Augmenting LoRaWAN Performance With Listen Before Talk

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***Abstract*— Standard LoRaWANs leverage pure ALOHA at the medium access control layer, which is proved to be a performance bottleneck as the network size scales up. Stimulated by this fact, this paper studies the applicability and the performance of listen before talk (LBT) medium access schemes in the context of LoRaWANs. We consider two different implementations of LBT: physical layer LBT based on energy detection only and MAC layer LBT based on layer 2 frame decoding, and we propose a Markovian framework to evaluate the performance of LoRaWANs under such setting in terms of data extraction rate and average delay experienced by transmitted uplink messages. The proposed framework is also leveraged to assess the performance of "mixed" LoRaWAN scenarios, where some devices access the channel according to the standard-compliant ALOHA protocol, while other devices transmit according to LBT.**

***Index Terms*— LoRa, LoRaWAN, unslotted ALOHA, listen before talk (LBT), low power wide area networks.**

I. Introduction

MOST of the application verticals within the Internet of Things (IoT) vision including smart cities and environmental monitoring are characterized by extremely dense networks of end devices capillary immersed in the reference environments. As an example, around 60,000 devices/km^2 are expected to be supported by Cellular IoT solutions currently being standardized within Third Generation Partnership Project (3GPP) and a *10x* increasing factor is foreseen for the upcoming 5G. Furthermore, such field devices have distinct traffic characteristics often working with extremely low duty cycles, intermittently generating very small amount of data. This new new traffic/network paradigm is often referred to as Machine-Type Communications (MTC) opposed to the classical human-to-human services offered by mobile cellular networks [1].

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To cope with such new traffic paradigm, mobile cellular networks are evolving towards MTC-compliant solutions including LTE-M, EC-GSM, NB-IoT and 5G; concurrently, IoT-specialized network operators are offering IoT connectivity through long-range, low-power wireless technologies like SigFox, LoRaWAN [2], Weightless and Ingenu, which often share the same value proposition of low energy consumption and Total Cost of Ownership (TCO), global reach and plug-and-play connectivity.

We focus here on LoRaWAN, which is characterized by an association-less star-of-stars topology in which end devices use single-hop spread spectrum wireless transmission to reach one or multiple gateways that relay messages towards a central network server in the backend. The performance of LoRaWAN networks in terms of coverage, end-to-end latency and Data Extraction Rate (DER) depend jointly on the network layout (number/position of the gateways), the configuration of the physical layer parameters (spreading factor, protection coding rate, channel bandwidth, emitted power) and, last but not least, by the efficiency of the Medium Access Control (MAC) scheme.

The MAC scheme adopted by LoRaWAN is based on pure ALOHA which is proved to be a major performance bottleneck as the network size scales up [3]. To this extent, we explore in this work the possibility of using Listen Before Talk (LBT) approaches in LoRaWANs to augment the network performance. We introduce a theoretical framework based on Markovian analysis to assess the performance of two LBT implementations based on pure energy detection at the physical layer and/or on frame decoding at the MAC layer, respectively. The aforementioned framework is then leveraged to evaluate the network performance in terms of DER and transmission delay under diverse network settings. Moreover, we also analyze network scenarios heterogeneous from the MAC point of view with end devices running the standard ALOHA-based access scheme and other end devices operating according to the aforementioned LBT approaches.

The main novel contributions of the present work with respect to the related literature can be summarized as follows:

- we evaluate the performance and feasibility of a hybrid LBT/ALOHA Medium Access Control scheme for LoRaWAN;
- we analyze the performance/complexity trade-off in implementing LBT either at the physical layer through energy detection or at the MAC layer through frame decoding;

- we propose a comprehensive theoretical framework to evaluate the performance of realistic LoRaWAN instances.

The manuscript is organized as follows: Section II reports on the related literature in the field of LoRaWAN performance evaluation; Section III gives a brief overview of LoRaWAN; Section IV describes the proposed modeling, whereas Section V reports and comments on the performance evaluation campaign. Concluding remarks are given in Section VI.

II. Related Work

The related work on the performance evaluation of LoRaWAN networks can be broadly grouped into two classes: work targeting coverage assessment of LoRa-based links and work focusing on system-level evaluation; the manuscripts in the first class all share a practical approach to evaluate coverage capabilities of LoRa-based links by assessing commercial LoRa-based transceivers under different physical layer configurations and different networking environments (indoor/outdoor, urban/rural) [4]–[6], and [7].

On the other hand, simulation-based system-level simulation is carried out in [3] and [8] by leveraging a simulator of LoRaWAN named LoRaSim. Simulation-based analysis of standard LoRaWANs is proposed also in [9] and [10]. Differently, the work in [11], [12] and [13] propose non-standard algorithms to improve LoRaWAN performance including a dynamic scheme to adjust the uplink LoRa data rate based, and a coding scheme to improve LoRa transmission robustness. The choice of the optimal gateway locations out of a set of candidate positions to achieve a specific performance goals is addressed in [14] as an optimization problem.

At the moment of writing, there are few works proposing theoretical models to capture LoRaWAN performances. Delobel *et al.* propose in [15] a Markovian analysis to assess the performance of Class B devices, whereas Georgious *et al.* propose in [16] a theoretical framework based on stochastic geometry to derive the uplink outage probability in LoRaWAN. Along the same lines, Zucchetto *et al.* analyze in [17] the performance of ALOHA and Listen Before Talk (LBT) approaches at the Medium Access Control of long-range technologies.

Regarding the modeling of pure LBT sensor networks (ZigBee networks), the vast majority of previous works relies on Markov models analyzing the behavior of slotted and unslotted 802.15.4 Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) [18]–[21]. The approach followed in all these works is similar: first, a Markov chain that models the inner behavior of a CSMA/CA device is used to obtain the probability of attempting a carrier sensing in a random time slot; then, the probability of finding the channel busy in a carrier sensing as well as the probability that the transmitted message encounters a collision are derived. These equations conform a set nonlinear system that can be solved to obtain different performance metrics. Whereas the inner behavior of the considered LBT devices in our theoretical framework (and its Markov chain modeling) is very similar to those that can be found on the aforementioned works, the coexistence of LBT and ALOHA devices makes the derivation of the collision probability (for both ALOHA and LBT devices) and of the probability of finding the channel busy in a carrier sensing (for LBT devices) much more complicated. These derivations are the main difference of our work with respect to previous ones assessing pure LBT sensor networks and its main theoretical novelty.

In our preliminary work [22], we focus on the performance evaluation of LBT techniques at the physical layer only; in the current work we provide the following novel contributions with respect to the reference literature: (i) we introduce a comprehensive theoretical framework based on Markovian analysis to assess the performance of Class A devices when running LBT at the physical layer (through energy detection) and at the MAC layer (through frame decoding); (ii) we capture the orthogonality loss among transmissions using different spreading factors (SF) and the capture effect for transmission using the same SF introducing a inter/intra-SF collision probability $p_{l,m}$, which indicates the probability that a transmission with SF l fails when there is a simultaneous transmission with SF m; (iii) we evaluate the complexity/performance trade-off involved in the use of the different LBT approaches; (iv) we provide a performance evaluation of LoRaWAN scenarios with Class A devices running ALOHA and LBT concurrently.

III. LoraWAN Overview

LoRaWAN building blocks are: *end devices* sensing and broadcasting field data in the uplink, *gateways* collecting the data from the end devices and forwarding them to a *network server* which is in charge of running the medium access control procedures, managing the radio frequency parameters and removing message duplicates from the end devices.

The LoRaWAN physical layer is based on Long Range (LoRa^{TM}), a proprietary Chirp Spread Spectrum (CSS) modulation technique developed by Semtech, operating in the unlicensed radio spectrum in the Sub-GHz Industrial, Scientific and Medical (ISM) bands with region-specific carrier frequencies and PHY parameter configurations.

The range and energy consumption of end devices mainly depend on four parameters at the physical layer:

- the channel bandwidth (BW): defines the amplitude in the frequency domain of the used channel; higher bandwidth leads to higher throughput but to lower sensitivity (because of integration of additional noise); BW can be chosen in the set [125kHz, 250kHz, 500kHz];
- the spreading factor (SF) is a configuration parameter of the modulation techniques defined as the ratio between the symbol rate and chip rate. The number of chips per symbol is calculated as 2^{SF}, thus the SF tells "how much" the reference signal is spread in time; the higher the spreading factor the longer the transmission range but the lower the transmission rate; indeed, each increase in SF halves the transmission rate and, hence, doubles transmission duration and ultimately energy consumption; LoRa specifications define a discrete set of usable spreading factors from SF $= 7$ to SF $= 12$.

- the coding rate (CR) defines the redundancy which can be optionally added to the LoRa messages by employing Forward Error Correction (FEC) codes; the higher the coding rate the higher protection against interference, but the lower the bit rate. CR can be chosen in the set [4/5, 4/6, 4/7, 4/8]
- transmission power (P_{tx}) which can be adjusted from -4dBm and $+20$dBm.

LoRa specifications define region-specific recommended combinations of the aforementioned physical layer parameters which are compliant with the local available spectrum and regulations. The proper configuration can be decided at design time and/or changed at run-time using automatic algorithms which are typically run by the network server in a centralized fashion.

The MAC level defines three classes of end devices: Class A devices transmit in the uplink using by standard a simple random ALOHA-based access protocol and can receive traffic in the downlink only after an uplink transmission. Class B devices can wake up periodically to receive scheduled downlink data traffic. Class C devices listen continuously and are typically mains-powered. Class A devices are, at the moment of writing, the ones with the highest diffusion in the market. To limit interference in the ISM band, Class A devices are mandated in Europe to operate with a duty cycle below 1% if running ALOHA access protocol, or, alternatively, adopting a Listen Before Talk approach with no limitations on the duty cycle.

IV. Problem Statement and Formulation

We consider a single-gateway LoRaWAN servicing a set of end devices generating messages according to a Poisson process with rate λ. Each end device uses a specific SF in transmission out of the available six ($M = 6$). An uplink transmission with SF l collides with another uplink transmission with SF m with probability $p_{l,m}$. The set of end devices includes devices running plain ALOHA access scheme and devices accessing the channel via a unslotted LBT scheme similar to the one used in the IEEE 802.15.4 standard [23].

According to this protocol, a device backs off transmission for a random number of backoff slots in the range $[0, 2^{BE}-1]$, being BE the backoff exponent that is initialized to m_{min}. When the backoff expires, the device senses the availability of the channel through energy detection or other Clear Channel Assessment (CCA) techniques. If the channel is perceived as busy, the BE is increased by 1 up to a maximum value (m_{max}) and the device backs off again for a period randomly generated with the new value of BE. This process is repeated until the number of failed CCAs exceeds the parameter m. In that case, the message is discarded. If the channel is clear, the device transmits the message.

Let L_l be the airtime in ms of a message transmitted with SF l (we assume fixed message size for all the end devices), and $N_{C,l}$ and $N_{A,l}$ the number of LBT and ALOHA end devices using SF l, respectively. We also denote with t_b, t_{CCA} and t_{TA} the duration of a backoff slot, of a CCA, and the turn-around time from the listening mode to the transmitting mode. For simplicity, we assume that $t_{CCA} \approx t_{TA} \approx t_b/2$. We also assume that the LBT function is implemented in two ways: (i) based on pure energy detection, (PHY CCA), which means that the channel is perceived as busy if one or more messages are being transmitted at CCA time regardless of their SF; (ii) based on layer-2 frame decoding (MAC CCA), which means that the channel is perceived as busy only if one or more messages are being transmitted at CCA time with the same SF of the end device performing CCA.

