Practical Exercise 10 – Algorithms Analysis and Big-O Notation

Overall Objective

To estimate algorithm efficiency using the Big-O notation and determine the complexity of various types of algorithms.

Description

1. Why is a constant factor ignored in the Big-O notation? Refer to Ch24 slide 17

The Big O notation estimates the execution time of an algorithm in relation to the input size.

If the time is not related to the input size, the algorithm is said to take *constant time* with the notation O(1).

For example, a method that retrieves an element at a given index in an array takes constant time, because it does not grow as the size of the array increases.

2. Why is a non-dominating term ignored in the Big-O notation? Refer to Ch24 slide 8

Consider the algorithm for finding the maximum number in an array of n elements.

If n is 2, it takes one comparison to find the maximum number.

If n is 3, it takes two comparisons to find the maximum number.

In general, it takes n-1 times of comparisons to find maximum number in a list of n elements.

Algorithm analysis is for large input size. If the input size is small, there is no significance to estimate an algorithm's efficiency.

As n grows larger, the n part in the expression n-1 dominates the complexity.

The Big O notation allows you to ignore the non-dominating part (e.g., -1 in the expression n-l) and highlight the important part (e.g., n in the expression n-l).

So, the complexity of this algorithm is O(n).

3. What is the order of each of the following functions? Refer to Additional slide 21&22

a.
$$\frac{(n^2+1)^2}{n} = \frac{(n^4+2n^2+1)}{n} = n^3+2n+\frac{1}{n} = n^3 = O(n^3)$$

(because n cube grow fastest)

b.
$$\frac{(n^2 + \log_2 n)^2}{n} = \frac{n^4 + 2n\log_2 n + (\log_2 n)^2}{n} = n^3 = O(n^3)$$
 (because n cube grow fastest)

c.
$$n^3 + 100n^2 + n = n^3 = O(n^3)$$
 (because n cube grow fastest)

d.
$$2^{n} + 100n^{2} + 45n = 2^{n} = O(2^{n})$$
 (because 2 power n grow fastest)

e.
$$n2^n + n^2 2^n = n^2 2^n = O(n^2 2^n)$$
 (because n square 2 power n grow fastest)

f.
$$3n^4 + 2n^2 + 10000 = 3n^4 = O(n^4)$$
 (**NOTE:** the 3 need to be omitted)

4. What is the number of iterations in the following loop?

```
int count = 5;

while (count < n) {

   count = count + 3;

}

Formula for loop

= \frac{n - startValue}{step}
= \frac{n - 5}{3}
= n
= O(n)
```

```
int sum = 1 + 2 + 3 ... + 100

Answer = O(1)
Because it is only calculated once.

Always pay attention on the loop. If without loop is always O(1).
```

5. Use the Big-O notation to estimate the time complexity of the following methods:

```
a. public static void method1(int n) {
    for(int i = 0; i < n; i += 2) {
        System.out.print(Math.random() + " ");
    }
}</pre>
```

Refer to Ch24 slide 11 or Additional slide 13, 21&22

Answer

$$= \frac{n - startValue}{step}$$

$$= \frac{n - 0}{2}$$

$$= \frac{n}{2}$$

$$= O(n)$$

Refer to Ch24 slide 13 or Additional slide 19, 20, 21&22

Answer

```
= outerLoop \times innerLoop
= \frac{n-0}{1} \times \frac{n-0}{1} (for inner loop take the worst case, which is n)
= n^2
= O(n^2)
```

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```
c. public static void method3(int[] m) {
    for(int i = 0; i < m.length; i += 2) {
        System.out.print(m[i] + " ");
    }

    for(int i = m.length - 1; i >= 0; i -= 2) {
        System.out.print(m[i] + " ");
    }
}
```

Refer to Ch24 slide 15

Answer

$$=loop_1 + loop_2$$

$$= \frac{n-0}{2} + \frac{n-0}{2}$$

$$= \frac{n}{2} + \frac{n}{2}$$

$$= n$$

$$= O(n)$$

6. Put the following growth functions in order:

$$\frac{5n^3}{4032}$$
, $44\log n$, 500 , $\frac{2^n}{45}$, $10n\log n$, $2n^2$, $3n$

Refer to Ch24 slide 30 or Additional slide 25

Answer:

500 ->
$$44 \log n$$
 -> $3n$ -> $10n \log n$ -> $2n^2$ -> $\frac{5n^3}{4032}$ -> $\frac{2^n}{45}$

- 7. Design/describe an algorithm for the following tasks, and analyse the time complexity of the algorithm.
 - a. Compute the sum of all numbers from n1 to n2 for (n1 < n2).

First method(use loop):

$$sum = 0$$

for(int i = n1; i <= n2; i++) $sum += i$
Complexity = $O(n)$

Second method (use formula):

$$sum(n_1, n_2) = \frac{n_2(n_2 - 1)}{2} - \frac{n_1(n_1 - 1)}{2}$$
Complexity = $O(1)$

b. Find the occurrence of the largest element in an array.

```
largest = xs[0]
for(i=1; i < xs.length; i++)
    if(largest < xs[i])
        largest = xs[i]

occurrence = 0
for(i=0; i < xs.length; i++)
    if(xs[i] == largest)
        occurrence++

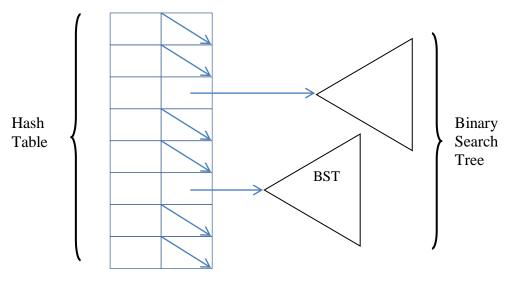
Complexity = n+n=2n=O(n)</pre>
```

c. Remove duplicate element in an array.

Complexity =
$$n \times n = n^2 = O(n^2)$$

8. Give a diagram of conceptual design of a student database of *N* records that supports the following search-time constraints:

Worst case search-time: O(log(N))Best and average search-time: O(1)



Hash table of N entries

9. What is dynamic programming? Give an example. (refer to recursive and non-recursive Fibonacci solutions)

Refer to Ch24 slide 36