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CS325  
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Assignment 2:

1. This cannot be written as a recursive function. This is because we are already at the base case and there needs to be no more recursive calls.
2. Recurrence Method

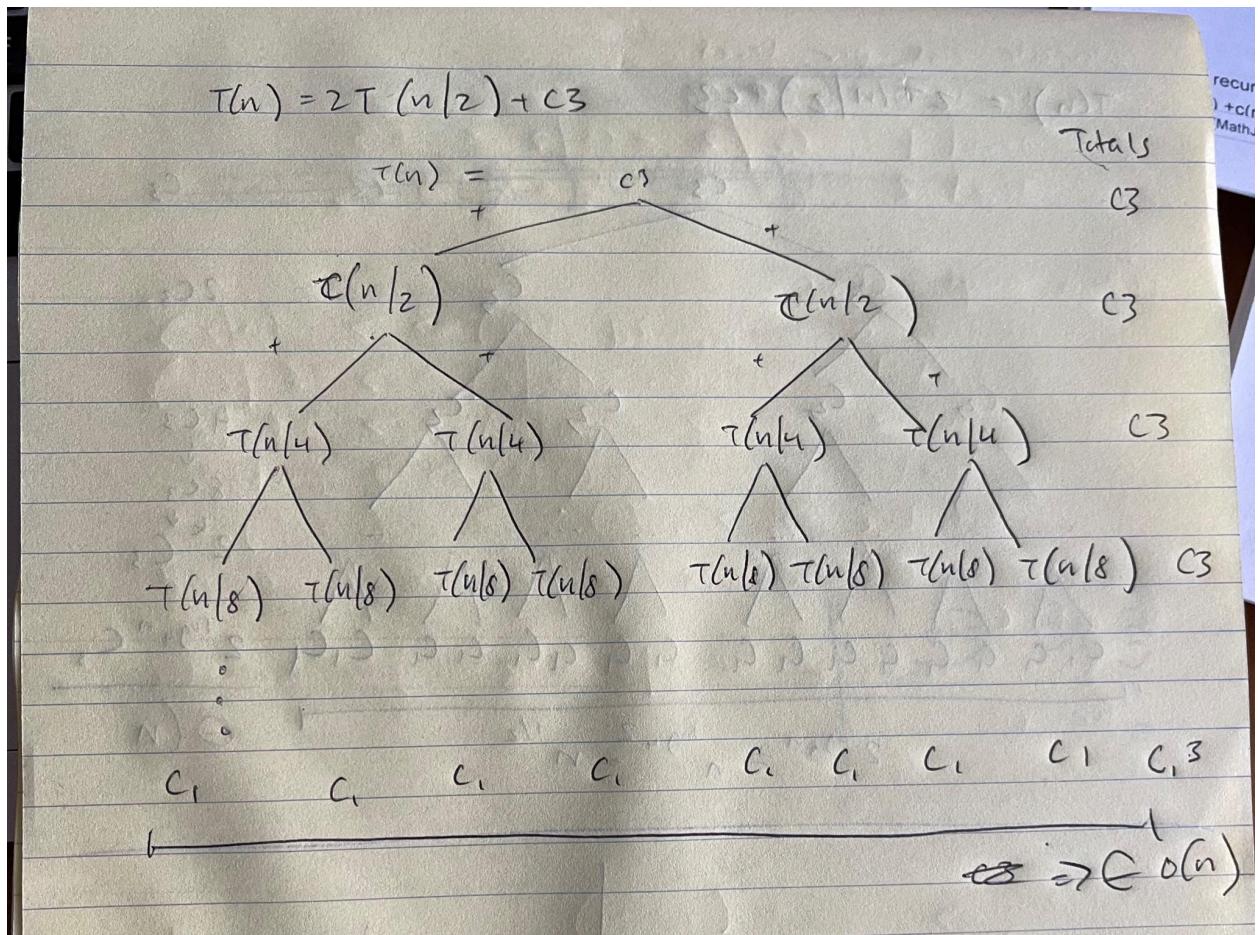
```
power2(x,n):
    if n==0:
        return 1
    if n==1:
        return x
    if (n%2)==0:
        return power2(x, n//2) * power2(x,n//2)
    else:
        return power2(x, n//2) * power2(x,n//2) * x
```

a.

