Comprehensive Throughput Analysis of Unslotted ALOHA for Low-Power Wide-Area Networks

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***Abstract*—Unslotted ALOHA has been often employed by several low-power wide-area networks (LPWANs) for Internet of Things (IoT) as a random access (RA) protocol. This work analyzes the performance of unslotted ALOHA systems in terms of throughput and RA delay, and investigates their optimization. Our analysis consists of: 1) two-*heterogeneous* user case, whose backoff rate and packet length are different; 2) *N-homogeneous* user case, whose backoff rate and packet length are identical; and 3) homogeneous users of infinite population model. In the two-user case, we investigate the throughput region of unslotted ALOHA by using a multiobjective optimization problem (MOOP) and derive the Laplace Stieltjes transform (LST) of the probability density function (PDF) of RA delay. For *N*-homogeneous user case, we show how the throughput behaves according to the population size, packet length, and backoff rate. Our work may provide a comprehensive analytical framework for unslotted ALOHA systems.**

***Index Terms*—Access delay, heterogeneous users, pure ALOHA, renewal theorem throughput, unslotted ALOHA.**

NOMENCLATURE

$\mathcal{E}_k$	kth event on the channel, i.e., the sum of the kth idle interval and channel busyness.
$\mathcal{I}_k$	kth idle interval.
$\mathcal{B}_k$	kth busyness; either success or collision.
$\mathcal{C}_i$	Event that user i's packet transmission is in collision.
N	Number of users in the system.
$C_{1,1}$	Length of a collision period when user 1's transmission is partially interfered by user 2's transmission.
$C_{1,2}$	Length of a collision period when user 2's packet transmission completely interfers with user 1's transmission.
C_2	Length of a collision period when user 2's transmission is interfered by user 1's transmission.
T_i	Packet transmission time of user i in two-user system.
β_i	User i's backoff rate in two-user system; $1/\beta_i$.
τ_i	Backoff interval drawn by user i in two-user system.
μ_i	User i's throughput in two-user system
μ_N	Throughput of N-user system.
ξ_i	Probability that user i's packet transmission is not interfered by the other user in two-user system.

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I. INTRODUCTION

A. Motivations

TO REALIZE the Internet of Things (IoT) over sparsely populated large regions for smart farming and agriculture, it is not cost effective to make use of wireless cellular networks such as long-term evolution (LTE) with licensed bands, because it not only increases capital expenditure (CAPEX) and operational expenditure (OPEX) but also raises the price of disposable IoT devices; that is, the protocols and hardwares seem heavy for IoT devices handling a low rate of data, e.g., a few hundreds bytes. To make matters worse, the expected revenue from the networks over sparsely populated regions may not be high enough to cover the overall costs. Therefore, a monetary value per unit data may become exorbitant, which can eventually increase the product cost based on IoT.

Recently, low-power wide-area networks (LPWANs) have emerged as a cost-effective solution for IoTs of smart agriculture, ecosystem monitoring, smart island, etc., [1], [2]. Using unlicensed bands, it targets the coverage area of 10 km for rural areas and keeps the cost of hardware below a few dollars. Among various LPWAN technologies, SIGFOX and long-range radio (LoRa) adopt an unslotted (pure) ALOHA system as a medium access control (MAC) protocol such that IoT devices transmit their data without being synchronized to the system, whereas the half-duplex mode is allowed in those systems. This can reduce the price of devices by getting rid of synchronization and time-tracking hardware, and simplifying radio-frequency (RF) parts. In addition to such LPWANs, it could be also expected to adopt unslotted ALOHA-type systems, wherever low-cost data networks with low data rates are demanded. It is thus essential to characterize and understand the fundamental limits of unslotted ALOHA systems to make most out of it. In what follows, we shall see that although unslotted ALOHA systems have been proposed a few decades ago, most of previous works for MAC layer developed approximate performance studies when dealing with a finite number of users. This motivates us to investigate the accurate performance modeling of unslotted ALOHA systems.

B. Related Works

Although unslotted ALOHA systems have been adopted in LPWANs, they have not drawn much research attentions since 1990s. As prior work, Abramson [4] analyzed the throughput of an unslotted ALOHA system using *Poisson approximation*, i.e., infinite population model. It was considered that users are divided into two groups of users and each group has its own arrival rate and packet transmission time and been modeled as a Poisson process. More approximate performance studies on unslotted ALOHA systems have been found in [5] and [7], when users transmit a packet whose length follows a probability distribution. Diffusion approximation for the process of backlogged users was applied in order to examine the stability of unslotted ALOHA systems in [8]. In contrast to unslotted ALOHA systems with error-free channels in [4] and [8], the effects of channel errors on the system performance have been examined in [9] and [10]. Assuming that packet arrivals follow an arbitrary distribution, Sant [12] investigated the throughput of unslotted ALOHA systems. Furthermore, Mcbridge [13] examined the systems with a finite population of users using the renewal theory, when all the users employ the same retransmission rate and transmit a constant length of packets. The output process of the unslotted ALOHA channel is analyzed in [11]. It is notable that an infinite population model, i.e., Poisson approximation, was used in [4] and [11] and an asymptotic approximation for a finite population model in [12] and [13] with the renewal theory. The approximate throughput of unslotted ALOHA systems with a multipacket reception (MPR) channel has been examined in [14].

Apart from the performance analysis in [4]–[14] and [19], backoff control algorithms have been proposed in order to control retransmissions from backlogged users optimally for throughput maximization in [15]–[17]. Especially, it has been shown in [17] that unslotted ALOHA systems with a finite population become *bistable*[1] depending on the population size, backoff rate, and packet arrival rate and the throughput-optimal control algorithm for retransmissions can eliminate such a bistability. Some LPWANs, e.g., LoRa systems, employ a half-duplex unslotted ALOHA system so that the cost for RF hardware can be lowered. However, since the access point (AP) cannot receive any packets from the uplink when it is in the transmitting mode, the effect of a half-duplex mode on the throughput is examined [18].

So far unslotted ALOHA systems have been investigated in the MAC layer in [4]–[8] and [11]–[18]. Code-division multiple access (CDMA) and chirp spread spectrum (CSS), on the other hand, have been considered as transmission technologies to leverage the physical layer technologies for a higher throughput. In CDMA-unslotted ALOHA systems [19], two classes of users have been considered; that is, one class has a higher access probability than the other, while some packets in collision can be successfully decoded when a multiple access interference (MAI) condition is met due to CDMA. Besides, in [20]–[22], LoRa systems with CSS have been extensively investigated with focus on how LoRa's own physical characteristics, such as CSS and its spreading factor, affect the MAC layer and spatial performances.

In comparison with [4]–[19], our work can be placed close to [4]–[7], [12], and [13] in the sense that a finite population model is assumed. Most of the previous works, however, made use of either Poisson approximation or some asymptotic analysis that takes N to ∞. In contrast, we investigate the throughput and RA delay of the systems with two heterogeneous users and throughput of N homogeneous users to see how the performance behaves depending on the population size.

Compared to [20]–[22], our work focuses on the MAC layer performance by simplifying physical layer technologies. This is not because of analytical convenience, but because of the independence of layers.

C. Contributions

The first contribution of this work is to characterize the performance of unslotted ALOHA systems with two heterogeneous users in terms of throughput and the Laplace Stieltjes transform (LST) of the probability density function (PDF) of the RA delay. According to two users' packet length and backoff rate, this work shows how packet length and backoff rate affect throughput and delay distribution. In particular, we explore the throughput region of unslotted ALOHA systems by formulating a multiobjective optimization problem (MOOP) for each user's throughput. We shall see that the throughput regions are in fact equivalent to the *Pareto frontier* of MOOP. This enables us to examine the effects of each user's backoff rate and packet length on the throughput region. The results from the two-user system show that no one gets any incentive of using a longer packet transmission time or shorter backoff interval than the other one. This motivates us to examine the system with N homogeneous users.

The second contribution is to analyze the throughput of the systems with N homogeneous users who have always a packet to send, i.e., saturated. The analysis is based on the renewal theory and order statistics by focusing on the average of busy periods formed by the users. It can show the behavior of the system throughput as the population size N and packet transmission time T vary. As a byproduct, the average RA delay can be also obtained.

Third, we further analyze the throughput of unslotted ALOHA systems with an infinite population model. By comparing with the finite population model, we show that infinite model shows quite conservative measure on the system performance for small population sizes.

[1]In a bistable random access (RA) system, the system switches back and forth from a low number of backlogged users to a high number so that high and low throughput states alternatively show up over time.

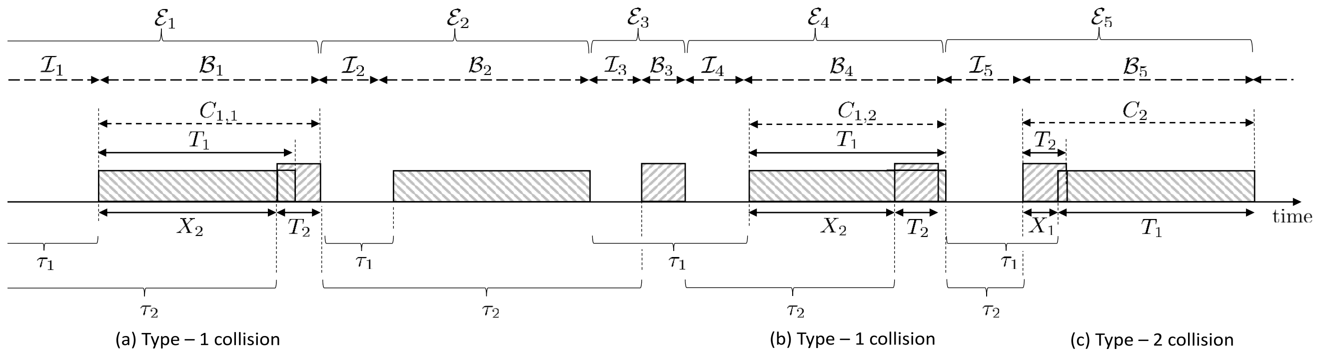

Fig. 1. Sample path of unslotted ALOHA systems. (a) and (b) Type-1 collision. (c) Type-2 collision.

