ s after the frame's reception, the following conditions must hold: (C1) E_i is a CU-ED, (C2) F_i survives fading, interference, and ACK collisions, and (C3) the starting time T_i of F_i satisfies

$$T - l_a + 3l_s - l_f - t_\delta \leq T_i \leq T - l_f - t_\delta. \quad (22)$$

Condition (C1) is satisfied with probability μ_c . Conditioning on $D = d$, condition (C2) is satisfied with probability $1 - P_F(d)$, found using (10). Let $\tilde{P}_{\text{st}}(x, y | d)$ be the probability that a CU-ED at distance d from the GW starts a transmission during $[x, y]$. The probability that condition (C3) is satisfied is $\tilde{P}_{\text{st}}(\nu_1, \nu_2 | d)$, where $\nu_1 = T - l_a + 3l_s - l_f - t_\delta$ and $\nu_2 = T - l_f - t_\delta$. Define $\delta_\nu = \nu_2 - \nu_1 = l_a - 3l_s$. Thus

the probability that F is lost due to collision with an ACK meant for E_i is

$$\begin{aligned} C_{\mathcal{A}}^{(i)} &= \mathbb{E}_D \left[\mu_c (1 - P_F(D)) \tilde{P}_{st}(\nu_1, \nu_2, D) \right] \\ &= \mu_c \mathbb{E}_D \left[(1 - P_F(D)) \lambda(\nu_2 - \nu_1) \frac{1 - (P_F(D))^{R_m+1}}{(1 - P_F(D))} \right] \\ &= \mu_c \lambda \delta_\nu \left(1 - \mathbb{E}_D [(P_F(D))^{R_m+1}] \right). \end{aligned} \quad (23)$$

Some manipulations yield

$$C_{\mathcal{A}}^{(i)} \approx \mu_c \lambda \delta_\nu \left(1 - \mathbb{E}_D \left[\left(1 - \mathbb{E}_A [\eta(A, D)] S_A \right)^{R_m+1} \right] \right), \quad (24)$$

where

$$\eta(A, D) = \theta_A(D) \left(1 - \mathcal{K}(\bar{R}) \mathbb{E}_{D'} \left[e^{-\xi_0^{-1} A(D/D')^{-\alpha}} \right] \right)^{n-1}. \quad (25)$$

Since there are $n - 1$ EDs other than the sender of F, each equally likely to elicit an ACK, $S_A = (1 - C_{\mathcal{A}}^{(i)})^{n-1}$. This provides a polynomial in S_A , which can be solved using a computer program to determine S_A . We can now compute $P_F(d_0)$ using (10), which lets us determine the MFP and ETC using (1) and (2), respectively.

V. RESULTS AND DISCUSSIONS

We verify our analysis using simulations in LoRaSim [13], into which we incorporated ACKs and retransmissions. Unless stated otherwise, the links have Rayleigh fading, and all EDs are CU-EDs. The transmit power in an uplink frame is 14 dBm. Hereafter, we use the term *non-bidirectional* to refer to a system without ACKs and transmissions. Each data point we plot is the average of 500 sessions. ED locations are randomly chosen at the beginning of each session. One session simulates network operations for $100\lambda^{-1}$ s, which means one ED transmits on the average 100 frames per session.

We first examine a bidirectional network in which ACKs are sent but no retransmissions are made. Fig. 1 shows the failure probabilities for an ED located 60 m from the GW. The EDs are uniformly distributed over a circle of radius 100 m, with the GW at the center, and transmit 5 payload bytes per frame with SF 7. The average inter-arrival time of fresh frames (i.e., λ^{-1}) from an ED is 10 s. The pathloss exponent is 3. The ACKs carry 1-byte payloads. Fig. 1 also shows the fraction of uplink frames that collide with ACKs and are lost (this occurs with probability $1 - S_A$). It is clear that ACKs are a major cause of frame loss. With 400 EDs, about 40% of the uplink frames experience collisions with ACKs. Fig. 1 also demonstrates the inaccuracy in the estimated MFP when all colliding frames are assumed to be lost (as in [4]) and when the impact of fading is ignored (as in [5] and [6]).

Fig. 2 shows the MFP for an ED located 60 m from the GW in a bidirectional network in which EDs make up to R_m retransmissions of unacknowledged frames. Results are shown for $R_m = 2$ and $R_m = 4$, and are compared with those for a non-bidirectional network. All network parameters are the same as those for Fig. 1. With fewer than 200 EDs, allowing up to 2 retransmissions reduce the MFP relative to the non-bidirectional network. But with more EDs, ACKs and

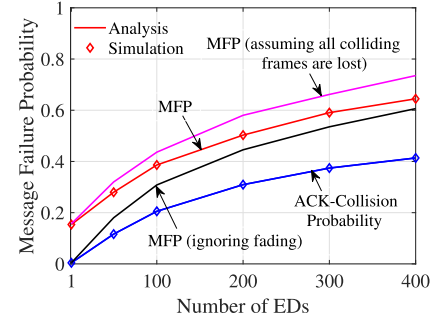


Fig. 1. Bidirectional LoRa with ACKs but no retransmissions.