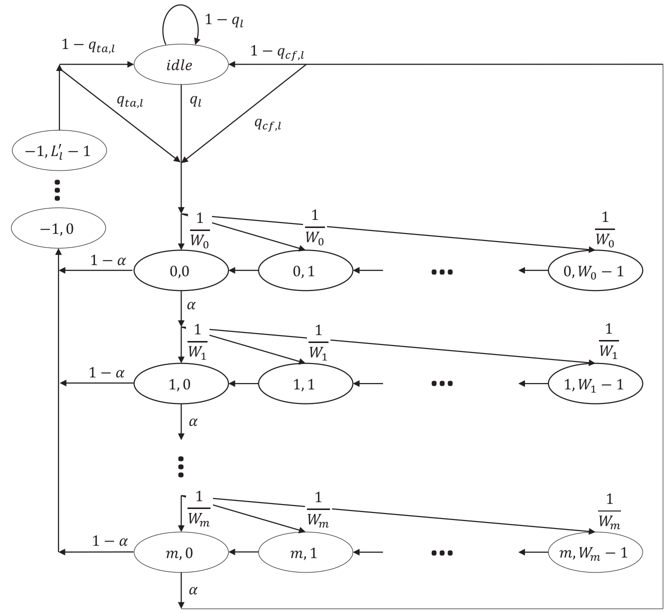


Fig. 1. Markov chain model of the unslotted LBT devices.

The performance metrics that we aim to obtain with the proposed model are the probability of a successful transmission for ALOHA and LBT devices and the average delay incurred for LBT devices (for ALOHA devices the delay is always the transmission time). In the following, we introduce a framework to assess the LoRaWAN performance with end devices using the PHY CCA and the MAC CCA, respectively.

A. PHY CCA

We rely on the Markov chain shown in Fig. 1 to model the backoff, sensing and transmitting states of the LBT devices. This approach has been widely used in the literature [18]–[20], [24]–[26], so we give here the minimum set of equations needed to model the inner state behavior of the LBT devices, without giving a complete derivation of them (Eqs. (1) and (2)).

A state is the tuple (i, j), with i the backoff stage and j the backoff counter ranging from 0 to $W_i = 2^{BE_i} - 1$, with BE_i the backoff exponent corresponding to the backoff stage i. In the states with $j = 0$, PHY CCA is performed. The probability that a device finds the channel busy when it performs a PHY CCA is α. Note that if the number of end

$$p_l(0,0) = \begin{cases} \left[\frac{1}{2}\left(\frac{1-(2\alpha)^{m+1}}{1-2\alpha}W_0+\frac{1-\alpha^{m+1}}{1-\alpha}\right)+L_l'\left(1-\alpha^{m+1}\right)+\frac{1-q_{cf,l}}{q_l}\alpha^{m+1}+\frac{1-q_{ta,l}}{q_l}\left(1-\alpha^{m+1}\right)\right]^{-1}, & \text{if } m<\hat{m} \\ \left[\frac{1}{2}\left(\frac{1-(2\alpha)^{\hat{m}+1}}{1-2\alpha}W_0+\frac{1-\alpha^{\hat{m}+1}}{1-\alpha}+\left(2^{m_b+1}+1\right)\alpha^{\hat{m}+1}\frac{1-\alpha^{m-\hat{m}}}{1-\alpha}\right)+L_l'\left(1-\alpha^{m+1}\right)\right. \\ \quad \left.+\frac{1-q_{cf,l}}{q_l}\alpha^{m+1}+\frac{1-q_{ta,l}}{q_l}\left(1-\alpha^{m+1}\right)\right]^{-1}, & \text{otherwise} \end{cases} \tag{2}$$

$$P\left(\mathcal{B}_{\mathcal{A},m}^C|\mathcal{B}_{\mathcal{C},l}\right) = \begin{cases} \frac{e^{-\lambda N_{A,m}t_{TA}}-e^{-\lambda N_{A,m}(L_l+t_{CCA}+t_{TA})}}{\lambda N_{A,m}(L_l+t_{CCA})}, & \text{if } L_m>L_l \\ \frac{(L_l-L_m+t_{TA})e^{-\lambda N_{A,m}(L_m+t_{CCA})}}{L_l+t_{CCA}}+\frac{e^{-\lambda N_{A,m}t_{TA}}-e^{-\lambda N_{A,m}(L_m+t_{CCA})}}{\lambda N_{A,m}(L_l+t_{CCA})}, & \text{otherwise} \end{cases} \tag{6}$$

devices is high (as typically happens on LoRaWAN, where there can be more than 500 end devices), we can safely assume that this probability is the same for all the LBT devices irrespective of their SF.

The states $(-1,j)$ represent the transmission of a message, with $0 \le j < L_l'$, and L_l' the duration in slots of a message transmitted using SF l including overhead and payload. The traffic generation of the devices is modeled with a message generation probability in idle state q_l. We also include in the model the probabilities of having a message ready to be transmitted after a channel access failure $q_{cf,l}$ and after a transmission attempt $q_{ta,l}$. The expressions of these probabilities are derived afterward.

The probability τ_l that a device attempts a PHY CCA in a randomly chosen time slot can be derived as in [20]:

$$\tau_l = \left(\frac{1-\alpha^{m+1}}{1-\alpha}\right)p(0,0), \tag{1}$$

where $p(0,0)$ is the steady state probability of the state $(0,0)$. The expression for $p(0,0)$ is given in Eq. (2), shown at the top of this page. In that expression, $\hat{m} = m_{max} - m_{min}$.

A channel can be found busy upon PHY CCA because occupied by ALOHA and/or LBT transmissions. The corresponding probability, α, can be written as

$$\alpha = P(\mathcal{B}_\mathcal{A} \cup \mathcal{B}_\mathcal{C}) = P(\mathcal{B}_\mathcal{A}) + P(\mathcal{B}_\mathcal{A}^C \cap \mathcal{B}_\mathcal{C}), \tag{3}$$

where $\mathcal{B}_\mathcal{A}$ indicates the event that the channel is found busy because of an ALOHA transmission and $\mathcal{B}_\mathcal{C}$ indicates the event that the channel is busy because of a LBT transmission.

The term $P(\mathcal{B}_\mathcal{A})$ is

$$P(\mathcal{B}_\mathcal{A}) = 1 - \prod_{l=1}^{M} e^{-\lambda N_{A,l}(L_l+t_{CCA})}, \tag{4}$$

i.e., the probability that any ALOHA device starts its transmission in the L_l+t_{CCA} seconds before the end of the PHY CCA (we assume that to detect the channel idle, there cannot be any ongoing transmission for the duration of the PHY CCA).

For $P(\mathcal{B}_\mathcal{A}^C \cap \mathcal{B}_\mathcal{C})$, we have the following result:

Proposition 1: Let $\mathcal{B}_{\mathcal{C},l}$ be the event that PHY CCA fails because of the transmission of a LBT device with SF l, and $\mathcal{B}_{\mathcal{A},m}$ the event that the PHY CCA fails because of an ALOHA transmission using SF m, then

$$P(\mathcal{B}_\mathcal{A}^C \cap \mathcal{B}_\mathcal{C}) = \sum_{l=1}^{M}\Bigg[L_l'(1-(1-\tau_l)^{N_{C,l}})(1-\alpha) \times \prod_{m=1}^{l-1}(1-\tau_m)^{N_{C,m}}\prod_{m=1}^{M}P\left(\mathcal{B}_{\mathcal{A},m}^C|\mathcal{B}_{\mathcal{C},l}\right)\Bigg], \tag{5}$$

with $P\left(\mathcal{B}_{\mathcal{A},m}^C|\mathcal{B}_{\mathcal{C},l}\right)$ computed with Eq. (6), shown at the top of this page.

Proof: See Appendix.

The term $L_l'(1-(1-\tau_l)^{N_{C,l}})(1-\alpha)$ in Eq. (5) corresponds to the probability that at least one device using SF l has accessed the channel and found it free in the L_l' previous time slots to the CCA, whereas $(1-\tau_m)^{N_{C,m}}$ corresponds to the probability that no LBT device using SF $m < l$ performs a CCA at the same time as the device using SF l that has occupy the channel. The term $P\left(\mathcal{B}_{\mathcal{A},m}^C|\mathcal{B}_{\mathcal{C},l}\right)$ is computed integrating on the time interval where the LBT device with SF l can start its transmission (hence causing the event $\mathcal{B}_{\mathcal{C},l}$), the probability that no ALOHA device using SF m is transmitting during the CCA.

Eqs. (1) (one for each SF) and (3) form a system of $M+1$ coupled nonlinear equations with variables τ_l and α that can be solved numerically to obtain the point of operation of the network.

From these variables different performance metrics can be obtained. First, the probability that a message transmitted by a LBT device using SF l collides is

$$\begin{aligned} P_{c,l,C} &= P\left(\mathcal{C}_{l,\mathcal{A}} \cup \mathcal{C}_{l,\mathcal{C}}\right) = P\left(\mathcal{C}_{l,\mathcal{A}}\right) + P\left(\mathcal{C}_{l,\mathcal{A}}^C \cap \mathcal{C}_{l,\mathcal{C}}\right) \\ &= P\left(\mathcal{C}_{l,\mathcal{A}}\right) + P\left(\mathcal{C}_{l,\mathcal{C}}\right)\left(1 - P\left(\mathcal{C}_{l,\mathcal{A}}\right)\right). \end{aligned} \tag{7}$$

In this expression, $\mathcal{C}_{l,\mathcal{A}}$ (or $\mathcal{C}_{l,\mathcal{C}}$) corresponds to the event that the LBT device using SF l collides with an ALOHA (or LBT) device. Note that the probabilities of colliding with another LBT device and with an ALOHA device are independent. The reason of this independence is that the event $\mathcal{C}_{l,\mathcal{C}}$ does not add any information to the event $\mathcal{C}_{l,\mathcal{A}}$ as the only possibility for a LBT device to collide with another LBT device is when both begin its transmission simultaneously.

If we denote as $\mathcal{C}_{l,\mathcal{A},m}$ the event that the LBT message using SF l collides with an ALOHA message using SF m, then the term $P\left(\mathcal{C}_{l,\mathcal{A}}\right)$ is

$$P\left(\mathcal{C}_{l,\mathcal{A}}\right) = 1 - P\left(\bigcap_{m=1}^{M} \mathcal{C}_{l,\mathcal{A},m}^{C}\right) = 1 - \prod_{m=1}^{M} P\left(\mathcal{C}_{l,\mathcal{A},m}^{C}\right)$$
$$= 1 - \prod_{m=1}^{M} e^{-p_{l,m}\lambda N_{A,m}(L_l + t_{TA})}. \quad (8)$$

Eq. (8) corresponds to the probability that there is at least one colliding transmission of an ALOHA device for the duration of the LBT message. When $l = m$, the probability $p_{l,m}$ allows introducing the capture effect between transmissions using the same SF (a message can be correctly received even if there is other transmissions using the same SF as long as its SIR is above a specific threshold). Similarly, it allows introducing the orthogonality loss between transmissions using different spreading factors when $l \neq m$, which depends again on the SIR between the colliding messages [27]. Note that the specific values of $p_{l,m}$ will depend on the topology of the network (positions and number of devices and gateways, transmitting power, path loss, etc.). For the ideal case of non-capture effect and perfect orthogonality between SFs, $p_{l,m} = 1$ for $l = m$ and 0 for $l \neq m$.

Likewise, we have the following expression for $P\left(\mathcal{C}_{l,\mathcal{C}}\right)$, whose derivation is obtained in the Appendix.

$$P\left(\mathcal{C}_{l,\mathcal{C}}\right) = 1-(1-p_{l,l}\tau_l)^{N_{C,l}-1} \prod_{\substack{m=1 \\ m \neq l}}^{M} (1-p_{l,m}\tau_m)^{N_{C,m}}. \quad (9)$$

From this expression, we see that colliding LBT devices can be seen as if they attempt a CCA with probability $p_{l,m}\tau_m$, and that $P\left(\mathcal{C}_{l,\mathcal{C}}\right)$ is the probability that any other colliding LBT device performs the CCA at the same time. Note also that in the case of orthogonality between SFs and non capture effect, $P\left(\mathcal{C}_{l,\mathcal{C}}\right)$ can be simplified to $1 - (1 - \tau_l)^{N_{C,l}-1}$.