Finally, we show that for the unsaturated condition, where users have a new packet according to a Poisson process, without controlling backoff rate, an unslotted ALOHA system is always unstable.

D. Organization and Notations

This article is organized as follows. In Section II, unslotted ALOHA systems with homogeneous and heterogeneous users are introduced. Our analysis is given in Section III. Section III-A presents the throughput region and the LST of RA delay distribution of unslotted ALOHA systems with two heterogeneous users. Throughput and RA delay of unslotted ALOHA systems with N homogeneous users are analyzed in Section III-B. Numerical studies are discussed in Section IV and concluding remarks are finally given in Section V.

When dealing with two-user systems in Section III-A, for the sake of exposition, we define an index $\bar{i}$ as

$$\bar{i} \triangleq i + (-1)^{i-1}. \tag{1}$$

If $i = 1$ (or 2), then $\bar{i} = 2$ (or 1). Therefore, when we mention user i, user $\bar{i}$ indicates the other user.

When the subscript of a notation is dropped, it denotes the parameter for the system with N homogeneous users and bears the same meaning unless otherwise specified. For example, when $1/\beta_i$ is the mean of exponentially distributed backoff interval of user i, $1/\beta$ denotes the mean of exponentially distributed backoff interval of a user in the N-user system.

II. Unslotted ALOHA System

We consider a system, where users are wirelessly connected with an AP and contend for the channel access to the AP. In this system, unslotted ALOHA is used as a RA protocol. We assume that each user always has a packet to transmit, i.e., *saturated* user. For this system with two heterogeneous users, say user 1 and user 2, whenever (re)transmitting a packet, user i for $i \in \{1, 2\}$ first draws a random backoff time, say τ_i, from an exponentially distributed random variable with mean $1/\beta_i$ (s) as shown in Fig. 1. It (re)transmits its packet after waiting for τ_i and its packet transmission takes T_i (s). In the system with N homogeneous users, all users take backoff times from the same exponential distribution with mean $1/\beta$ (s) and their packet transmission time is T (s) long.

If one user's ongoing packet (re)transmission is interfered by at least another user's packet (re)transmission, a collision occurs. The AP can decode none of packets (re)transmitted in a collision. The collision period is defined as a time interval from the beginning of the packet transmitted earliest in a collision to the end of the last packet transmission time. At the end of a collision period, the AP informs the users of the collision by a broadcast message such that the (re)transmitting users can reschedule their packet (re)transmission by drawing new backoff times again. For instance, Fig. 1 illustrates the system with two heterogeneous users. If user 2 (or user 1) would not (re)transmit during user 1's (or user 2') packet (re)transmission, user 1's (or user 2's) packet transmission becomes successful. Upon success, the AP broadcasts the transmission result, i.e., success, right after the end of the packet transmission such that the user in (re)transmission can be aware of its channel outcome and reschedule the next packet transmission.

We assume an ideal channel coding scheme in the physical layer that can protect a single user's packet transmission over the uplink and the feedback information over the downlink from channel errors. Thus, they could be always success. We, however, assume that a partial overlap of the packets transmitted by other users leads to an overall decoding failure, i.e., collision.

III. Throughput Region Analysis

A. Unslotted ALOHA With Two Users

1) Throughput Region: Let μ_i denote user i's throughput of unslotted ALOHA systems as well. With reference to Fig. 1, the channel is said to be busy if at least one user (re)transmits; otherwise, idle. A busy channel can turn out to be success or collision. In Fig. 1, $\mathcal{I}_k$ and $\mathcal{B}_k$ for $k = 1, 2, \ldots$ denote the length of the idle period and the busy period of event $\mathcal{E}_k$, respectively, while $\mathcal{E}_k$ denotes an event that busy period $\mathcal{B}_k$ follows idle period $\mathcal{I}_k$ on the channel.

We first examine the probability that user i for $i \in \{1, 2\}$ makes a successful packet (re)transmission, denoted by $\Pr[\mathcal{S}_i]$. First, ξ_i is defined as the probability that user i is the only (re)transmitting user in a busy period; that is, its transmission is not interfered by user $\bar{i}$'s packet transmission. Note that the events regarding ξ_i correspond to $\mathcal{E}_2$ for user 1 and $\mathcal{E}_3$ for user 2 in Fig. 1, respectively. Let $\Pr[\mathcal{C}_i]$ denote the probability that user i's packet transmission is in collision. Using ξ_i, we can find $\Pr[\mathcal{S}_i]$.

Lemma 1: The probability that user i makes a successful packet transmission $\Pr[\mathcal{S}_i]$ is obtained as

$$\Pr[\mathcal{S}_i] = \frac{\xi_i}{\xi_i + \Pr[\mathcal{C}_i]} = \frac{\beta_i e^{-\beta_{\bar{i}} T_i}}{\beta_1 + \beta_2 - \beta_{\bar{i}} e^{-\beta_i T_{\bar{i}}}} \tag{2}$$

where $\bar{i}$ is defined in (1) and the expression of ξ_i is given in Appendix A. It is true that $\Pr[\mathcal{S}_i]$ implies the ratio of the number of packets transmitted by user i without user $\bar{i}$ interfering to the total number of packets transmitted by it.

Proof: See Appendix A. ■

Theorem 1: Based on the renewal reward theorem [23], the throughput of user i is expressed as

$$\mu_i = \frac{\xi_i}{\mathbb{E}[\mathcal{Z}]} \tag{3}$$

where the denominator $\mathbb{E}[\mathcal{Z}]$ in (3) denotes the average of renewal periods, i.e., the sum of the average busy and idle periods. If $T_i > T_{\bar{i}}$ for $i \in \{1, 2\}$, we have

$$\begin{aligned}\mathbb{E}[\mathcal{Z}] &= \mathbb{E}[\mathcal{B}] + \mathbb{E}[\mathcal{I}] \\ &= T_i + \frac{1}{\sum_{i=1}^{2} \beta_i}\Bigg[1 + \frac{\beta_{\bar{i}}}{\beta_i}\left(1 - e^{-\beta_i T_{\bar{i}}}\right) \\ &\qquad + \frac{\beta_i}{\beta_{\bar{i}}} e^{-\beta_{\bar{i}} T_i}\left(e^{\beta_{\bar{i}} T_{\bar{i}}} - 1\right) - \sum_{i=1}^{2} \beta_{\bar{i}} T_i e^{-\beta_i T_{\bar{i}}}\Bigg]\end{aligned} \tag{4}$$

where we drop the subscripts of $\mathcal{B}$ and $\mathcal{I}$ for convenience. The detailed expressions of $\mathbb{E}[\mathcal{B}]$ and $\mathbb{E}[\mathcal{I}]$ are given in the proof.

Proof: See Appendix B. ■

Making use of Theorem 1, we can find the throughput region of unslotted ALOHA systems by solving the following problem:

$$\begin{aligned}&\underset{\beta_1, \beta_2}{\text{minimize}} \quad [-\mu_1, -\mu_2] \\ &\text{subject to} \quad \beta_i \geq 0, \quad i = 1, 2.\end{aligned} \tag{5}$$

The Lagrangian function of (5) can be expressed as

$$\mathcal{L} = -\sum_{i=1}^{2} \alpha_i \mu_i - \sum_{i=1}^{2} \gamma_i \beta_i \tag{6}$$

and the Fritz John (FJ) conditions can be found as

$$\frac{\partial \mathcal{L}}{\partial \beta_k} = -\sum_{i=1}^{2} \alpha_i \frac{\partial \mu_i}{\partial \beta_k} - \gamma_k = 0. \tag{7}$$

From the complementary slackness (CS) conditions, i.e., $\gamma_k \beta_k = 0$, we have $\gamma_k = 0$ for $\beta_k > 0$. As in (8), we have

$$\begin{bmatrix} \frac{\partial \mu_1}{\partial \beta_1} & \frac{\partial \mu_2}{\partial \beta_1} \\ \frac{\partial \mu_1}{\partial \beta_2} & \frac{\partial \mu_2}{\partial \beta_2} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{8}$$

Accordingly, the maximizer β_i in (5) should satisfy

$$\frac{\partial \mu_1}{\partial \beta_1} \frac{\partial \mu_2}{\partial \beta_2} - \frac{\partial \mu_2}{\partial \beta_1} \frac{\partial \mu_1}{\partial \beta_2} = 0. \tag{9}$$

Although the expression of (9) is so lengthy that it cannot be simplified, it can be numerically found that (9) has a unique pair of roots β_1 and β_2. Substituting this pair of β_1 and β_2 into the expressions of μ_1 and μ_2, we get the envelope of throughput region.

As special cases, let $\beta_1 \to 0$ ($[1/\beta_1] \to \infty$), i.e., long interretransmission interval so that user 1 will not (re)transmit. Then, the throughput of each user can be expressed as

$$\mu_1 = 0 \quad \text{and} \quad \mu_2 = \frac{1}{T_2 + \frac{1}{\beta_2}}. \tag{10}$$

On the other hand, if $\beta_2 \to 0$, we have

$$\mu_1 = \frac{1}{T_1 + \frac{1}{\beta_1}} \quad \text{and} \quad \mu_2 = 0. \tag{11}$$

When all the parameters are identical, i.e., $\beta_1 = \beta_2 = \beta$ and $T_1 = T_2 = T$, the throughput $\mu_1 = \mu_2 = \mu$ is reduced to

$$\mu = \frac{\beta e^{-\beta T}}{3 + 2\left[T\beta - (1 + T\beta)e^{-\beta T}\right]}. \tag{12}$$

Note that $(\partial \mu / \partial \beta) = -[([2(T\beta)^2 + 3(T\beta - 1)]e^{T\beta} + 2)/([(3 + 2T\beta)e^{T\beta} - 2(T\beta + 1)]^2)]$. From $(\partial \mu / \partial \beta) = 0$, the maximizer β in (12) should satisfy

$$2T^2\beta^2 + 3T\beta - 3 = -2e^{-T\beta}. \tag{13}$$

The solution of (13) can be numerically found as

$$\beta^* \approx \frac{0.4413}{T}. \tag{14}$$

Using (14), each user obtains the throughput μ^* as

$$\mu^* \approx \frac{0.1399}{T}. \tag{15}$$

2) LST of Access Delay: Let us derive the LST of the PDF of the RA delay of each user so that the moments of the access delay can be obtained from it. Let $\mathcal{G}_i(s)$ denote the LST of the PDF for the time taken for user i to transmit its packet successfully because user $\bar{i}$ does not (re)transmit for user i's packet transmission time, which occurs with probability ξ_i. With reference to Fig. 1, when user 1 (or 2) (re)transmits its packet successfully, $\mathcal{G}_1(s)$ [or $\mathcal{G}_2(s)$] corresponds to the length of event $\mathcal{E}_2$ (or $\mathcal{E}_3$), which takes idle time $\mathcal{I}_2$ (or $\mathcal{I}_3$) and one packet transmission time T_i. In Appendix C, $G_i(s)$ is found in (62).

