An ARQ Scheme with Packet Combining

Shyam S. Chakraborty, Erkki Yli-Juuti, and Markku Liinaharja

***Abstract*— In an automatic repeat request (ARQ) scheme, a packet is retransmitted if it gets corrupted due to transmission errors caused by the channel. Here we describe a ready-to-implement ARQ scheme with packet combining. An analytical description of the scheme in random error channel shows excellent agreement with simulation results. An upper bound for type-II schemes is defined. For smaller packet sizes, throughput of the proposed scheme is sufficiently close to the upper bound till a very high bit error rate.**

***Index Terms*— Bit inversion, hybrid type-I and type-II ARQ schemes, SRJ, XOR.**

I. Introduction

COMBINATIONS of automatic repeat request (ARQ) and forward error correction coding (FEC) are called hybrid ARQ schemes. In a type-I strategy, when an FEC-ed packet is received and still found to contain errors after the error correction procedure, a retransmission is requested. In type-II strategies [1], the data packet is coded in two parts: the first part contains data and parity bits for error detection, and the second part contains parity bits for error correction, computed by using an invertible code. If the first transmission is found erroneous, the parity bits are sent in the retransmission. The retransmitted block, if reaches the receiver without error, can retrieve the original data itself. However, if it also gets corrupted by channel error, then it is combined with the previous block to form a 1/2 rate code [2], [3], [5]. Further improvements of this scheme involve in combining successive erroneous copies to form a $1/m$ rate code to retrieve the correct packet [6]. Performances of these schemes are usually analyzed in a binary symmetric channel (BSC) or in a Gaussian noise (AWGN) channel.

A simple idea close to the type-II schemes was presented by Sindhu [4], where erroneous copies are XORed to locate the errors in a combined copy. The correct packet can then be retrieved by an exhaustive search method. This concept has the advantage that since only error detection is necessary, existing transmitters which only append frame check sequence (FCS) can very well be used. Also, unlike a number of type-II schemes which employ soft decision (e.g., [7, and references therein], [3], [8], [10]), hard decision can be used here and only software modification to a conventional receiver would often be sufficient. However, [4] concerns bursty error channel only, and study of a working ARQ scheme with packet combining in BSC channel seems important.

In this letter, we provide a ready-to-implement algorithm, which is a modification of ARQ (hereafter called the extended ARQ or EARQ) with packet combining, and an approximate analytical description of it in BSC channel. This is further compared with simulation results and the upper bound of type-II ARQ schemes. It is assumed that the round-trip delay is negligible and that FCS provides perfect error detection [9].

II. The EARQ Scheme

Of the three basic ARQ schemes, stop-and-wait (SW), go-back-N (GBN) and selective reject (SRJ), SRJ provides the best throughput,[1] which (normalized w.r.t. packet payload) in a random error channel is given by $T(p)$:

$$T(p) = 1 - B(p) = (1 - p_e)^n \tag{1}$$

where $B(p) = 1 - (1 - p_e)^n$ is the packet-error rate (PER), p_e is the bit-error rate (BER) and n is the packet size in bits.

In EARQ, the transmitter remains identical to the SRJ scheme. However, unlike a conventional SRJ, here the receiver stores an erroneously received packet before requesting a retransmission. A correctly received second copy is accepted as usual, and the stored copy is discarded. However, if the second copy is also received erroneously, then these two copies are XORed to locate the errors in both blocks together. The decision process then involves a brute-force bit-by-bit inversion of the located bit error positions and checking for correctness using the FCS. This operation fails if, "when two copies are erroneous, there is at least one bit position in which both copies have an error" (henceforth termed as "double error"). In this case, further retransmissions are requested and the procedure is repeated until the correct packet is retrieved. The probability of a successful retrieval at the Lth attempt is

$$P(L) = \left[1 - \sum_{r=1}^{L-1} P(r)\right] \cdot [T(p) + (1 - T(p)) \cdot (1 - \alpha_d(L))] \tag{2}$$

where $\alpha_d(L)$ is the conditional probability that, provided the Lth copy is erred, it has double errors with all the preceding

[1] If the round-trip delay is negligible, both SW and GBN behave identically to SRJ.

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S. S. Chakraborty was with the Communications Laboratory, Department of Electrical and Communications Engineering, Helsinki University of Technology, Espoo, FIN-02015 HUT, Finland. He is now with TKK, Telecommunications Laboratory, Espoo 02150, Finland (e-mail: shyam.chakraborty@hut.fi).

E. Yli-Juuti is with Nokia Telecommunications, FIN-00045 NOKIA GROUP, Finland.

M. Liinaharja is with the Communications Laboratory, Department of Electrical and Communications Engineering, Helsinki University of Technology, Espoo 02015, Finland.

