Problem Set 3: Quantitative Economics (ECON 8185-002)

Teerat Wongrattanapiboon

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Endogenous labor supply

We now incorporate endogenous labor supply in our utility function as follows:

$$U(c,l) = \frac{(c^{\eta} l^{1-\eta})^{1-\mu}}{1-\mu}$$

where l represents leisure. The budget constraint now becomes

$$c + a' \leq \epsilon w(1-l) + (1+r)a.$$

To implement endogenous grid method, as in the case of exogenous labor supply, we start by guessing r and solve for w with

$$w(r) = (1-\theta)\left(\frac{r+\delta}{\theta}\right)^{\frac{\theta}{\theta-1}}.$$

Then we guess $c^j(a', \epsilon') = ra' + w\epsilon'$ as before and use it to solve for $\bar{l}(c^j)$ from

$$\frac{u_l(c,l)}{u_c(c,l)} = w\epsilon.$$

This is an equation of only one unknown, l, given a guess for c. Then, for all (a'_k, ϵ_j), we use $\bar{l}(c)$ and c^j to solve for

$$\bar{c}(a'_k, \epsilon_j) = U_c^{-1}\left[\beta(1+r)\sum_{\epsilon'} P(\epsilon'|\epsilon) \cdot U_c\left[c^j(a'_k, \epsilon'), \bar{l}(c)\right]\right]$$

$$\bar{a}(a'_k, \epsilon_j) = \frac{\bar{c}(a'_k, \epsilon_j) + a'_k - w\epsilon_j(1-\bar{l})}{1+r}.$$

Then we update $c^{j+1}(a, \epsilon)$ as follows:

$$c^{j+1}(a,\epsilon) = (1+r)a + w\epsilon \text{ for all } a \leq \bar{a}(a'_0, \epsilon)$$

$$c^{j+1}(a,\epsilon) = \text{ Interpolate } [\bar{c}(a'_k, \epsilon), \bar{c}(a'_{k+1}, \epsilon)] \text{ when } a \in [\bar{a}(a'_k, \epsilon), \bar{a}(a'_{k+1}, \epsilon)]$$