Let $\mathcal{F}_i(s)$ be the LST of the PDF of the length of type-i collision plus idle period $\mathcal{I}_k$. In Fig. 1, $\mathcal{F}_1(s)$ corresponds to the length of events $\mathcal{E}_1$ and $\mathcal{E}_4$, and $\mathcal{F}_2(s)$ for $\mathcal{E}_5$, respectively.

Theorem 2: Let $H_i(s)$ denote the LST of the PDF of the time taken for user i to transmit its packet successfully. It can be obtained by

$$H_i(s) = \frac{\mathcal{G}_i(s)\xi_i}{1 - \left(\mathcal{G}_{\bar{i}}(s)\xi_{\bar{i}} + \sum_{i=1}^{2} \mathcal{F}_i(s)\omega_i\right)} \tag{16}$$

where ω_i denotes the probability of type-i collision; its expression is given in Appendix A.

Proof: See Appendix C. ■

Theorem 2 can be verified by Theorem 1 as follows. Note that the mean of the packet transmission time is obtained by $\overline{h}_1 = -H_1'(0)$, and $\mathcal{G}_i(0) = \mathcal{F}_i(0) = 1$. Using $[\mathcal{N}(s)/\mathcal{D}(s)]' =$

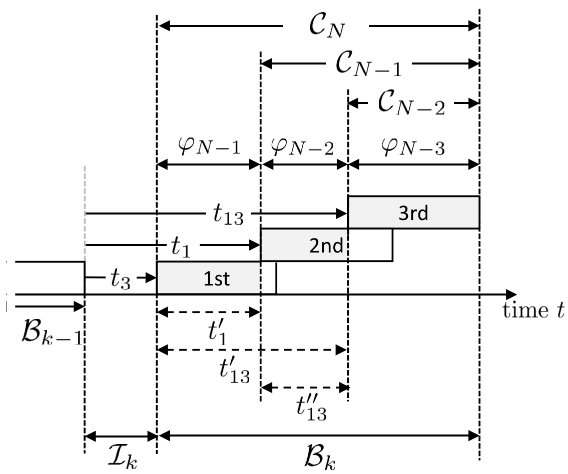

Fig. 2. Timing diagram of unslotted ALOHA system with N homogeneous users.

$[(\mathcal{N}'(s)\mathcal{D}(s) - \mathcal{N}(s)\mathcal{D}'(s))/(\mathcal{D}(s)^2)]$, the mean of the service time for a packet $\overline{h}_1$ is obtained as

$$\begin{aligned}\overline{h}_1 &= -\frac{\xi_1}{(1-(\xi_2+\sum_{i=1}^2 \omega_i))^2}\Bigg[\mathcal{G}_1'(0)\Bigg(1-\xi_2-\sum_{i=1}^{2}\omega_i\Bigg) \\ &\qquad + \mathcal{G}_2'(0)\xi_2 + \sum_{i=1}^{2}\mathcal{F}_i'(0)\omega_i\Bigg] \\ &= -\frac{\left(\sum_{i=1}^2 \mathcal{G}_i'(0)\xi_i + \sum_{i=1}^2 \mathcal{F}_i'(0)\omega_i\right)}{\xi_1} \end{aligned} \tag{17}$$

where we have used $\xi_1 + \xi_2 + \omega_1 + \omega_2 = 1$. We find $\mathcal{G}_i'(0)$ and $\mathcal{F}_i'(0)$ as

$$\mathcal{G}_i'(0) = -\left(\tfrac{1}{\beta_1+\beta_2} + T_i\right) \tag{18}$$

and

$$\mathcal{F}_i'(0) = -\left(\frac{1}{\beta_1+\beta_2} + \overline{C}_i\right). \tag{19}$$

By plugging (18) and (19) into (17), we can find that $\overline{h}_1 = 1/\mu_1$, which is true for the saturated condition.

B. Unslotted ALOHA With N Homogeneous Users

Let us consider pure ALOHA systems with N homogeneous users, where each user adopts exponentially distributed backoff times with the same mean $1/\beta$ (s) and transmit a packet of T (s).

As shown in Fig. 2, $\mathcal{I}_k$ and $\mathcal{B}_k$ denote the kth idle and busy period, respectively. For a user transmitted in a busy period, it will wait for the feedback from the AP. Therefore, it will not retransmit the packet within the same busy period. Let τ_i be the backoff interval of user i, whose cumulative distribution function (CDF) and PDF are denoted by

$$F_\tau(x) = 1 - e^{-\beta x} \quad \text{and} \quad f_\tau(x) = \beta e^{-\beta x}. \tag{20}$$

For backoff intervals τ_1, τ_3, τ_5, and τ_{13} drawn by users 1, 3, 5, and 13, let t_i be the remaining (or residual) time of backoff interval τ_i after the end of B_{k-1}. The first packet transmission by user 3 occurs first, since t_3 is the minimum of them. It is important to notice that t_i's follow the same exponential distribution with mean $1/\beta$ due to the memoryless property of exponential distribution [24]. Let us consider the order statistics of these remaining backoff intervals. Let $t_{(r)}$ denote the rth order of them; that is, $t_{(1)}$ is the minimum of t_1, t_3, t_5, t_{13}, and $t_{(4)}$ (in this example) the maximum of them, which corresponds to t_3 and t_5, respectively. If $\tau_{(r)}$ denotes the rth order of τ_i, the distributions of $t_{(r)}$ and $\tau_{(r)}$ are same, because t_i follows the same distribution of τ_i. If there are a total of N independently and identically distributed exponentially backoff intervals (chosen by N users), the CDF and PDF of the rth order backoff interval and remaining backoff interval are, respectively, expressed as

$$F_{(r)}(x; N) = \sum_{n=r}^{N}\binom{N}{n}[F_\tau(x)]^n[1-F_\tau(x)]^{N-n} \tag{21}$$

and

$$f_{(r)}(x; N) = \frac{N! f_\tau(x)[F_\tau(x)]^{r-1}[1-F_\tau(x)]^{N-r}}{(r-1)!(N-r)!}. \tag{22}$$

Lemma 2: At the beginning epoch of the jth order transmission in a busy period, let q_j denote the probability that none of the remaining $N-j$ users (re)transmit during the time duration of the jth order transmission, i.e., T (s)

$$q_j = e^{-(N-j)\beta T}. \tag{23}$$

Proof: From (21), we can see that $F_{(1)}(T; N-j)$ is the probability that the first (re)transmission among $N-j$ users during interval T is made. The probability that no one among $N-j$ users (re)transmits during T is then expressed as

$$\begin{aligned} q_j &= 1 - F_{(1)}(T; N-j) \\ &= 1 - \sum_{n=1}^{N-j}\binom{N-j}{n}[F_\tau(t)]^n[1-F_\tau(T)]^{N-j-n} \\ &= 1 - [1-F_\tau(t)]^{N-j}\left[\sum_{n=0}^{N-j}\binom{N-j}{n}\left[\frac{F_\tau(T)}{1-F_\tau(T)}\right]^n - 1\right] \\ &= 1 - [1-F_\tau(t)]^{N-j}\left[\left(1+\frac{F_\tau(T)}{1-F_\tau(T)}\right)^{N-j} - 1\right] \\ &= (1-F_\tau(t))^{N-j} = e^{-(N-j)\beta T} \end{aligned} \tag{24}$$

which completes the proof. ■

Let ξ denote the probability that a user makes a successful (re)transmission. The following corollary finds ξ.

Corollary 1: The first transmission in a busy period becomes successful with probability ξ

$$\xi = q_1 = e^{-(N-1)\beta T}. \tag{25}$$

Proof: Suppose that a tagged user is (re)transmitting. Its (re)transmission is successful if $N-1$ other users would not (re)transmit any packets during the tagged user's packet transmission time. In other words, the backoff intervals of $N-1$ users are longer than the tagged user's packet transmission time. So we have (25). This is also found by Lemma 2 as well. ■

Lemma 3: The average busy period in Fig. 2 can be obtained as (we drop the subscript of $\mathcal{B}_k$)

$$\mathbb{E}[\mathcal{B}] = T + \sum_{j=1}^{N-1} \mathbb{E}\big[\varphi_{N-j}\big] \prod_{i=1}^{j} (1 - q_i) \tag{26}$$

where $\mathbb{E}[\varphi_{N-j}]$ is the average duration between the beginning epoches of the jth and the $(j+1)$th transmissions in a busy period and is expressed as

$$\mathbb{E}\big[\varphi_{N-j}\big] = \frac{1 - \big[1 + (N-j)\beta T\big] e^{-(N-j)\beta T}}{\big(1 - e^{-(N-j)\beta T}\big)(N-j)\beta}. \tag{27}$$

Proof: See Appendix D. ■

Theorem 3: The throughput of pure ALOHA systems with N homogeneous users, denoted by μ_N, can be expressed as

$$\mu_N = \frac{e^{-(N-1)\beta T}}{\frac{1}{N\beta} + \mathbb{E}[\mathcal{B}]} \tag{28}$$

where $\mathbb{E}[\mathcal{B}]$ is given in Lemma 3.