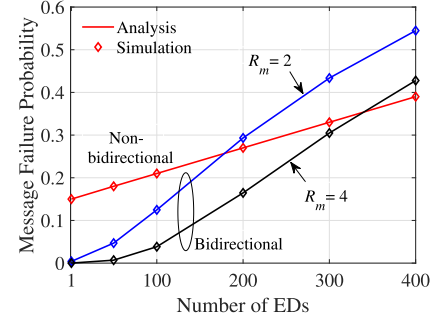


Fig. 2. Bidirectional vs. non-bidirectional LoRa.

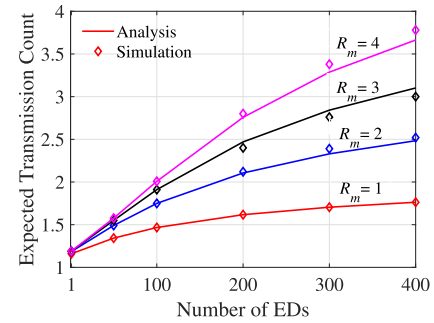


Fig. 3. ETC for bidirectional LoRa.

retransmissions become counterproductive, and their impact becomes progressively worse with increasing node density. With up to 4 retransmissions permitted, reliability improvements are seen if the number of EDs is below 350.

Fig. 3 illustrates the energy cost of retransmissions in terms of the ETC. Note that for the non-bidirectional network, ETC = 1. In a bidirectional network, the energy consumption per message per ED for a given R_m increases with increasing node density as a result of more transmission failures. With the number of EDs fixed, the energy expenditure increases with R_m . The ETC for a 400-node network with $R_m = 4$ is about 3.7, which signifies a 270% increase in the energy consumption in the uplink compared to the non-bidirectional network. As seen from Fig. 2, the MFP with bidirectional traffic for the 400-node network is about 0.44, as opposed to 0.38 for the non-bidirectional network. By contrast, with 50 EDs, using $R_m = 4$ reduces the MFP from 0.18 to 0.006 while approximately doubling the energy expenditure relative to the non-bidirectional network.

Fig. 4 plots the analytically obtained MFP against R_m for an ED located 100 m from the GW in a network of 300 EDs distributed over a circle of 200 m radius. The EDs transmit 1-byte payloads with SF 8. The solid and dashed curves are

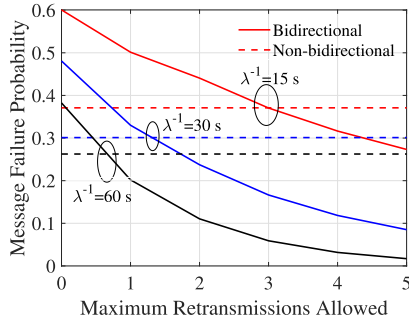


Fig. 4. MFP vs. maximum permitted retransmissions.

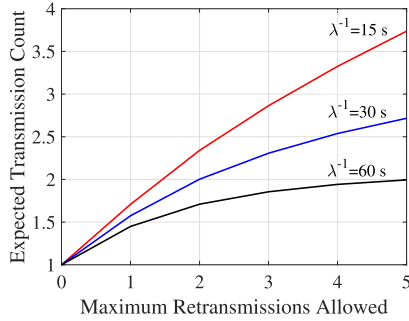


Fig. 5. ETC vs. maximum permitted retransmissions.

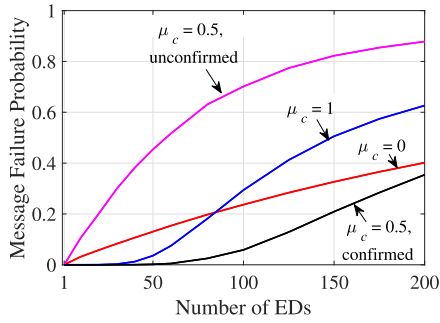


Fig. 6. MFP in the presence of log-normal shadowing.

for a bidirectional and non-bidirectional network, respectively. The MFP is observed to rise with increasing uplink arrival rates (decreasing λ^{-1}). The point of intersection between the solid and dashed curves for a given λ^{-1} gives the minimum R_m that provides higher reliability than a non-bidirectional system. The minimum R_m increases with decreasing λ^{-1} . Note that at $R_m = 0$, the gap between the corresponding solid and dashed lines indicates the increase in the MFP due to the losses that occur because a frame arrives during an ACK transmission.

Fig. 5 plots ETC for the setup of Fig. 4. Increasing R_m results in higher ETC—and consequently greater energy consumption. For sparser uplink traffic (larger λ^{-1}), the ETC saturates faster compared to when uplink frames arrive more frequently (smaller λ^{-1}). From Figs. 4 and 5, we observe that for $\lambda^{-1} = 60$ s, increasing R_m from 2 to 5 decreases the MFP by 84% (from 0.11 to 0.017) at the cost of a 16% increase in the ETC (from 1.71 to 1.99). For $\lambda^{-1} = 15$ s, the same increase in R_m reduces the MFP by 38% (from 0.44 to 0.27) while increasing the ETC by 62% (from 2.3 to 3.73).

