Packet Combining Systems Based on the Viterbi Decoder

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Abstract— Type-I hybrid-ARQ protocols can be used to construct powerful adaptive rate algorithms through the use of packet combining techniques. In this paper several packet combining schemes are presented for use in conjunction with the Viterbi decoder over stationary and time-varying channels. The first technique presented is an averaged diversity combiner, which is shown to be identical in performance to an interleaved code combiner over an AWGN channel. The averaged diversity combiner is then generalized to make use of packet weights based on either ideal channel state information or weights derived from side information generated by the Viterbi decoder. It is shown that the weighted diversity combiner using decoder side information performs almost as well as the system using ideal channel state information. All of the packet combining schemes discussed in this paper provide improved throughput and reliability performance relative to that provided by the standard type-I hybrid-ARQ or FEC systems. This performance improvement is obtained at the expense of negligible to moderate modifications to the transmitter and receiver. Performance bounds are derived for each of the combining schemes and their tightness verified through simulation results.

I. Introduction

Many communication channels exhibit widely varying noise and attenuation levels. These variations may be due to changes in the weather, the number of active users on the channel, or any of a number of other channel parameters. In all cases, however, the error control system designer is presented with a formidable problem. He or she may proceed under the assumption that the channel will exhibit its worst behavior at all times. The resulting error control system provides excellent reliability performance, but the throughput performance is poor due to the transmission of unnecessary redundancy when the channel conditions are relatively good. The ideal solution is an adaptive system that carefully matches the rate of the error control code to the channel conditions.

The type-II hybrid-ARQ protocol [1, 2, 3, 4, 5] can be viewed as a simple adaptive rate system. As channel conditions vary, the instantaneous code rate of the type-II system varies between a base rate R and a lower rate R/2. If the channel can be modeled as a two-state Markov process (each state corresponding to a stationary channel), then the type-II system provides an excellent error control solution. If, on the other hand, the channel tends to exhibit three or more distinct noise/attenuation levels, or even a wide continuum of such levels, a more complex approach must be taken to optimize throughput and reliability.

In any communication system that uses a feedback channel for retransmission requests, throughput and reliability performance can be improved through the use of packet combining. In packet combining systems the receiver forms increasingly reliable estimates of the transmitted data by combining all received packets prior to decoding, improving the reliability of the decoded data and decreasing the probability of further retransmission requests. Through the combination of an arbitrary number of packets, the error control system is able to effect multiple code rates, providing a higher level of adaptivity to changing channel conditions than the basic type-II hybrid-ARQ protocol. Benelli [6, 7], Bruneel and Moeneclaey [8], Lau and Leung [9], Metzner and Chang [10], Sindhu [11], and Wicker [12] have proposed methods for combining packets using diversity (symbol-by-symbol) combining. Chase [13], Krishna, Morgera, and Odoul [14, 15], Hagenauer [2], Kallel and Haccoun [3, 16, 17], and Wicker [18, 19] have proposed methods for combining packets using code (codeword-by-codeword) combining.

Hagenauer [2] was the first to note that a type-II hybrid-ARQ protocol based on rate-compatible punctured convolutional (RCPC) codes could be improved through the application of the code combining techniques developed by Chase [13]. Kallel and Haccoun [3, 16] have explored this idea in detail by interleaving multiple received convolutionally encoded packets to form a single code word from a convolutional repetition code. In this paper the approach of Kallel and Haccoun is modified and then generalized for use over time-varying channels. The modification consists of the use of the Yamamoto and Itoh algorithm [22] for error detection in the Viterbi decoder as opposed to the use of CRC error detection after Viterbi error correction. The modified Viterbi decoder implements a type-I hybrid-ARQ protocol as defined by Lin, Costello, and Miller [5]. A brief review of the Yamamoto and Itoh algorithm and its performance is provided in Section 2.

In many cases it is possible to use combinatorial techniques in conjunction with the subject code's weight enumerator to obtain exact performance expressions for packet combining systems (for example, this is the case with systems based on MDS codes [18, 19]). In convolutionally encoded systems, however, this approach is impracticable for much the same reason that the performance of convolutionally encoded FEC systems is discussed in terms of upper and lower bounds: the union bound cannot be avoided in the analysis [23] without an excursion into an intractable series of combinatorial exercises. In Section 3 the Kallel-Haccoun performance bounds for packet combining systems [3, 16, 25] are reviewed and extended. This paper makes extensive use of the Kallel-Haccoun bounds, and through simulation results, shows them to be quite tight in the examples treated here.

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In Section 4 several packet combining techniques based on the Viterbi decoder are presented and analyzed. The first system is a simple averaged diversity combiner (ADC) in which the average soft decision value is obtained for all received copies of each bit in the received packet and the resulting combined packet decoded. The ADC is shown here to provide exactly the same performance as the interleaved code combining system of Kallel and Haccoun [16, 17] over an AWGN channel. The ADC system is then generalized for use with time-varying channels through the use of packet weighting schemes. The first scheme uses ideal channel information to create weights in the manner suggested by Chase [13]. The second scheme derives packet weights from side information generated by the Viterbi decoder. It is shown in Section 5 that the latter scheme performs almost as well as the former.

In Section 5 the performance bounds for the various packet combining schemes discussed in this paper are corroborated through the use of simulation results. In all cases it is shown that the use of packet combining substantially improves throughput performance in comparison to the type-I hybrid-ARQ protocol based on the Viterbi decoder.

II. Type-I Hybrid-ARQ Viterbi Decoders

The key to modifying an FEC decoder for use in a type-I hybrid-ARQ protocol is the identification of a source of reliability information within the decoding process [20, 21]. This information is used to estimate the reliability of received packets, indicating whether a retransmission request is in order. The path metrics calculated during Viterbi decoding provide just such a source of information. In the Yamamoto-Itoh algorithm [22], the surviving path and the best non-surviving path are compared at each node at each stage in the decoding process. If the difference in the path metrics of the two paths falls below a threshold u, then the survivor is declared unreliable. If all paths are declared unreliable before decoding is completed, then a retransmission request is generated.

As noted in the introduction, the use of the Yamamoto-Itoh algorithm for error detection in a packet combining system is a departure from the use of CRC error detection as exhibited previously in the literature (for example, see [3, 16]). There are two benefits that arise from the use of the Yamamoto-Itoh algorithm: first, there is no longer any need for the CRC encoder/decoder pair. Though CRC systems are extremely simple to implement, they do introduce significant delay. Second, the Yamamoto-Itoh algorithm allows for the dynamic reallocation of error correction and error detection through control of the parameter u. The CRC approach does, however, offer the advantage of greater flexibility in the initial selection of an error detection system (and therefore its performance) independent of the error correction capacity of the code combining system.

The performance analysis for the Yamamoto-Itoh algorithm using soft decision decoding follows a method developed by Viterbi [23]. It is assumed that binary code symbols are transmitted by binary phase-shift keyed (PSK) modulation over an AWGN channel with one-sided noise spectral density N_o (W/Hz). Each symbol is transmitted with an energy of E (Joules) and the receiver demodulates the received signal using a phase coherent demodulator. The normalized demodulator output y_l is given by $y_l = x_l + n_l$, where $x_l \in \{+1, -1\}$ and n_l is a zero mean Gaussian random variable with variance $N_o/2E$ [1].

The probability of retransmission is upper bounded by [24]

$$P_x = 1 - (1 - P_{x1})^{\frac{N}{n}}, \tag{1}$$

where

$$P_{x1} \leq \sum_{i=d_{free}}^{\infty} a_i P_x(i), \tag{2}$$

$$P_x(i) \leq Q\left[\frac{2i - u}{2\sqrt{i\frac{N_o}{2E}}}\right], \tag{3}$$

a_i is the number of error paths of weight i for the convolutional code used, n is the number of output bits per code trellis branch, N is the length of the transmitted packet, and u is the retransmission threshold.

The decoded bit error rate for the Yamamoto-Itoh algorithm is bounded above by [24]

$$P_b \leq \frac{1}{m} \sum_{i=d_{free}}^{\infty} c_i P_e(i) \tag{4}$$

where

$$P_e(i) = Q\left[\frac{2i + u}{2\sqrt{i\frac{N_o}{2E}}}\right], \tag{5}$$

m is the number of information bits per code trellis branch, and c_i is the total number of nonzero information bits associated with all code words of weight i.