Proof: As we have done with the system with two users, the throughput of the system with N users can be obtained as

$$\mu_N = \frac{\xi}{\mathbb{E}[\mathcal{Z}]} = \frac{\xi}{\mathbb{E}[\mathcal{I}] + \mathbb{E}[\mathcal{B}]} \tag{29}$$

where ξ is given in Corollary 1 and $\mathbb{E}[\mathcal{B}]$ in Lemma 3. To obtain $\mathbb{E}[\mathcal{I}]$, let us recall that N users adopt an identical backoff rate β independently. Therefore, the idle period in (29) is the mean of the minimum of N independently identically distributed exponential random variables with mean $1/\beta$

$$\mathbb{E}[\mathcal{I}] = \frac{1}{N\beta} \tag{30}$$

which completes the proof. ■

Corollary 2: The access delay of a user in the system with N homogeneous users is obtained as

$$E[D] = \frac{N}{\mu_N}. \tag{31}$$

Proof: See Appendix E. ■

Theorem 4: If $N\beta$ converges to a constant G when $N \to \infty$ in (28), the throughput of the system with the finite population model becomes

$$\mu_\infty = Ge^{-2GT} \tag{32}$$

where $G = N\beta$.

Proof: Let us return to Lemma 3 and Theorem 3 and show that Theorem 3 can be linked to Theorem 4 as the population size grows, i.e., $N \to \infty$. With $\mathcal{C}_N$ denoting the a busy period starting with N users, from (68) we have

$$\begin{aligned}\lim_{N\to\infty} \mathbb{E}[\mathcal{C}_N] &= \lim_{N\to\infty} \big[Tq_1 + \big(\mathbb{E}\big[\varphi_{N-1}\big] + \mathbb{E}\big[\mathcal{C}_{N-1}\big]\big)(1 - q_1)\big] \\ &= Te^{-GT} + \frac{1 - (1 + GT)e^{-GT}}{G} \\ &\quad + \Big(1 - e^{-GT}\Big) \lim_{N\to\infty} \mathbb{E}\big[\mathcal{C}_{N-1}\big]\end{aligned} \tag{33}$$

where we have applied the fact that $\lim_{N\to\infty} q_1 = e^{-GT}$ and $\lim_{N\to\infty} \mathbb{E}[\varphi_{N-1}] = [(1 - (1 + GT)e^{-GT})/(G(1 - e^{-GT}))]$. Note that $\mathbb{E}[\varphi_{N-1}]$ is given in (27).

In order to obtain the expression of $\lim_{N\to\infty} \mathbb{E}[\mathcal{C}_N]$, let us first prove $\lim_{N\to\infty} \mathbb{E}[\mathcal{C}_N] = \lim_{N\to\infty} \mathbb{E}[\mathcal{C}_{N-1}]$. Replacing N by $N+1$ in (33), we have

$$\begin{aligned}\lim_{N\to\infty} \mathbb{E}\big[\mathcal{C}_{N+1}\big] &= Te^{-GT} + \frac{1 - (1 + GT)e^{-GT}}{G} \\ &\quad + \Big(1 - e^{-GT}\Big) \lim_{N\to\infty} \mathbb{E}[\mathcal{C}_N].\end{aligned} \tag{34}$$

Subtracting (34) from (33), we have

$$\begin{aligned}&\lim_{N\to\infty} \big(\mathbb{E}\big[\mathcal{C}_{N+1}\big] - \mathbb{E}[\mathcal{C}_N]\big) \\ &= \Big(1 - e^{-GT}\Big) \lim_{N\to\infty} \big(\mathbb{E}[\mathcal{C}_N] - \mathbb{E}\big[\mathcal{C}_{N-1}\big]\big).\end{aligned} \tag{35}$$

By recursively applying the above procedure, for any positive integer k we can have

$$\begin{aligned}&\lim_{N\to\infty} \big(\mathbb{E}\big[\mathcal{C}_{N+k}\big] - \mathbb{E}\big[\mathcal{C}_{N+k-1}\big]\big) \\ &= \Big(1 - e^{-GT}\Big)^k \lim_{N\to\infty} \big(\mathbb{E}[\mathcal{C}_N] - \mathbb{E}\big[\mathcal{C}_{N-1}\big]\big).\end{aligned} \tag{36}$$

Since $(\mathbb{E}[\mathcal{C}_N] - \mathbb{E}[\mathcal{C}_{N-1}])$ is finite and $1 - e^{-GT} < 1$, if we make k tend to infinity, we have

$$\lim_{k\to\infty} \lim_{N\to\infty} \big(\mathbb{E}\big[\mathcal{C}_{N+k}\big] - \mathbb{E}\big[\mathcal{C}_{N+k-1}\big]\big) = 0 \tag{37}$$

which can be simplified as

$$\lim_{N\to\infty} \big(\mathbb{E}[\mathcal{C}_N] - \mathbb{E}\big[\mathcal{C}_{N-1}\big]\big) = 0. \tag{38}$$

By plugging the result of (38) into (33), we can further derive

$$\lim_{N\to\infty} \mathbb{E}[\mathcal{C}_N] = \frac{e^{GT} - 1}{G}. \tag{39}$$

Consequently, when $N \to \infty$, the system throughput in (28) can be expressed as

$$\begin{aligned}\mu_\infty = \lim_{N\to\infty} \mu_N &= \lim_{N\to\infty} \frac{e^{-(N-1)\beta T}}{\frac{1}{N\beta} + \mathbb{E}[\mathcal{C}_N]} \\ &= \frac{e^{-GT}}{\frac{1}{G} + \frac{e^{GT}-1}{G}} = Ge^{-2GT}.\end{aligned} \tag{40}$$

This completes the proof. ■

Note that this is identical toAbramson's result, i.e., in [4, eq. (2)]. His derivation is based on that the vulnerable period of unslotted ALOHA is two packet transmission times, during which no packet should be (re)transmitted. The throughput of his infinite population model is a product of the offered load G and the packet transmission success probability e^{-2GT}.

C. Unsaturated System

Let us consider unslotted ALOHA systems with the unsaturated condition, i.e., the number of users having a new packet to transmit follows a Poisson process with rate λ. They become backlogged and the backlogged users (re)transmit their packets according to the exponentially distributed backoff rate as before. Each user has a unit-sized buffer to store a packet and upon successful transmission of the packet, the user becomes nonbacklogged.

Theorem 5: An unslotted ALOHA system under the unsaturated condition always becomes unstable if β is fixed.

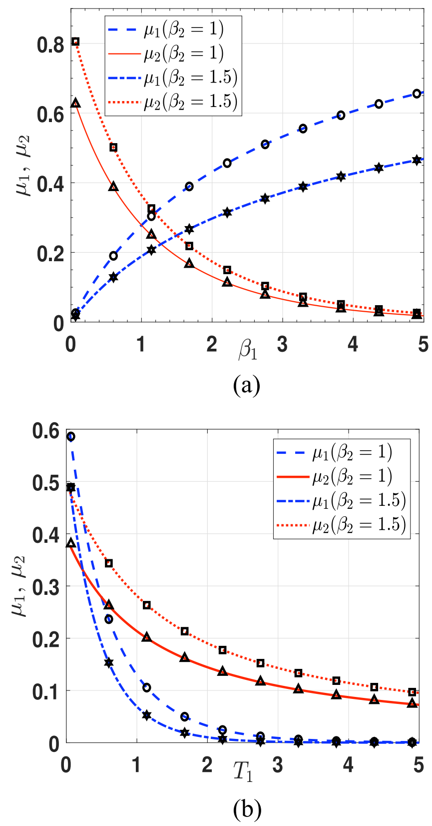

Fig. 3. Throughput with various β_1's or T_1's. (a) Throughput with $T_1 = T_2 = 0.5$. (b) Throughput with $T_2 = 0.5$ and $\beta_1 = 1$.

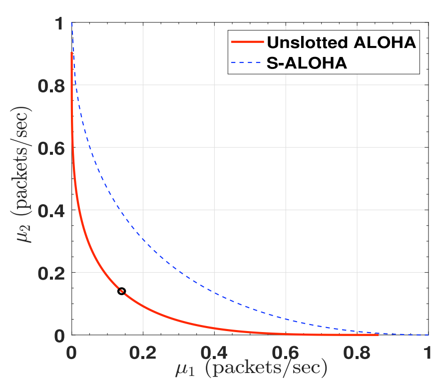

Fig. 4. Throughput region of unslotted ALOHA and S-ALOHA systems with $T_1 = T_2 = 1$.

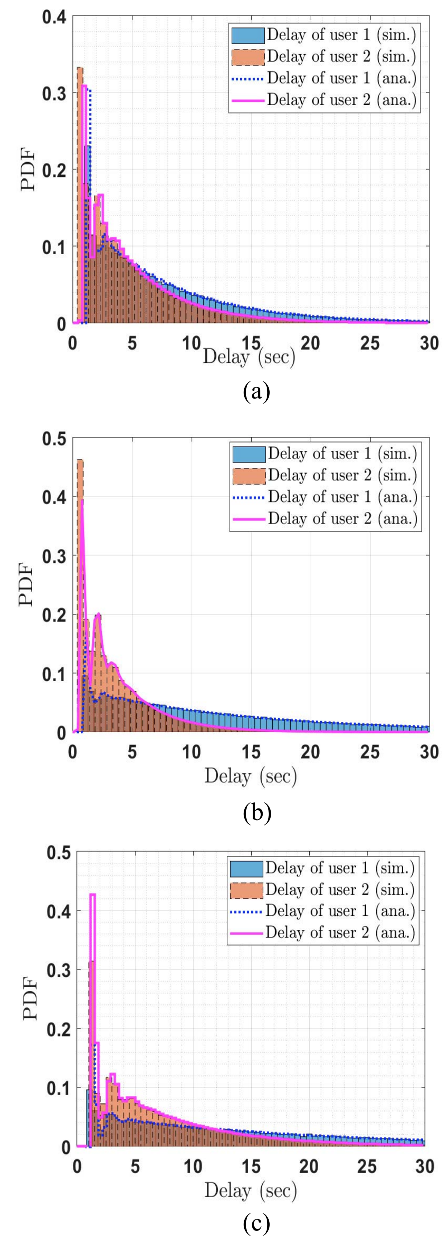

Fig. 5. PDFs of RA delay in two-user systems. (a) $T_1 = 1, T_2 = 0.5, \beta_1 = 1$, and $\beta_2 = 1$. (b) $T_1 = 1, T_2 = 0.5, \beta_1 = 1$, and $\beta_2 = 1.5$. (c) $T_1 = 1, T_2 = 1, \beta_1 = 1$, and $\beta_2 = 1.5$.

