Problem Set 3: Quantitative Economics (ECON 8185-002)

Teerat Wongrattanapiboon

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Exogenous labor supply

Assume utility function, its first derivative, and the inverse of the first derivative are

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

$$Uc(c) = c^{-\gamma},$$

$$Uc^{-1}(x) = x^{-\frac{1}{\gamma}}.$$

Household's budget constraint and the skill process are

$$c_t + a_{t+1} \leq e_i w + (1+r)a_t.$$

$$a_{t+1} \geq \underline{a} = 0.$$

The skill process is

$$\log(e_{i,t}) = \rho \log(e_{i,t-1}) + \sigma \epsilon_{i,t}.$$

The production function is

$$Y_t = F(Kt, N_t) = AK_t^\theta N_t^{1-\theta}$$

It is useful to set A such that steady state $Y = 1$. We will use endogenous grid method to solve for stationary equilibrium for this economy. The algorithm is as follows:

1) Guess r

2) Solve for w with

$$w(r) = (1-\theta)\left(\frac{r+\delta}{\theta}\right)^{\frac{\theta}{\theta-1}}.$$

3) Given a grid for (a, ϵ), we first guess $c^j(a, \epsilon) = ra + w\epsilon$.

4) Then, for all (a'_k, ϵ_j), we substitute a guess for the consumption tomorrow, $c^j(a'_k, \epsilon_j)$, in the Euler equation to solve for current consumption

$$\bar{c}(a'_k, \epsilon_j) = U_c^{-1}\left[\beta(1+r)\sum_{\epsilon'} P(\epsilon'|\epsilon_j) \cdot U_c\left[c^j(a'_k, \epsilon')\right]\right].$$

We then obtain current assets given consumption today defined on asset grid tomorrow as follows:

$$\bar{a}(a'_k, \epsilon_j) = \frac{\bar{c}(a'_k, \epsilon_j) + a'_k - w\epsilon_j}{1+r}.$$

5) Then we update $c^{j+1}(a, \epsilon)$ as follows:

$$c^{j+1}(a, \epsilon) = (1+r)a + w\epsilon \text{ for all } a \leq \bar{a}(a'_1, \epsilon)$$

$$c^{j+1}(a, \epsilon) = \text{ Interpolate } [\bar{c}(a'_k, \epsilon), \bar{c}(a'_{k+1}, \epsilon)] \text{ when } a \in [\bar{a}(a'_k, \epsilon), \bar{a}(a'_{k+1}, \epsilon)]$$

Note here that (a'_k, ϵ_j) are on the same grid as (a, ϵ). The first case is when the borrowing constraint is binding.

6) Repeat step 4 and 5 until $c^j(a, \epsilon)$ converged. Observe that we no longer need a root finding procedure, but still need to interpolate the optimal policy on our defined grids in this EGM algorithm.

Endogenous labor supply

We now incorporate endogenous labor supply in our utility function as follows:

$$U(c, l) = \frac{(c^\eta l^{1-\eta})^{1-\mu}}{1-\mu}$$

where l represents leisure. The budget constraint now becomes

$$c + a' \leq \epsilon w(1 - l) + (1 + r)a.$$

To implement endogenous grid method, as in the case of exogenous labor supply, we start by

1) Guess r

2) Solve for w with

$$w(r) = (1 - \theta) \left(\frac{r + \delta}{\theta} \right)^{\frac{\theta}{\theta - 1}}.$$

3) Then we guess $c^j(a, \epsilon) = ra + w\epsilon$ as before.

4) Use this guess to solve for $\bar{l}(a, \epsilon)$ from

$$\frac{u_l(c^j(a, \epsilon), l)}{u_c(c^j(a, \epsilon), l)} = \frac{(c^\eta l^{1-\eta})^{-\mu} c^\eta (1 - \eta) l^{-\eta}}{(c^\eta l^{1-\eta})^{-\mu} l^{1-\eta}(\eta) c^{\eta-1}} = \left(\frac{1 - \eta}{\eta} \right) \cdot \frac{c}{l} = w\epsilon.$$

Note that above is an equation of only one unknown, l, given a guess for c, $c^j(a, \epsilon)$.

5) Then, for all (a'_k, ϵ_j), we use $\bar{l}(a'_k, \epsilon_j)$ and $c^j(a'_k, \epsilon_j)$ to solve for

$$\bar{c}(a'_k, \epsilon_j) = U_c^{-1} \left[\beta(1 + r) \sum_{\epsilon'} P(\epsilon'|\epsilon_j) \cdot U_c \left[c^j(a'_k, \epsilon'), \bar{l}(a'_k, \epsilon') \right] \right]$$

$$\bar{a}(a'_k, \epsilon_j) = \frac{\bar{c}(a'_k, \epsilon_j) + a'_k - w\epsilon_j(1 - \bar{l}(a'_k, \epsilon_j))}{1 + r}.$$

Note here that (a'_k, ϵ_j) are on the same grid as (a, ϵ).

6) Then we update $c^{j+1}(a, \epsilon)$ as follows:

$$c^{j+1}(a, \epsilon) = (1 + r)a + w\epsilon(1 - \bar{l}(a, \epsilon)) \text{ for all } a \leq \bar{a}(a'_1, \epsilon)$$

$$c^{j+1}(a, \epsilon) = \text{ Interpolate } [\bar{c}(a'_k, \epsilon), \bar{c}(a'_{k+1}, \epsilon)] \text{ when } a \in [\bar{a}(a'_k, \epsilon), \bar{a}(a'_{k+1}, \epsilon)]$$