III. Performance Bounds for Packet Combining Systems

One of the principle difficulties encountered in the analysis of code combining schemes is that packets that cause the generation of retransmission requests are "noisier" on the average than those that have not. Kallel and Haccoun have developed a series of expressions that characterize this phenomenon [3, 16, 25, 17].

Let Tr be the expected number of transmission attempts that must be made before a packet is accepted by the receiver (whether correctly or incorrectly). Kallel and Haccoun showed that Tr is bounded above and below by

$$1 + \sum_{L=1}^{\infty} \prod_{j=1}^{L} P(R_j) \leq Tr \leq 1 + \sum_{L=1}^{\infty} P(R_L), \tag{6}$$

where $P(R_L)$ is the probability of generating a retransmission request while decoding the packet formed by combining L received copies of the packet. Bounds on the expected throughput T of the adaptive rate system using an ideal selective repeat protocol are then obtained through the expression $T = R/Tr$, where R is the code rate.

[1]Note that in the derivation of the soft decision performance bounds provided by Yamamoto and Itoh in [22], the demodulator output is normalized such that the random component n_l has unit variance.

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Upper and lower bounds on the decoded bit error rate P_B provided by a packet combining system can be derived in a similar manner [24]:

$$P_B \leq P(B_1) + \sum_{L=1}^{\infty}\left(\prod_{j=1}^{L} P(R_j)\right)[P(B_{L+1}) - P(B_L)] \quad (7)$$

and

$$P_B \geq P(B_1) + \sum_{L=1}^{\infty} P(R_L)[P(B_{L+1}) - P(B_L)], \quad (8)$$

where $P(B_L)$ is the probability of bit error in the decoded data after L received copies are combined and the result accepted by the decoder (i.e. there are no further retransmission requests). In the following analyses the principal objective is the derivation of values for $P(R_L)$ and $P(B_L)$. Once these values are known, the above expressions can be used to bound the overall throughput and reliability performance for the various packet combining Viterbi decoders.

IV. Packet Combining Systems for the Viterbi Decoder

In this section several packet combining techniques are presented and analyzed. These techniques are actually successive generalizations of the same basic idea. The first technique discussed is an averaged diversity combiner (ADC) that combines packets bit-by-bit by averaging their soft-decision values. It is shown that this technique is identical to the interleaved code combining technique presented by Kallel and Haccoun [3, 16] when both techniques are used over an AWGN channel. The ADC is generalized for use with time-varying channels through the addition of packet-by-packet weighting. The first weighting scheme uses ideal channel information in the manner suggested by Chase [13]. The second scheme uses weights derived from side information generated by the Viterbi decoder. It is shown later in this paper that the two weighting schemes provide almost identical results.

The packet combining systems presented here are analyzed for both stationary and time-varying channels. In the stationary and very slowly-varying cases, it is assumed that the channel noise and attenuation level is constant during the transmission of all packets involved in a given packet combining operation. As packet weighting is superfluous in this case, analytical and simulation results are only provided for the ADC. A geosynchronous satellite channel provides a good example of a slowly-varying channel. Weather conditions cause long-term changes in the channel, but do not occur fast enough to cause a significant difference between the packets in a given combining operation. The ADC adapts to long-term changes in the channel by varying the number of packets combined prior to accepting the results of a decoding operation. As will be shown in the examples in Section 5, the ADC thus extends the operating range over which an acceptable level of throughput can be obtained in comparison to the basic type-I hybrid-ARQ protocol.

In the case of the"moderately-varying" channel it is assumed that the channel noise and attenuation level is constant during the transmission of a packet, but may vary from packet to packet. This typically occurs in channels in which the propagation delay is long compared to the transmission time (e.g. some meteor-burst channels). Bursty channels can also be placed under this heading, for a burst may affect one packet during its transmission without affecting the next. In either case retransmitted packets may encounter a substantially different channel than that seen by the intitial transmission. Moderately-varying channels are to be characterized as follows.

Channel conditions are described by the linear SNR with probability density function (pdf) f_{SNR}, the SNR in decibels $\mathrm{SNR}_{\mathrm{dB}}$ with pdf f_{dB}, the variance σ^2 with PDF f_{σ^2}, or the channel bit error rate p with pdf f_p. The effective channel pdf $f_{U,L}(x)$ shall be used to characterize the noise corrupting the symbols in a packet formed through the combination of L received packets. The units of x are indicated by the subscript U, where

$$U \in \{\mathrm{SNR}, \mathrm{dB}, \sigma^2, p\}. \quad (9)$$

$f_{U,L}(x)$ is derived for each combining method presented in this section.

The probability of packet acceptance on a given transmission as a function of the noise on the channel is given as $P_{A,U}(x)$. The probability of decoded bit error as a function of x given that the packet is accepted is similarly given by $P_{B,U}(x)$. $P_{A,U}(x)$ and $P_{B,U}(x)$ are functions of the performance of the Yamamoto-Itoh algorithm.

The probability that a packet is accepted after exactly L copies of the packet have been received is seen to be

$$P(A_L) = \int_{-\infty}^{\infty} f_{U,L}(x) P_{A,U}(x) dx. \quad (10)$$

The probability of decoded bit error given that L copies of the packet have been received and the packet is accepted is

$$P(B_L) = \int_{-\infty}^{\infty} P_{B,U}(x)\mathrm{Prob.}(x| \; L \text{ copies received, packet accepted}) dx \quad (11)$$

where

$$\begin{aligned}\mathrm{Prob.}(x|L \text{ copies received, packet accepted}) &= \frac{f_{U,L}(x)\mathrm{Prob.}(\mathrm{Accepted}|x)}{\int_{-\infty}^{\infty} f_{U,L}(x)P_{A,U}(x)dx} \\ &= \frac{f_{U,L}(x)P_{A,U}(x)}{P(A_L)}. \quad (12)\end{aligned}$$

The probability of bit error in the decoded packet is thus

$$P(B_L) = \frac{1}{P(A_L)} \int_{-\infty}^{\infty} P_{B,U}(x) f_{U,L}(x) P_{A,U}(x) dx. \quad (13)$$

In the following subsections, the effective probability density functions for the averaged diversity combining scheme and the weighted diversity combining schemes are derived. Equations (10) and (13) are then used to calculate $P(R_L) = 1 - P(A_L)$ and $P(B_L)$. System performance bounds are then calculated using the general adaptive rate bounds.

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A. Interleaved Code Combining and Averaged Diversity Combining

Interleaved code combining is, as its name implies, a code combining method where the symbols in the received copies of a packet are interleaved to form a single packet at the receiver. For example suppose that L copies of a packets have been received. Let $V_i = (y_{i1} y_{i2} y_{i3} \cdots y_{iN})$ be the i^{th} received packet, where y_{ij} is the j^{th} bit of the i^{th} packet. The combined packet is

$$Y_I = (y_{11} y_{21} \cdots y_{L1} y_{12} y_{22} \cdots y_{L2} \cdots y_{1k} y_{2k} \cdots y_{LN}),$$

a noise corrupted code word from a convolutional repetition code. If the original encoder has a code rate R and minimum free distance d_{free}, then the combined packet has an effective code rate of R/L and minimum free distance of Ld_{free} [2].

Averaged diversity combining reflects a different approach to packet combining. Suppose that L packets have been received. The L packets are combined into a single packet *of the same length* by averaging the soft decision values of the copies of each code word coordinate. If $V_i = (y_{i1} y_{i2} y_{i3} \cdots y_{iN})$ is the i^{th} received packet, the new packet is $Y_D = (z_1 z_2 \ldots z_N)$, where

$$z_j = \frac{1}{L} \sum_{i=1}^{L} y_{ij}. \tag{14}$$

The decoder itself need not be modified beyond the type-I hybrid-ARQ modification to be able to decode the combined packet, for the combined packet is a noise corrupted code word from the same code used for each of the individual packet transmissions.