Proof: See Appendix F. ■

To run the (unsaturated) system stably, we need a backoff algorithm of controlling β dynamically such that $\lambda < \mu_m$ for $m > m^*$ and m^* is the first state, at which the mean arrival rate is less than the throughput. This is discussed in Appendix F.

Corollary 3: If the system controls $\beta = (1/[2mT])$ by estimating the number of backlogged users m in real time, the system becomes stable as long as the input rate satisfies $\lambda < (1/2T)e^{-1}$.

Proof: It is important to notice that maximizing μ_m is equivalent to minimizing the negative drift in (74). ■

Notice that the estimation algorithms for m are found in [16] and [17].

IV. NUMERICAL RESULTS

In all the figures in this section, the lines and symbols depict the analytical and simulation results, respectively. We build a simulation program with MATLAB. The runtime of simulation is set to 3×10^6 (s) and its time average is obtained.

By varying β_1, we present the throughput μ_i (packets/s) with $T_1 = T_2 = 0.5$ and $\beta_2 = 1, 1.5$ in Fig. 3(a). The analytical results agree with simulations. If the throughput in unit of bits per sec is interested, the total number of bits carried by a packet should be multiplied to μ_i. As user 1's backoff rate β_1 increases, its throughput is improved and user 2's throughput drops. When β_2 is raised up to 1.5, it can be seen that user 2's throughput increases and user 1's throughput decreases compared to the case with $\beta_1 = 1$. If β_2 becomes smaller

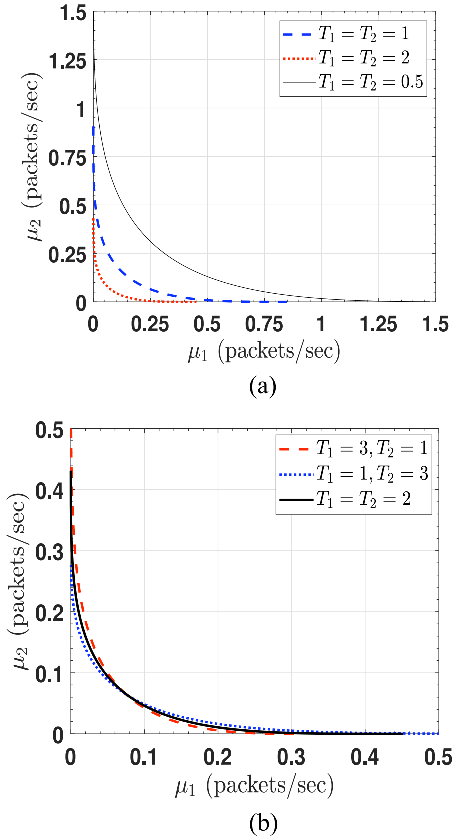

Fig. 6. Throughput region of unslotted ALOHA systems. (a) Throughput region with various packet lengths and $T_1 = T_2$. (b) Throughput region with $T_1 \neq T_2$.

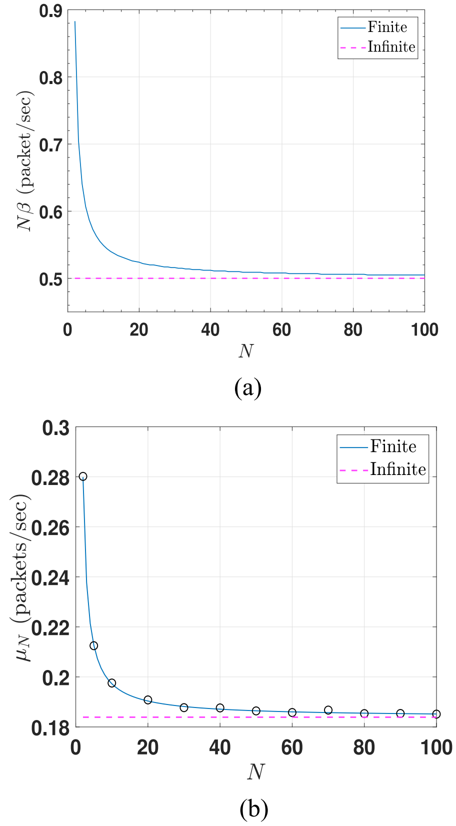

Fig. 7. Throughput and optimal control with packet length $T = 1$. (a) Optimal $N\beta$ of pure ALOHA system. (b) Maximum throughput with N users.

than one, the opposite result can be envisioned; that is, user 2's throughput decreases and user 1's throughput increases. Additionally, when two users' parameters are identical, i.e., $\beta_1 = \beta_2$ and $T_1 = T_2$, they share the same throughput $\mu_1 = \mu_2$. Although not presented here, $\mu_1 = \mu_2$ are maximized when $\mu_1 = \mu_2 = 0.4413/0.5 = 0.8826$. As one user tries to get a higher throughput by increasing its backoff rate, it hurts the other user's throughput. Fig. 3(b) depicts the throughput as user 1's packet transmission time T_1 is raised. It can be observed that the longer the user 1's packet transmission time (user 1 wants to send more information upon a success), the smaller throughput two users get. However, notice that user 1's longer packet transmission eventually hurts itself more than user 2. When user 1 shortens its packet transmission time, he gets a higher throughput than user 2. However, the amount of the information in its packet becomes smaller. For $\beta_1 \neq \beta_2$, two users get the same throughput at a smaller T_1 (≈ 0.25).

In Fig. 4 the throughput region of unslotted ALOHA systems is compared to that of the slotted ALOHA (S-ALOHA) system. Note that the stability region of S-ALOHA system is found in [25]

$$\sqrt{\mu_1} + \sqrt{\mu_2} = 1 \tag{41}$$

where $\mu_i = p_i(1-p_{\bar{i}})$ and p_i is the (re)transmission probability of user i each slot for the S-ALOHA system.

For fair comparison, we assume a unit packet length for two systems, i.e., $T_1 = T_2 = 1$. As expected, S-ALOHA systems have a much larger throughput region. The small circle on the boundary curve of unslotted ALOHA systems represents when two users have the same throughput, i.e., $\mu_1 = \mu_2 = 0.1399$ in (15). The sum throughput of S-ALOHA systems is 0.5, when $\mu_1 = \mu_2$ in (41), while that of unslotted ALOHA systems in (15) is 0.2798 for $T = 1$. The boundary curve of unslotted ALOHA systems in Fig. 4 is best approximated by $\lambda_1^{(1/2.85)} + \lambda_2^{(1/2.85)} = 1$.

Fig. 5(a)–(c) compares the PDFs of RA delay of a two-user system against the simulations that the bar graphs show. We perform a numerical inversion of the Laplace transform of the PDF of the RA delay in (16) using the method in [27]. Compared to Fig. 5(a), where we set different packet sizes ($T_1 = 1$ and $T_2 = 0.5$), and equal backoff intervals ($\beta_1 = 1$ and $\beta_2 = 1$), user 2 has a shorter backoff interval in Fig. 5(b). This makes the tail of user 1's PDF longer, while user 2 has a shorter tail and higher probabilities for small RA delay. It means that user 2 takes advantage of reducing the mean backoff interval. From Fig. 5(b) and (c), we make two users' packet length equal. Then, user 2's PDF has a long tail as user 1's, whereas it can be observed that user 2 cannot make a successful RA for a short time with a high probability.

Fig. 6(a) depicts the throughput region of unslotted ALOHA systems with two users having the same packet length, while the packet length varies. We assume that the bit duration is fixed so that a longer packet can carry more bits. It can be seen that the longer the packet length, the smaller the throughput (packets/s) is observed. In contrast, in Fig. 6(b), we consider the system, where one user's packet length can be much longer than the other's. Furthermore, a system with the same

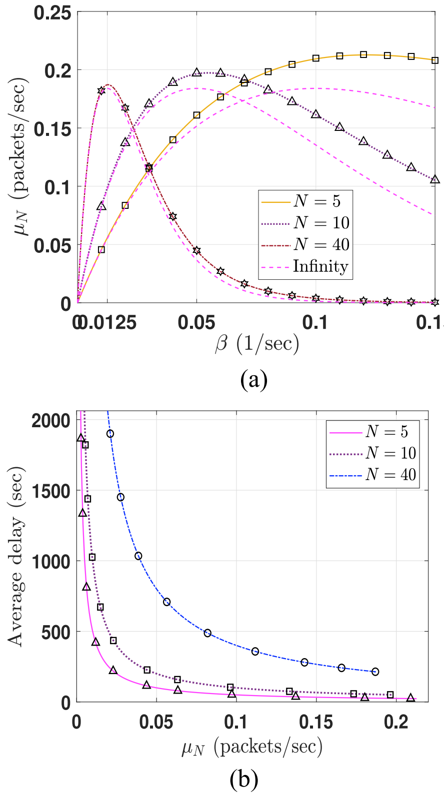

Fig. 8. Throughput and the average RA delay with N users. (a) Throughput with N users. (b) Average RA delay with N users.

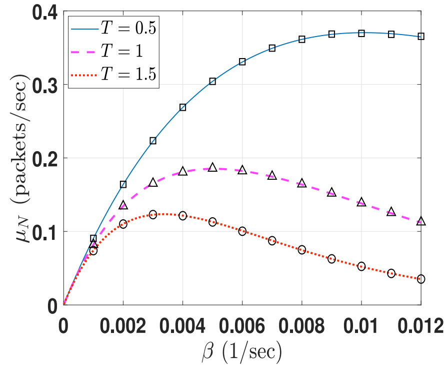

Fig. 9. Throughput with various packet lengths and $N = 100$.