If we also define ξ_l as the probability that the transmission suffers a wireless channel error (i. e., that the message is not received in any of the gateways with a SNR higher than the SNR threshold of SF l), the DER of a LBT device using SF l corresponds to the probability of a successful transmission

$$DER_{l,C} = (1 - P_{c,l,C})\left(1 - \alpha^{m+1}\right)(1 - \xi_l), \quad (10)$$

i.e., the probability that the LBT device finds the channel free in any of the CCA attempts times the probability that the message does not collide given that it is transmitted times the probability that there is not a wireless channel error.

We compute now the average delay experienced by a LBT message. We distinguish between two different cases: (i) when the channel has been found idle in any of the $m + 1$ allowed CCAs and the message has been transmitted and (ii) when the message is discarded because of a channel access failure (i.e., the channel has been found busy in the $m + 1$ CCAs). To compute these metrics, we consider only the time from the instant the message is ready to be transmitted (i.e., we do not include any queuing time).

In the first case, the average delay is

$$\mathrm{E}\left[T_{ta,l}\right] = L_l + \mathrm{E}\left[T_b\right], \quad (11)$$

being T_b the random time that a device spends in backoff or sensing states during the LBT mechanism. The expected value of T_b is

$$\mathrm{E}\left[T_b\right] = \sum_{i=0}^{m} P(\mathcal{D}_i)\mathrm{E}\left[T_{b,i}\right], \quad (12)$$

where $P(\mathcal{D}_i)$ is the probability of finding the channel idle at the $i+1$th CCA attempt, given that the channel has been found busy in the preceding i attempts and the message has not been discarded due to a channel access failure; and $\mathrm{E}\left[T_{b,i}\right]$ is the expected time a device spends in backoff or sensing states given the event $\mathcal{D}_i$. $P(\mathcal{D}_i)$ can be calculated as

$$P(\mathcal{D}_i) = \frac{\alpha_l^i}{\sum_{k=0}^{m} \alpha_l^k}\alpha_l^i = \frac{1 - \alpha_l}{1 - \alpha_l^{m+1}}, \quad (13)$$

while

$$\mathrm{E}\left[T_{b,i}\right] = (i+1)t_{CCA} + \sum_{k=0}^{i} \mathrm{E}\left[B_k\right] t_b$$
$$= (i+1)t_{CCA} + \sum_{k=0}^{i} t_b \frac{W_k - 1}{2}, \quad (14)$$

with $B_k = \mathcal{U}(0, W_k)$ a discrete uniform random variable indicating the backoff outcome at backoff stage k.

The delay suffered by a message when it is discarded due to a channel access failure is

$$\mathrm{E}\left[T_{cf,l}\right] = \sum_{k=0}^{m} t_b \frac{W_k - 1}{2}, \quad (15)$$

which is independent of the SF l used by the LBT device.

Finally, the probabilities of having a message ready to be transmitted in idle state is $q_l = 1 - e^{-\lambda t_b}$ (i.e., the probability of a message arrival in one time slot). The probabilities of having a message ready to be transmitted after a transmission attempt and after a channel access failure can be approximated with the busy server probability of a non saturated system, $q_{ta,l} = \lambda\mathrm{E}\left[T_{ta,l}\right]$ and $q_{cf,l} = \lambda\mathrm{E}\left[T_{cf,l}\right]$.

The collision probability for an ALOHA device using SF l is

$$P_{c,l,A} = P\left(\mathcal{A}_{l,\mathcal{A}}\right) + P\left(\mathcal{A}_{l,\mathcal{A}}^{C} \cap \mathcal{A}_{l,\mathcal{C}}\right), \quad (16)$$

where $\mathcal{A}_{l,\mathcal{A}}$ (or $\mathcal{A}_{l,\mathcal{C}}$) is the event that the ALOHA device collides with another ALOHA (or LBT) device.

The term $P(\mathcal{A}_{l,\mathcal{A}})$ is computed similarly to $P\left(\mathcal{C}_{l,\mathcal{A}}\right)$ but considering that the timespan where other transmissions will produce a collision is $L_l + L_m$,

$$P\left(\mathcal{A}_{l,\mathcal{A}}\right) = 1 - \prod_{m=1}^{M} P\left(\mathcal{A}_{l,\mathcal{A},m}^{C}\right), \quad (17)$$

with

$$P\left(\mathcal{A}_{l,\mathcal{A},m}^{C}\right) = \begin{cases} e^{-2p_{l,l}\lambda(N_{A,l}-1)L_l} & \text{if } l = m \\ e^{-p_{l,m}\lambda N_{A,m}(L_l + L_m)} & \text{if } l \neq m. \end{cases} \quad (18)$$

Whereas for $P(\mathcal{A}_{l,\mathcal{A}}^{C} \cap \mathcal{A}_{l,\mathcal{C}})$, we have the following result:

$$P(\mathcal{A}^C_{l,\mathcal{A},n}|\mathcal{A}_{l,\mathcal{C},m}) = \begin{cases} \dfrac{e^{-p_{l,n}\lambda N_{A,n}L_l} - e^{-p_{l,n}\lambda N_{A,n}(L_m+L_l+t_{TA})}}{p_{l,n}\lambda N_{A,n}(L_m+t_{TA})}, & \text{if } L_m > L_n \\ \dfrac{(L_m - L_n + t_{TA})e^{-p_{l,n}\lambda N_{A,n}(L_n+L_l)}}{L_m+t_{TA}} + \dfrac{e^{-p_{l,n}\lambda N_{A,n}L_l} - e^{-p_{l,n}\lambda N_{A,n}(L_n+L_l)}}{p_{l,n}\lambda N_{A,n}(L_m+t_{TA})}, & \text{otherwise} \end{cases} \quad (20)$$

Proposition 2: Let $\mathcal{A}_{l,\mathcal{C},m}$ the event that the ALOHA device using SF l collides with a message of a LBT device using SF m, $\mathcal{A}_{l,\mathcal{A},m}$ the event that the ALOHA device using SF l collides with a message of a ALOHA device using SF m, then

$$\begin{aligned} P(\mathcal{A}^C_{l,\mathcal{A}} \cap \mathcal{A}_{l,\mathcal{C}}) = \sum_{m=1}^{M} \Bigg[& \left(1-(1-p_{l,m}\tau_m)^{N_{C,m}}\right)(1-\alpha) \\ & \times \left(L'_m + \frac{t_{TA}}{t_b}\right) \prod_{n=1}^{m-1} (1-p_{l,n}\tau_n)^{N_{C,n}} \\ & \times \prod_{n=1}^{M} P\left(\mathcal{A}^C_{l,\mathcal{A},n} \middle| \mathcal{A}_{l,\mathcal{C},m}\right)\Bigg], \end{aligned} \quad (19)$$

with $P(\mathcal{A}^C_{l,\mathcal{A},n}|\mathcal{A}_{l,\mathcal{C},m})$ computed with Eq. (20), shown at the top of this page. In that expression, $N_{A,n}$ is substituted by $(N_{A,l}-1)$ for $n = l$.

Proof: See Appendix.

The term $(1-(1-p_{l,m}\tau_m)^{N_{C,m}})(1-\alpha)(L'_m + t_{TA}/t_b)$ in Eq. (19) corresponds to the probability that at least one colliding device using SF m has accessed the channel and found it free in the $L'_m + t_{TA}/t_b$ previous time slots to the beginning of the ALOHA transmission, whereas $(1-p_{l,n}\tau_n)^{N_{C,n}}$ corresponds to the probability that no colliding LBT device using SF $n < m$ performs a CCA at the same time as the colliding LBT device using SF m. The term $P\left(\mathcal{A}^C_{l,\mathcal{A},n}|\mathcal{A}_{l,\mathcal{C},m}\right)$ is computed integrating on the time interval where the LBT device with SF m can start its transmission (hence causing the event $\mathcal{A}_{l,\mathcal{C},m}$), the probability that no ALOHA device using SF n collides with the ALOHA transmission using SF l.

Finally, the DER of a ALOHA device using SF l corresponds to the probability of a successful transmission

$$DER_{l,A} = (1 - P_{c,l,A})(1-\xi_l). \quad (21)$$

B. MAC CCA

The behavior of LBT devices when the CCA is performed at the MAC layer can also be modeled with the Markov chain of Fig. 1, but considering that in this case the probability of finding the channel busy on a MAC CCA will depend on the SF of the device. Therefore, we need to substitute in the Markov chain the term α with α_l, being l the SF of the LBT device. The derivation of α_l is also done with Eq. (3), but restricting the events that cause a failed MAC CCA to the transmissions using SF l, i.e.

$$\begin{aligned} \alpha_l = P(\mathcal{B}_{\mathcal{A},l} \cup \mathcal{B}_{\mathcal{C},l}) &= P(\mathcal{B}_{\mathcal{A},l}) + P(\mathcal{B}^C_{\mathcal{A},l} \cap \mathcal{B}_{\mathcal{C},l}) \\ &= P(\mathcal{B}_{\mathcal{A},l}) + P(\mathcal{B}^C_{\mathcal{A},l}|\mathcal{B}_{\mathcal{C},l})P(\mathcal{B}_{\mathcal{C},l}), \end{aligned} \quad (22)$$

$P(\mathcal{B}_{\mathcal{A},l})$ is the probability that any ALOHA device using SF l starts its transmission in the $L_l + t_{CCA}$ seconds before the end of the MAC CCA,

$$P(\mathcal{B}_{\mathcal{A},l}) = 1 - e^{-\lambda N_{A,l}(L_l + t_{CCA})}. \quad (23)$$

$P(\mathcal{B}_{\mathcal{C},l})$ corresponds to the probability that at least another LBT device using SF l has performed a MAC CCA in the previous L_l seconds and it has found the channel empty,

$$P(\mathcal{B}_{\mathcal{C},l}) = \left(1-(1-\tau_l)^{N_{C,l}-1}\right)(1-\alpha_l)L'_l. \quad (24)$$

Note that this expression is similar to the term $(1-(1-\tau_l)^{N_{C,l}})(1-\alpha)$ in Eq. (5), but substituting $N_{C,l}$ by $N_{C,l}-1$. With this, we exclude the device performing the MAC CCA from the total number of LBT devices that can cause the failed MAC CCA.

The conditional probability $P(\mathcal{B}^C_{\mathcal{A},l}|\mathcal{B}_{\mathcal{C},l})$ is computed with Eq. (48) in the Appendix with $L_m = L_l$, leading to

$$\begin{aligned} P(\mathcal{B}^C_{\mathcal{A},l}|\mathcal{B}_{\mathcal{C},l}) = & \frac{t_{TA}e^{-\lambda N_{A,l}(L_l+t_{CCA})}}{L_l + t_{CCA}} \\ & + \frac{e^{-\lambda N_{A,l}t_{TA}} - e^{-\lambda N_{A,l}(L_l+t_{CCA})}}{\lambda N_{A,l}(L_l+t_{CCA})}. \end{aligned} \quad (25)$$

The point of operation of the network is obtained solving the M (one for each SF) systems of 2 coupled nonlinear equations with variables τ_l and α_l and Eqs. (1) and (22).

The probability that a message transmitted by a LBT device using SF l collides is

$$\begin{aligned} P_{c,l,C} = P\left(\mathcal{C}_{l,\mathcal{A}} \cup \mathcal{C}_{l,\mathcal{C}}\right) &= P\left(\bigcup_{m=1}^{M} (\mathcal{C}_{l,\mathcal{A},m} \cup \mathcal{C}_{l,\mathcal{C},m})\right) \\ &= 1 - P\left(\bigcap_{m=1}^{M} (\mathcal{C}_{l,\mathcal{A},m} \cup \mathcal{C}_{l,\mathcal{C},m})^C\right) \\ &= 1 - \prod_{m=1}^{M} \left[1 - P\left(\mathcal{C}_{l,\mathcal{A},m} \cup \mathcal{C}_{l,\mathcal{C},m}\right)\right] \\ &= 1 - \prod_{m=1}^{M} \left[P\left(\mathcal{C}^C_{l,\mathcal{A},m}\right) - P(\mathcal{C}^C_{l,\mathcal{A},m} \cap \mathcal{C}_{l,\mathcal{C},m})\right]. \end{aligned} \quad (26)$$

Note that the penultimate step can be done as the probability of colliding with devices using other SF is independent between different SFs; ALOHA devices transmit irrespective of the channel occupancy, whereas LBT devices only detect transmissions happening in their SF to decide whether they can transmit or not.

