

Matthew Wong
CS325
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Assignment 2:

1. This cannot be written as a recursive function. This is because we are already at the base case and there needs to be no more recursive calls.
2. Recurrence Method

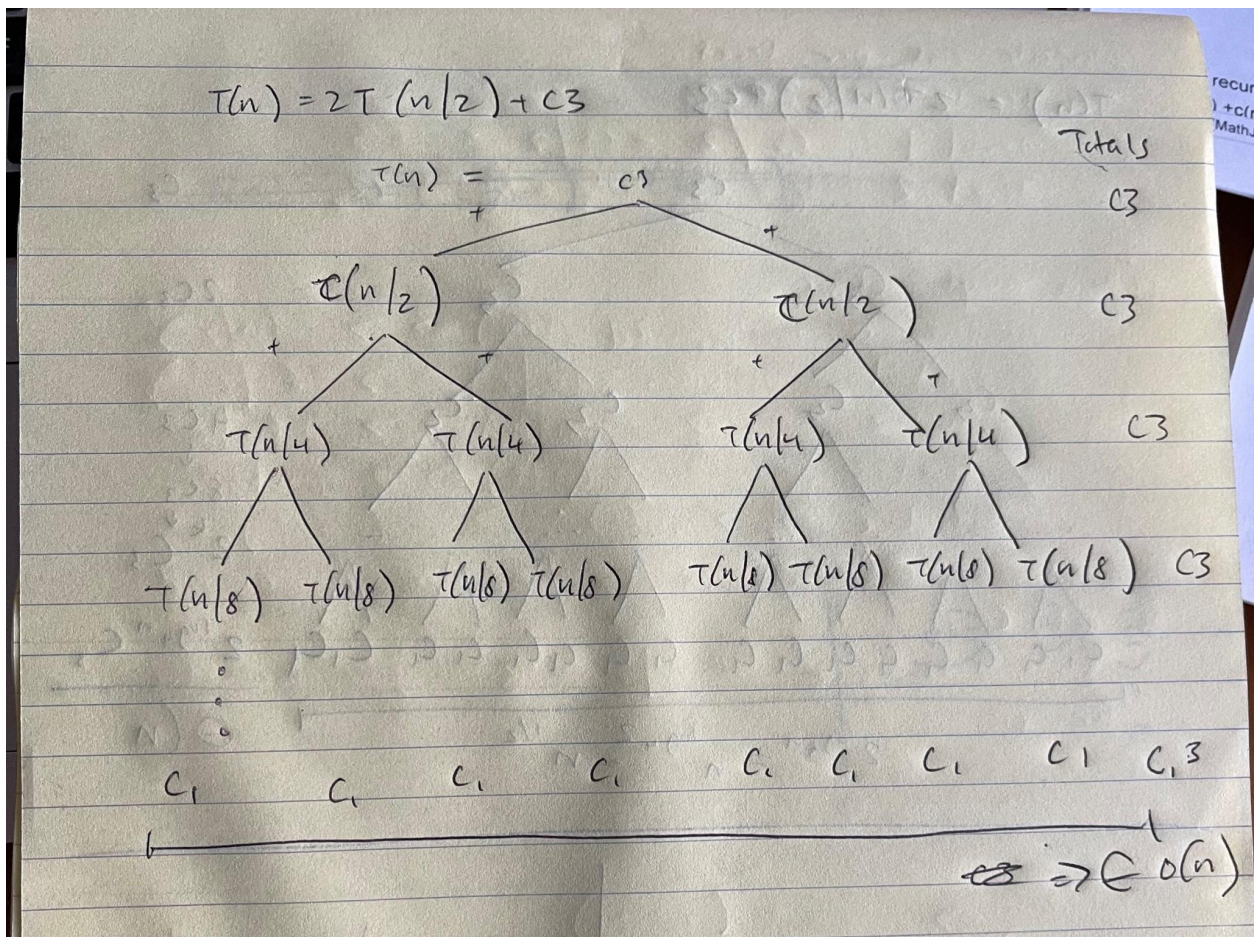
```
power2(x,n):  
    if n==0:  
        return 1  
    if n==1:  
        return x  
    if (n%2)==0:  
        return power2(x, n//2) * power2(x,n//2)  
    else:  
        return power2(x, n//2) * power2(x,n//2) * x
```

a.

i. **Substitution method:**

$T(n) = 2T(n/2) + c_3$
 $\Rightarrow T(n/2) = 2T(n/4) + c_3$ ##all we did was /2
 $\Rightarrow T(n) = 2[2T(n/4) + c_3] + c_3$ ##substitute back in to $t(n)$
 $\Rightarrow 4T(n/4) + 3c_3$ ##simplify
 $\Rightarrow T(n/4) = 2T(n/8) + c_3$ ##we need $n/4$ same as $n/2$
 $\Rightarrow T(n) = 4[2T(n/8) + c_3] + 3c_3$ #substitute into our equation
 $\Rightarrow 8T(n/8) + 7c_3$ #simplified from above
 $\Rightarrow 8[2T(n/16) + c_3] + 7c_3$ #same as above for $n/16$
 $\Rightarrow 16T(n/16) + c_3] + 15c_3$ #simplified out
 $\Rightarrow 32T(n/32) + c_3] + 31c_3 \dots$ #same as above for $n/32$
 $\Rightarrow 2^k T(n/2^k) + (2^k - 1)c_3$ ##our base case
 $\Rightarrow T(0) = c_1$
 $\Rightarrow T(1) = c_2$
 $\Rightarrow T(n) = 2^k T(n/2^k) + (2^k - 1)c_3$
If $n/2^k = 1$
Then $n = 2^k \log n$ which is also $= k$
 $\Rightarrow T(n) = 2^{\log n} T(n/2^{\log n}) + (2^{\log n} - 1)c_3$ #sub $\log n$ as k for base
 $\Rightarrow nT(n/n) + (n-1)c_3$ # $2^{\log n} = n$ then $2^{\log n} - 1 = n - 1$
 $\Rightarrow nT(n/n) + (n-1)c_3$
 $\Rightarrow nc_2 + (n-1)c_3$
 $\in \Theta(n)$
 $\Rightarrow T(n) = \Theta(n)$

ii. Recurrence tree method:



iii. Master method

Case 1: $a = b^k$

$a=3, b=2, k=0$

Time complexity: $\Theta(n^{\log(\text{base } b)a}) = \Theta(n^{\log 2}) = \Theta(n)$

b. $T(n) = 4T(n/2) + n$
 $= 4[4T(n/4) + n/2] + n$
 $= 16T(n/4) + n/2 + n$
 $= 16[4T(n/8) + n/4] + 2n + n$
 $= 64T(n/8) + 4n + 2n + n$
 $= \dots$
 $= 4^{\log n} T(n/2^{\log n}) + \dots + 4n + 2n + n$
 $= n^{\log 4} T(n/n^{\log 2}) + \dots + 4n + 2n + n$
 $= n^2 T(1) + \dots + n(1 + 2 + 3 + \dots)$
 $= O(n^2)$

= time complexity $\Theta(n^2)$

3. A

a. Pseudocode

Declare array = []

#use loop and user input portion grab user input and store into array

Declare temp variable

#Store length of array into temp

#divide portion

Take index of array

All even numbers add up

All odd numbers add up

#combine and conquer

Now add up all and divide by index round up

b. Since divide and conquer always halves it should be: $O(n \log n)$