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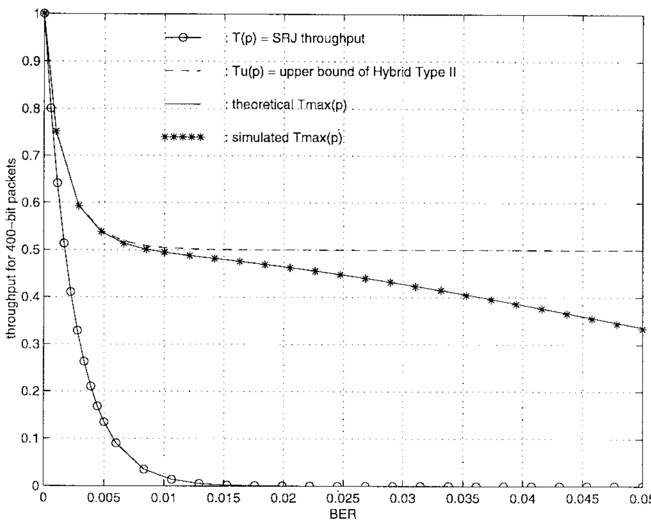

Fig. 1. Comparison of the SRJ throughput, the upper bound of hybrid type-II schemes and the maximum throughput achievable by the EARQ scheme (400-bit packet size).

$L-1$ copies in the buffer. Denoting the expected value of L as $\langle L\rangle$, the throughput is given by

$$T_e(p) = \frac{1}{\langle L\rangle}. \quad (3)$$

If $\alpha_d(L)$ is considered negligible (i.e., no double errors occur), we have

$$T_U(p) = \frac{1}{2-T(p)} = \frac{1}{1+B(p)}. \quad (4)$$

To note that, $T_U(p)$ is the upper bound of type-II ARQ schemes, and thus also forms a basis of comparison.

III. Maximum Throughput Achievable by EARQ

Assuming that occurrences of double errors in different pairs of copies are independent events and using combinatorics, it can be shown that

$$\alpha_d(L) = \frac{\displaystyle\sum_{k_1=1}^{n}\cdots\sum_{k_L=1}^{n} P(k_1,\cdots,k_L)\cdot \prod_{\substack{1\le i,j\le L\\ i<j}} P_{i,j}}{\displaystyle [B(p)]^L \cdot \prod_{l=2}^{L-1}\alpha_d(l)} \quad (5)$$

where

$$P(k_1,k_2,\cdots,k_L) = P(k_1)\cdot P(k_2)\cdots P(k_L) \quad (6)$$

is the joint probability of the L copies having $k_1,\cdots,k_L$ errors, respectively. The numbers of errors in the copies are independent, identically distributed (i.i.d.) random variables with binomial distributions:

$$P(k_i) = \binom{n}{k_i}\cdot p_e^{k_i}\cdot(1-p_e)^{n-k_i}. \quad (7)$$

Further, $P_{i,j}$ is the probability that two copies with k_i and k_j errors, respectively, have a double error, and is given by

$$P_{i,j} = 1 - \frac{(n-k_i)!\,(n-k_j)!}{n!\,(n-k_i-k_j)!}. \quad (8)$$

Using $\alpha_d(L)$ in (2) and (3), we have $T_{\max}(p)$: the maximum throughput achievable by the EARQ scheme. The above algorithm has been simulated with packet size of 400 bits, for 100 000 correctly received packets. Fig. 1 provides the simulation results (marked with asterisks), along with $T(p)$, $T_U(p)$, and $T_{\max}(p)$ obtained by using $\alpha_d(4)$. The general agreement of simulation results with $T_{\max}(p)$ is excellent till BER as high as 0.05, and further iteration was felt not necessary. This also demonstrates that the aforementioned assumption of independence is fairly accurate till a significantly high BER.

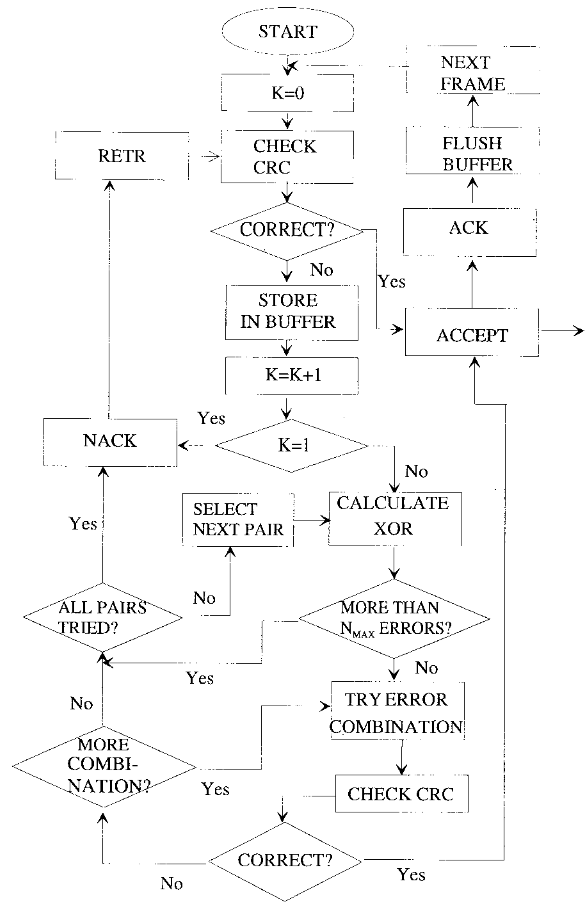

Fig. 2. The receiver decision model in EARQ.