The probability of not colliding with ALOHA devices $P(\mathcal{C}^C_{l,\mathcal{A},m})$ is

$$P\left(\mathcal{C}^C_{l,\mathcal{A},m}\right) = \begin{cases} e^{-p_{l,l}\lambda N_{A,l}(L_l+t_{TA})} & \text{if } l = m \\ e^{-p_{l,m}\lambda N_{A,m}(L_l+L_m)} & \text{if } l \neq m. \end{cases} \quad (27)$$

For $m = l$, the probabilities of colliding with another LBT device and with an ALOHA device are independent as the only possibility for a LBT device to collide with another LBT device is when both devices performs the CCA simultaneously. With this, the computation of $P(\mathcal{C}^C_{l,\mathcal{A},l} \cap \mathcal{C}_{l,\mathcal{C},l})$ simplifies to

$$\begin{aligned} P(\mathcal{C}^C_{l,\mathcal{A},l} \cap \mathcal{C}_{l,\mathcal{C},l}) &= P\left(\mathcal{C}^C_{l,\mathcal{A},l}\right) P\left(\mathcal{C}_{l,\mathcal{C},l}\right) \\ &= e^{-p_{l,l}\lambda N_{A,l}(L_l+t_{TA})} \left(1-p_{l,l}\tau_l\right)^{N_{C,l}-1}. \end{aligned} \quad (28)$$

The computation of $P(\mathcal{C}^C_{l,\mathcal{A},m} \cap \mathcal{C}_{l,\mathcal{C},m})$ when $l \neq m$ is more difficult, since the collisions with other LBT devices using different SFs can happen at any time during the transmission of the message. For this, we split it into the different slots that form the message of the LBT device using SF l,

$$P(\mathcal{C}^C_{l,\mathcal{A},m} \cap \mathcal{C}_{l,\mathcal{C},m}) = \sum_{k=0}^{L'_l} P(\mathcal{C}^{(k),C}_{l,\mathcal{A},m} | \mathcal{C}^{(k)}_{l,\mathcal{C},m}) P(\mathcal{C}^{(k)}_{l,\mathcal{C},m}). \quad (29)$$

In this expression, the superscript (k) indicates the occurrence of the events at the end of time slot k (except for $k = 0$, that indicates the occurrence at the beginning of the LBT transmission). Therefore, $P(\mathcal{C}^{(0)}_{l,\mathcal{C},m})$ indicates the probability that there is at least one LBT transmission with SF m that collides with the LBT device using SF l when this begins its transmission. This probability is

$$P(\mathcal{C}^{(0)}_{l,\mathcal{C},m}) = \left(1-\left(1-p_{l,m}\tau_m\right)^{N_{C,m}}\right)(1-\alpha_m)L'_m. \quad (30)$$

Similarly $P(\mathcal{C}^{(k)}_{l,\mathcal{C},m})$ corresponds to the probability that a colliding LBT device using SF m performs a CCA in the k slot finding the channel empty and no colliding LBT devices using SF m performs a CCA in the previous $k-1$ slots,

$$\begin{aligned} P(\mathcal{C}^{(k)}_{l,\mathcal{C},m}) &= \left(1-\left(1-p_{l,m}\tau_m\right)^{N_{C,m}}\right) \\ &\quad \times (1-\alpha_m)(1-p_{l,m}\tau_m)^{(k-1)N_{C,m}}. \end{aligned} \quad (31)$$

To compute $P(\mathcal{C}^{(k),C}_{l,\mathcal{A},m} | \mathcal{C}^{(k)}_{l,\mathcal{C},m})$ we rely on the fact that LBT devices performing MAC CCA do not detect the channel as busy when there are transmissions on other SFs. Thus, we can consider that they behave as ALOHA devices from the point of view of the collisions with messages using other SFs and derive $P(\mathcal{C}^{(0),C}_{l,\mathcal{A},m} | \mathcal{C}^{(0)}_{l,\mathcal{C},m})$ from Eq. (62) of the Appendix obtaining

$$\begin{aligned} P(\mathcal{C}^{(0),C}_{l,\mathcal{A},m} | \mathcal{C}^{(0)}_{l,\mathcal{C},m}) &= \frac{t_{TA} e^{-p_{l,m}\lambda N_{A,m}(L_m+L_l)}}{L_m + t_{TA}} \\ &\quad + \frac{e^{-p_{l,m}\lambda N_{A,m} L_l} - e^{-p_{l,m}\lambda N_{A,m}(L_m+L_l)}}{p_{l,m}\lambda N_{A,m}(L_m + t_{TA})}. \end{aligned} \quad (32)$$

Likewise, for $k \neq 0$ $P(\mathcal{C}^{(k),C}_{l,\mathcal{A},m} | \mathcal{C}^{(k)}_{l,\mathcal{C},m})$ corresponds to the probability that no colliding ALOHA transmission with SF m occurs from the instant where the collision with the LBT device begins until the end of the LBT transmission with SF l,

$$P\left(\mathcal{C}^{(k),C}_{l,\mathcal{A},m} \middle| \mathcal{C}^{(k)}_{l,\mathcal{C},m}\right) = e^{-p_{l,m}\lambda N_{A,m}(L_l - kS_b + t_{TA})}. \quad (33)$$

With this, the DER for LBT devices using MAC CCA is

$$DER_{l,C} = (1 - P_{c,l,C})\left(1 - \alpha_l^{m+1}\right)(1 - \xi_l). \quad (34)$$

Proceeding as in Eq. (26), the collision probability for ALOHA devices using SF l is

$$P_{c,l,A} = 1 - \prod_{m=1}^{M} \left[P\left(\mathcal{A}^C_{l,\mathcal{A},m}\right) - P\left(\mathcal{A}^C_{l,\mathcal{A},m} \cap \mathcal{A}_{l,\mathcal{C},m}\right)\right]. \quad (35)$$

The probability of not colliding with other ALOHA devices $P(\mathcal{A}^C_{l,\mathcal{A},m})$ is the same as in the PHY CCA case and is computed with Eq. (18). For $m = l$, the computation of $P(\mathcal{A}^C_{l,\mathcal{A},m} \cap \mathcal{A}_{l,\mathcal{C},m})$ is

$$P(\mathcal{A}^C_{l,\mathcal{A},l} \cap \mathcal{A}_{l,\mathcal{C},l}) = P\left(\mathcal{A}^C_{l,\mathcal{A},l} | \mathcal{A}_{l,\mathcal{C},l}\right) P\left(\mathcal{A}_{l,\mathcal{C},l}\right), \quad (36)$$

with $P(\mathcal{A}_{l,\mathcal{C},l})$ the probability that at least one LBT device using SF l has begun a transmission in the $L_l + t_{TA}$ seconds before the transmission attempt of the ALOHA device, which is

$$P(\mathcal{A}_{l,\mathcal{C},l}) = \left(1-\left(1-p_{l,l}\tau_l\right)^{N_{C,l}}\right)(1-\alpha_l)\left(L'_l + \frac{t_{TA}}{t_b}\right). \quad (37)$$

Likewise, the probability $P(\mathcal{A}^C_{l,\mathcal{A},l} | \mathcal{A}_{l,\mathcal{C},l})$ can be computed with Eq. (62) in the Appendix obtaining

$$\begin{aligned} P(\mathcal{A}^C_{l,\mathcal{A},l} | \mathcal{A}_{l,\mathcal{C},l}) &= \frac{t_{TA} e^{-2p_{l,l}\lambda(N_{A,l}-1)L_l}}{L_l + t_{TA}} \\ &\quad + \frac{e^{-p_{l,l}\lambda(N_{A,l}-1)L_l} - e^{-2p_{l,l}\lambda(N_{A,l}-1)L_l}}{p_{l,l}\lambda(N_{A,l}-1)(L_l + t_{TA})}. \end{aligned} \quad (38)$$

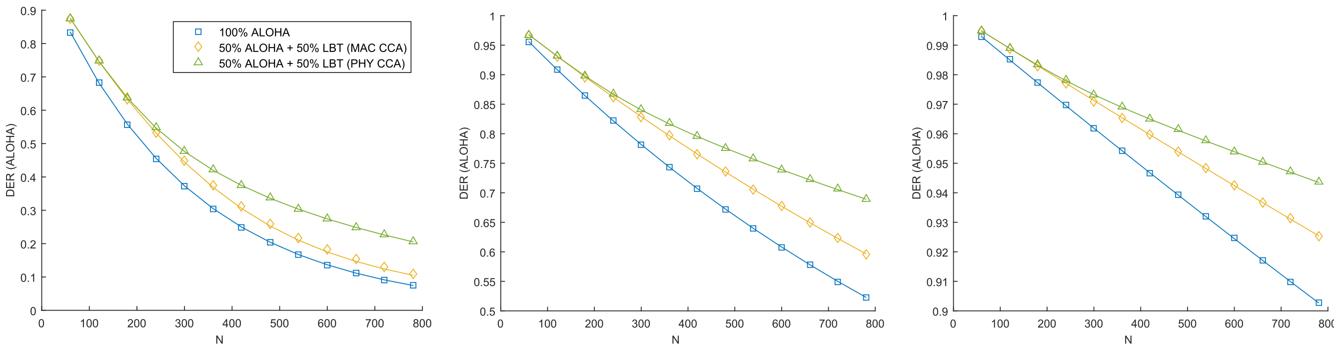
On the other hand, when $l \neq m$, it holds that $P(\mathcal{C}^C_{l,\mathcal{A},m} \cap \mathcal{C}_{l,\mathcal{C},m}) = P(\mathcal{A}^C_{l,\mathcal{A},m} \cap \mathcal{A}_{l,\mathcal{C},m})$, since a LBT device using SF l behaves as an ALOHA device for transmissions on other SF. Therefore, the computations of these probabilities can be done through Eqs. (29)-(33).

Finally, the DER of ALOHA devices is computed with Eq. (21) but using the expression of $P_{c,l,A}$ given in Eq. (35).

C. Discussion on the Models

We can derive the following insights on the models derived in the previous subsections:

1) In all the cases, the DER of both ALOHA and LBT devices decreases with: (i) the total number of ALOHA/LBT devices, (ii) the message generation rate, (iii) the message size and (iv) the probabilities $p_{l,m}$.
2) For the PHY CCA case, a LBT device finds the channel busy whenever there is another (LBT or ALOHA) device transmitting irrespective of its SF. A LBT transmission may collide (depending on the probabilities $p_{l,m}$) if: (i) another LBT device starts its transmission at the same time, or (ii) an ALOHA device starts transmitting after it. On the other hand, an ALOHA transmission may collide (again depending on $p_{l,m}$) if: (i) there is another LBT/ALOHA transmitting at the moment it starts its transmission, or (ii) another ALOHA device starts transmitting after it. Therefore, it holds that $P_{c,l,C} < P_{c,l,A}$. Similarly, it holds that $P_{c,l,C} < \alpha$. Finally, there cannot be established an order relationship between α and $P_{c,l,A}$ (and therefore between the DER of ALOHA and LBT devices).



(a) SF = 12 (b) SF = 10 (c) SF = 7

Fig. 2. DER for ALOHA devices as a function of the number of devices.