Given a received packet corrupted by transmission over an AWGN channel, an interleaved code combining Viterbi decoder and an averaged diversity combining Viterbi decoder always select the same code word. This can be shown through a comparison of the path metrics calculated by the Viterbi decoder for each of the combining methods. Assume that the convolutional code has rate $\frac{m}{n}$ and that the individual noise processes affecting the received symbols are statistically independent. Without code or diversity combining, the partial path metric for the t^{th} trellis branch is

$$M \propto \sum_{j=(t-1)n+1}^{tn} \log p(y_j | x_j), \tag{15}$$

where y_j is the j^{th} received symbol and x_j is the j^{th} transmitted code symbol. Assume that L code words are combined using interleaved code combining. If y_{ij} is the j^{th} received symbol from the i^{th} received packet, then the partial path metric for the interleaved word over the t^{th} transition is

$$M_I \propto \sum_{i=1}^{L} \sum_{j=(t-1)n+1}^{tn} \log p(y_{ij} | x_j). \tag{16}$$

If the channel is AWGN, the inner product provides the proper metric [23], giving the following branch metric for the interleaved code combiner.

$$M_I = \sum_{i=1}^{L} \sum_{j=(t-1)n+1}^{tn} y_{ij} x_j \tag{17}$$

The symbols for the new code word formed by averaged diversity combining are

$$z_j = \frac{1}{L} \sum_{i=1}^{L} y_{ij}. \tag{18}$$

The partial path metric over the t^{th} transition is then

$$M_A \propto \sum_{j=(t-1)n+1}^{tn} \log p(z_j | x_j). \tag{19}$$

Assuming an AWGN channel, it is then clear that

$$\begin{aligned} M_A &= \sum_{j=(t-1)n+1}^{tn} z_j x_j = \sum_{j=(t-1)n+1}^{tn} \frac{1}{L} \sum_{i=1}^{L} y_{ij} x_j \\ &= \frac{1}{L} \sum_{i=1}^{L} \sum_{j=(t-1)n+1}^{tn} y_{ij} x_j = \frac{1}{L} M_I \end{aligned} \tag{20}$$

Since the branch metrics for the interleaved code combiner and the averaged diversity combiner always differ by a fixed factor L, the decisions made at the nodes of the trellis diagram are identical. The Viterbi decoder chooses the same code word regardless of which combining method is used.

The ADC and the interleaved code combiner thus provide the same performance over an AWGN channel. It is important to note, however, that these two techniques reflect two different approaches to packet combining. In general, code combining systems offer better throughput and reliability performance than diversity combining systems (as noted by Chase [13]) while diversity combining systems are easier to implement. In this particular instance the two approaches converge.

The ADC system is implemented here using a Viterbi decoder that incorporates the Yamamoto-Itoh algorithm. Equations (1) and (4) provide the probability P_b of bit error in the decoded packets and the probability P_x that retransmission request is generated. The variance of a symbol resulting from the combination of L symbols by the averaged diversity combiner is

$$\sigma^2(L) = \frac{N_o}{2EL} = \frac{\sigma^2}{L}, \tag{21}$$

where σ^2 is the variance of the noise process corrupting the individual demodulated signals before combining. The probability of bit error for a packet that has been accepted after L copies have been combined is then

$$P(B_L) \leq \frac{1}{m} \sum_{i=d_{free}}^{\infty} c_i Q \left[\frac{2i + u}{2\sqrt{i \frac{\sigma^2}{L}}} \right]. \tag{22}$$

The probability of a retransmission request after L packet have been combined is

$$P(R_L) = P_x(L) = 1 - [1 - P_{x1}(L)]^{\frac{N}{n}}, \tag{23}$$

where

$$P_{x1}(L) \leq \sum_{i=d_{free}}^{\infty} a_i Q \left[\frac{2i - u}{2\sqrt{i \frac{\sigma^2}{L}}} \right]. \tag{24}$$

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Upper bounds on the expected number of transmissions and the probability of decoded bit error for the averaged diversity combiner system can now be calculated using the bounds for a general adaptive rate system in equations (6) and (7).

To derive performance bounds for the ADC system over a non-stationary channel it is necessary to find the effective pdf $f_{X,L}$ for the symbols in the combined packet, the probability of packet acceptance $P_{A,X}(x)$, and the probability of decoded bit error $P_{B,X}(x)$ as a function of the channel noise variance $X = \sigma^2$.

Let $X = \{x_1, x_2, x_3, \ldots, x_N\}$, where $x_i \in \{1, -1\}$, be the packet that the transmitter is attempting to send over the channel. Assume that L packets $Y_1, Y_2, Y_3, \ldots, Y_L$ have been received with corresponding noise variances $\sigma_1^2, \sigma_2^2, \sigma_3^2, \ldots, \sigma_L^2$. Let y_{ij} be the demodulator output resulting from the reception of the j^{th} bit of the i^{th} received packet. The combined packet $Z = \{z_1, z_2, z_3, \ldots, z_N\}$ is formed using

$$z_j = \frac{1}{L}\sum_{i=1}^{L} y_{ij}. \tag{25}$$

The expected value of z_j is $E\{z_j|x_j\} = x_j$ and the variance of z_j is

$$V\{z_j\} = \frac{1}{L^2}\sum_{i=1}^{L}\sigma_i^2. \tag{26}$$

Let $f_{\sigma^2}(x)$ be the pdf of the noise variance of the channel. The pdf of the variance of the combined packet $V\{z\}$ is

$$f_{\sigma^2,L}(x) = f\mathrm{sum}_{,L}(xL^2), \tag{27}$$

where

$$f\mathrm{sum}_{,L}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi^L(w)e^{-jwx}dw \tag{28}$$

is the pdf of the sum of the L variances and

$$\Phi(w) = \int_{-\infty}^{\infty} f_{\sigma^2}(x)e^{jwx}dx \tag{29}$$

is the characteristic function of the channel noise variance.

The functions $P_{A,\sigma^2}(x)$ and $P_{B,\sigma^2}(x)$, where x is in units of noise variance, are also needed to derive the performance bounds. Using the bounds on the performance of the Yamamoto and Itoh algorithm, these functions are found to be

$$P_{B,\sigma^2}(x) \leq \frac{1}{m}\sum_{i=d_{free}}^{\infty} c_i Q\left[\frac{2i+u}{2\sqrt{ix}}\right] \tag{30}$$

$$P_{A,\sigma^2}(x) = [1 - P_{x1}(x)]^{\frac{N}{n}}, \tag{31}$$

where

$$P_{x1}(x) \leq \sum_{i=d_{free}}^{\infty} a_i\, Q\left[\frac{2i-u}{2\sqrt{ix}}\right], \tag{32}$$

and the coefficients a_i and c_i are derived from the generating function of the convolutional code.

Upper bounds on the expected number of transmissions and the probability of decoded bit error are now derived using equations (10), (13), and the general adaptive rate bounds.

B. Reliability Weighting Using Perfect Side Information

Suppose that perfect channel information is available during the transmission of each packet. For a binary symmetric channel (BSC) the desired information is the channel bit error rate p. For an additive white Gaussian noise (AWGN) channel the desired information is the signal-to-noise ratio SNR, the signal-to-noise ratio in dB $\mathrm{SNR_{dB}}$, or the channel noise variance σ^2. Using this information, weighting factors can be derived for the maximum likelihood decoding of multiple packets [13].

For an AWGN channel the weighting factors depend on the variance of the noise during the transmission of each packet. Assume that L copies of a packet of length N have been received. Chase showed that

$$\log p(Y|X_m) \propto \sum_{i=1}^{L}\frac{1}{\sigma_i^2}\sum_{j=1}^{N} y_{ij}x_{m_j}, \tag{33}$$

where $x_{m_j} = \pm 1$, $y_{ij} = a_j + n_{ij}$, $a_j = \pm 1$ is the transmitted bit, and n_{ij} is a Gaussian random variable with variance σ_i^2 [13].

To obtain the weights for a diversity combining system one need only interchange the summations in the expression above and include the factor $\frac{1}{L}$.

$$\log p(Y|X_m) \propto \sum_{j=1}^{N} x_{m_j}\left[\frac{1}{L}\sum_{i=1}^{L}\frac{1}{\sigma_i^2}y_{ij}\right] \tag{34}$$

It should be noted that even if hard decision reception is employed on each packet, a soft decision Viterbi decoder must be used to to implement weighted diversity combining. The performance analysis is thus pursued only for the soft decision AWGN channel case[2].

