Problem Set 3: Quantitative Economics (ECON 8185-002)

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Exogenous labor supply

Assume utility function, its first derivative, and the inverse of the first derivative are

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma},$$
$$Uc(c) = c^{-\gamma},$$
$$Uc^{-1}(x) = x^{-\frac{1}{\gamma}}.$$

Household's budget constraint and the skill process are

$$c_t + a_{t+1} \leq e_i w + (1+r)a_t.$$
$$a_{t+1} \geq \underline{a} = 0.$$

The skill process is

$$\log(e_{i,t}) = \rho \log(e_{i,t-1}) + \sigma \epsilon_{i,t}.$$

The production function is

$$Y_t = F(Kt, N_t) = AK_t^{\theta} N_t^{1-\theta}$$

It is useful to set A such that steady state $Y = 1$. We will use endogenous grid method to solve for stationary equilibrium for this economy. The algorithm is as follows:

1) Guess r

2) Solve for w with

$$w(r) = (1-\theta)\left(\frac{r+\delta}{\theta}\right)^{\frac{\theta}{\theta-1}}.$$

3) Given a grid for (a, ϵ), we first guess $c^j(a, \epsilon) = ra + w\epsilon$.

4) Then, for all (a'_k, ϵ_j), we substitute a guess for the consumption tomorrow, $c^j(a'_k, \epsilon_j)$, in the Euler equation to solve for current consumption

$$\bar{c}(a'_k, \epsilon_j) = U_c^{-1}\left[\beta(1+r)\sum_{\epsilon'} P(\epsilon'|\epsilon_j) \cdot U_c\left[c^j(a'_k, \epsilon')\right]\right].$$

We then obtain current assets given consumption today defined on asset grid tomorrow as follows:

$$\bar{a}(a'_k, \epsilon_j) = \frac{\bar{c}(a'_k, \epsilon_j) + a'_k - w\epsilon_j}{1+r}.$$

5) Then we update $c^{j+1}(a, \epsilon)$ as follows:

$$c^{j+1}(a, \epsilon) = (1+r)a + w\epsilon \text{ for all } a \leq \bar{a}(a'_1, \epsilon)$$
$$c^{j+1}(a, \epsilon) = \text{ Interpolate } [\bar{c}(a'_k, \epsilon), \bar{c}(a'_{k+1}, \epsilon)] \text{ when } a \in [\bar{a}(a'_k, \epsilon), \bar{a}(a'_{k+1}, \epsilon)]$$

Note here that (a'_k, ϵ_j) are on the same grid as (a, ϵ). The first case is when the borrowing constraint is binding.

6) Repeat step 4 and 5 until $c^j(a, \epsilon)$ converged. Observe that we no longer need a root finding procedure, but still need to interpolate the optimal policy on our defined grids in this EGM algorithm.

Endogenous labor supply

We now incorporate endogenous labor supply in our utility function as follows:

$$U(c, l) = \frac{c^{1-\gamma}}{1-\gamma} - \phi\frac{l^{1+\eta}}{1+\eta}$$

$$Uc(c) = c^{-\gamma}$$

$$Uc^{-1}(x) = x^{-\frac{1}{\gamma}}$$

$$Ul(l) = \phi l^{\eta}$$

$$Ul^{-1}(x) = (x/\phi)^{1/\eta}$$

where l represents labor. The budget constraint now becomes

$$c + a' \leq \epsilon wl + (1 + r)a.$$

To implement endogenous grid method, as in the case of exogenous labor supply, we start by

1) Guess r

2) Solve for w with

$$w(r) = A(1-\theta)\left(\frac{r+\delta}{A\theta}\right)^{\frac{\theta}{\theta-1}}.$$

3) Then we guess $c^j(a, \epsilon) = ra + w\epsilon$ as before.

4) Then, for all (a'_k, ϵ_j), we use $c^j(a'_k, \epsilon_j)$ to solve for

$$\bar{c}(a'_k, \epsilon_j) = U_c^{-1}\left[\beta(1+r)\sum_{\epsilon'} P(\epsilon'|\epsilon_j) \cdot U_c\left[c^j(a'_k, \epsilon')\right]\right]$$

5) Use $\bar{c}(a'_k, \epsilon_j)$ to solve for $\bar{l}(a_k, \epsilon_j)$ from

$$\frac{u_l(l)}{u_c(\bar{c}(a'_k, \epsilon_j))} = \frac{\phi l^{\eta}}{\bar{c}(a'_k, \epsilon_j)^{-\gamma}} = w\epsilon_j.$$

Note that above is an equation of only one unknown, l, given $\bar{c}(a'_k, \epsilon_j)$.

6) Then, for all (a'_k, ϵ_j), we use $\bar{c}(a'_k, \epsilon_j)$ and $\bar{l}(a'_k, \epsilon_j)$ to solve for

$$\bar{a}(a'_k, \epsilon_j) = \frac{\bar{c}(a'_k, \epsilon_j) + a'_k - w\epsilon_j\bar{l}(a'_k, \epsilon_j)}{1+r}.$$

Note here that (a'_k, ϵ_j) are on the same grid as (a, ϵ).

