

## **Cover sheet for Assignment 3**

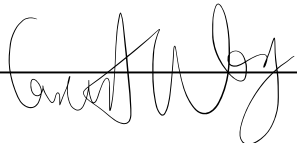
**Due Friday Nov. 6, 11:00pm**

Complete this page and attach it to the front of your assignment.

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(Underline your last name)

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I declare that this assignment is solely my own work, and is in accordance with the University of Toronto Code of Behavior on Academic Matters.

**Signature:** 

# CSC457: Written Assignment 3

Grant Wong

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## Question 1

- (a) When there are  $m$  packets in the buffer then a new packet is dropped and lost with the probability

$$p_m = \frac{1 - \rho}{1 - \rho^{m+1}} \rho^m$$

meaning the probability that a new packet is lost is  $p_m$  and the rate that new packets are dropped is given by  $p_m \lambda$ .

- (b) Throughput is defined by the arrival rate minus the loss rate, that is,

$$\lambda - p_m \lambda = \lambda(1 - p_m)$$

- (c) It is such that  $p_m \ll 1$  if and only if  $\rho^{m+1} \ll 1$  which implies for part (a) the result

$$p_m = (1 - \rho) \rho^m$$

- (d) From Little's formula the expected delay  $T = \frac{N}{\lambda'}$  and with  $\lambda' = (1 - \rho)\lambda$  as well as  $N = \sum_{n=1}^m n p_n$  implies that

$$T = \frac{\sum_{n=1}^m n p_n}{(1 - \rho)\lambda}$$

## Question 2

- (a) Each packet is comprised of  $N$  bits and each bit has a bit transfer error probability of  $P_{\text{bit}}$  so the probability  $P_{\text{packet}}$  that a packet from host A arrives error-free at switch C is

$$P_{\text{packet}} = (1 - P_{\text{bit}})^N$$

- (b) If host A takes  $k$  transmission attempts to have the  $k$ th packet accepted at switch C then the first  $k-1$ th packets arrive with an error and the  $k$ th packet arrives error-free which is expressed as

$$P_k = (1 - P_{\text{packet}})^{k-1} P_{\text{packet}} = \left(1 - (1 - P_{\text{bit}})^N\right)^{k-1} (1 - P_{\text{bit}})^N$$

- (c) Denote  $E[T_{AC}]$  to be the total delay for one hop from host A to switch C and denote for  $E[T_{CB}]$  in an analogous way. Note that from our assumptions that  $E[T_{AC}] = E[T_{CB}]$ . Now, expressing  $E[T_{AC}]$  we have

$$E[T_{AC}] = \sum_{k=1}^{\infty} E[T|k] P_k = \sum_{k=1}^{\infty} k \frac{N}{R} P_k = \sum_{k=1}^{\infty} k \frac{N}{R} (1 - P_{\text{packet}})^{k-1} P_{\text{packet}}$$

then using the infinite series result  $\sum_{i=1}^{\infty} i(1-a)^{i-1} = \frac{1}{a^2}$  for  $|1-a| < 1$

$$= \frac{N}{R} \frac{P_{\text{packet}}}{P_{\text{packet}}^2} = \frac{N}{R} \frac{1}{P_{\text{packet}}} = \frac{N}{R} \frac{1}{(1 - P_{\text{bit}})^N}$$

The total delay  $E[T]$  is given by

$$E[T] = E[T_{AB}] + E[T_{CB}] = 2 \frac{N}{R} \frac{1}{(1 - P_{\text{bit}})^N}$$

- (d) In end-to-end ARQ transmitting a packet from host A to host B error-free has the probability  $P'_{\text{packet}} = (1 - P_{\text{bit}})^N (1 - P_{\text{bit}})^N = (1 - P_{\text{bit}})^{2N}$ . The average delay is then

$$E[T'] = 2 \frac{N}{R} \frac{1}{P'_{\text{packet}}} = 2 \frac{N}{R} \frac{1}{(1 - P_{\text{bit}})^{2N}}$$

- (e) In computing  $E[T']$  we note that  $P'_{\text{packet}} = P_{\text{packet}}^2$ .

	$P_{\text{packet}} = 0.99$	$P_{\text{packet}} = 0.2$	$P_{\text{packet}} = 0.002$
$E[T]$	$2.02N/R$	$10N/R$	$1000N/R$
$E[T']$	$2.04N/R$	$50N/R$	$500000N/R$

- (f) From the result of (e) we can determine which type of ARQ, i.e. hop-by-hop or end-to-end, to use based on the bit error probability  $P_{\text{bit}}$ . If it is high (as is in the case of a wireless transmission) then hop-by-hop significantly reduces the expected delay, however, if it is low (as is in the case of optical transmission) then end-to-end yields only a small increase in expected delay. Meaning, one should use hop-by-hop with a high  $P_{\text{bit}}$  but end-to-end with a low  $P_{\text{bit}}$ .