The performance analysis for the weighted diversity combiner is very similar to the analysis of the averaged diversity combiner. The combined packet $Z = \{z_1, z_2, z_3, \ldots, z_N\}$ is formed using the following expression:

$$z_j = \left[\sum_{i=1}^{L}\frac{1}{\sigma_i^2}\right]^{-1}\sum_{i=1}^{L}\frac{1}{\sigma_i^2}y_{ij}. \tag{35}$$

The conditional mean of z_j is $E\{z_j|x_j\} = x_j$ where $x_j = \pm 1$ is the transmitted symbol. The variance of the combined symbols is

$$V\{z_j\} = \sigma_Z^2 = \left[\sum_{i=1}^{L}\frac{1}{\sigma_i^2}\right]^{-1}. \tag{36}$$

By letting $\mathrm{SNR}_i = 1/(2\sigma_i^2)$ be the signal-to-noise ratio associated with the i^{th} copy of the packet, the effective signal-to-noise ratio of the combined packet SNR_Z becomes

$$\mathrm{SNR}_Z = \sum_{i=1}^{L}\mathrm{SNR}_i. \tag{37}$$

Let $f_{\mathrm{SNR}}(x)$ be the pdf of the signal-to-noise ratio associated with a single received packet. The pdf of the effective signal-to-noise of a packet formed through the combination of L packets

[2]The performance of the BSC case with weighted diversity combining can be estimated from the AWGN channel with weighted diversity combining by applying a 2 dB [31] or a 3 dB [4] loss in SNR.

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is then

$$f_{\mathrm{SNR_z,L}}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi^L(w)e^{-jwx}dw, \tag{38}$$

where

$$\Phi(w) = \int_{-\infty}^{\infty} f_{\mathrm{SNR}}(x)e^{jwx}dx \tag{39}$$

is the characteristic function of the signal-to-noise ratio of the channel.

The probability of packet acceptance after L copies of the packet have been received $P_{\mathrm{A,SNR}}(x)$ and the probability of bit error $P_{\mathrm{B,SNR}}(x)$ are easily found from the Yamamoto-Itoh performance bounds. These are

$$P_{\mathrm{B,SNR}}(x) \leq \frac{1}{m}\sum_{i=d_{free}}^{\infty} c_i Q\left[\frac{2i+u}{2\sqrt{i\frac{1}{2x}}}\right] \tag{40}$$

$$P_{\mathrm{A,SNR}}(x) = [1 - P_{x1}(x)]^{\frac{N}{n}}, \tag{41}$$

where

$$P_{x1}(x) \leq \sum_{i=d_{free}}^{\infty} a_i Q\left[\frac{2i-u}{2\sqrt{i\frac{1}{2x}}}\right], \tag{42}$$

and the coefficients a_i and c_i are derived from the generating function of the selected convolutional code.

Upper bounds on the expected number of transmissions and the probability of decoded bit error are now derived using equations (10), (13), and the general adaptive rate bounds.

C. Reliability Weighting Using Decoder Side Information

In the previous section the channel condition during the transmission of each packet was provided by a "perfect" outside source. In this section a method of estimating channel conditions using information generated by a soft decision Viterbi decoder is proposed and analyzed.

Assume that the i^{th} received packet $Y_i = \{y_{i1}, y_{i2}, \ldots, y_{iN}\}$ was transmitted with a signal-to-noise ratio SNR_i. This is equivalent to a noise variance of $\sigma_i^2 = 1/(2\mathrm{SNR}_i)$. Assume also that the packet was decoded correctly by the Viterbi decoder (no combining at this point). The probability that this assumption is correct is $1 - P_e$, where P_e is the probability of decoder error. For a rate m/n convolutional code, each transition of the Viterbi trellis corresponds to n coded bits and m information bits. The partial path metric of the correct path over the k^{th} transition of the Viterbi trellis is

$$M_k = \sum_{j=(k-1)n+1}^{kn} x_j \cdot y_{ij} \tag{43}$$

where $X = \{x_1, x_2, \ldots, x_N\}$ is the transmitted packet, $x_i \in \{+1, -1\}$, $y_{ij} = x_j + n_{ij}$, and n_{ij} is a zero-mean Gaussian random variable with variance σ_i^2. M_k is thus the sum of n Gaussian random variables with unit mean and variance σ_i^2 and thus has mean n and variance $n\sigma_i^2$.

An estimate of the noise variance on the channel during the transmission of the packet X_i is found by summing the values $(M_k - n)^2$ for $k = 1, 2, \ldots, N/n$ and dividing the result by N. To see this, consider the function

$$\Omega = \frac{\sum_{k=1}^{N/n}(M_k - n)^2}{n\sigma_i^2}. \tag{44}$$

The random variable Ω has a chi-square (χ^2) distribution with $\frac{N}{n}$ degrees of freedom [32]. Ω thus has a mean of $\frac{N}{n}$ and a variance of $2\frac{N}{n}$. An estimate for σ_i^2 is obtained by multiplying Ω by the quantity $\sigma_i^2\frac{n}{N}$ to get

$$\Lambda_i = \sigma_i^2\frac{n}{N}\Omega = \frac{1}{N}\sum_{k=1}^{N/n}(M_k - n)^2. \tag{45}$$

The mean of estimate Λ_i is σ_i^2 and the variance is $2\frac{n}{N}\left(\sigma_i^2\right)^2$. If a decoding error does occur, only a relatively small number of trellis branches are likely to be affected and the estimate will remain good.

The estimate of the channel noise variance formed in this manner is extremely good for moderately large packets ($N/n \gg 10$). The central limit theorem indicates that the distribution of the estimate Λ_i is essentially Gaussian. In addition, for channels with $\mathrm{SNR_{dB}} >$ -3 dB (most practical channels), the channel noise variance σ_i^2 is always less than one. Therefore

$$\left(\sigma_i^2\right)^2 < \sigma_i^2,$$

and hence

$$Var(\Lambda_i) = \frac{2n}{N}\left(\sigma_i^2\right)^2 \ll \sigma_i^2.$$

The variance of the estimate is much less than the variance of the received symbols within the packet.

The quality of the estimate of the variance is quantified using two approximations developed by Papoulis [33, page 154]. Papoulis showed that if the function of two random variables $g(x, y)$ is sufficiently smooth near the point (η_x, η_y), where η_x and η_y are the means of x and y respectively, then the mean η_g and variance σ_g^2 of $g(x, y)$ can be approximated using the following expressions:

$$\eta_g \simeq g + \frac{1}{2}\left(\frac{\partial^2 g}{\partial x^2}\sigma_x^2 + 2\frac{\partial^2 g}{\partial x \partial y} r\sigma_x\sigma_y + \frac{\partial^2 g}{\partial y^2}\sigma_y^2\right) \tag{46}$$

$$\sigma_g^2 \simeq \left(\frac{\partial g}{\partial x}\right)^2\sigma_x^2 + 2\left(\frac{\partial g}{\partial x}\right)\left(\frac{\partial g}{\partial y}\right) r\sigma_x\sigma_y + \left(\frac{\partial g}{\partial y}\right)^2\sigma_y^2, \tag{47}$$

where σ_x^2 and σ_y^2 are the variances of x and y respectively, r is the correlation coefficient between x and y, and the function $g(x, y)$ and its derivatives are evaluated at $x = \eta_x$ and $y = \eta_y$.