Fig. 6 shows the MFP for log-normal shadowing on the links—which is a scenario of practical interest [14]—and uplink transmissions with SF 12 and $\lambda^{-1} = 10$ s. Three

cases are investigated: only unconfirmed traffic (i.e., a non-bidirectional network, obtained for $\mu_c = 0$), only confirmed traffic ($\mu_c = 1$), and a mixed traffic scenario in which 50% of the nodes send CU frames ($\mu_c = 0.5$). A comparison of confirmed-only and unconfirmed-only scenarios reveals a familiar pattern in which CU is beneficial for lower node densities. In the mixed scenario, we observe that the CU-EDs are able to achieve a low MFP at the cost of performance deterioration for the EDs that do not employ CU. The ETC for the CU-EDs in these networks showed similar patterns as for the Rayleigh-fading scenarios reported earlier.

VI. CONCLUSION

We analyzed the reliability and energy consumption of LoRa networks that employ acknowledgements and retransmissions. Characterizing the role acknowledgements play in increasing the energy consumption at the end devices, we demonstrated that the maximum number of times an end device is permitted to retransmit an unacknowledged message is a key design parameter, whose judicious selection is crucial to achieve desired delivery and energy performance. Our analytical framework provides a way to make such selections. It helps determine whether the energy budget at the EDs justifies the use of bidirectional traffic, and if so, it helps to find a suitable cap on the retransmissions per message to satisfy (energy constrained) reliability requirements.

REFERENCES

- [1] J. M. Marais, A. M. Abu-Mahfouz, and G. P. Hancke, “A survey on the viability of confirmed traffic in a LoRaWAN,” *IEEE Access*, vol. 8, pp. 9296–9311, Jan. 2020.
- [2] M. Capuzzo, D. Magrin, and A. Zanella, “Confirmed traffic in LoRaWAN: Pitfalls and countermeasures,” in *Proc. 17th Annu. Medit. Ad Hoc Networking Workshop (Med-Hoc-Net)*, Jun. 2018, pp. 1–7.
- [3] A.-I. Pop, U. Raza, P. Kulkarni, and M. Sooriyabandara, “Does bidirectional traffic do more harm than good in LoRaWAN based LPWA networks?” in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2017, pp. 1–6.
- [4] G. Ferre, “Collision and packet loss analysis in a LoRaWAN network,” in *Proc. 25th Eur. Signal Process. Conf. (EUSIPCO)*, Aug. 2017, pp. 2586–2590.
- [5] D. Bankov, E. Khorov, and A. Lyakhov, “Mathematical model of LoRaWAN channel access with capture effect,” in *Proc. IEEE 28th Annu. Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, Oct. 2017, pp. 1–5.
- [6] M. Capuzzo, D. Magrin, and A. Zanella, “Mathematical modeling of LoRa WAN performance with bi-directional traffic,” in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2018, pp. 206–212.
- [7] O. Georgiou and U. Raza, “Low power wide area network analysis: Can LoRa scale?” *IEEE Wireless Commun. Lett.*, vol. 6, no. 2, pp. 162–165, Apr. 2017.
- [8] M. Bor, U. Roedig, T. Voigt, and J. Alonso, “Do LoRa low-power wide-area networks scale?” in *Proc. 19th ACM Int. Conf. Modeling, Anal. Simulation Wireless Mobile Syst.*, 2016, pp. 59–67.
- [9] W. Gao, W. Du, Z. Zhao, G. Min, and M. Singhal, “Towards energy-fairness in LoRa networks,” in *Proc. IEEE 39th Int. Conf. Distrib. Comput. Syst. (ICDCS)*, Jul. 2019, pp. 788–798.
- [10] *SX1272/3/6/7/8: LoRa Modem Designer’s Guide AN1200.13*, Semtech Corp., Camarillo, CA, USA, Jul. 2013.
- [11] *AN1200.22 LoRa Modulation Basics*, Semtech Corp., Camarillo, CA, USA, May 2015.
- [12] V. Toro-Betancur, G. Premsankar, M. Slabicki, and M. Di Francesco, “Modeling communication reliability in LoRa networks with device-level accuracy,” in *Proc. IEEE Conf. Comput. Commun. (INFOCOM)*, May 2021, pp. 1–10.
- [13] *LoRaSim*. Accessed: May 2011. [Online]. Available: <https://www.lancaster.ac.uk/scs/sites/lora/lorasim.html>
- [14] D. Magrin, M. Capuzzo, and A. Zanella, “A thorough study of LoRaWAN performance under different parameter settings,” *IEEE Internet Things J.*, vol. 7, no. 1, pp. 116–127, Jan. 2020.