REVISION

2016年12月8日 12:06

1. Replicating portfolio (same payoff)
2. Law of one-price (same payoff -> same price at any previous time)
3. Brownian Motion
   a. $W_0 = 0$
   b. $W_s - W_t \sim N(0,\ s-t)$
   c. For $r < s \le t < u$, $W_u - W_t\ independent\ W_s - W_r$
   d. Fix ω, $t \mapsto W_t(\omega)$ continuous trajectory(function)
   Additionally, fix t, $\omega \mapsto W_t(\omega)$ is a RV
4. Ito isometry:
   $\mathbb{E}\left[\left(\int_0^T g_s dW_s\right)^2\right] = \mathbb{E}\left[\int_0^T |g_s|^2 ds\right]$
5. SDE:
   $dX_t = U_t dt + V_t dW_t,\ X_0 = x_0$
6. Ito formula:
   $Y_t := f(t,\ X_t)$
   $dY_t = \left[\frac{\partial f}{\partial t}(t,\ X_t) + U_t \frac{\partial f}{\partial x}(t,\ X_t) + V_t^2 \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\left(t,\ X_t\right)\right]dt + \left[V_t \frac{\partial f}{\partial x}(t,\ X_t)\right]dW_t$
   Then integrate
7. 用Ito formula proof identity: LHS=積分 RHS=多項式
   Construct f(t,x) = 多項式有W_t嗰項
8. Geometric Brownian Motion:
   a. $dS_t = \mu S_t dt + \sigma S_t dW_t,\ S_0 = S_0$
   b. $f(t,\ x) = ln\, x,\ Y_t := ln\, S_t$
   c. Using Ito formula, compute $dY_t = \left(r - \frac{\sigma^2}{2}\right)dt + \sigma dB_t$
   d. Integration
   e. $S_t = e^{Y_t} = S_0 \cdot exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\}$
   f. This is gBM
9. Z~N(0,1), $\mathbb{E}\left[Z^2\right] = 1$
10. Indicator function:
    $\mathbf{1}_{S_T > K} = \begin{cases} 1,\ S_T > K \\ 0,\ otherwise \end{cases}$
    $W_T \sim N(0,\ T)$
    $\mathbb{E}\left[\mathbf{1}_{S_T > K}\right] = \mathbb{E}\left[\mathbf{1}_{W_t > \frac{ln \frac{K}{S_0} - \left(r - \frac{\sigma^2}{2}\right)T}{\sigma}}\right] = N\left(\frac{ln \frac{K}{S_0} + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) = N\left(d_-\right)$
11. 計exponential嘅expectation要用completing the square
12. Pricing an option: $\Pi(0; option) = e^{-rT}\mathbb{E}\left[\Phi(S_T)\right]$
    Eg: $\Pi(0; Call) = e^{-rT}\mathbb{E}\left[\Phi(S_T)\right] = e^{-rT}\mathbb{E}\left[(S_T - K)\cdot \mathbf{1}_{S_T > K}\right] = S_0 N(d_+) - Ke^{-rT}N(d_-)$
13. $E[XY] = E[X]E[Y] + cov(X,\ Y)$
    $Var(X) = E\left[(X - E(X))^2\right] = E\left[X^2\right] - (E[X])^2$
    $cov(X,\ X) = Var(X)$
14. Put-call parity: $c_0 + Ke^{-rt} = p_0 + S_0$