When the combined packet is formed using perfect channel information and weighted diversity combining the following expression is used:

$$z_j = \left[\sum_{i=1}^{L}\frac{1}{\sigma_i^2}\right]^{-1}\sum_{i=1}^{L}\frac{1}{\sigma_i^2}y_{ij}. \tag{48}$$

The summands $\frac{1}{\sigma_i^2}y_{ij}$ are Gaussian random variables with mean x_j/σ_i^2 and variance $1/\sigma_i^2$. When the estimate Λ_i of the channel

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variance is used instead of the actual variance the combining formula becomes

$$z_i = \left[\sum_{i=1}^{L} \frac{1}{\Lambda_i}\right]^{-1} \sum_{i=1}^{L} \frac{1}{\Lambda_i} y_{ij}. \tag{49}$$

The mean of z_j is still x_j, but the relative weighting of the packets is affected slightly. Note that the summands $\frac{1}{\Lambda_i} y_{ij}$ are now functions $g(y_{ij}, \Lambda_i)$ of two random variables. The estimate Λ_i is actually a function of y_{ij}, but for a moderately long packet the correlation coefficient r is approximately zero. Using Papoulis' formulae the mean and variance of the summands become

$$\eta_g \simeq \frac{x_j}{\sigma_i^2}\left(1 + \frac{2n}{N}\right), \tag{50}$$

$$\sigma_g^2 \simeq \frac{1}{\sigma_i^2}\left(1 + \frac{2n}{N\sigma_i^2}\right). \tag{51}$$

For example, for a rate 1/2 code ($n = 2$) with packet length $N = 1000$, the error $\epsilon = \frac{n}{N\sigma_j^2}$ in the mean and variance of the summands is less than 0.02 (2%) if $\sigma_j^2 > 0.1$ ($\mathrm{SNR_{dB}} < 7$ dB). A channel with $\mathrm{SNR_{dB}} > 7$ dB allows for reliable decoding without combining and thus does not present a problem. The estimate of the channel variance is thus very good and is considered nearly optimal for low SNR's where combining generates the most benefit.

To obtain an estimate of the channel condition, an attempt must be made to decode the current copy of the packet before combining it with the previously received copies of the packet. If the current copy of the packet is decoded reliably then the decoded packet is accepted and no combining is used. The performance of the weighted diversity combiner using side information thus differs slightly from the ideal weighted diversity combining scheme.

The expected number of transmissions of a packet for the weighted diversity combining system using decoder side information is

$$\begin{aligned} Tr = 1 &+ P(R_1') + P(R_1', R_2') + \cdots \\ &+ P(R_1', R_2', \ldots, R_n') + \cdots, \end{aligned} \tag{52}$$

where R_n' is the event that a retransmission is requested by the decoder when decoding either the n^{th} copy of the packet or the n combined copies of the packet. Therefore $P(R_n') = P(R_1, R_n)$ and $P(R_1') = P(R_1)$. It is easily shown that

$$P(R_1', R_2', \ldots, R_n') \geq P(R_1')P(R_2') \cdots P(R_n'). \tag{53}$$

By noting that

$$\begin{aligned} P(R_j') = P(R_1, R_j) &= P(R_1)P(R_j|R_1) \\ &\geq P(R_1)P(R_j), \end{aligned} \tag{54}$$

it is seen that

$$P(R_1', R_2', \ldots, R_n') \geq [P(R_1)]^{n-1} \prod_{i=1}^{n} P(R_i). \tag{55}$$

A lower bound on Tr is thus provided by

$$Tr \geq 1 + \sum_{n=1}^{\infty} [P(R_1)]^{n-1} \prod_{i=1}^{n} P(R_i). \tag{56}$$

An upper bound on $P(R_1', R_2', \ldots, R_n')$ is found in the following manner:

$$\begin{aligned} P(R_1', \ldots, R_n') &= P(R_n')P(R_1', \ldots, R_{n-1}'|R_n') \\ &\leq P(R_n') \\ &= P(R_1, R_n) \\ &= P(R_n)P(R_1|R_n) \\ &\leq P(R_n). \end{aligned} \tag{57}$$

Thus the expected number of retransmissions is upper bounded by

$$Tr \leq 1 + \sum_{n=1}^{\infty} P(R_n). \tag{58}$$

The probability of decoded bit error is also slightly affected by the implementation of the system using decoder side information. After each copy of a packet is received there is a possibility that the packet is decoded using only the most recently received copy of the packet. The probability of decoded bit error given the packet is decoded after the n^{th} copy has been received is

$$\begin{aligned} P(B_n') &= \frac{P(B_1)P(A_1) + P(B_n)P(A_n)}{P(A_1) + P(A_n)} \\ &= \frac{P(B_1)[1 - P(R_1)] + P(B_n)[1 - P(R_n)]}{2 - P(R_1) - P(R_n)}, \end{aligned} \tag{59}$$

where $P(B_n)$ is given by equation (13). The bounds on total probability of decoded bit error for the system using decoder side information are [24]

$$\begin{aligned} P_B \leq P(B_1) &+ \sum_{i=1}^{\infty} [P(B_{i+1}) - P(B_i)] \\ &\cdot P^{i-1}(R_1) \prod_{j=1}^{i} P(R_j) \end{aligned} \tag{60}$$

and

$$P_B \geq P(B_1) + \sum_{i=1}^{\infty} P(R_i)[P(B_{i+1}) - P(B_i)]. \tag{61}$$

The weighted combining system using decoder side information has a lower expected number of retransmissions (higher throughput) and greater probability of bit error than the weighted combining system using perfect channel information. This difference is due to the initial decoding attempt on each received packet copy in the former system. The impact on throughput is shown in the next section to be very small.

V. Analytical and Simulation Results

To determine the tightness of the bounds derived in the previous section, simulation results have been compiled for the two combining systems. To simplify the comparison of the results, the same (2,1,3) convolutional code (rate 1/2 , memory length 3) is used in each of the examples. The packet length is fixed at 1,000 coded bits for all systems considered. The generating function for the (2,1,3) code is

$$T(X, Y, Z) = \frac{X^6Y^2Z^5 + X^7YZ^4 - X8Y^2Z^5}{\begin{pmatrix} 1 - XY(Z + Z^2) - X^2Y^2(Z^4 - Z^3) \\ -X^3YZ^3 - X^4Y^2(Z^3 - Z^4) \end{pmatrix}}. \tag{62}$$

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The parameters of the code needed to calculate the bounds derived in the previous two sections are:

- $d_{free} = 6$,
- $a(i : i = d_{free}, d_{free}+1, d_{free}+2, \ldots) = \{1, 3, 5, 11, 25, 55, 121, 267, 589, 1299, 2865, 6319, 13937, 30739, 67797, 145649, 298455, 572683, 942999, 1216286, \ldots\}$,
- $c(i : i = d_{free}, d_{free}+1, d_{free}+2, \ldots) = \{2, 7, 18, 49, 130, 333, 836, 2069, 5060, 12255, 29444, 70267, 166726, 393635, 925334, 2100931, 4502972, 8956629, 15139128, 19818994, \ldots\}$.

The code above was selected because it offered a moderate amount of error correction/detection capacity while still being amenable to a thorough analysis without resort to simplifying assumptions. Several other codes were simulated as well as the (2,1,3) code, and all showed the same basic tendencies discussed in the following pages.

The results provided in the following discussion result from the simulated transmission of 10,000 packets of 1,000 bits each for the stationary or slowly varying channel cases, and 1,000 packets of 1,000 bits each for the moderately-varying channel case. The software implementation of the Viterbi decoder completed the decoding of an entire packet before releasing decoded bits, so there were no truncation effects. Quantization effects were also minimized through the use of eight bit vectors to represent the soft-decision values of the received bits.

A. Averaged Diversity Combining Over Stationary and Slowly-Varying Channels

It is assumed that averaged diversity combining is incorporated into a soft-decision Viterbi decoder implementing the Yamamoto-Itoh algorithm. The following computed and simulated results are for the AWGN channel case.

Lower bounds for the throughput of the averaged diversity combining system are plotted in Fig. 1. The simulation results and the lower bounds for the throughput of the Yamamoto-Itoh algorithm without combining are also plotted for comparison. Note that the systems with combining provide significant throughput far below the signal-to-noise ratios at which the throughput for the systems without combining becomes negligible.

Fig. 2 shows the upper bounds on the decoded BER for systems using the Yamamoto-Itoh hybrid-ARQ algorithm with and without averaged diversity combining. The bounds on the decoded BER for systems using the combining algorithm are plotted only for SNR's where a good approximation of the value of the probability of retransmission is available. From the throughput results for the Yamamoto-Itoh algorithm without combining in Fig. 1, it is found that the upper bounds on the probability of retransmission approximate the actual probability of retransmission for SNR's greater than or equal to 0.5 dB, 1.5 dB, and 2.5 dB for thresholds of 1, 3, and 5 respectively. Note that the bounds on the decoded BER for the averaged diversity combining system are always less than or equal to the decoded BER bounds of the system without combining. The increased throughput performance resulting from the averaged diversity combining does not increase the decoded BER of the system.