7) Then we update $c^{j+1}(a, \epsilon)$ as follows:

$$c^{j+1}(a, \epsilon) = (1+r)a + w\epsilon\bar{l}(a, \epsilon) \text{ for all } a \leq \bar{a}(a'_1, \epsilon)$$

$$c^{j+1}(a, \epsilon) = \text{ Interpolate } [\bar{c}(a'_k, \epsilon), \bar{c}(a'_{k+1}, \epsilon)] \text{ when } a \in [\bar{a}(a'_k, \epsilon), \bar{a}(a'_{k+1}, \epsilon)]$$

8) Check whether $c^{j+1}(a, \epsilon)$ and $c^j(a, \epsilon)$ are close enough. If no, repeat step (4) down again with $c^{j+1}(a, \epsilon)$. If yes, use the updated $c^{j+1}(a, \epsilon)$ to back out

$$l(a_i, \epsilon_j) = Ul^{-1}(w \cdot \epsilon_j \cdot Uc(c^{j+1}(a_i, \epsilon_j)))$$

$$a'(a_i, \epsilon_j) = (1+r)a_i + w\epsilon_j l - c^{j+1}(a_i, \epsilon_j)$$

Then we solve for stationary distribution λ. With this, we then solve for equilibrium r.

1) Guess r_0.

2) Solve for policy functions $a'(a,\epsilon), c(a,\epsilon)$ and $l(a,\epsilon)$.

3) Use $a'(a,\epsilon)$ to compute $\lambda(a,\epsilon)$.

4) Compute aggregate supply of capital (savings) $K_s(r_0) = \int a_i di$ and aggregate labor supply $N(r_0) = \int \epsilon_i l_i di$.

5) Compute $r_s = A\theta\left(\frac{K_s(r_0)}{N(r_0)}\right)^{\theta-1} - \delta$. This is an interest rate as implied by aggregate supply of capital.

6) Compare r_0 and r_s. If they are close enough, then we solve for an equilibrium interest rate $r = \frac{r_0+r_s}{2}$. Otherwise, set

$$r_0 = 0.8 \cdot r_0 + 0.2 \cdot r_s$$

and repeat from (1) through (6) until r converged.

Add Government

We assume $\tau = 0.4$, $T/Y = 0.13$ and $G/Y = 0.20$, and the budget constraint now becomes

$$c + a' \leq (1-\tau)\epsilon w l + T + (1+(1-\tau)r)a.$$

Note that we also have government budget constraint as follows:

$$G + T + rB = B' - B + \tau(wN + rA),$$

where N is the aggregate labor supply and A is the aggregate asset supply. For EGM, steps (1),(2),(3),(4) are the same (or change (3) as well) ???:

5) Use $\bar{c}(a'_k, \epsilon_j)$ to solve for $\bar{l}(a_k, \epsilon_j)$ from

$$\frac{u_l(l)}{u_c(\bar{c}(a'_k,\epsilon_j))} = \frac{\phi l^\eta}{\bar{c}(a'_k,\epsilon_j)^{-\gamma}} = (1-\tau)w\epsilon_j.$$

6) Then, for all (a'_k, ϵ_j), we use $\bar{c}(a'_k,\epsilon_j)$ and $\bar{l}(a'_k,\epsilon_j)$ to solve for

$$\bar{a}(a'_k,\epsilon_j) = \frac{\bar{c}(a'_k,\epsilon_j) + a'_k - (1-\tau)w\epsilon_j\bar{l}(a'_k,\epsilon_j) + T}{1+(1-\tau)r}.$$

Note here that (a'_k, ϵ_j) are on the same grid as (a,ϵ).

7) Then we update $c^{j+1}(a,\epsilon)$ as follows:

$$c^{j+1}(a,\epsilon) = (1-\tau)w\epsilon\bar{l}(a,\epsilon) + T + (1+(1-\tau)r)a \text{ for all } a \leq \bar{a}(a'_1,\epsilon)$$
$$c^{j+1}(a,\epsilon) = \text{ Interpolate } [\bar{c}(a'_k,\epsilon), \bar{c}(a'_{k+1},\epsilon)] \text{ when } a \in [\bar{a}(a'_k,\epsilon), \bar{a}(a'_{k+1},\epsilon)]$$

8) Check whether $c^{j+1}(a,\epsilon)$ and $c^j(a,\epsilon)$ are close enough. If no, repeat step (4) down again with $c^{j+1}(a,\epsilon)$. If yes, use the updated $c^{j+1}(a,\epsilon)$ to back out

$$l(a_i,\epsilon_j) = Ul^{-1}((1-\tau)\cdot w \cdot \epsilon_j \cdot Uc(c^{j+1}(a_i,\epsilon_j)))$$
$$a'(a_i,\epsilon_j) = (1-\tau)w\epsilon_j l + T + (1+(1-\tau)r)a_i - c^{j+1}(a_i,\epsilon_j)$$