IV. Consideration of Computational Complexity in EARQ

The complexity of the bit inversion procedure is

$$N = 2^{2n\cdot p_e} - 2. \quad (9)$$

Thus, even for medium-sized packets and at moderate BER, random fluctuations of the channel may make the bit inversion procedure extremely complex. To make the EARQ algorithm implementable, an upper limit of computational complexity is defined. If the total number of "1's" in an XORed pair of erroneous copies exceeds $N_{\max}$, the pair is discarded, and either a new pair is chosen or a retransmission is sought. In this case, $P_{i,j}$ is given by

$$P_{i,j} = \begin{cases} 1 - \dfrac{(n-k_i)!\,(n-k_j)!}{n!\,(n-k_i-k_j)!}, & \text{if } k_i+k_j \le N_{\max} \\ 1, & \text{if } k_i+k_j > N_{\max}. \end{cases} \quad (10)$$

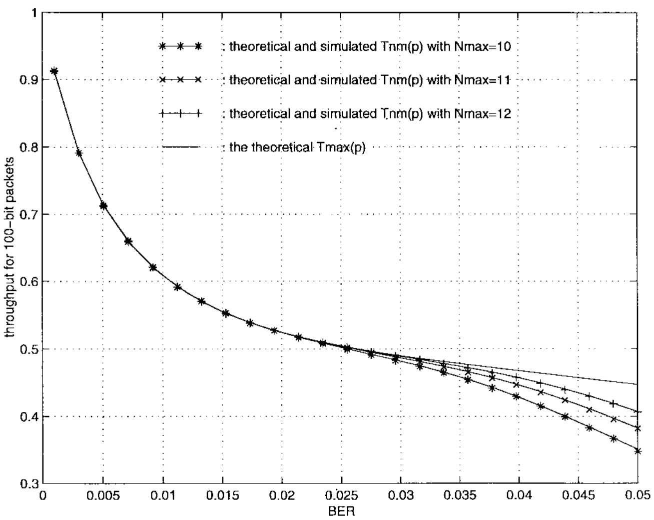

Fig. 3. The EARQ throughput $T_{nm}(p)$ with three values of $N_{\max}$ and $T_{\max}(p)$, the maximum throughput achievable by EARQ (100-bit packet size).

The complete EARQ algorithm is given in Fig. 2. Using (2), (5), and (10) in (3), we have $T_{nm}(p)$ as the throughput of the EARQ scheme. Fig. 3 provides both the analytical and simulation results for $T_{nm}(p)$, along with $T_{\max}(p)$ for comparison. Limitations provided by $N_{\max}$ clearly increase the number of retransmissions, which is more evident for bigger packets. However, computational complexity in determining $\alpha_d(L)$ also increases drastically with L. Hence, we have chosen the 100-bit packet size, along with the $N_{\max}$ values of 10, 11, and 12, and $\alpha_d(L)$ has been computed up to $L = 5$. The general agreement between analytical expression and simulation results is excellent, confirming the validity of the analysis.

V. Conclusions

The discussions above suggest the following.

- Since the complexity of the EARQ scheme grows exponentially with the packet size, the scheme is more suitable for smaller packet size, for example, the GSM normal burst of 114 bits. It can also be shown that in this scheme, transmission efficiency is better for short packet sizes.
- Since FCS is the minimum overhead required for reliable transmission over a noisy channel, $T(p)$, i.e., SRJ throughput given by (1), forms the lower bound of any hybrid type-II scheme. So, comparison of a hybrid ARQ scheme with $T(p)$ (e.g., [10]) does not reveal enough information.
- On the other hand, since any type-II ARQ scheme would need at least one retransmission to repair a corrupt first transmission, $T_U(p)$ given by (4), forms the upper bound and hence provides a better basis of comparison. It is seen from Fig. 1 that, till a BER of 0.02, throughput of EARQ is only 6% lower than $T_U(p)$. So unless BER is still higher, any other scheme (e.g., [2], [6], [8]) with however added complexity, would at best be only 6% more efficient than the EARQ scheme.
- The above conclusion is particularly important if the channel is time varying and constituted of a fairly long good and a relatively short bad state, for example, as described in [5]. In such cases, the difference in performance between any other type-II scheme and EARQ will become still smaller.
- Further, while the limitation posed by $N_{\max}$ makes the EARQ scheme implementable, it is also seen that, for small packet size and till a sufficiently high BER, $T_{nm}(p)$ provides results comparable with $T_{\max}(p)$.

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