As expected, the addition of averaged diversity combining to a

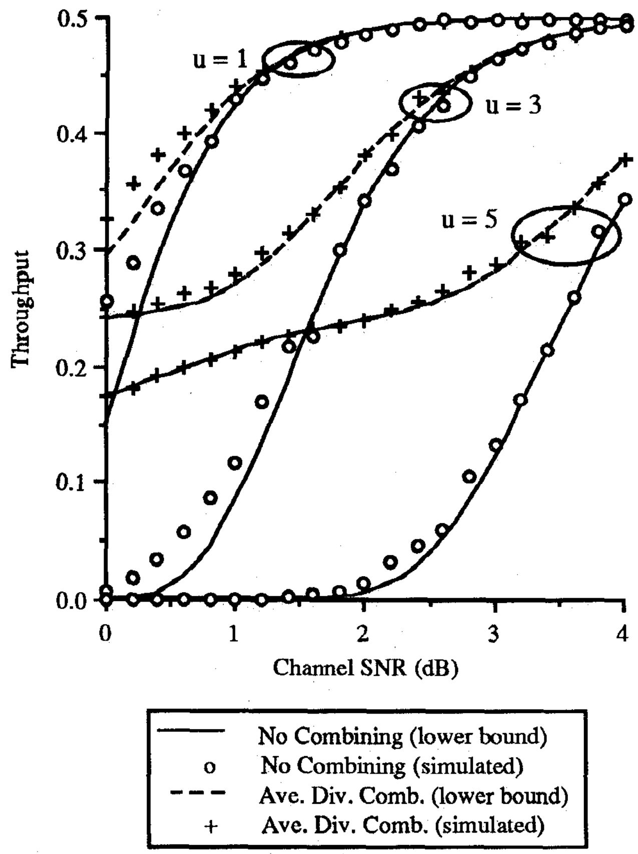

Figure 1: Throughput for Averaged Diversity Combining over a Stationary AWGN Channel

soft decision Viterbi decoder with type-I hybrid-ARQ capability greatly improves the performance of the system. For example, assuming a threshold u of 5, the system employing averaged diversity combing maintains a throughput greater then 0.2 for SNR's greater than or equal to 1 dB. The throughput of the system without code combining has negligible throughput over channels with SNR's less than 2 dB. The upper bounds on the decoded BER in Fig. 2 also indicate that the decoded BER for systems employing code combining is much lower than systems without code combining for low channel SNR's.

B. Moderately-Varying Channels

Four time-varying channel models were used to establish the performance characteristics of the combining algorithms defined in Section 4. Each of the channel models are described in terms of the pdf of the channel's SNR in dB. The pdf of the first channel is *uniformly* distributed between 1 and 7 dB. The pdf of the second channel is *Gaussian* with a mean of 3 dB and a variance of 2 dB. An offset *Rayleigh* distribution is used for the third channel. The offset Rayleigh distribution is a simple Rayleigh distribution with $\sigma^2 = 2$ dB that is offset such that the minimum is 1 dB (instead of 0 dB). The pdf of the fourth channel is bimodal with equal impulses at 0 and 4 dB. Sketches of each of these channel models

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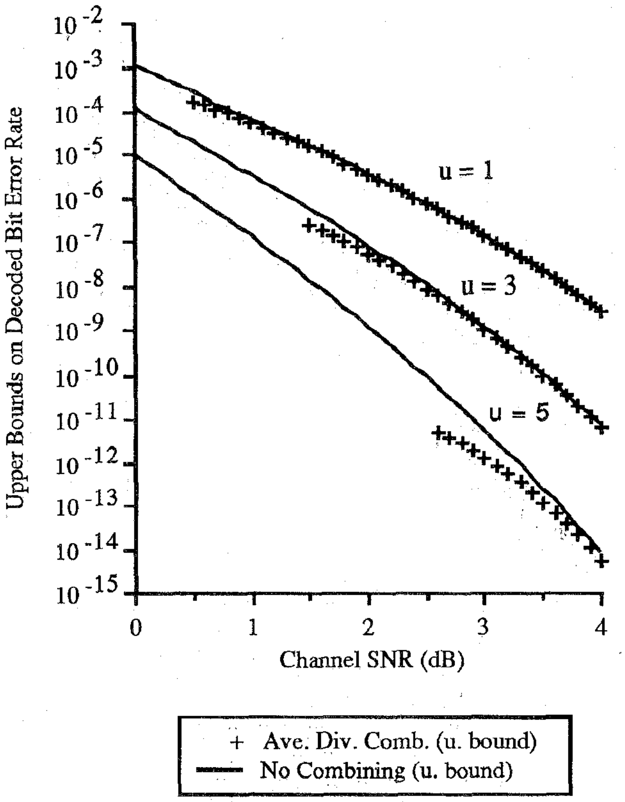

Figure 2: Upper Bounds on Decoded BER for Averaged Diversity Combining over a Stationary AWGN Channel

are provided in Fig. 3. The performance bound computations for the systems using averaged diversity combining and weighted diversity combining require that the pdf of the channel models be in terms of the channel noise variance and the linear SNR of the channel, respectively. The pdf's for the channel's linear SNR and noise variance are derived and used in the computation of the performance bounds.

The four channel models were selected so as to be tractable analytically, but highly disparate in character. For example, the Gaussian distributed channel is compact and has a very definite mean whose approximate value is frequently assumed by the channel. The bimodal channel, on the other hand, is highly spread out and has a mean that is never assumed by the channel. This distinction has an interesting impact on some of the results that follow.

The performance bounds are computed in the following manner. The pdf of the effective noise variance or the linear SNR for L combined packets is calculated using the inverse Fourier transform of the L^{th} power of the Fourier transform of the single packet pdf. The probability of packet acceptance given L combined packets is then found by numerical integration of equation (10). The probability of decoded bit error for L combined packets is found by numerical integration on equation (13). The

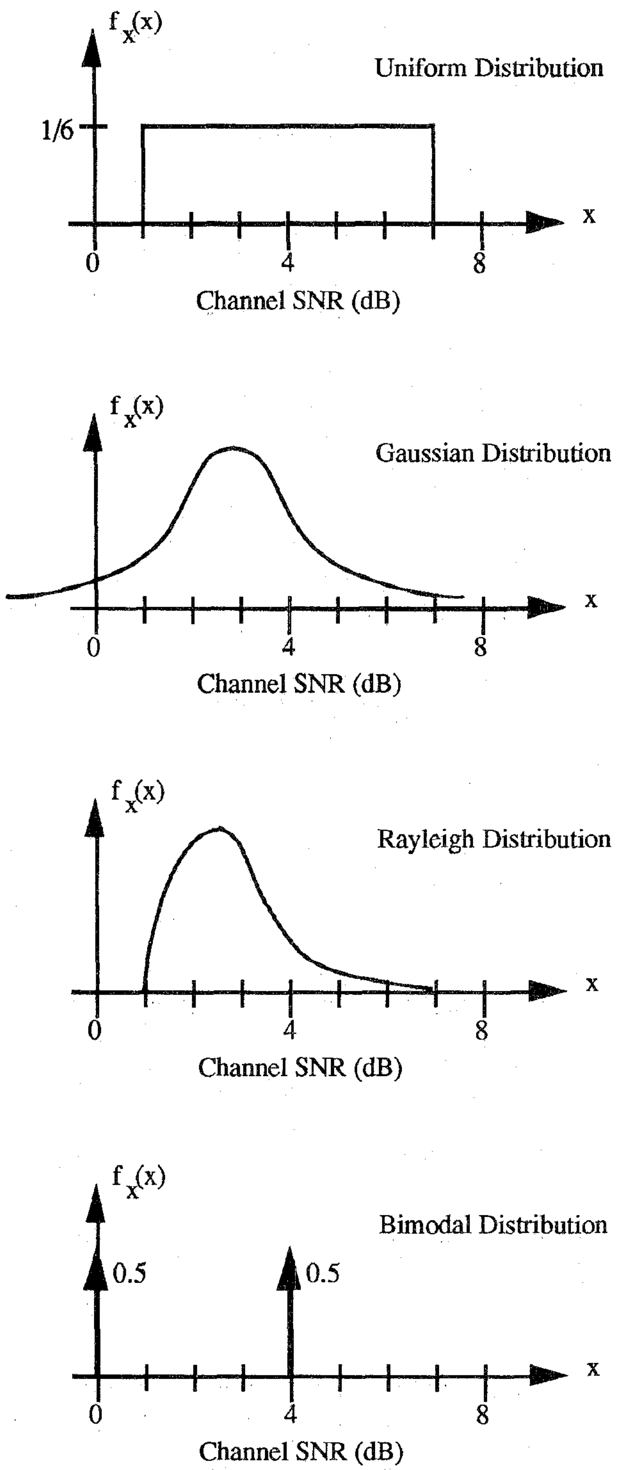

Figure 3: Distributions of Non-Stationary Channels

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number of points used in each numerical integration and the maximum frequency of the characteristic function used in the calculation of the effective PDFs were selected through trial and error to give a 3 decimal place accuracy in the expected throughput and a 2 digit accuracy in the decoded BER.