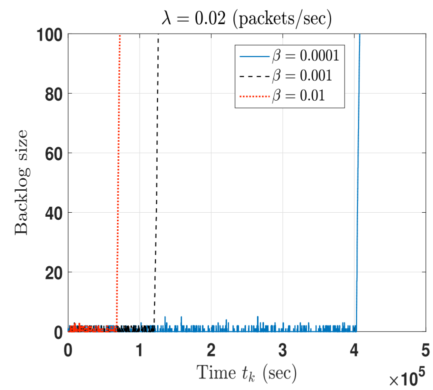

Fig. 10. Instability of unslotted ALOHA system with fixed β's.

packet length is compared. In the three systems presented, we have $T_1 + T_2 = 4$. If one user employs a longer packet, the other user's throughput is improved. As in Fig. 3(b), if one user increases its packet length, in the end it hurts itself. Accordingly, the best strategy that both users employ is to have the same packet length.

Before discussing Fig. 7, let us remind that $N\beta$ is the average traffic load to the system, whereas μ_N is the throughput. The "finite" in Fig. 7(a) shows $N\beta$ to maximize throughput μ_N of finite population, whereas the "infinite" means a maximizer for Theorem 4, i.e., infinite population. Each one is numerically obtained from Theorems 3 and 4 as a maximizer. Fig. 7(b) depicts the maximum throughput achieved by $N\beta$ in Fig. 7(a). For any T, $N\beta$ for finite population can be approximated by $N\beta = (N/[(2N - 1.782)T])$, whereas we have $N\beta = 1/2$ for infinite population. As N grows larger than 40, it can be seen that $N\beta$ of finite population quickly converges to 0.5. This implies that the optimal backoff rate β is set to $1/(2N)$ for N backlogged users. Let us recall that the sum throughput is close to 0.28 in Fig. 5, when $\beta_i = \beta$ and $T_i = T = 1$ in the system with two heterogeneous users. This can be also observed in Fig. 7(b), where Theorem 3 yields the same result for $N = 2$. For $N \geq 40$, the maximum throughput gets close to $0.5e^{-1} \approx 0.184$, which can be obtained with infinite population assumption.

Fig. 8(a) depicts the throughput with various population sizes as β increases, where the packet size T is set to one. Thus, for $N = 40$, we can see that the optimal β is $1/(2N) = 0.0125$, which is found in Fig. 7(a). As the population size increases, the throughput gets shrunk rapidly. However, if β is controlled in a way of maximizing μ_N according to N, it seems that the system gets μ_N as good as 0.184. One important observation in Figs. 7(a) and (b) and 8(a) throughput performance with infinite population seems quite inaccurate for $N \leq 40$. In Fig. 8(b), we depict the average RA delay corresponding to μ_N presented in Fig. 8(a). As expected, increasing μ_N reduces the RA delay and a large population size increases it.

According to packet length T, the throughput is depicted for $N = 100$ in Fig. 9, where the optimal backoff rate is 0.005 for $T = 1$. Notice that the bit duration is assumed to be fixed. Therefore, the shorter the packet size T, the smaller the number of bits in a packet. Due to large N, i.e., 100, the throughput can be approximated by $\mu_N \approx \mu N e^{-2T\mu N}$. As the length of packet (the amount of information in a packet) is reduced, the throughput is significantly improved. Therefore, depending on IoT applications, an optimal packet size can be chosen.

Finally, Fig. 10 shows some sample paths of the number of backlogged users, when the system employs a fixed β. Even though a very low rate of packet arrivals is considered, it can be seen that the backlog size eventually grows without a bound as time goes by.

V. Conclusion

This work investigated the throughput and RA delay of unslotted ALOHA systems extensively, from the system with two heterogeneous users to the one with N homogeneous users. Furthermore, we also developed an approximation with the infinite population model and showed instability condition of the unsaturated system.

For the two-user system with the same packet length, if a user increases its backoff rate, he can get more throughput selfishly, while it hurts the other user. On the other hand, if a user increases its packet transmission time to deliver more information upon success, it will hurt itself more than the other user. Therefore, the users cannot be motivated to use a packet transmission time longer than the others. Although the users may have some incentive of attempting to increase its backoff rate, it could be guessed that their collective behavior eventually shall hurt themselves. This advised to use equal backoff rate.

For the system with N homogeneous users who use an equal backoff rate, it was shown that throughput can be higher than $0.5e^{-1}$ (infinite population case) if the number of (backlogged) users is less than around 25. For the throughput maximization, the optimal backoff rate was numerically found as a function of population size N, i.e., the number of backlogged users. Infinite population model is inaccurate when the number of backlogged users N is less than 40. However, the finite-user system became asymptotically an infinite-user system otherwise.

In this work, for the unsaturated systems discussed in this article, we have considered users who can hold only one packet. As future work, we are interested in the stability region of unslotted ALOHA systems, where users have an infinite queue for incoming packets.

Appendix A
Proof of Lemma 1

The denominator $\xi_i + \Pr[\mathcal{C}_i]$ in (2) means the probability that user i (re)transmits a packet, regardless of whether it is success or collision; thus, $1 - (\xi_i + \Pr[\mathcal{C}_i])$ is the probability that user i does not transmit a packet.

In order to find ξ_i in (2), let us denote by $f_{\tau_i}(x)$ the PDF of user i's backoff time τ_i, i.e., $f_{\tau_i}(x) = \beta_i e^{-\beta_i x}$. Suppose that user 1's packet is (re)transmitted at time τ_1 after two users draw their backoff times at the beginning of the system. Its (re)transmission would not be interfered if user 2 does not (re)transmit during $\tau_1 + T_1$. Thus, ξ_1 is expressed as

$$\begin{aligned}\xi_1 &= \Pr[(\tau_1 < \tau_2) \cap (\tau_2 > \tau_1 + T_1)] \\ &= \Pr[\tau_2 > \tau_1 + T_1 | \tau_2 > \tau_1] \Pr[\tau_2 > \tau_1]. \end{aligned} \tag{42}$$

Each term in the second line of (42) is explained as follows. The first term is the probability that user 2's (re)transmission takes place after user 1's packet transmission time is over, given that user 1 (re)transmits earlier than user 2. Using $f_{\tau_2}(x) = \beta_2 e^{-\beta_2 x}$, we have

$$\Pr[\tau_2 > \tau_1 + T_1 | \tau_2 > \tau_1] = e^{-\beta_2 T_1}. \tag{43}$$

The second term of (42) indicates the conditioning probability that user 1 (re)transmits earlier than user 2; that is, user 1's backoff time τ_1 is shorter than user 2's. It can be expressed as

$$\begin{aligned}\Pr[\tau_2 > \tau_1] &= \int_0^\infty \Pr[\tau_2 > x | \tau_1 = x] f_{\tau_1}(x) dx \\ &= \frac{\beta_1}{\beta_1 + \beta_2}. \end{aligned} \tag{44}$$

Accordingly, from (43) and (44), ξ_1 is obtained as

$$\xi_1 = \frac{\beta_1}{\beta_1 + \beta_2} e^{-\beta_2 T_1}. \tag{45}$$

Similarly, ξ_2 denotes the probability that user 2 (re)transmits its packet successfully without user 1's interference. As done in (42), it can be expressed as

$$\begin{aligned}\xi_2 &= \Pr[(\tau_2 < \tau_1) \cap (\tau_1 > \tau_2 + T_2)] \\ &= \Pr[\tau_2 > \tau_1 + T_1 | \tau_2 > \tau_1] \Pr[\tau_2 > \tau_1] = \frac{\beta_2}{\beta_1 + \beta_2} e^{-\beta_1 T_2}. \end{aligned} \tag{46}$$

Let us derive the collision probability $\Pr[\mathcal{C}_i]$ in (2). Referring to Fig. 1, it can be observed that user i goes through two types of collisions. The first type is that user 1's packet (re)transmission gets collided due to user 2's interference. Let $C_{1,1}$ and $C_{1,2}$ be the length of a collision period that user 1's transmission gets collided by user 2. But $C_{1,1}$ is used when user 2's transmission is partially overlapped with user 1's transmission so that the collision period is longer than T_1, while $C_{1,2}$ implies that user 2's transmission is completely overlapped with user 1's transmission. For the events $\mathcal{E}_1$ and $\mathcal{E}_4$ in Fig. 1, user 1's packet transmission gets collided by user 2's transmission, each of which lasts for $C_{1,1}$ and $C_{1,2}$ (s), respectively. This is called type-1 collision. The second type of collisions is called type-2 collision, which occurs when user 2 gets interfered by user 1's transmission. For instance, the event $\mathcal{E}_5$ in Fig. 1 describes that user 1's transmission interferes user 2's packet transmission. The length of type-2 collision periods is denoted by C_2 (s). Let ω_i be the probability of type-i collision, respectively. We then write $\Pr[\mathcal{C}_i]$ as

$$\Pr[\mathcal{C}_i] = \omega_1 + \omega_2. \tag{47}$$

Notice that $\Pr[\mathcal{C}_1] = \Pr[\mathcal{C}_2]$, since ω_i and $\omega_{\bar{i}}$ are the probability that user i gets collided and that user i gives rise to a collision to user $\bar{i}$, respectively.

In order to find the expression of ω_1 in (47), it is important to notice that provided that user 1 (re)transmits before user 2 does, user 1's (re)transmission turns out to be a failure if user 2's (re)transmission takes place in the middle of user 1's transmission. Thus, ω_1 can be expressed as

$$\begin{aligned}\omega_1 &= \Pr[(\tau_1 < \tau_2) \cap (\tau_2 < \tau_1 + T_1)] \\ &= \int_0^\infty \Pr[\tau_1 < \tau_2 < \tau_1 + T_1 | \tau_1 = x] f_{\tau_1}(x) dx \\ &= \int_0^\infty \left(1 - e^{-\beta_2 T_1}\right) e^{-\beta_2 x} \beta_1 e^{-\beta_1 x} dx \\ &= \frac{\beta_1}{\beta_1 + \beta_2} \left(1 - e^{-\beta_2 T_1}\right) \end{aligned} \tag{48}$$

where the following has been used in the second line:

$$\Pr[\tau_1 < \tau_2 < \tau_1 + T_1 | \tau_1 = x] = e^{-\beta_2 x} - e^{-\beta_2 (x+T_1)}.$$

Applying the same argument to user 2's collision due to user 1's later (re)transmission, we get ω_2 as

$$\begin{aligned}\omega_2 &= \Pr[(\tau_2 < \tau_1) \cap (\tau_1 < \tau_2 + T_2)] \\ &= \frac{\beta_2}{\beta_1 + \beta_2}\left(1 - e^{-\beta_1 T_2}\right).\end{aligned} \tag{49}$$

Using (45)–(49), we can get (2).

