Introduction

1. Survival func: $S(t) = 1 - F(t) = P(T > t) = \int_t^\infty f(u)\mathrm{d}u$
   $f(u) = -\frac{\mathrm{d}}{\mathrm{d}u}S(u)$, $S(0) = 1$, $S(\infty) = 0$
2. Empirical survival func: $\hat{S}_n(t) = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} I_{(t,\infty)}(T_i)$, $n\hat{S}_n(t) \sim \mathrm{Bin}(n, S(t))$
3. Hazard func: $h(t) = \dfrac{f(t)}{S(t)}$, $S(t) = \exp\left(-\displaystyle\int_0^t h(u)\mathrm{d}u\right) = \exp(-H(x))$, $E[t] = \displaystyle\int_0^\infty S(t)\mathrm{d}t$

Mortality tables

4. Prob of dying: $q_x = \frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x}$
5. Prob of surviving: $p_x = 1 - q_x = \frac{l_{x+1}}{l_x}$
6. $e_x = \displaystyle\sum_{i=1}^{\infty} iP[\text{death in year } (x+i)] = \dfrac{\sum_{n=x+1}^{\infty} l_n}{l_x}$
7. Force of motality: $\mu_x = -\frac{1}{l}\frac{\mathrm{d}l_x}{\mathrm{d}x} = -\log(p_x)$
8. Mortality rate: $m_x = \dfrac{\text{number of deaths}}{\text{person-time at risk}} = \dfrac{d_x}{\int_x^{x+1} l_t \mathrm{d}t} = \dfrac{d_x}{\int_0^1 l_{x+t}\mathrm{d}t}$
   Central mortality rate: $m_x = \dfrac{d_x}{l_x - \frac{d_x}{2}}$
9. Prob that die within n yrs: ${}_nq_x = \frac{l_x - l_{x+n}}{l_x}$
10. Prob of surviving n yrs: ${}_np_x = \frac{l_{x+n}}{l_n} = 1 - {}_nq_x$
11. Prob that die in age $x + m \sim x + m + n$: ${}_{m|n}q_x = \dfrac{l_{x+m} - l_{x+m+n}}{l_x} = {}_mp_x \cdot {}_nq_{x+m}$
12. ${}_np_x = \exp\left(-\displaystyle\int_0^n \mu_{x+t}\mathrm{d}t\right)$, $l_x = \exp\left(-\displaystyle\int_0^x \mu_t \mathrm{d}t\right)$
13. Linear interpolation (increasing μ, deaths Uniformly distributed)
    ${}_tl_x = l_x + t(l_{x+1} - l_x) = l_x - td_x$, ${}_tq_x = tq_x$, $\mu_{x+t} = \frac{q_x}{1 - tq_x}$
14. Exponential interpolation (constant μ)
    ${}_tl_x = l_x\left(\dfrac{l_{x+1}}{l_x}\right)^t = l_x p_x^t$, $\mu_{x+t} = -\log\left(\dfrac{l_{x+1}}{l_x}\right) = -\log(p_x)$
15. Hyperbolic interpolation (decreasing μ)
    ${}_tl_x = \dfrac{l_x p_x}{p_x + tq_x}$, ${}_tp_x = \dfrac{p_x}{p_x + tq_x}$, $\mu_{x+t} = \dfrac{q_x}{p_x + tq_x} = \dfrac{q_x}{1 - q_x(1-t)}$
16. Select table: $l_{[x+1]} = l_x \cdot p'_x$ (forward, then calc all new ls), $l_{[x]}$(for backward only)

Parametric models and estimation

17. de Moivre hazard: $h(t) = \frac{1}{\omega - t}$, where $l_\omega = 0$ or negligible
    $f(t) = \frac{1}{\omega}$ (deaths Uniformly distributed)

18. Weibull hazard: $h(t) = \gamma t^{(\gamma-1)}; t > 0, \gamma > 0$, $H(t) = t^\gamma$
$\gamma < 1$: decreasing; $\gamma = 1$: constant, $\gamma > 1$: increasing
Generalised with a scale parameter α and a location parameter μ: $h(t) = \dfrac{\gamma}{\alpha}\left(\dfrac{t-\mu}{\alpha}\right)^{\gamma-1}$

19. Gompertz hazard: $h(t) = Bc^t$, logarithm is linear in t
$H(t) = -\left(c^t - 1\right)\ln g$, where $\ln g = \dfrac{-B}{\ln c}$
Gompertz-Makeham hazard: $h(t) = A + Bc^t$ (adult mortality in developed countries)

20. Log-linear models: $\log(T) = \alpha + \sigma\varepsilon$

21. 1

Non-par and dist-free approaches

22. Kaplan-Meier estimator: $\hat{S}(t) = \displaystyle\prod_{\{j:t_j \le t\}} \left(1 - \frac{d_j}{n_j}\right)$
Variance:

23. Nelson-Aalen estimator: $\hat{H}(t) = \displaystyle\sum_{\{j:t_j \le t\}} \left(\frac{d_j}{n_j}\right)$
Breslow estimator: $\hat{S}(t) = \exp\left(-\hat{H}(t)\right)$
(greater than K-M estimator by Taylor series, asymptotically equivalent)

24. Logrank test:

Semi-par surv modelling

25. Cox PH model: $h(t, X) = h_0(t)\psi(X;\beta)$, commonly $\psi(X;\beta) = \exp\left(\beta^T X\right)$ log-linear form
Likelihood $L(\beta) = \displaystyle\prod_{j=1}^{d} \frac{\psi(i_j)}{\sum_{k \in R(\tau_j)} \psi(k)}$, $I(\hat{\beta}) = -\dfrac{\partial^2 \ln L(\hat{\beta})}{\partial \beta^2}$,
95% confidence interval for β: $\hat{\beta} \pm z_{\alpha/2}\sqrt{\dfrac{1}{I(\hat{\beta})}}$