i. **Substitution method:**

$$\begin{aligned} T(n) &= 2T(n/2) + c_3 \\ \Rightarrow T(n/2) &= 2T(n/4) + c_3 && \text{\#all we did was } /2 \\ \Rightarrow T(n) &= 2[2T(n/4) + c_3]c_3 && \text{\#substitute back in to } t(n) \\ \Rightarrow 4T(n/4) &+ 3c_3 && \text{\#simplify} \\ \Rightarrow T(n/4) &= 2T(n/8) + c_3 && \text{\#we need } n/4 \text{ same as } n/2 \\ \Rightarrow T(n) &= 4[2T(n/8) + c_3] + 3c_3 && \text{\#substitute into our equation} \\ \Rightarrow &= 8T(n/8) + 7c_3 && \text{\#simplified from above} \\ \Rightarrow &= 8[2T(n/16) + c_3] + 7c_3 && \text{\#same as above for } n/16 \\ \Rightarrow &= 16T(n/16) + c_3 + 15c_3 && \text{\#simplified out} \\ \Rightarrow &= 32T(n/32) + c_3 + 31c_3 \dots && \text{\#same as above for } n/32 \\ \Rightarrow &= 2kT(n/2k) + (2k-1)c_3 && \text{\#our base case} \\ \Rightarrow T(0) &= c_1 \\ \Rightarrow T(1) &= c_2 \\ \Rightarrow T(n) &= 2kT(n/2k) + (2k-1)c_3 \\ \text{If } n/2k &= 1 \\ \text{Then } n &= 2k\log n \text{ which is also } = k \\ \Rightarrow T(n) &= 2\log n T(n/2\log n) + (2\log n - 1)c_3 \text{ #sub } \log n \text{ as } k \text{ for base} \\ \Rightarrow nT(n/n) &+ (n-1)c_3 && \text{\#}2k\log(n)=k \text{ then } 2\log(n) = 1 \\ \Rightarrow nT(n/n) &+ (n-1)c_3 \\ \Rightarrow nc_2 &+ (n-1)c_3 \\ \in \Theta(n) & \\ \Rightarrow T(n) &= \Theta(n) \end{aligned}$$

## ii. Recurrence tree method:



## iii. Master method

$$\text{Case 1: } a = b^k$$

$$a=3, b=2, k=0$$

$$\text{Time complexity: } \Theta(n^{\log_{\text{base}b}a}) = \Theta(n^{\log_2 3}) = \Theta(n)$$

$$\begin{aligned}
 b. \quad T(n) &= 4T(n/2) + n \\
 &= 4[4T(n/4)+n/2] + n \\
 &= 16T(n/4) + n/2 + n \\
 &= 16[4T(n/8)+n/4] + 2n+n \\
 &= 64(n/8)+4n+2n+n \\
 &= \dots \\
 &= 4^{\log n} T(n/2^{\log n}) + \dots + 4n + 2n + n \\
 &= n^{\log 4} T(n/n^{\log 2}) + \dots + 4n + 2n + n \\
 &= n^2 T(1) + \dots + n(1+2+3+\dots) \\
 &= \Theta(n^2)
 \end{aligned}$$

= time complexity  $\Theta(n^2)$

3. A

a. **Pseudocode**

Declare array = [ ]

#use loop and user input portion grab user input and store into array

Declare temp variable

#Store length of array into temp

#divide portion

Take index of array

All even numbers add up

All odd numbers add up

#combine and conquer

Now add up all and divide by index round up

b. Since divide and conquer always halves it should be:  $O(n \log n)$