Appendix B
Proof of Theorem 1

Observing the sample path of unslotted ALOHA systems in Fig. 1, we can see that the channel alternates its state between idleness and busyness. Let us consider $\mathbb{E}[\mathcal{I}]$ in (4) first. Idle periods $\mathcal{I}_k$ for $k = 1, 2, \ldots$ are the minimum of two users' backoff times; that is, $\mathcal{I} = \min(\tau_1, \tau_2)$, where τ_i is an exponential random variable with mean $1/\beta_i$. Let $f_{\mathcal{I}}(t)$ be the PDF of $\mathcal{I}$. We can get $f_{\mathcal{I}}(t) = (\beta_1 + \beta_2)e^{-(\beta_1+\beta_2)t}$, i.e., exponential distribution with mean $[1/(\beta_1 + \beta_2)]$. Therefore, $\mathbb{E}[\mathcal{I}]$ is given by

$$\mathbb{E}[\mathcal{I}] = \frac{1}{\beta_1 + \beta_2}. \tag{50}$$

Notice that we are assuming the system that user 1's packet transmission time T_1 is longer than or equal to that of user 2, i.e., T_2. Thus, the expression of $\mathbb{E}[\mathcal{B}]$ that we shall derive is valid for $T_1 \geq T_2$. If one is interested in the system for $T_1 < T_2$, one may switch all the parameters of user 1 with those user 2 in the results.

Let us recall that the type-1 collision in Fig. 1 is the case that user 1 gets collided, while the type-2 collision is the case that user 1 causes a collision. The type-1 collision can be divided into two cases in Fig. 1(a) and (b), whose length is denoted by $C_{1,1}$ and $C_{1,2}$, respectively. In Fig. 1(a), user 1's transmission gets collided so that the resulting collision period $C_{1,1}$ is longer than its packet transmission time T_1

$$C_{1,1} = X_2 + T_2 \tag{51}$$

where X_2 denotes the portion of user 1's packet transmission time that is not corrupted *prior to* user 2's packet transmission. Let $g_{X_i}(x)$ denote the PDF of random variable X_i, i.e., how much the user $\bar{i}$'s packet transmission time is not overlapped by user i's transmission provided that user i's packet transmission occurs in the middle of user $\bar{i}$'s packet transmission $T_{\bar{i}}$. From Fig. 1 and by applying the memoryless property of exponential distributions, we can find $g_{X_i}(x)$ as

$$g_{X_i}(x) = \frac{\beta_i e^{-\beta_i x}}{1 - e^{-\beta_i T_{\bar{i}}}} \tag{52}$$

where the denominator is the probability that user i's transmission happens during user $\bar{i}$'s transmission.

On the other hand, in Fig. 1(b), user 2's packet transmission time is completely overlapped with user 1's packet transmission time, which results in the collision period

$$C_{1,2} = T_1. \tag{53}$$

Then, we can get $\overline{C}_1$ as

$$\begin{aligned}\overline{C}_1 &= \int_0^{T_1} C_1 g_{X_2}(x)dx \\ &= \underbrace{\int_{T_1-T_2}^{T_1} C_{1,1} g_{X_2}(x)dx}_{\widetilde{C}_{1,1}} + \underbrace{\int_0^{T_1-T_2} C_{1,2} g_{X_2}(x)dx}_{\widetilde{C}_{1,2}}.\end{aligned} \tag{54}$$

In the above, $\widetilde{C}_{1,1}$ is obtained as

$$\begin{aligned}\widetilde{C}_{1,1} &= \int_{T_1-T_2}^{T_1} (x + T_2) g_{X_2}(x)dx \\ &= \int_{T_1-T_2}^{T_1} x g_{X_2}(x)dx + T_2 \frac{e^{-\beta_2(T_1-T_2)} - e^{-\beta_2 T_1}}{1 - e^{-\beta_2 T_1}} \\ &= \frac{1}{1 - e^{-\beta_2 T_1}}\Bigg[e^{-\beta_2 T_1}\left((T_1 - T_2)e^{\beta_2 T_2} - T_1\right) \\ &\qquad + \left(\frac{1}{\beta_2} + T_2\right)e^{-\beta_2 T_1}\left(e^{\beta_2 T_2} - 1\right)\Bigg]\end{aligned} \tag{55}$$

where we have used the following:

$$\begin{aligned}\int_a^b x g_{X_i}(x)dx = \frac{1}{1 - e^{-\beta_i T_{\bar{i}}}}\Bigg[a e^{-\beta_i a} - b e^{-\beta_i b} \\ + \frac{1}{\beta_i}\left(e^{-\beta_i a} - e^{-\beta_i b}\right)\Bigg].\end{aligned} \tag{56}$$

In (54), we have

$$\begin{aligned}\widetilde{C}_{1,2} &= \int_0^{T_1-T_2} T_1 g_{X_2}(x)dx \\ &= T_1 \frac{1 - e^{-\beta_2(T_1-T_2)}}{1 - e^{-\beta_2 T_1}}.\end{aligned} \tag{57}$$

Equation (54) can be rearranged as

$$\begin{aligned}\overline{C}_1 &= \widetilde{C}_{1,1} + \widetilde{C}_{1,2} = T_1 \\ &\quad + \frac{1}{1 - e^{-\beta_2 T_1}}\left[\frac{1}{\beta_2}e^{-\beta_2(T_1-T_2)} - \left(\frac{1}{\beta_2} + T_2\right)e^{-\beta_2 T_1}\right].\end{aligned} \tag{58}$$

When type-2 collision occurs in Fig. 1(c), the collision period C_2 can be between T_1 and $T_1 + T_2$. It can be observed that collision period C_2 is the sum of T_1 and x for $0 \leq x \leq T_2$

$$C_2 = X_1 + T_1. \tag{59}$$

We obtain $\overline{C}_2$ with (52) and (59) as

$$\begin{aligned}\overline{C}_2 &= \int_0^{T_2} (x + T_1) g_{X_1}(x)dx \\ &= T_1 + \frac{1}{\beta_1} - \frac{T_2 e^{-\beta_1 T_2}}{1 - e^{-\beta_1 T_2}}\end{aligned} \tag{60}$$

where (56) has been used.

According to Fig. 1, we can write $\mathbb{E}[\mathcal{B}]$ as

$$\mathbb{E}[\mathcal{B}] = \sum_{i=1}^{2} \omega_i \overline{C}_i + \sum_{i=1}^{2} \xi_i T_i \tag{61}$$

where type-i collision occurs with probability ω_i. When user i makes a successful transmission with probability ξ_i, it takes T_i (s). This completes the proof.

APPENDIX C
PROOF OF THEOREM 2

Observing events $\mathcal{E}_2$ and $\mathcal{E}_3$, we can see that an idle period $\mathcal{I}_k$ for $k = 2, 3$ is followed by a packet transmission time T_i, when user i makes a successful transmission with probability ξ_i. We can write $\mathcal{G}_i(s)$ by

$$\mathcal{G}_i(s) = \mathbb{E}\Big[e^{-s(\mathcal{I}+T_i)}\Big] = \frac{\beta_1+\beta_2}{s+\beta_1+\beta_2}e^{-sT_i}. \tag{62}$$

Such an idle period precedes each type-i collision period C_i in Fig. 1. $\mathcal{F}_1(s)$ can be obtained as

$$\begin{aligned}\mathcal{F}_1(s) &= \mathbb{E}\Big[e^{-s(\mathcal{I}+C_1)}\Big] = \mathbb{E}\Big[e^{-s\mathcal{I}}\Big]\mathbb{E}\Big[e^{-sC_1}\Big]\\ &= \frac{\beta_1+\beta_2}{s+\beta_1+\beta_2}\int_0^{T_1} e^{-sC_1} g_{X_2}(x)dx \end{aligned}\tag{63}$$

where we calculate

$$\begin{aligned}\int_0^{T_1} e^{-sC_1} g_{X_2}(x)dx &= \int_0^{T_1} e^{-sC_1}\frac{\beta_2 e^{-\beta_2 x}}{1-e^{-\beta_2 T_1}}dx\\ &= \frac{\beta_2}{1-e^{-\beta_2 T_1}}\Bigg[\int_0^{T_1-T_2} e^{-sT_1}e^{-\beta_2 x}dx\\ &\qquad + \int_{T_1-T_2}^{T_1} e^{-s(x+T_2)}e^{-\beta_2 x}dx\Bigg]\\ &= \frac{\beta_2}{1-e^{-\beta_2 T_1}}\\ &\quad\times\Bigg[\frac{e^{-sT_1}}{\beta_2}\Big(1-e^{-\beta_2(T_1-T_2)}\Big)\\ &\qquad + \frac{e^{-sT_2}e^{-(s+\beta_2)T_1}}{s+\beta_2}\Big(e^{(s+\beta_2)T_2}-1\Big)\Bigg]. \end{aligned}\tag{64}$$

We also get $\mathcal{F}_2(s)$ as

$$\begin{aligned}\mathcal{F}_2(s) &= \mathbb{E}\Big[e^{-s(\mathcal{I}+C_2)}\Big]\\ &= \mathbb{E}\Big[e^{-s\mathcal{I}}\Big]\cdot\int_0^{T_2} e^{-s(x+T_1)}g_{X_1}(x)dx\\ &= \frac{\beta_1+\beta_2}{s+\beta_1+\beta_2}\cdot\frac{e^{-sT_1}\beta_1\big(1-e^{-(s+\beta_1)T_2}\big)}{\big(1-e^{-\beta_1 T_2}\big)(s+\beta_1)}. \end{aligned}\tag{65}$$

