Class notes: Advanced Topics in Macroeconomics

Topic: Adding Fiscal Policies

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In the first class (9/13), we discussed empirical relevance of our theories and the idea that we would be matching them to NIPA data from the BEA. An important factor was missing from the framework we spelled out, namely, fiscal policy.

The next step of the course involves solving models with taxes, transfers, and government purchases. More specifically, assume that households choose paths for per capita consumption c_t and leisure ℓ_t to solve:

$$\begin{aligned}
\max_{\{c_t,\ell_t,s_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, \ell_t\right) N_t \\
\text{subject to } \sum_{t=0}^{\infty} p_t \{(1+\tau_{ct})\, c_t + v_t\left(s_{t+1} - s_t\right)\} \\
\leq \sum_{t=0}^{\infty} p_t \{(1-\tau_{dt})\, d_t s_t + (1-\tau_{ht})\, w_t h_t + \kappa_t\}
\end{aligned}$$

where $N_t = (1+\gamma_n)^t$ is the size of the population, v_t is the price of firm shares, s_t is the quantity of shares held, d_t is the amount of per capita distributions paid per share to the shareholders (i.e., households), w_t is the pre-tax wage rate paid to labor, and $h_t = 1 - \ell_t$. Relative to the first class, we are now adding taxes on consumption at rate τ_{ct}, taxes on dividends at rate τ_{dt}, taxes on labor at rate τ_{ht}, and government transfers κ_t. As before, the price used when summing expenditures and incomes is p_t, which is the Arrow-Debreu price (and equal to the household marginal utility in equilibrium and the rate at which firms discount future distributions).

For businesses, we assume they maximize the present value of aggregate distributions

D_t to households after subtracting corporate profits taxes:

$$
\begin{aligned}
& E_0 \sum_{t=0}^{\infty} p_t D_t \\
& \text{s.t.} \quad K_{t+1} = (1-\delta) K_t + X_t \\
& \qquad D_t = F(K_t, Z_t H_t) - w_t H_t - X_t \\
& \qquad\qquad - \tau_{pt} \left(F(K_t, Z_t H_t) - w_t H_t - \delta K_t \right)
\end{aligned}
$$

where, again, p_t, is the discount factor for the firm shareholders, K_t is the capital stock, X_t is gross investment, H_t is the total labor input, τ_{pt} is the corporate tax rate, and $Z_t = z_t(1+\gamma_z)^t$ is the technology parameter with

$$
\log z_{t+1} = \rho \log z_t + \epsilon_{t+1}
$$

and $\epsilon \sim N(0, \sigma^2)$. Notice that taxes are paid on *accounting profits*, which are equal to returns to capital owners less depreciation. The U.S. system does not tax corporate net investment $K_{t+1} - K_t$.

As we did earlier, we can map the two problems into one more convenient one (at least for computational purposes):

$$
\begin{aligned}
& \max_{\{c_t, x_t, \ell_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t) N_t \\
& \text{subj. to} \quad c_t + x_t = r_t k_t + w_t h_t + \kappa_t \\
& \qquad\qquad - \tau_{ct} c_t - \tau_{ht} w_t h_t - \tau_{pt} (r_t k_t - \delta k_t) \\
& \qquad\qquad - \tau_{dt} \left(r_t k_t - x_t - \tau_{pt} (r_t k_t - \delta k_t) \right) \\
& \qquad N_{t+1} k_{t+1} = [(1-\delta) k_t + x_t] N_t \\
& \qquad h_t + \ell_t = 1 \\
& \qquad S_t = P S_{t-1} + Q \epsilon_t, \quad S_t = [\log z_t, \tau_{ct}, \tau_{ht}, \tau_{dt}, \tau_{pt}, \log g_t] \\
& \qquad c_t, x_t \geq 0 \quad \text{in all states,}
\end{aligned}
$$

where $g_t = G_t / N_t$ is the per-capita government purchases and the prices r_t and w_t are marginal products for the technology $Y_t = F(K_t, Z_t H_t)$.

In equilibrium, resources must add up:

$$C_t + X_t + G_t = Y_t$$

and the government must satisfy the period-by-period budget

$$\begin{aligned} G_t + N_t \kappa_t = {} & \tau_{ct} C_t + \tau_{ht} w_t H_t + \tau_{pt} \left(r_t K_t - \delta K_t \right) \\ & + \tau_{dt} \left(r_t K_t - X_t - \tau_{pt} \left(r_t K_t - \delta K_t \right) \right) \end{aligned}$$

since we have not allowed for debt here.