Simulations were used to verify the throughput bounds calculated for the Yamamoto and Itoh system without combining, with averaged diversity combining, with weighted diversity combining with perfect side information, and with weighted diversity combining using decoder side information. Due to the extremely good reliability performance provided by the hybrid-ARQ systems (with and without packet combining), none of the simulations had a sufficient number of decoded bit errors to obtain a value for the decoded BER.

Tables 1 – 8 list the results of the computation of the lower bounds on throughput, simulated throughput, and the computation of upper bounds on decoded BER for each of the channel models and coding systems. The following abbreviations are used in the tables to indicate the type of error control system used:

- YI – Yamamoto-Itoh type-I hybrid-ARQ algorithm without any combining,
- AD – YI with averaged diversity combining,
- WD – YI with weighted diversity combining with perfect side information,
- IWD – YI with imperfect weighted diversity combining using decoder side information.

In Tables 1, 3, 5 and 7 the simulated throughput is listed in parenthesis under the calculated lower bound. For the cases where the lower bound on expected throughput is less than 0.100, the simulations were prohibitively time consuming and were therefore not completed. Sufficient numbers of simulations were, however, completed to verify the bounds derived in the previous sections.

Several interesting results become apparent when viewing the throughput results in Tables 1, 3, 5 and 7. First, averaged diversity combining, as in the case of the stationary and slowly-varying channels, provides large increases in throughput when used over moderately-varying channels. For instance, the throughput for the Yamamoto-Itoh algorithm with threshold $u = 5$ over a channel with Gaussian distributed SNR results in a simulated throughput of 0.180 (see Table 3). The addition of averaged diversity combining improves the system's simulated throughput performance to 0.294. This is an improvement of 68 percent over the system without combining. Therefore the simple averaged diversity combining protocol provides a considerable increase in throughput over hybrid-ARQ systems with relatively low throughput without relying on any channel side information.

Another important result is the relative performance between the averaged diversity combining systems and the systems using weighted diversity combining with perfect side information. Chase [13] determined that weighted diversity combining was optimum when the channel conditions were varying. The analyses and simulations performed here verify Chase's conclusions, but for many cases weighted diversity combining provides small or even negligible increases in throughput over averaged diversity combining. In fact in all the cases where the throughput for the systems using averaged diversity scheme is at least 0.250, the increase in throughput afforded by systems using weighted diversity combining is less than 2 percent.

The reason for the small improvement provided by weighted diversity combining can be seen by considering the effective SNR of the packet formed by combining two copies of a packet transmitted over channels with different SNR's. Assume that one copy is transmitted over a channel with SNR = 0 dB and the second copy over a channel with 4 dB SNR. If the two copies are combined using averaged diversity combining, then the effective SNR of the combined packet is 4.56 dB. The effective SNR of the combination using weighted diversity combining is 5.46 dB. If for example the threshold u is set at 5, then the probability of accepting the packet combined using averaged diversity combining is approximately 0.9 and the probability of accepting the packet combined using weighted diversity combining is approximately 0.98. Both have a very high probability of accepting the two packets combined and, in an adaptive rate system where the acceptance of the first copy is independent of the combining method used, the two combining systems have nearly identical throughput. In Table 7 it can be seen that for the case $u = 5$ the throughput of the weighted diversity combining system has only 1.5 percent higher throughput than the system using averaged diversity combining.

The systems using weighted diversity combining do have a significantly higher throughput than systems using averaged diversity combining when the throughput becomes relatively low. For example, in Table 7 the system with $u = 7$ using weighted diversity combining has a 10 percent higher throughput than the system using averaged diversity combining over a channel with bimodal SNR. For the case of a system with $u = 7$ over a channel with Rayleigh distributed SNR, the throughput of the weighted system is 6 percent higher than the averaged combining system. In each of these cases, the throughput for the averaged diversity combining systems is less than 0.250 and thus there is a significant probability that 3 or more copies of a packet will need to be transmitted before the packet is accepted. The advantage of weighted diversity combining over averaged diversity combining is greater when more copies of each packet are combined.

The throughput of the weighted diversity combining protocol using decoder side information performed nearly the same as the protocol using perfect side information. In all cases the throughput for the systems using decoder side information is within 2.5 percent of the throughput for the systems using perfect side information (the worst case is seen in Table 7 for $u = 5$). The use of estimates for the channel SNR or noise variance derived from decoder side information thus does not appear to appreciably degrade the performance of the weighted combining systems.

The upper bounds on decoded BER for most of the cases listed in Tables 2, 4, 6 and 8 reflect the improvement in decoded BER encountered in ARQ protocols when the number of retransmission requests is reduced while all other parameters remain fixed . The only unexpected results are for the bimodal channel in Table 8. For the systems using averaged diversity combining with $u = 4$, 5 or 6, the upper bound on the decoded BER was higher than the corresponding systems without code combining. At first the results seem incongruous, but upon further reflection they can be explained. For each of these cases the probability

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of accepting the first copy of the packet when transmitted over a channel with 0 dB SNR is very small. But, if 2 copies transmitted over a channel with 0 dB SNR are combined, then there is a significant probability that the combined packet is accepted. The combination of two copies of a packet transmitted over a channel with 0 dB SNR has an effective SNR of 3 dB. Therefore the system using combining has a significant probability of accepting a combined packet with 3 dB effective SNR while the system without combining almost always waits for a packet transmitted over a channel with 4 dB SNR. Accepting a combined packet with an effective SNR of 3 dB results in a higher decoded BER than accepting a packet transmitted over a channel with 4 dB SNR. It is thus possible to have a higher decoded BER for a combining system than for a system without combining. ADC systems are the only examples in Table 8 for which combining systems are outperformed by non-combining systems, but it should be noted that the same situation can arise with weighted diversity combining systems. Note also that these are only bounds and that the actual decoded BER's may not differ as dramatically as indicated by the bounds.

VI. Conclusions

A type-I hybrid-ARQ protocol can provide better reliability performance than an FEC system using the same code. The cost of the increased reliability is a reduction in throughput caused by retransmissions as the communication channel degrades. Packet combining recovers a considerable portion of the lost throughput by varying the effective code rate of the error control system. As each successive copy of a packet is received it is combined with all previously received copies to form a new code word which, in most cases, has a greater effective SNR (equivalent to a lower effective channel BER) than any of the received copies by themselves.

The combining protocols described here are designed such that only a single decoder is needed to decode any received packet or any combination of received copies of the packet. The implementation of any of these protocols requires only minor modifications of the transmitting and receiving systems. The majority logic combining system extends the operating range of hard decision Viterbi decoders to BER's greater than 0.1, while the upper bounds on the decoded BER for the majority logic combining system remain lower than the bounds for the system using hybrid-ARQ alone.

Averaged diversity combining increases the range of stationary and slowly-varying channel SNRs over which significant throughput can be achieved by type-I hybrid-ARQ protocols using soft decision Viterbi decoders. At low channel SNR's the averaged diversity combiner provides significantly better throughput performance than simple hybrid-ARQ systems while providing reliability performance that is at least as good.

Over moderately-varying channels the simple averaged diversity combiner provides a great improvement in throughput without the added complexity of channel state estimation. In several simulated examples the averaged diversity combining improved the throughput by over 50 percent when compared to type-I hybrid-ARQ alone.