Let $\mathcal{G}_i^{(k)}(s)$ denote the LST of the PDF of the time when user i's successful (re)transmission occurs at the kth event. The $\mathcal{G}_1^{(k)}(s)$ can be expressed as

$$\begin{aligned}\mathcal{G}_1^{(k)}(s) &= \mathcal{G}_1(s)\xi_1\sum_{k_1=0}^{k-1}\sum_{k_2=0}^{k-1-k_1}\frac{(k-1)!}{k_1!k_2!(k-1-k_1-k_2)!}\\ &\qquad(\mathcal{G}_2(s)\xi_2)^{k_1}(\mathcal{F}_1(s)\omega_1)^{k_2}(\mathcal{F}_2(s)\omega_2)^{k-1-k_1-k_2}\\ &= \mathcal{G}_1(s)\xi_1\left[\mathcal{G}_2(s)\xi_2+\sum_{i=1}^{2}\mathcal{F}_i(s)\omega_i\right]^{k-1} \end{aligned}\tag{66}$$

where the multinomial theorem has been used; that is, $(\sum_{i=1}^n x_i)^k = \sum_{k_1+\cdots+k_n=n}[n!/(k_1!\cdots k_n!)]x_1^{k_1}\cdots x_n^{k_n}$. It can read that among $k-1$ events, there are k_1 successful transmissions of user 2's packet, k_2 collisions of type-1, and $k-1-k_1-k_2$ collisions of type-2, whereas the kth event is user 1's successful transmission with probability ξ_1. We write $H_1(s)$ as

$$\begin{aligned}H_1(s) &= \sum_{k=1}^{\infty}\mathcal{G}_1^{(k)}(s)\\ &= \mathcal{G}_1(s)\xi_1\sum_{k=1}^{\infty}\left[\mathcal{G}_2(s)\xi_2+\sum_{i=1}^{2}\mathcal{F}_i(s)\omega_i\right]^{k-1}\\ &= \frac{\mathcal{G}_1(s)\xi_1}{1-\Big(\mathcal{G}_2(s)\xi_2+\sum_{i=1}^2\mathcal{F}_i(s)\omega_i\Big)}. \end{aligned}\tag{67}$$

Similarly, we can also find $H_2(s)$ by switching $\mathcal{G}_1(s)\xi_1$ with $\mathcal{G}_2(s)\xi_2$ in (67). This completes the proof.

APPENDIX D
PROOF OF LEMMA 3

In characterizing the length of busy periods, let us define φ_{N-j} as a time interval from the beginning of the jth packet transmission to that of the next packet, i.e., $(j+1)$th packet in $\mathcal{B}_k$. For example, φ_{N-2} denotes the time interval from the beginning of the second packet to that of the third one in Fig. 2. Notice that φ_{N-j} is the part of the jth packet, which is not corrupted by the $(j+1)$th *colliding* packet. In addition, $\mathcal{C}_{N+1-j}$ denotes a time interval from the beginning of the jth transmission to the end of busy period $\mathcal{B}_k$. As shown in Fig. 2, if $j=1$, $\mathcal{C}_N$ is the beginning of the first packet transmission to the end of the busy period, i.e., the entire busy period. Referring to Fig. 2, we can find that $\mathbb{E}[\mathcal{C}_{N+1-j}]$ for $j\in\{1,2,\ldots,N-1\}$ satisfies the following recursion:

$$\mathbb{E}\big[\mathcal{C}_{N+1-j}\big] = Tq_j + \big(\mathbb{E}[\varphi_{N-j}]+\mathbb{E}\big[\mathcal{C}_{N-j}\big]\big)\big(1-q_j\big) \tag{68}$$

where q_j in Lemma 2 is the probability that no users begin to (re)transmit during the jth packet transmission time in a busy period; that is, the jth packet is the last one in the busy period. In particular, we have $\mathbb{E}[\mathcal{C}_1] = T$, which corresponds to the fourth packet transmission time in Fig. 2. Notice in (68) that in $\mathbb{E}[\mathcal{C}_N]$ it takes one packet transmission time with probability q_1, whereas $\mathbb{E}[\varphi_{N-1}]$ and $\mathbb{E}[\mathcal{C}_{N-1}]$ subsequently follow if any one (re)transmits during the first packet transmission time with probability $1-q_1$.

Now, we need to find $\mathbb{E}[\varphi_{N-j}]$ for $j\in\{1,\ldots,N-1\}$ so that (68) can be recursively calculated. Since one user has (re)transmitted in Fig. 2, φ_{N-1} is in fact the minimum of backoff intervals made by $N-1$ users, given that the minimum backoff interval occurs during the previous packet transmission time T (s). Let t_i' in Fig. 2 denote the remaining time of backoff interval of user i, once t_3 passes; that is, how long user i's backoff interval will take after t_3. Due to the memoryless property of exponentially distributed τ_i, t_i' follows the same distribution of τ_i. Let $\tau_{(r)}^{(N-j)}$ be the rth backoff time among t_1', t_{13}', and t_5' in Fig. 2, when $N-j$ users shall (re)transmit. Because t_i' follows the same distribution of τ_i, the CDF of $\tau_{(r)}^{(N-j)}$ follows (21). Then, $\mathbb{E}[\varphi_{N-1}]$ is expressed as

$$\mathbb{E}\big[\varphi_{N-1}\big] = \mathbb{E}\Big[\tau_{(1)}^{(N-1)}\Big|\tau_{(1)}^{(N-1)}\le T\Big]. \tag{69}$$

It can be seen that provided that the minimum backoff interval among $N-1$ users, i.e., $\tau_{(1)}^{(N-1)}$, falls into the ongoing packet transmission time, φ_{N-1} is equal to t_1.

As before, it can be observed that φ_{N-2} is the minimum of backoff intervals made by $N-2$ users (since two users have already (re)transmitted), given that it takes place during the second packet transmission time T (s). After t_1 passes, let t''_{13} and t''_5 indicate the remaining times of t'_{13} and t'_5, respectively. They follow the same exponential distribution of t'_{13} and t'_5, too. By the same token for (69), $\mathbb{E}[\varphi_{N-j}]$ can be expressed as

$$\mathbb{E}\left[\varphi_{N-j}\right] = \mathbb{E}\left[\tau_{(1)}^{(N-j)}\middle|\tau_{(1)}^{(N-j)} \le T\right]. \tag{70}$$

Using (21) and (22), we evaluate (70) as

$$\begin{aligned}\mathbb{E}\left[\varphi_{N-j}\right] &= \frac{\int_0^T x\cdot f_{(1)}(x; N-j)dx}{F_{(1)}(T; N-j)}\\ &= \frac{1}{1-e^{-(N-j)\beta T}}\int_0^T x\cdot (N-j)\beta e^{-(N-j)\beta x}dx\\ &= \frac{1-\left[1+(N-j)\beta T\right]e^{-(N-j)\beta T}}{\left[1-e^{-(N-j)\beta T}\right](N-j)\beta}.\end{aligned} \tag{71}$$

Finally, notice that $\mathbb{E}[\mathcal{B}] = \mathbb{E}[\mathcal{C}_N]$. Using (68) and (70), we get (26). This completes the proof.

Appendix E
Proof of Corollary 2

Let X_t be the number of backlogged users at time epoch upon a successful packet transmission. In addition, B_t and $A_{\Delta t}$ denote the number of backlogged users who make successful transmissions, and the number of new packet arrivals during Δt (s). At time $t+\Delta t$, the evolution of X_t can be expressed as

$$X_{t+\Delta t} = X_t - B_t + A_{\Delta t} \tag{72}$$

which can be rewritten as

$$\frac{X_{t+\Delta t}-X_t}{\Delta t} = -\frac{B_t}{\Delta t} + \frac{A_{\Delta t}}{\Delta t}. \tag{73}$$

As $\Delta t \to 0$ and $t \to \infty$, i.e., being in the steady state, it can be expected that the rate of successfully transmitted packets, i.e., throughput μ_N, is equal to that of arriving packets, i.e., $[(X_{t+\Delta t}-X_t)/(\Delta t)] \to 0$. Since there are always N backlogged users in the saturated system, we can divide it by the throughput to find the average delay. This completes the proof.

Appendix F
Proof of Theorem 5

Recall the definitions of X_t, B_t, and $A_{\Delta t}$ given in Appendix V. Let us introduce the definition of stability by the Foster–Lyapunov theorem [24].

Definition 1: Suppose an irreducible, aperiodic, continuous-time Markov process $X_t \in \mathbb{Z}^+$. Define a drift $\mathcal{D}_m$ for each state m as

$$\mathcal{D}_m = \mathbb{E}\left[X_{t+\Delta t} - X_t|X_t = m\right] < \infty. \tag{74}$$

The system is said to be stable (or the Markov process of X_t is ergodic) if there exists a state (or integer) m^* such that we have $\mathcal{D}_m < 0$ for $m > m^*$.

Plugging (72) into (74), we get

$$\begin{aligned}\mathbb{E}\left[X_{t+\Delta t} - X_t|X_t = m\right] &= \mathbb{E}[-B_t + A_{\Delta t}|X_t = m]\\ &= -\mathbb{E}[B_t|X_t = m]\\ &\quad + \mathbb{E}[A_{\Delta t}|X_t = m] < 0.\end{aligned} \tag{75}$$

In fact, $\mathbb{E}[B_t|X_t = m]$ is the average number of successful transmissions given m backlogged users

$$\mathbb{E}[B_t|X_t = m] = \xi. \tag{76}$$

Moreover, $\mathbb{E}[A_{\Delta t}|X_t = m]$ is the mean packet arrivals to the system during Δt

$$\mathbb{E}[A_{\Delta t}|X_t = m] = \lambda\mathbb{E}[\mathcal{Z}] \tag{77}$$

where λ (packets/T) is the average packet arrival rate to the system. Plugging (76) and (77) into (74), for the system to be stable, we should have

$$\lambda < \frac{\xi}{\mathbb{E}[\mathcal{Z}]} = \mu_m. \tag{78}$$

Theorem 3 can be used for μ_m. As $m \to \infty$, we can see $\mu_m \to 0$. Since all the states of this Markov process communicate, once the process hits a high state, it would never come back to a lower state. Then, $X_{t+\Delta t}$ does not have a steady-state probability distribution. This completes the proof.

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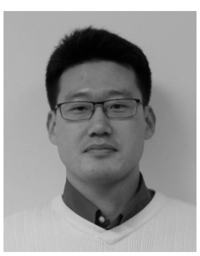

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