3) For the MAC CCA case, a LBT device finds the channel busy whenever there is another (LBT or ALOHA) device transmitting on its SF. The collision conditions for ALOHA and LBT transmissions are the same as in the PHY CCA case. Similarly, the relationships between $P_{c,l,C}$, $P_{c,l,A}$ and α_l also hold. For the specific case of perfect orthogonality between SFs and no capture effect, it can be verified the additional relationship $P_{c,l,C} < \alpha_l < P_{c,l,A}$ as well.
4) Comparing both CCA methods and if we assume that the probabilities τ_l are approximately the same irrespective of the CCA method (which is not completely true as the point of operation of the network will vary), it can be verified that the probability of finding the channel busy is higher for the PHY CCA than for the MAC CCA. On the contrary, the collision probabilities for LBT and ALOHA devices are higher if the MAC CCA method is used (since a LBT device may collide with other LBT/ALOHA transmissions on other SFs that has not detected when performing the CCA). For the specific case of perfect orthogonality between SFs and no capture effect, the expressions of $P_{c,l,C}$ are $P_{c,l,A}$ are the same for both methods (which does not imply that the probabilities are the same, as the point of operation of the network will differ).
5) Although it has been assumed that the LoRaWAN consist of an unique gateway, the model is valid for multiple gateways as long as it can be assumed that the LBT nodes are capable of detecting any transmission in the network when performing their CCA. If this condition is met, the effect of multiple gateways would be included directly in the probabilities ξ_l and $p_{l,m}$, which depend on the specific physical network deployment.

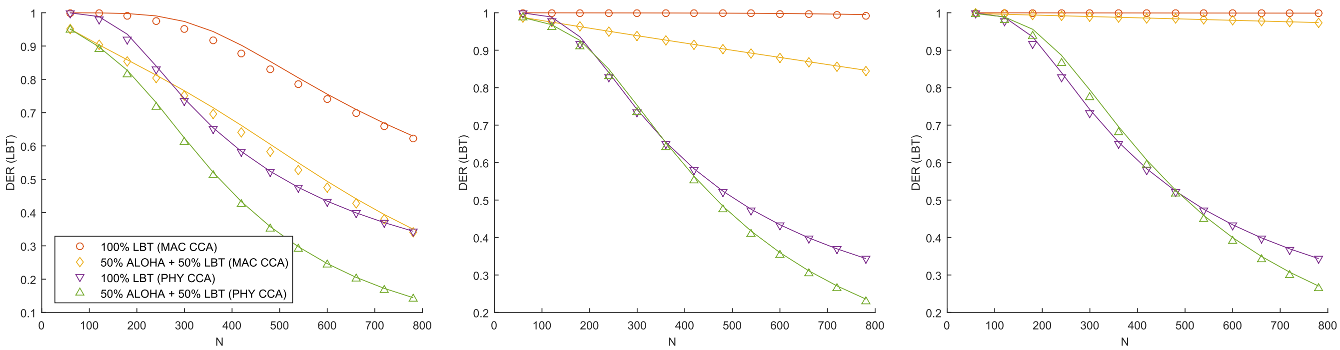
V. Numerical Results

To validate the proposed model and the goodness of using a LBT strategy in LoRaWAN, we have run two sets of simulations. In the first one, we have neglected the effect of the wireless channel errors and the capture effect between messages transmitted using the same SF. In these simulations the channel conditions of each device are not considered and the devices are assumed to be uniformly distributed among the different SFs. The second set of simulations consider two more realistic deployments, with the presence of wireless channel errors, capture effect and inter-SF collisions probabilities that depend on the received signal strength of the messages at the gateway(s). In these simulations, the allocation of SFs to devices is based on the average SNR experienced by each device. In both cases, the simulations have been done through a system-level discrete event simulator written in C++ (we have used our own simulator as at the moment there is not an official implementation of LoRaWAN in common simulation frameworks such as NS-3). The simulation results are obtained by averaging 20 simulation runs consisting each of $4 \cdot 10^7$ messages transmissions.

A. Simulations With Ideal Channel Conditions

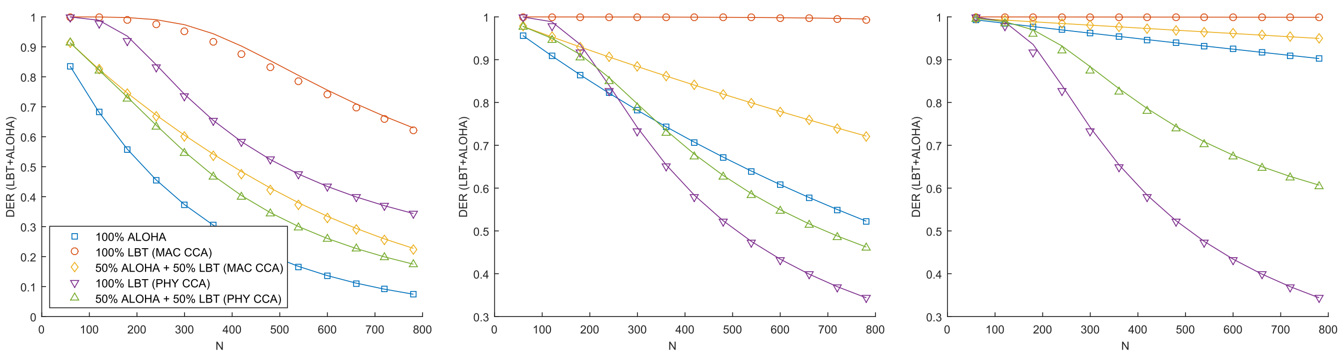
We consider a system with a varying number N of end devices (from 60 to 780) served by a single gateway. The devices are distributed uniformly across the different SFs, so that the number of devices using each SF is $N/6$. Each device sends 20 byte messages generated following a Poisson process of rate $\lambda = 1/180\ \mathrm{s}^{-1}$. Assuming that the LoRaWAN header and the preamble are 13 and 8 bytes respectively, the coding rate is $4/5$ and the bandwidth is 125 kHz, the SF-dependent total airtime to send a message is 71.94 ms (SF 7), 133.63 ms (SF 8), 246.78 ms (SF 9), 452.60 ms (SF 10), 987.13 ms (SF 11) and 1810.43 ms (SF 12).

The MAC parameters of LBT devices are $m_{min} = m_{max} = 12$ and $m = 4$. We have set the duration of a backoff slot to 1.4 ms, which corresponds to the time required to transmit the 8 bytes of the LoRaWAN physical layer preamble using SF 7 with a bandwidth of 125 kHz. The election of these backoff exponents is due to the long duration of the messages transmitted using large SFs (for instance, the duration of a message transmitted with SF 12 is 1293 slots). If they were not that large, the probability of finding the channel busy in successive CCAs given that it was busy in the first CCA would be high since the transmitted message causing the first failed CCA may not has ended in the next CCA. In all these simulations we have considered that $\xi_l = 0$ (no wireless channel errors), $p_{l,l} = 1$ (no capture effect), and $p_{l,m} = 0$ for $l \neq m$ (no inter-SF collision probability). We do not



(a) SF = 12 (b) SF = 10 (c) SF = 7

Fig. 3. DER for LBT devices as a function of the number of devices.



(a) SF = 12 (b) SF = 10 (c) SF = 7

Fig. 4. DER for all devices irrespective of their channel access method as a function of the number of devices.

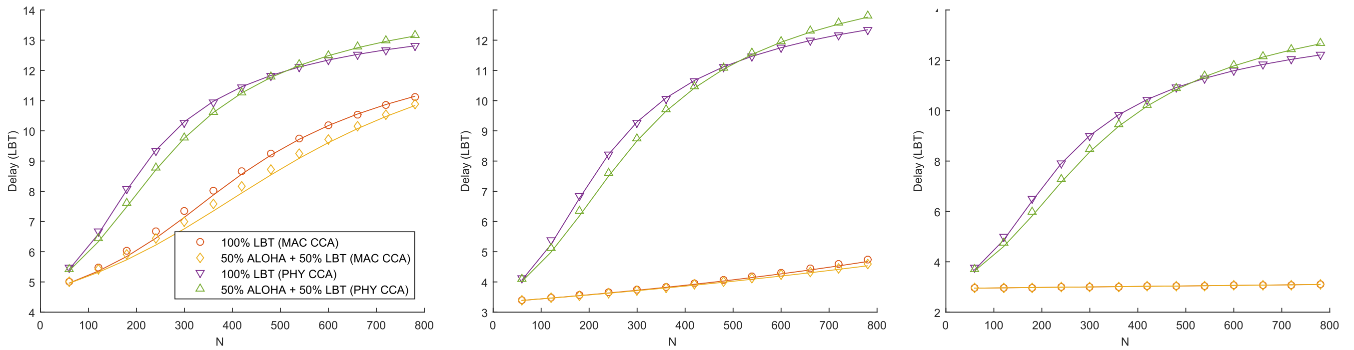
represent the 95%-confidence intervals as their relative size is well below 1%.

Fig. 2 shows the DER of ALOHA devices as a function of the total number of devices in the network for three different SFs. In this and the remaining figures, we depict with points the results from the discrete event simulator and with lines the values from our analytical model. Each line corresponds to a different configuration of ALOHA and LBT devices. For instance, the '50% ALOHA +50% LBT (MAC CCA)' line indicates that 50% of devices in each SF are ALOHA and 50% LBT performing MAC CCA.

As can be seen, the inclusion of LBT devices in a LoRaWAN network impacts positively on the DER of ALOHA devices in all the cases, independently of the number of devices and their specific SF. The reason behind this behavior is evident: if there is an ALOHA transmission on the channel, LBT devices will detect it and will refrain from transmitting, thus decreasing the collision probability of ALOHA messages. If we compare DER CCA with PHY CCA, we realize that the DER is moderately higher when PHY CCA is used. The reason of this behavior is clear; when PHY CCA is employed, LBT devices refrain from transmitting much more often than with MAC CCA, which decreases the collision probability of ALOHA devices.

Fig. 3 shows the same results but for LBT devices. For PHY CCA, the DER of LBT devices using large SFs (SF = 12) increases as the percentage of ALOHA devices decreases. On the contrary, for low SFs (SF = 7), the mix of ALOHA and LBT devices leading to the optimal DER depends on the number of devices; when it is low, it is better to have a high percentage of ALOHA devices, whereas when it is large, it is better to have more LBT devices. The reason behind this behavior is the following: the prevailing source of error for LBT devices using PHY CCA with low values of SFs is channel access failures, which depends on the probability of finding the channel busy. When the total number of devices in the network is low, the probability of a channel access failure is higher when the percentage of LBT devices is also high, as they delay their transmissions until the channel is empty. While this strategy minimizes collisions, the percentage of time that the channel is busy is higher, which increases the number of channel access failures. On the contrary, when the total number of devices in the network is large, the percentage of time that the channel is busy is higher when the percentage of LBT devices is low, as ALOHA devices transmit irrespective of the channel state (on the contrary, LBT devices would not transmit after reaching the maximum number of CCAs). We do not see this effect for high SFs (SF = 12), as in this case the prevailing source of error is collisions, which decrease when the percentage of ALOHA devices decreases regardless of the total number of devices in the network.

For MAC CCA, the results for LBT devices are much better since in this case the probability of finding the channel busy is much lower, leading consequently to fewer channel



(a) SF = 12 (b) SF = 10 (c) SF = 7

Fig. 5. Average delay in seconds for LBT devices as a function of the number of devices.

access failures. It is also worth noting that the DER amongst the different SF is more similar for LBT devices using PHY CCA, which is due to the fact that the probability of having a channel access failure is the same for all LBT devices in this case irrespective of its SF.

Fig. 4 depicts the DER of the devices (regardless of their channel access method) as a function of the total number of devices in the network for three different SFs. Several conclusions can be drawn; first, MAC CCA perform always much better than PHY CCA (and than ALOHA) as the number of channel access failures is much larger with PHY CCA. When the SF is high (SF $=$ 12), the use of LBT (PHY CCA or MAC CCA) always improves the performance with respect to ALOHA, being this improvement the highest when all the devices are LBT. For low SF (SF $=$ 7), only MAC CCA is better than ALOHA. The reason PHY CCA works worse than ALOHA is that the probability of a channel access failure for LBT devices is much higher than the collision probability of ALOHA devices (since the transmission time using SF $=$ 7 is the shortest, ALOHA devices do not collide much with this SF). For intermediate SFs, the choice of ALOHA or PHY CCA depends on the number of devices: PHY CCA is better only when the number of devices is low.