Over moderately-varying channels where there is a significant probability that 3 or more copies of a packet will be required before reliable decoding is possible, weighted diversity combining provides improvement in throughput over averaged diversity combining. Optimal weighting factors are derived from ideal channel information. These weights increase the probability of acceptance of the combined copies of a packet over that of a packet formed through averaged diversity combining.

Weighting factors for weighted diversity combining can be derived from the path metrics generated by the Viterbi decoder. The simulation results show that the weighted combining systems using decoder side information have throughputs very near that of systems which have access to perfect channel state information. A weighted diversity combining can thus be implemented in a convolutionally encoded system using a Viterbi decoder with type-I hybrid-ARQ without the additional complexity of channel monitoring hardware.

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Table 1: Throughput Lower Bounds and Simulation Results for Uniformly Distributed Channel SNR

u	Throughput Lower Bounds (simulated)			
	YI	AD	WD	IWD
1	0.493 (0.494)	0.494 (0.494)	0.494 (0.494)	0.494 (0.494)
2	0.477 (0.477)	0.478 (0.478)	0.478 (0.478)	0.478 (0.478)
3	0.435 (0.436)	0.442 (0.441)	0.442 (0.441)	0.442 (0.441)
4	0.365 (0.367)	0.393 (0.394)	0.393 (0.393)	0.393 (0.390)
5	0.278 (0.282)	0.342 (0.342)	0.343 (0.349)	0.343 (0.349)
6	0.174 (0.186)	0.283 (0.291)	0.288 (0.291)	0.288 (0.296)
7	0.023 (*)	0.212 (0.218)	0.223 (0.230)	0.223 (0.223)

* - not simulated.

Table 2: Upper Bounds on Decoded BER for Uniformly Distributed Channel SNR

u	Decoded BER Upper Bounds			
	YI	AD	WD	IWD
1	3.6×10^{-6}	3.5×10^{-6}	3.5×10^{-6}	3.6×10^{-6}
2	5.9×10^{-7}	5.6×10^{-7}	5.6×10^{-7}	5.8×10^{-7}
3	5.4×10^{-8}	4.7×10^{-8}	4.7×10^{-7}	5.3×10^{-8}
4	1.3×10^{-9}	9.4×10^{-10}	9.4×10^{-10}	1.2×10^{-9}
5	3.8×10^{-12}	2.1×10^{-12}	2.1×10^{-12}	3.3×10^{-12}
6	9.9×10^{-16}	3.5×10^{-16}	3.5×10^{-16}	6.9×10^{-16}
7	7.4×10^{-21}	9.4×10^{-22}	9.4×10^{-22}	2.6×10^{-21}

Table 3: Throughput Lower Bounds and Simulation Results for Gaussian Distributed Channel SNR

u	Throughput Lower Bounds (simulated)			
	YI	AD	WD	IWD
1	0.476 (0.483)	0.477 (0.483)	0.477 (0.483)	0.477 (0.483)
2	0.448 (0.454)	0.453 (0.458)	0.453 (0.458)	0.453 (0.458)
3	0.392 (0.400)	0.411 (0.417)	0.411 (0.417)	0.411 (0.419)
4	0.299 (0.313)	0.354 (0.362)	0.355 (0.361)	0.355 (0.361)
5	0.175 (0.180)	0.293 (0.294)	0.296 (0.299)	0.296 (0.301)
6	0.064 (*)	0.235 (0.241)	0.241 (0.246)	0.241 (0.244)
7	0.010 (*)	0.176 (0.181)	0.185 (0.187)	0.185 (0.186)

* - not simulated.

Table 4: Upper Bounds on Decoded BER for Gaussian Distributed Channel SNR

u	Decoded BER Upper Bounds			
	YI	AD	WD	IWD
1	1.8×10^{-5}	1.7×10^{-5}	1.7×10^{-5}	1.8×10^{-5}
2	1.7×10^{-6}	1.6×10^{-6}	1.7×10^{-6}	1.7×10^{-6}
3	8.8×10^{-8}	6.9×10^{-8}	6.9×10^{-8}	8.5×10^{-8}
4	1.7×10^{-9}	1.0×10^{-9}	1.0×10^{-9}	1.5×10^{-9}
5	8.2×10^{-12}	2.9×10^{-12}	2.9×10^{-12}	5.6×10^{-12}
6	4.4×10^{-15}	5.7×10^{-16}	5.7×10^{-16}	1.5×10^{-15}
7	6.7×10^{-20}	1.6×10^{-20}	1.5×10^{-21}	5.0×10^{-21}

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Table 5: Throughput Lower Bounds and Simulation Results for Rayleigh Distributed Channel SNR

u	Throughput Lower Bounds (simulated)			
	YI	AD	WD	IWD
1	0.493 (0.496)	0.493 (0.496)	0.493 (0.496)	0.493 (0.496)
2	0.471 (0.484)	0.472 (0.484)	0.472 (0.484)	0.472 (0.484)
3	0.405 (0.456)	0.420 (0.458)	0.420 (0.461)	0.420 (0.456)
4	0.272 (0.375)	0.343 (0.398)	0.344 (0.400)	0.344 (0.399)
5	0.119 (0.246)	0.278 (0.327)	0.282 (0.332)	0.282 (0.330)
6	0.027 (*)	0.228 (0.273)	0.233 (0.272)	0.233 (0.270)
7	0.002 (*)	0.171 (0.209)	0.176 (0.211)	0.176 (0.215)

* - not simulated.

Table 6: Upper Bounds on Decoded BER for Rayleigh Distributed Channel SNR

u	Decoded BER Upper Bounds			
	YI	AD	WD	IWD
1	3.3×10^{-6}	3.2×10^{-6}	3.2×10^{-6}	3.3×10^{-6}
2	5.3×10^{-7}	5.3×10^{-7}	5.0×10^{-7}	5.3×10^{-7}
3	5.8×10^{-8}	4.7×10^{-8}	4.7×10^{-8}	5.6×10^{-8}
4	2.5×10^{-9}	1.3×10^{-9}	1.3×10^{-9}	2.1×10^{-9}
5	2.0×10^{-11}	4.7×10^{-12}	4.7×10^{-12}	1.0×10^{-11}
6	1.5×10^{-14}	7.9×10^{-16}	7.9×10^{-16}	2.3×10^{-15}
7	3.2×10^{-19}	1.3×10^{-21}	1.3×10^{-21}	4.9×10^{-21}

Table 7: Throughput Lower Bounds and Simulation Results for Bimodal Distributed Channel SNR

u	Throughput Lower Bounds (simulated)			
	YI	AD	WD	IWD
1	0.324 (0.368)	0.270 (0.395)	0.270 (0.395)	0.270 (0.395)
2	0.259 (0.290)	0.252 (0.352)	0.252 (0.352)	0.252 (0.348)
3	0.248 (0.257)	0.247 (0.327)	0.249 (0.327)	0.249 (0.329)
4	0.233 (0.237)	0.236 (0.310)	0.246 (0.309)	0.246 (0.308)
5	0.170 (0.168)	0.204 (0.264)	0.229 (0.268)	0.229 (0.262)
6	0.041 (*)	0.159 (0.200)	0.194 (0.212)	0.194 (0.209)
7	0.000 (*)	0.125 (0.145)	0.154 (0.160)	0.154 (0.162)

* - not simulated.

Table 8: Upper Bounds on Decoded BER for Bimodal Distributed Channel SNR

u	Decoded BER Upper Bounds			
	YI	AD	WD	IWD
1	2.9×10^{-4}	1.9×10^{-4}	1.9×10^{-4}	2.7×10^{-4}
2	1.5×10^{-5}	7.6×10^{-6}	7.6×10^{-6}	1.3×10^{-5}
3	2.6×10^{-8}	1.3×10^{-8}	1.3×10^{-8}	2.2×10^{-8}
4	3.1×10^{-13}	7.9×10^{-12}	1.4×10^{-13}	2.5×10^{-13}
5	8.2×10^{-15}	2.1×10^{-13}	2.8×10^{-15}	5.8×10^{-15}
6	2.1×10^{-16}	4.3×10^{-16}	1.7×10^{-17}	5.1×10^{-17}
7	4.3×10^{-18}	2.2×10^{-21}	1.7×10^{-21}	6.9×10^{-21}

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