Fig. 5 shows the average delay suffered by a LBT message from the moment it initiates the backoff process until it is received (wrongly or not) or discarded due to a channel access failure, computed as $T_l = \alpha^{m+1}\mathrm{E}\left[T_{ta,l}\right] + \left(1-\alpha^{m+1}\right)\mathrm{E}\left[T_{cf,l}\right]$. For PHY CCA, the average delay is similar for all the SFs, as this delay depends mainly on α, which is the same for all the SFs. It is also worth noting that the mix of ALOHA and LBT devices achieving the lowest delay depends on the number of devices. The reason to this behavior is similar to the one given to explain the DER of LBT devices in Fig. 3c. The average delay depends on the average number of times that a LBT device finds the channel busy when performing a CCA, which in turn depends on α (the probability of an unsuccessful CCA). When the number of devices is low, α grows with the percentage of LBT devices as they delay their transmissions until the channel is empty, thus increasing the occupancy of the channel. On the contrary, when the number of devices in the network is large, the occupancy of the channel is higher (and therefore α) when the percentage of LBT devices is low, as ALOHA devices transmit irrespective of the channel state and LBT devices would not transmit after reaching the maximum number of CCAs. For MAC CCA, the average delay highly depends on the SF, as in this case the probability of finding the channel busy upon CCA, α_l, varies for the different SFs.

As a final remark, it is worth mentioning that our model fits closely the results obtained with simulations in all the cases, demonstrating its accurateness and utility to obtain performance results with a low computational cost.

B. Simulations With Realistic Channel Conditions

We consider now a simulation scenario consisting of one gateway and a varying number of end devices (from 50 to 950) randomly scattered following an uniform distribution in a 20 [km] $\times$ 20 [km] square area. The gateway is located at the center of this area. Each device sends a 20 byte packet after a time period drawn from an exponential distribution of mean 180 s. The transmission power of end devices is set to 14 dBm (the maximum power allowed in Europe for LoRa) and the propagation losses are modeled using a log-distance path loss model with shadowing,

$$PL(d) = PL(d_0) + 10\alpha\log(d/d_0) + X_\sigma, \tag{39}$$

where $PL(d_0)$ is the path loss value at a reference distance d_0, α is the path loss gain and X_{sigma} is a Gaussian variable with zero mean and σ_{shad} standard deviation representing a log-normal shadowing. The parameters of the aforementioned model are set to $d_0 = 1000$ m, $PL(d_0) = 128.95$ dB, $\alpha = 2.32$ and $\sigma_{shad} = 7.08$ dB, according to the experimental campaign described in [28]. Noise power at the gateway is computed with $\sigma^2_{noise} = -174 + \mathrm{NF} + 10\log\mathrm{BW}$ dBm, with NF the noise figure of the receiver (here assumed to be 6 dB) and BW $=$ 125 kHz.

The SF used by each device is determined according to its average SNR, so that the lowest SF whose SNR threshold is lower than the average SNR experienced by the device minus a SNR margin of 5 dB is selected (to account for the shadowing). Following this strategy, each device transmits with the minimum airtime allowed by its channel conditions. The SNR thresholds are shown in Table I [29]. If the average

TABLE I

LIST OF VARIABLES AND PARAMETERS

Parameter	Definition
λ	Message generation rate
M	Number of Spreading Factors
$p_{l,m}$	Probability that a transmission on SF l collides with a simultaneous transmission on SF m
BE	Backoff exponent of LBT devices
m_{min}, m_{max}	Initial backoff exponent / Maximum backoff exponent
L_l, L_l'	Airtime of a message transmitted with SF l in seconds/slots
$N_{C,l}, N_{A,l}$	Number of LBT/ALOHA devices using SF l
t_b	Duration of a backoff slot (for LBT devices)
t_{CCA}	Duration of a Clear Channel Assessment (for LBT devices)
t_{TA}	Turnaround time form the listening to the transmitting mode (for LBT devices)
α	Probability that a LBT device finds the channel busy when performing a PHY CCA
α_l	Probability that a LBT device using SF l finds the channel busy when performing a MAC CCA
q_l	Message generation probability in idle state (for LBT devices)
$q_{cf,l}$	Prob. of having a message ready to be transmitted after a channel access failure (for LBT devices)
$q_{ta,l}$	Prob. of having a message ready to be transmitted after a transmission attempt (for LBT devices)
τ_l	Probability that a LBT devices attempts a CCA in a randomly chosen time slot
$\mathcal{B}_{\mathcal{A}}, \mathcal{B}_{\mathcal{C}}$	Event that the channel is found busy because of an ALOHA/LBT transmission
$\mathcal{B}_{\mathcal{A},m}, \mathcal{B}_{\mathcal{C},m}$	Event that the channel is found busy because of an ALOHA/LBT transmission using SF m
$P_{c,l,A}, P_{c,l,C}$	Collision probability of messages transmitted by ALOHA/LBT devices using SF l
$\mathcal{C}_{l,\mathcal{A}}, \mathcal{C}_{l,\mathcal{C}}$	Event that a LBT device using SF l collides with an ALOHA/LBT device
$\mathcal{C}_{l,\mathcal{A},m}, \mathcal{C}_{l,\mathcal{C},m}$	Event that a LBT device using SF l collides with an ALOHA/LBT device using SF m
$\mathcal{C}_{l,\mathcal{C},m}^{(k)}, \mathcal{C}_{l,\mathcal{A},m}^{(k)}$	Event that a LBT device using SF l collides with an ALOHA/LBT device using SF m at time slot k
$DER_{l,A}, DER_{l,C}$	Data Extraction Rate of ALOHA/LBT devices using SF l
ξ_l	Probability that a transmission on SF l suffers a wireless channel error
$T_{ta,l}$	Delay experienced by a message using SF l when it has been transmitted
$T_{cf,l}$	Delay suffered by a message when it is discarded due to a channel access failure
T_b	Time that a device spends in backoff or sensing states during LBT
$\mathcal{D}_i$	Event that the channel is found idle a the $i+1$th CCA attempt
$T_{b,i}$	Delay that a device spends in backoff or sensing states during LBT given event $\mathcal{D}_i$
$\mathcal{A}_{l,\mathcal{A}}, \mathcal{A}_{l,\mathcal{C}}$	Event that an ALOHA device using SF l collides with an ALOHA/LBT device
$\mathcal{A}_{l,\mathcal{A},m}, \mathcal{A}_{l,\mathcal{C},m}$	Event that an ALOHA device using SF l collides with an ALOHA/LBT device using SF m

TABLE II

SNR THRESHOLDS FOR DIFFERENT SFS

SF	7	8	9	10	11	12
SNR_{th} [dB]	-7.5	-10	-12.5	-15	-17.5	-20

TABLE III

FRACTION OF DEVICES USING EACH SF

SF	7	8	9	10	11	12
1 GW	8.33	5.4	8.81	14.4	23.9	39
2 GW	11.81	6.02	9.29	14.35	22.53	36

SNR of a device minus the SNR margin is below the SNR threshold of SF 12, the device is discarded from the simulation.

To model the partial orthogonality property of different SFs and the capture effect when there are simultaneous transmissions, we use the following SIR threshold matrix [27]

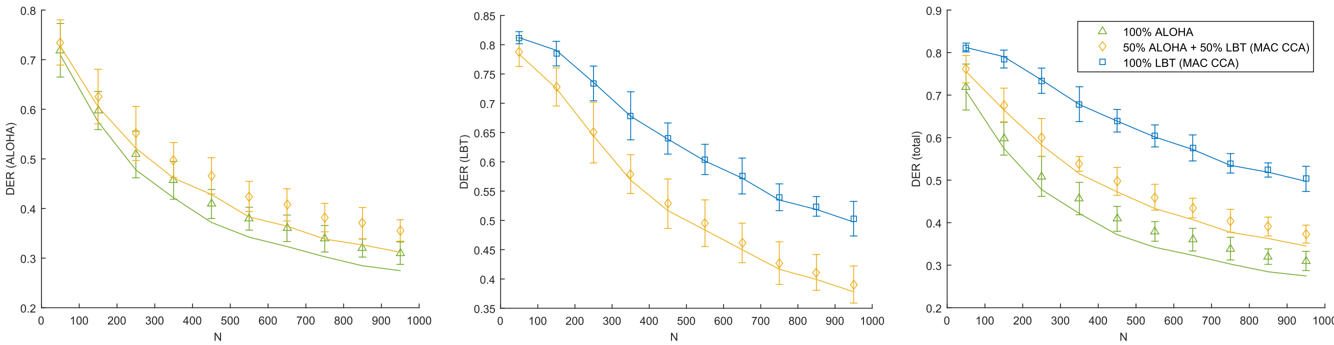
$$T_{l,m} = \begin{pmatrix} 6 & -16 & -18 & -19 & -19 & -20 \\ -24 & 6 & -20 & -22 & -22 & -22 \\ -27 & -27 & 6 & -23 & -25 & -25 \\ -30 & -30 & -30 & 6 & -26 & -28 \\ -33 & -33 & -33 & -33 & 6 & -29 \\ -36 & -36 & -36 & -36 & -36 & 6 \end{pmatrix} \quad (40)$$

The element $T_{l,m}$ is the SIR margin in dB that a message sent with SF $= l+6$ must have so that it is correctly decoded in the interfering message has SF $= m+6$. With this, a message is received correctly if: (i) its SNR is above the SNR threshold of its corresponding SF, (ii) the received power is above the SIR margin for all the interfering messages.

Under the considered scenario, we obtain by simulation that the probability of a wireless channel error is $\xi_l \approx 0.194$ for SF $= \{8, \ldots, 12\}$ and $\xi_l \approx 0.113$ for SF $= 7$. Similarly, $p_{l,l} \approx 0.724$ for SF $= \{8, \ldots, 12\}$ and $p_{l,l} \approx 0.692$ for SF $= 7$. For $l \neq m$, $p_{l,m}$ ranges from 0.062 to $3.18 \cdot 10^{-4}$. Additionally, the fraction of devices using each device is given in the second row of Table III.

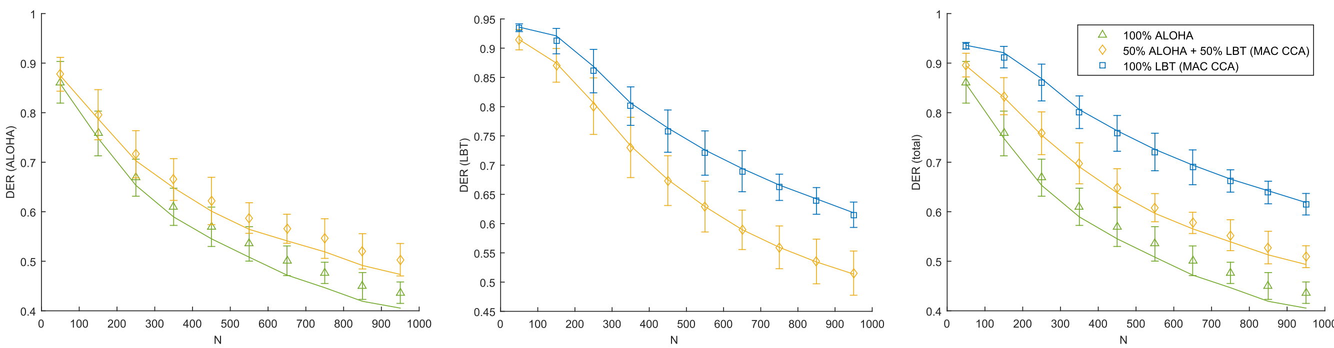
In we introduce these probabilities as well as the distribution of devices among SFs of Table III into our modeling, we obtain the results depicted in Fig. 6. In this graphs, the lines correspond to the outcome of our analytical model and the points to the results of our simulator. We also show the 95% confidence intervals for the results of the simulator. For simplicity, we have considered only the MAC CCA scheme since its performance is superior to the PHY CCA. As can be seen, the introduction of LBT devices into LoRaWAN has a very positive impact on the average DER, which allows increasing meaningfully the device density of the network (for instance, if we want to ensure an average DER of 0.7, the use of LBT makes it possible to triple the number of devices). This positive effect is observed in both ALOHA and LBT devices.

We have also tested the performance of using LBT devices in the case where two gateways are deployed in the network. The use of several gateways adds diversity to the network, which in turn increases its performance. In this case, we have considered a 24 [km] $\times$ 20 [km] square area with the gateways



(a) ALOHA (b) LBT (c) Total (ALOHA+LBT)

Fig. 6. DER averaged across SF as a function of the number of devices (scenario with 1 gateway).



(a) ALOHA (b) LBT (c) Total (ALOHA+LBT)

Fig. 7. DER averaged across SF as a function of the number of devices (scenario with 2 gateways).

separated 2 km (positions (−1,0) [km] and (1,0) [km] from the center of the area). The allocation of SF to devices is done as in the case of a single gateway, but considering the gateway with the best average SNR. In this case, a message is received correctly if the two conditions described in the case of a single gateway (SNR above the SNR threshold of its SF and received power above the SIR margin) hold in at least one of the two gateways.

In this case, we obtain by simulation that the probability of a wireless channel error is $\xi_l \approx 4.7 \cdot 10^{-2}$ for SF $= 7$ and ranging between $5.9 \cdot 10^{-2}$ and $8.8 \cdot 10^{-2}$ for SF $= \{8, \ldots, 12\}$. Similarly, $p_{l,l} \approx 0.5$ for SF $= \{8, \ldots, 12\}$ and $p_{l,l} \approx 0.43$ for SF $= 7$. For $l \neq m$, $p_{l,m}$ ranges from $1.5 \cdot 10^{-3}$ to $7.26 \cdot 10^{-7}$. Finally, the fraction of devices using each device is given in the third row of Table III.

The results of this scenario are shown in Fig. 7. Again, the use of LBT devices is positive on the average DER and allows increasing the number of users of the network (for instance, if we want to ensure an average DER of 0.7, the use of LBT makes it possible to go from 200 devices to 600). Indeed, the improvement is similar with both one and two gateways.

Finally, unlike the results obtained with ideal channel conditions, in this case our model does not fit perfectly with the results of the simulator. The reason behind this behavior is the assumption of independence between the probability of having a wireless channel error (ξ_l) and the probability of colliding with other simultaneous transmission $p_{l,m}$, which does not hold completely. However, the difference between the simulator and the model is not significant and the validity of our model remains unchanged.

VI. Concluding Remarks

Stimulated by the fact that pure ALOHA performs poorly for LoRaWAN as the network size scales up, we advocated the use of Listen before Talk approaches to augment LoRaWAN performance. To this extent, we proposed a theoretical framework to evaluate medium access control performance of LoRaWAN under two LBT approaches: a physical layer LBT approach based on energy detection only, and a MAC layer LBT based on layer-2 frame decoding; we leveraged the proposed framework to assess the performance of LoRaWAN scenarios in terms of data extraction rate and average transmission delay, further considering "mixed" scenarios where ALOHA and LBT end devices coexist in the same network.

Appendix

A. Proof of Proposition 1

Using basic set theory, the term $P(\mathcal{B}_{\mathcal{A}}^C \cap \mathcal{B}_{\mathcal{C}})$ can be expressed as

$$
\begin{aligned}
&P(\mathcal{B}_{\mathcal{A}}^C \cap \mathcal{B}_{\mathcal{C}}) \\
&\quad = P\left(\mathcal{B}_{\mathcal{A}}^C \bigcap \left(\bigcup_{l=1}^{M} \mathcal{B}_{\mathcal{C},l}\right)\right) \\
&\quad = P\left(\mathcal{B}_{\mathcal{A}}^C \bigcap \left(\bigcup_{l=1}^{M} \left(\mathcal{B}_{\mathcal{C},l} - \bigcup_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}\right)\right)\right)
\end{aligned}
$$

$$= \sum_{l=1}^{M} P\left(\mathcal{B}_{\mathcal{A}}^{C} \bigcap \left(\mathcal{B}_{\mathcal{C},l} - \bigcup_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}\right)\right)$$
$$= \sum_{l=1}^{M} P\left(\mathcal{B}_{\mathcal{A}}^{C} \bigcap \mathcal{B}_{\mathcal{C},l} \bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C}\right)$$
$$= \sum_{l=1}^{M} \left[P\left(\mathcal{B}_{\mathcal{A}}^{C} \middle| \mathcal{B}_{\mathcal{C},l} \bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C}\right) P\left(\mathcal{B}_{\mathcal{C},l} \bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C}\right)\right], \quad (41)$$

where $\bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C} = \Omega$ when $l = 1$. Now we derive the two probabilities on the previous expression. First, we can rewrite

$$P\left(\mathcal{B}_{\mathcal{C},l} \bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C}\right) = P\left(\bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C} \middle| \mathcal{B}_{\mathcal{C},l}\right) P\left(\mathcal{B}_{\mathcal{C},l}\right). \quad (42)$$

The term $P\left(\mathcal{B}_{\mathcal{C},l}\right)$ is related with the probability that at least one device using SF l has accessed the channel and found it free multiplied with the duration of the message it transmits (this term corresponds to the probability to sense busy in a LBT network without ALOHA devices [18]). Therefore,

$$P\left(\mathcal{B}_{\mathcal{C},l}\right) = (1 - (1 - \tau_l)^{N_{C,l}})(1 - \alpha)L_l'. \quad (43)$$

Leveraging the commutativity of set union, we can assume that $l = 1$ corresponds to the SF for which the messages last longer and that the different SFs are ordered in descending order of its duration (i.e $L_i < L_j$ for $j < i$). Additionally, if the PHY CCA fails because of the transmission of some LBT device using SF l, this LBT device must have found the channel empty on its PHY CCA. This implies that no other device can be transmitting before this PHY CCA. Consequently, the only case where a LBT device using SF $m < l$ could also transmit and occupy the channel is if it performs its PHY CCA at the same time as the LBT device using SF l. Therefore

$$\left(\bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C} \middle| \mathcal{B}_{\mathcal{C},l}\right) = \prod_{m=1}^{l-1} (1 - \tau_m)^{N_{C,m}}. \quad (44)$$

i.e., the probability that no LBT device using SF $m < l$ performs a CCA at the same time as the device using SF l that has occupy the channel.

The same assumption ($L_i < L_j$ for $j < i$) also allows simplifying the first probability in Eq. (41). The event $\mathcal{B}_{\mathcal{C},l}$ implies that no other device is transmitting when the LBT device causing the failed PHY CCA performs its CCA, and the event $\bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C}$ implies that no other LBT device with SF $m < l$ performs the PHY CCA at the same time as the LBT device causing the event $\mathcal{B}_{\mathcal{C},l}$. From the point of view of ALOHA transmissions, the only meaningful event is $\mathcal{B}_{\mathcal{C},l}$ as it forces that there cannot be an ALOHA transmission prior to the PHY CCA of the LBT device occupying the channel. Therefore

$$P\left(\mathcal{B}_{\mathcal{A}}^{C} \middle| \mathcal{B}_{\mathcal{C},l} \bigcap_{m=1}^{l-1} \mathcal{B}_{\mathcal{C},m}^{C}\right) = P\left(\mathcal{B}_{\mathcal{A}}^{C} | \mathcal{B}_{\mathcal{C},l}\right). \quad (45)$$

Given that ALOHA transmissions are independent amongst them, the previous expression can be rewritten as

$$P\left(\mathcal{B}_{\mathcal{A}}^{C} | \mathcal{B}_{\mathcal{C},l}\right) = \prod_{m=1}^{M} P\left(\mathcal{B}_{\mathcal{A},m}^{C} | \mathcal{B}_{\mathcal{C},l}\right). \quad (46)$$

with $\mathcal{B}_{\mathcal{A},m}$ the event that the PHY CCA fails because of an ALOHA transmission using SF m. As event $\mathcal{B}_{\mathcal{C},l}$ ensures that there is no ALOHA transmissions before the PHY CCA of the transmission that has occupied the channel, the term $P(\mathcal{B}_{\mathcal{A},m}^{C} | \mathcal{B}_{\mathcal{C},l})$ has to consider only transmissions after that PHY CCA. In order to compute this probability, we consider two cases. If $L_m > L_l$, $P(\mathcal{B}_{\mathcal{A},m}^{C} | \mathcal{B}_{\mathcal{C},l})$ corresponds to the probability that no ALOHA transmission using SF m starts from the beginning of the turn-around time of the LBT transmission using SF l to the end of the PHY CCA. As all the transmitting states in the Markov chain of Fig. 1 have the same probability, we can assume that the beginning of that LBT transmission is uniformly distributed in $[-L_l, t_{CCA}]$ (considering that the LBT device that finds the channel busy starts its PHY CCA at time instant 0). With this, we have

$$P(\mathcal{B}_{\mathcal{A},m}^{C} | \mathcal{B}_{\mathcal{C},l}) = \int_{-L_l}^{t_{CCA}} \frac{e^{-\lambda N_{A,m}(t_{CCA} - t + t_{TA})}}{L_l + t_{CCA}} dt$$
$$= \frac{e^{-\lambda N_{A,m} t_{TA}} - e^{-\lambda N_{A,m}(L_l + t_{CCA} + t_{TA})}}{\lambda N_{A,m}(L_l + t_{CCA})}. \quad (47)$$

The integrand of the previous equation indicates that if the LBT transmission with SF l that has occupied the channel has begun at time instant $t \in [-L_l, t_{CCA}]$, there cannot be any ALOHA transmission with SF m in the time period $[t - t_{TA}, t_{CCA}]$.

If $L_m \leq L_l$, we have to consider two cases depending on the instant at which the transmission that has produced the event $\mathcal{B}_{\mathcal{C},l}$ starts. If it has begun at $t \in [-L_l, -L_m + t_{TA}]$, event $\mathcal{B}_{\mathcal{A},m}^{C} | \mathcal{B}_{\mathcal{C},l}$ implies that there has not been any ALOHA transmission with SF m in $[-L_m, t_{CCA}]$. On the contrary, if it has begun at $t \in [-L_m + t_{TA}, t_{CCA}]$, then there cannot have been any ALOHA transmission in $[t - t_{TA}, t_{CCA}]$. Note that $P(\mathcal{B}_{\mathcal{A},m}^{C} | \mathcal{B}_{\mathcal{C},l}) = 1$ in eqs. (47) and (48) if $N_{A,m}$ is zero.

$$P(\mathcal{B}_{\mathcal{A},m}^{C} | \mathcal{B}_{\mathcal{C},l}) = \int_{-L_l}^{-L_m + t_{TA}} \frac{e^{-\lambda N_{A,m}(L_m + t_{CCA})}}{L_l + t_{CCA}} dt + \int_{-L_m + t_{TA}}^{t_{CCA}} \frac{e^{-\lambda N_{A,m}(t_{CCA} - t + t_{TA})}}{L_l + t_{CCA}} dt$$
$$= \frac{(L_l - L_m + t_{TA}) e^{-\lambda N_{A,m}(L_m + t_{CCA})}}{L_l + t_{CCA}} + \frac{e^{-\lambda N_{A,m} t_{TA}} - e^{-\lambda N_{A,m}(L_m + t_{CCA})}}{\lambda N_{A,m}(L_l + t_{CCA})}. \quad (48)$$

Substituting Eqs. (42)-(48) into Eq. (41), we prove the proposition.

B. Derivation of Eq. (9)

Using basic set theory, the term $P\left(\mathcal{C}_{l,\mathcal{C}}\right)$ can be expressed as

$$P\left(\mathcal{C}_{l,\mathcal{C}}\right) = 1 - P\left(\bigcap_{m=1}^{M} \mathcal{C}_{l,\mathcal{C},m}^{C}\right) = 1 - \prod_{m=1}^{M} P\left(\mathcal{C}_{l,\mathcal{C},m}^{C}\right), \quad (49)$$

with $\mathcal{C}_{l,\mathcal{C},m}$ the event that the LBT device using SF l collides with a LBT device using SF m. For $l \neq m$, the probability of this event is

$$P\left(\mathcal{C}_{l,\mathcal{C},m}\right) = \sum_{k=1}^{N_{C,m}} P\left(\mathcal{C}_{l,\mathcal{C},m} | \mathcal{S}_{\mathcal{C},m} = k\right) P\left(\mathcal{S}_{\mathcal{C},m} = k\right), \quad (50)$$

where $P\left(\mathcal{S}_{\mathcal{C},m} = k\right)$ is the probability that k LBT devices using SF m perform the CCA at the same time as the LBT device with SF l for which we are computing the collision probability. The probability that at least one of the k messages transmitted with SF m collides with the packet with SF l is

$$P\left(\mathcal{C}_{l,\mathcal{C},m}|\mathcal{S}_{\mathcal{C},m}=k\right)=1-(1-p_{l,m})^k, \tag{51}$$

On the other hand,

$$P\left(\mathcal{S}_{\mathcal{C},m}=k\right)=\binom{N_{C,m}}{k}\tau_m^k\left(1-\tau_m\right)^{N_{C,m}-k}. \tag{52}$$

Substituting (51) and (52) into (50) and using the binomial theorem we have

$$\begin{aligned}&P\left(\mathcal{C}_{l,\mathcal{C},m}\right)\\&=\sum_{k=1}^{N_{C,m}}\binom{N_{C,m}}{k}\tau_m^k\left(1-\tau_m\right)^{N_{C,m}-k}\\&\quad-\sum_{k=1}^{N_{C,m}}\binom{N_{C,m}}{k}\left((1-p_{l,m})\tau_m\right)^k\left(1-\tau_m\right)^{N_{C,m}-k}\\&=1-\left(1-\tau_m\right)^{N_{C,m}}-\left((1-p_{l,m})\tau_m+(1-\tau_m)\right)^{N_{C,m}}\\&\quad+\left(1-\tau_m\right)^{N_{C,m}}\\&=1-\left(1-p_{l,m}\tau_m\right)^{N_{C,m}}. \end{aligned}\tag{53}$$

For $m=l$, we can proceed in the same way by substituting $N_{C,m}$ with $N_{C,l}-1$. Introducing (53) into (49), we obtain Eq. (9).

C. Proof of Proposition 2

Operating as in Eqs. (41) and (42), we obtain

$$\begin{aligned}P(\mathcal{A}_{l,\mathcal{A}}^C\cap\mathcal{A}_{l,\mathcal{C}})=\sum_{m=1}^{M}\Bigg[&P\left(\mathcal{A}_{l,\mathcal{A}}^C\middle|\mathcal{A}_{l,\mathcal{C},m}\bigcap_{n=1}^{m-1}\mathcal{A}_{l,\mathcal{C},n}^C\right)\\&\times P\left(\bigcap_{n=1}^{m-1}\mathcal{A}_{l,\mathcal{C},n}^C\middle|\mathcal{A}_{l,\mathcal{C},m}\right)P\left(\mathcal{A}_{l,\mathcal{C},m}\right)\Bigg],\end{aligned}\tag{54}$$

We derive now each of the three probabilities in the previous expression. First, $P(\mathcal{A}_{l,\mathcal{C},m})$ is the probability that, when the ALOHA device begins its transmission, there is at least one message in the channel from a LBT device using SF m that collides with it. The probability of this event is

$$P(\mathcal{A}_{l,\mathcal{C},m})=\sum_{k=1}^{N_{C,m}}P\left(\mathcal{A}_{l,\mathcal{C},m}|\mathcal{R}_{\mathcal{C},m}=k\right)P(\mathcal{R}_{\mathcal{C},m}=k), \tag{55}$$

where $P\left(\mathcal{R}_{\mathcal{C},m}=k\right)$ is the probability that k LBT devices using SF m has accessed the channel and found it free multiplied with the duration of a message using that SF,

$$\begin{aligned}P\left(\mathcal{R}_{\mathcal{C},m}=k\right)=&\binom{N_{C,m}}{k}\tau_m^k\left(1-\tau_m\right)^{N_{C,m}-k}\\&\times(1-\alpha)\left(L_l'+\frac{t_{TA}}{t_b}\right).\end{aligned}\tag{56}$$

Note that the turnaround time of the LBT device is also introduced to account for the collisions with ALOHA transmissions beginning during that time. The probability that at least one of the k messages transmitted with SF m collides with the packet with SF l is the same as Eq. (51), and proceeding as in Eq. (53), we obtain

$$P(\mathcal{A}_{l,\mathcal{C},m})=\left(1-(1-p_{l,m}\tau_m)^{N_{C,m}}\right)(1-\alpha)\left(L_m'+\frac{t_{TA}}{t_b}\right). \tag{57}$$

Note that this probability is similar to $P\left(\mathcal{B}_{\mathcal{C},l}\right)$ defined previously (with the exception of the term $p_{l,m}$), but while a LBT device would find the channel busy failing its CCA, an ALOHA device would transmit causing a collision.

The computation of $P(\bigcap_{n=1}^{m-1}\mathcal{A}_{l,\mathcal{C},n}^C|\mathcal{A}_{l,\mathcal{C},m})$ also relies on the assumption that $L_i<L_j$ for $j<i$. With this assumption, if the ALOHA device using SF l collides with a LBT device using SF m, this LBT device must have found the channel empty when it has performed its CCA and no other device can be transmitting before this CCA. Therefore, the only case where other LBT devices using SF $n<m$ could also transmit and collide with the ALOHA device is if they start transmitting at the same time as the LBT device using SF m. Thus

$$P\left(\bigcap_{n=1}^{m-1}\mathcal{A}_{l,\mathcal{C},n}^C\middle|\mathcal{A}_{l,\mathcal{C},m}\right)=\prod_{n=1}^{m-1}P\left(\mathcal{A}_{l,\mathcal{C},n}^C|\mathcal{A}_{l,\mathcal{C},m}\right), \tag{58}$$

with

$$P\left(\mathcal{A}_{l,\mathcal{C},n}^C|\mathcal{A}_{l,\mathcal{C},m}\right)=(1-p_{l,n}\tau_n)^{N_{C,n}}, \tag{59}$$

i.e., the probability that no device using SF n performs a CCA at the same time as a device using SF m and collides with the ALOHA device using SF l.

Likewise, the conditional probability of not colliding with another ALOHA device conditioned on the event $\mathcal{A}_{l,\mathcal{C},m}\bigcap_{n=1}^{m-1}\mathcal{A}_{l,\mathcal{C},n}^C$ only depends on the event $\mathcal{A}_{l,\mathcal{C},m}$ since it forces that there cannot be an ALOHA transmission prior to the CCA of the LBT device using SF m that has collided with the ALOHA device using SF l. Therefore

$$\begin{aligned}P\left(\mathcal{A}_{l,\mathcal{A}}^C\middle|\mathcal{A}_{l,\mathcal{C},m}\bigcap_{n=1}^{m-1}\mathcal{A}_{l,\mathcal{C},n}^C\right)&=P\left(\mathcal{A}_{l,\mathcal{A}}^C\middle|\mathcal{A}_{l,\mathcal{C},m}\right)\\&=\prod_{n=1}^{M}P\left(\mathcal{A}_{l,\mathcal{A},n}^C\middle|\mathcal{A}_{l,\mathcal{C},m}\right),\end{aligned}\tag{60}$$

with $\mathcal{A}_{l,\mathcal{A},n}$ the event that ALOHA device transmitting with SF l collides with an ALOHA device using SF n. Note that the last step of the previous expression can be done since ALOHA transmissions are independent amongst them.

The computation of $P(\mathcal{A}_{l,\mathcal{A},n}^C|\mathcal{A}_{l,\mathcal{C},m})$ depends on the relationship between L_m and L_n and on whether $n=l$. In all the cases, the event $\mathcal{A}_{l,\mathcal{C},m}$ ensures that there is no ALOHA transmissions before the CCA of the transmission that has caused the collision, therefore we have to consider only transmissions after that CCA. If $L_n>L_m$, $P(\mathcal{A}_{l,\mathcal{A},n}^C|\mathcal{A}_{l,\mathcal{C},m})$ is the probability that no ALOHA transmission using SF n starts from the beginning of the turn-around time of the LBT transmission using SF m to the end of the collided ALOHA transmission using SF l. As the beginning of that transmission is uniformly distributed in $[-L_l,t_{TA}]$ (considering that the beginning of the collided ALOHA transmission using SF l occurs at time instant 0), we have for $n\neq l$

$$\begin{aligned}&P(\mathcal{A}_{l,\mathcal{A},n}^C|\mathcal{A}_{l,\mathcal{C},m})\\&\quad=\int_{-L_m}^{t_{TA}}\frac{e^{-p_{l,n}\lambda N_{A,n}(L_l-t+t_{TA})}}{L_m+t_{TA}}dt\\&\quad=\frac{e^{-p_{l,n}\lambda N_{A,n}L_l}-e^{-p_{l,n}\lambda N_{A,n}(L_m+L_l+t_{TA})}}{p_{l,n}\lambda N_{A,n}(L_m+t_{TA})}.\end{aligned}\tag{61}$$

The integrand of the previous equation indicates that if the LBT transmission with SF m that has caused the collision has begun at time instant $t \in [-L_m, t_{TA}]$, there cannot be any ALOHA transmission with SF n in the time period $[t - t_{TA}, L_l]$.

If $L_n \leq L_m$, we have to divide the computation into two parts depending on the instant at which the transmission that has produced the event $\mathcal{A}_{l,\mathcal{C},m}$ starts. If it has begun at $t \in [-L_m, -L_n + t_{TA}]$, event $\mathcal{A}^C_{l,\mathcal{A},n}|\mathcal{A}_{l,\mathcal{C},m}$ implies that there has not been any ALOHA transmission with SF n in $[-L_n, L_l]$. On the contrary, if it has begun at $t \in [-L_n + t_{TA}, t_{TA}]$, then there cannot have been any ALOHA transmission in $[t - t_{TA}, L_l]$. Note that $P(\mathcal{A}^C_{l,\mathcal{A},n}|\mathcal{A}_{l,\mathcal{C},m}) = 1$ in eqs. (61) and (62) if $N_{A,n}$ or $p_{l,n}$ are zero.

$$\begin{aligned} P(\mathcal{A}^C_{l,\mathcal{A},n}|\mathcal{A}_{l,\mathcal{C},m}) &= \int_{-L_m}^{-L_n+t_{TA}} \frac{e^{-p_{l,n}\lambda N_{A,n}(L_l+L_n)}}{L_m + t_{TA}} dt \\ &\quad + \int_{-L_n+t_{TA}}^{t_{TA}} \frac{e^{-p_{l,n}\lambda N_{A,n}(L_l-t+t_{TA})}}{L_m + t_{TA}} dt \\ &= \frac{(L_m - L_n + t_{TA})e^{-p_{l,n}\lambda N_{A,n}(L_n+L_l)}}{L_m + t_{TA}} \\ &\quad + \frac{e^{-p_{l,n}\lambda N_{A,n}L_l} - e^{-p_{l,n}\lambda N_{A,n}(L_n+L_l)}}{p_{l,n}\lambda N_{A,n}(L_m + t_{TA})}. \end{aligned} \tag{62}$$

The expressions for $n = l$ are exactly the same substituting $N_{A,n}$ with $(N_{A,l} - 1)$ every time it appears on (61) and (62). Finally, substituting Eqs. (55) - (62) into Eq. (54), we prove the proposition.

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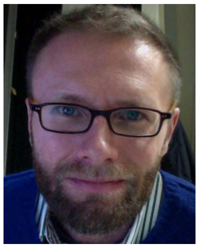

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